Many Lemmas Extracted from Prover9 Proofs I (Monster Draft)

CXSbZP8nCbob49P892dvsj5A1Dsmrg7AvKn6zYTkABum C. TaULp

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Axiom 1. (*cf. [1]*) $e \cdot x = x$

Axiom 2. (*cf. [1]*) $x \cdot e = x$

Axiom 3. (*cf. [1]*) $x \backslash (x \cdot y) = y$

Axiom 4. (*cf. [1]*) $x \cdot (x \backslash y) = y$

Axiom 5. (*cf. [1]*) $(x \cdot y)/y = x$

Axiom 6. (*cf. [1]*) $(x/y) \cdot y = x$

Definition 1. (*cf. [1, 2]*) *$a(x,y,z)$ is defined to be $(x \cdot (y \cdot z)) \backslash ((x \cdot y) \cdot z)$.*

Definition 2. (*cf. [1, 2]*) *$K(x,y)$ is defined to be $(y \cdot x) \backslash (x \cdot y)$.*

Definition 3. (*cf. [1, 2]*) *$T(u,x)$ is defined to be $x \backslash (u \cdot x)$.*

Definition 4. (*cf. [1, 2]*) *$L(u,x,y)$ is defined to be $(y \cdot x) \backslash (y \cdot (x \cdot u))$.*

Definition 5. (*cf. [1, 2]*) *$R(u,x,y)$ is defined to be $((u \cdot x) \cdot y)/(x \cdot y)$.*

Axiom 7. (*cf. [1]*) $T(T(u,x),y) = T(T(u,y),x)$

Axiom 8. (*cf. [1]*) $T(L(u,x,y),z) = L(T(u,z),x,y)$

Axiom 9. (*cf. [1]*) $T(R(u,x,y),z) = R(T(u,z),x,y)$

Axiom 10. (*cf. [1]*) $L(R(u,x,y),z,w) = R(L(u,z,w),x,y)$

Axiom 11. (*cf. [1]*) $L(L(u,x,y),z,w) = L(L(u,z,w),x,y)$

Axiom 12. (*cf. [1]*) $R(R(u,x,y),z,w) = R(R(u,z,w),x,y)$

Proposition 1. (*cf. [6]*) *$x \cdot y = z \rightarrow z/y = x$.*

Proposition 2. (*cf. [6]*) *$x \cdot y = z \rightarrow x \backslash z = y$.*

Proposition 3. (*cf. [6]*) *$z/y = x \rightarrow x \cdot y = z$.*

Proposition 4. (*cf. [6]*) *$z/y = x \rightarrow x \backslash z = y$.*

Proposition 5. (*cf. [6]*) *$x \backslash z = y \rightarrow x \cdot y = z$.*

Proposition 6. (*cf. [6]*) *$x \backslash z = y \rightarrow z/y = x$.*

Proposition 7. (*cf. [3]*) *$x \cdot y = u \wedge x \cdot z = u \rightarrow y = z$.*

Proposition 8. (*cf. [3]*) *$y \cdot x = u \wedge z \cdot x = u \rightarrow y = z$.*

Proposition 9. *(cf. [3])* $x \cdot y = x \cdot z \to y = z$.

Proposition 10. *(cf. [3])* $y \cdot x = z \cdot x \to y = z$.

Proposition 11. *(cf. [3])* $x/y = u \wedge x/z = u \to y = z$.

Proposition 12. *(cf. [3])* $y/x = u \wedge z/x = u \to y = z$.

Proposition 13. *(cf. [3])* $x/y = x/z \to y = z$.

Proposition 14. *(cf. [3])* $y/x = z/x \to y = z$.

Proposition 15. *(cf. [3])* $x\backslash y = u \wedge x\backslash z = u \to y = z$.

Proposition 16. *(cf. [3])* $y\backslash x = u \wedge z\backslash x = u \to y = z$.

Proposition 17. *(cf. [3])* $x\backslash y = x\backslash z \to y = z$.

Proposition 18. *(cf. [3])* $y\backslash x = z\backslash x \to y = z$.

Proposition 19. *(cf. [4])* $a(x, y, z) = e \to L(z, y, x) = z$.

Proposition 20. *(cf. [4])* $L(z, y, x) = z \to a(x, y, z) = e$.

Proposition 21. *(cf. [4])* $T(x, y) = x \to T(y, x) = y$.

Proposition 22. *(cf. [4])* $T(x, y) = x \to K(x, y) = e$.

Proposition 23. *(cf. [4])* $K(x, y) = e \to T(x, y) = x$.

Proposition 24. *(cf. [5])* $y/(x\backslash y) = x$.

Proposition 25. *(cf. [5])* $(y/x)\backslash y = x$.

Proposition 26. *(cf. [5])* $e\backslash x = x$.

Proposition 27. *(cf. [5])* $x/e = x$.

Proposition 28. *(cf. [5])* $x\backslash x = e$.

Proposition 29. *(cf. [5])* $x/x = e$.

Proposition 30. *(cf. [5])* $x\backslash y = e \to x = y$.

Proposition 31. *(cf. [5])* $x/y = e \to x = y$.

Proposition 32. *(cf. [5])* $x \cdot (y \cdot z) = (x \cdot y) \cdot z \to a(x, y, z) = e$.

Proposition 33. *(cf. [5])* $a(x, y, z) = e \to x \cdot (y \cdot z) = (x \cdot y) \cdot z$.

Proposition 34. *(cf. [5])* $x \cdot y = y \cdot x \to K(x, y) = e$.

Proposition 35. *(cf. [5])* $K(x, y) = e \to x \cdot y = y \cdot x$.

Definition 6. *(cf. [7])* $F(u, x)$ *is defined to be* $(x \cdot u)/x$.

Proposition 36. *(cf. [7])* $T(z, x) = T(u, x) \to z = u$.

Proposition 37. *(cf. [7])* $L(z, x, y) = L(u, x, y) \to z = u$.

Proposition 38. *(cf. [7])* $R(z, x, y) = R(u, x, y) \to z = u$.

Proposition 39. *(cf. [7])* $F(z, x) = F(u, x) \to z = u$.

Proposition 40. *(cf. [7])* $T(F(u, x), x) = u$.

Proposition 41. *(cf. [7])* $F(T(u,x),x) = u$.

Proposition 42. *(cf. [7])* $F(F(u,x),y) = F(F(u,y),x)$.

Proposition 43. *(cf. [7])* $F(L(u,x,y),z) = L(F(u,z),x,y)$.

Proposition 44. *(cf. [7])* $F(R(u,x,y),z) = R(F(u,z),x,y)$.

Proposition 45. *(cf. [7])* $F(T(u,x),y) = T(F(u,y),x)$.

Proposition 46. *(cf. [11])* $x \cdot T(y,x) = y \cdot x$.

Proposition 47. *(cf. [11])* $T(x/y,y) = y\backslash x$.

Proposition 48. *(cf. [11])* $(x \cdot T(y,x))/x = y$.

Proposition 49. *(cf. [11])* $T(x,x\backslash y) = (x\backslash y)\backslash y$.

Proposition 50. *(cf. [11])* $x \cdot T(T(y,x),z) = T(y,z) \cdot x$.

Proposition 51. *(cf. [11])* $T(T(x/y,z),y) = T(y\backslash x,z)$.

Proposition 52. *(cf. [11])* $(x \cdot y) \cdot L(z,y,x) = x \cdot (y \cdot z)$.

Proposition 53. *(cf. [11])* $L(x\backslash y,x,z) = (z \cdot x)\backslash(z \cdot y)$.

Proposition 54. *(cf. [11])* $R(x,y,z) \cdot (y \cdot z) = (x \cdot y) \cdot z$.

Proposition 55. *(cf. [11])* $R(x/y,y,z) = (x \cdot z)/(y \cdot z)$.

Proposition 56. *(cf. [11])* $x \cdot ((x\backslash e) \cdot y) = L(y,x\backslash e,x)$.

Proposition 57. *(cf. [11])* $(x\backslash e) \cdot y = x\backslash L(y,x\backslash e,x)$.

Proposition 58. *(cf. [11])* $x \cdot L(T(y,x),z,w) = L(y,z,w) \cdot x$.

Proposition 59. *(cf. [11])* $x \cdot R(T(y,x),z,w) = R(y,z,w) \cdot x$.

Proposition 60. *(cf. [11])* $T(R(x/y,z,w),y) = R(y\backslash x,z,w)$.

Proposition 61. *(cf. [11])* $T(x/y,z) \cdot y = y \cdot T(y\backslash x,z)$.

Proposition 62. *(cf. [11])* $(x \cdot y)\backslash(x \cdot (z \cdot y)) = L(T(z,y),y,x)$.

Proposition 63. *(cf. [11])* $T(x,x \cdot y) = L(T(x,y),y,x)$.

Proposition 64. *(cf. [11])* $T((x \cdot y)/(z \cdot y),z) = R(z\backslash x,z,y)$.

Proposition 65. *(cf. [11])* $T(R(x,y,z),w) \cdot (y \cdot z) = (T(x,w) \cdot y) \cdot z$.

Proposition 66. *(cf. [10])* $R(x,x\backslash e,y) = y/((x\backslash e) \cdot y)$.

Proposition 67. *(cf. [10])* $(e/x) \cdot (x \cdot y) = L(y,x,e/x)$.

Proposition 68. *(cf. [10])* $R(x,y,y\backslash e) = (x \cdot y) \cdot (y\backslash e)$.

Proposition 69. *(cf. [10])* $R(x,e/y,y) = (x \cdot (e/y)) \cdot y$.

Proposition 70. *(cf. [10])* $L(x\backslash(y\backslash z),x,y) = (y \cdot x)\backslash z$.

Proposition 71. *(cf. [10])* $R(x/y,y,y\backslash e) = x \cdot (y\backslash e)$.

Proposition 72. *(cf. [10])* $T(L(x/y,z,w),y) = L(y\backslash x,z,w)$.

Proposition 73. *(cf. [10])* $L(x\backslash T(y,z),x,z) = (z \cdot x)\backslash(y \cdot z)$.

Proposition 74. (*cf. [10]*) $R((x/y)/z, z, y) = x/(z \cdot y)$.

Proposition 75. (*cf. [10]*) $L((x\backslash y)/z, z, x) = (z \cdot ((x \cdot z)\backslash y))/z$.

Proposition 76. (*cf. [10]*) $K(x\backslash e, x) = (x\backslash e) \cdot x$.

Proposition 77. (*cf. [10]*) $K(x, e/x) = x \cdot (e/x)$.

Proposition 78. (*cf. [10]*) $L(x\backslash e, x, y) = (y \cdot x)\backslash y$.

Proposition 79. (*cf. [10]*) $R(e/x, x, y) = y/(x \cdot y)$.

Proposition 80. (*cf. [10]*) $R(e/x, x, y)\backslash y = x \cdot y$.

Proposition 81. (*cf. [10]*) $(x \cdot y)\backslash((z \cdot x) \cdot y) = R(T(z, x \cdot y), x, y)$.

Proposition 82. (*cf. [8]*) $(x \cdot y) \cdot K(y, x) = y \cdot x$.

Proposition 83. (*cf. [8]*) $R(x/y, z, w) \cdot y = y \cdot R(y\backslash x, z, w)$.

Proposition 84. (*cf. [8]*) $(x \cdot y) \cdot R((x \cdot y)\backslash y, z, w) = (x \cdot R(x\backslash e, z, w)) \cdot y$.

Proposition 85. (*cf. [8]*) $L(R(x\backslash(y\backslash z), w, u), x, y) = R((y \cdot x)\backslash z, w, u)$.

Proposition 86. (*cf. [8]*) $x \cdot (y \cdot R(y\backslash(x\backslash y), z, w)) = (x \cdot R(x\backslash e, z, w)) \cdot y$.

Proposition 87. (*cf. [8]*) $(x \cdot y) \cdot R((x \cdot y)\backslash z, w, u) = x \cdot (y \cdot R(y\backslash(x\backslash z), w, u))$.

Proposition 88. (*cf. [8]*) $(x\backslash y) \cdot R((x\backslash y)\backslash y, z, w) = R(x, z, w) \cdot (x\backslash y)$.

Proposition 89. (*cf. [8]*) $x\backslash R(x, y, z) = R(x, y, z) \cdot (x\backslash e)$.

Proposition 90. (*cf. [8]*) $K(x\backslash e, x) = (e/x)\backslash(x\backslash e)$.

Theorem 1. $a(x, y/z, z) = (x \cdot y)\backslash((x \cdot (y/z)) \cdot z)$.

Proof. We have $a(x, y/z, z) = (x \cdot ((y/z) \cdot z))\backslash((x \cdot (y/z)) \cdot z)$ by Definition 1. Hence we are done by Axiom 6. □

Theorem 2. $K(y\backslash x, y) = x\backslash((y\backslash x) \cdot y)$.

Proof. We have $K(y\backslash x, y) = (y \cdot (y\backslash x))\backslash((y\backslash x) \cdot y)$ by Definition 2. Hence we are done by Axiom 4. □

Theorem 3. $K(y, x/y) = x\backslash(y \cdot (x/y))$.

Proof. We have $K(y, x/y) = ((x/y) \cdot y)\backslash(y \cdot (x/y))$ by Definition 2. Hence we are done by Axiom 6. □

Theorem 4. $L(z, y, x/y) = x\backslash((x/y) \cdot (y \cdot z))$.

Proof. We have $L(z, y, x/y) = ((x/y) \cdot y)\backslash((x/y) \cdot (y \cdot z))$ by Definition 4. Hence we are done by Axiom 6. □

Theorem 5. $(x \cdot y)/T(x, y) = y$.

Proof. We have $(x \cdot y)/(y\backslash(x \cdot y)) = y$ by Proposition 24. Hence we are done by Definition 3. □

Theorem 6. $R(x, y, z)\backslash((x \cdot y) \cdot z) = y \cdot z$.

Proof. We have $(((x \cdot y) \cdot z)/(y \cdot z))\backslash((x \cdot y) \cdot z) = y \cdot z$ by Proposition 25. Hence we are done by Definition 5. □

Theorem 7. $T((x \cdot y)/x, x) = y$.

Proof. We have

$$x \cdot T((x \cdot y)/x, x) = ((x \cdot y)/x) \cdot x \tag{1}$$

by Proposition 46. We have $x \cdot y = ((x \cdot y)/x) \cdot x$ by Axiom 6. Hence we are done by (1) and Proposition 7. □

Theorem 8. $(x\backslash y) \cdot T(x, x\backslash y) = y$.

Proof. We have $x \cdot (x\backslash y) = y$ by Axiom 4. Hence we are done by Proposition 46. □

Theorem 9. $z \cdot (x \cdot (x\backslash T(y, z))) = y \cdot z$.

Proof. We have $z \cdot T(y, z) = y \cdot z$ by Proposition 46. Hence we are done by Axiom 4. □

Theorem 10. $x \cdot T(y/x, x) = y$.

Proof. We have $(y/x) \cdot x = y$ by Axiom 6. Hence we are done by Proposition 46. □

Theorem 11. $x \cdot y = y \cdot z \to T(x, y) = z$.

Proof. Assume

$$x \cdot y = y \cdot z. \tag{2}$$

We have $y \cdot T(x, y) = x \cdot y$ by Proposition 46. Then $y \cdot T(x, y) = y \cdot z$ by (2). Hence we are done by Proposition 9. □

Theorem 12. $K(T(x, y), y) = (x \cdot y)\backslash(T(x, y) \cdot y)$.

Proof. We have $K(T(x, y), y) = (y \cdot T(x, y))\backslash(T(x, y) \cdot y)$ by Definition 2. Hence we are done by Proposition 46. □

Theorem 13. $T(y, T(x, y)) = T(x, y)\backslash(x \cdot y)$.

Proof. We have $T(y, T(x, y)) = T(x, y)\backslash(y \cdot T(x, y))$ by Definition 3. Hence we are done by Proposition 46. □

Theorem 14. $T(x/(x/y), x/y) = y$.

Proof. We have $(x/y)\backslash x = y$ by Proposition 25. Hence we are done by Proposition 47. □

Theorem 15. $y/T(x, x\backslash y) = x\backslash y$.

Proof. We have $(x \cdot (x\backslash y))/T(x, x\backslash y) = x\backslash y$ by Theorem 5. Hence we are done by Axiom 4. □

Theorem 16. $(x \cdot T(T(y, x), z))/x = T(y, z)$.

Proof. We have $(x \cdot T(T(y, z), x))/x = T(y, z)$ by Proposition 48. Hence we are done by Axiom 7. □

Theorem 17. $x \cdot K(y\backslash x, y) = (y\backslash x) \cdot y$.

Proof. We have $(y \cdot (y\backslash x)) \cdot K(y\backslash x, y) = (y\backslash x) \cdot y$ by Proposition 82. Hence we are done by Axiom 4. □

Theorem 18. $T((y\backslash z)\backslash z, x) = T(T(y, x), y\backslash z)$.

Proof. We have $T(T(y, y\backslash z), x) = T(T(y, x), y\backslash z)$ by Axiom 7. Hence we are done by Proposition 49. □

Theorem 19. $(x/z)\backslash x = y \cdot (y\backslash z)$.

Proof. We have $y \cdot (y\backslash z) = z$ by Axiom 4. Then

$$((y \cdot (y\backslash z))/e) \cdot e = z \tag{3}$$

by Axiom 6. We have $((y\cdot(y\backslash z))/e)\cdot e = y\cdot(y\backslash z)$ by Axiom 6. Then $(x/(((y\cdot(y\backslash z))/e)\cdot e))\backslash x = y\cdot(y\backslash z)$ by Proposition 25. Hence we are done by (3). ☐

Theorem 20. *$y = (x\backslash y) \cdot z \to T(x, x\backslash y) = z$.*

Proof. Assume

$$y = (x\backslash y) \cdot z. \tag{4}$$

We have $(x\backslash y) \cdot T(x, x\backslash y) = y$ by Theorem 8. Then $(x\backslash y) \cdot T(x, x\backslash y) = (x\backslash y) \cdot z$ by (4). Hence we are done by Proposition 9. ☐

Theorem 21. *$(x\backslash e)\backslash K(x\backslash e, x) = x$.*

Proof. We have $(x\backslash e) \cdot x = K(x\backslash e, x)$ by Proposition 76. Hence we are done by Proposition 2. ☐

Theorem 22. *$x\backslash K(x, e/x) = e/x$.*

Proof. We have $x \cdot (e/x) = K(x, e/x)$ by Proposition 77. Hence we are done by Proposition 2. ☐

Theorem 23. *$R(x, y/z, z) \cdot y = (x \cdot (y/z)) \cdot z$.*

Proof. We have $e \cdot y = y$ by Axiom 1. Then

$$((x \cdot (y/z)) \cdot z) \cdot (((x \cdot (y/z)) \cdot z)\backslash(e \cdot y)) = y \tag{5}$$

by Axiom 4. We have $((x \cdot (y/z)) \cdot z) \cdot (((x \cdot (y/z)) \cdot z)\backslash(e \cdot y)) = e \cdot y$ by Axiom 4. Then $((((x \cdot (y/z)) \cdot z) \cdot (((x \cdot (y/z)) \cdot z)\backslash(e \cdot y)))/z) \cdot z = e \cdot y$ by Axiom 6. Then

$$(y/z) \cdot z = e \cdot y \tag{6}$$

by (5). We have

$$e \cdot y = (((x \cdot (y/z)) \cdot z)/(e \cdot y))\backslash((x \cdot (y/z)) \cdot z) \tag{7}$$

by Proposition 25. We have $e \cdot ((((x \cdot (y/z)) \cdot z)/(e \cdot y))\backslash((x \cdot (y/z)) \cdot z)) = (((x \cdot (y/z)) \cdot z)/(e \cdot y))\backslash((x \cdot (y/z)) \cdot z)$ by Axiom 1. Then

$$y = (((x \cdot (y/z)) \cdot z)/(e \cdot y))\backslash((x \cdot (y/z)) \cdot z) \tag{8}$$

by (7) and Proposition 7. We have $(((x \cdot (y/z)) \cdot z)/(e \cdot y)) \cdot ((((x \cdot (y/z)) \cdot z)/(e \cdot y))\backslash((x \cdot (y/z)) \cdot z)) = (x \cdot (y/z)) \cdot z$ by Axiom 4. Then $(((x \cdot (y/z)) \cdot z)/(e \cdot y)) \cdot y = (x \cdot (y/z)) \cdot z$ by (8). Then $(((x \cdot (y/z)) \cdot z)/((y/z) \cdot z)) \cdot y = (x \cdot (y/z)) \cdot z$ by (6). Hence we are done by Definition 5. ☐

Theorem 24. *$x/(z\backslash((x/y) \cdot y)) = z$.*

Proof. We have $((x/(z\backslash((x/y) \cdot y))) \cdot (z\backslash((x/y) \cdot y)))/(z\backslash((x/y) \cdot y)) = x/(z\backslash((x/y) \cdot y))$ by Axiom 5. Then $((((x/(z\backslash((x/y)\cdot y)))\cdot(z\backslash((x/y)\cdot y)))/y)\cdot y)/(z\backslash((x/y)\cdot y)) = x/(z\backslash((x/y)\cdot y))$ by Axiom 6. Then

$$((x/y) \cdot y)/(z\backslash((x/y) \cdot y)) = x/(z\backslash((x/y) \cdot y)) \tag{9}$$

by Axiom 6. We have $((x/y) \cdot y)/(z\backslash((x/y) \cdot y)) = z$ by Proposition 24. Hence we are done by (9). ☐

Theorem 25. *$R(x, y, y\backslash z) \cdot z = (x \cdot y) \cdot (y\backslash z)$.*

Proof. We have $(((x\cdot y)\cdot(y\backslash z))/(y\cdot(y\backslash z)))\cdot((((x\cdot y)\cdot(y\backslash z))/(y\cdot(y\backslash z)))\backslash((((x\cdot y)\cdot(y\backslash z))/(y\cdot(y\backslash z)))\cdot z)) = (((x\cdot y)\cdot(y\backslash z))/(y\cdot(y\backslash z)))\cdot z$ by Axiom 4. Then $(((x\cdot y)\cdot(y\backslash z))/(y\cdot(y\backslash z)))\cdot(y\cdot(y\backslash((((x\cdot y)\cdot(y\backslash z))/(y\cdot(y\backslash z)))\backslash((((x\cdot y)\cdot(y\backslash z))/(y\cdot(y\backslash z)))\cdot z)))) = (((x\cdot y)\cdot(y\backslash z))/(y\cdot(y\backslash z)))\cdot z$ by Axiom 4. Then

$$(((x\cdot y)\cdot(y\backslash z))/(y\cdot(y\backslash z)))\cdot(y\cdot(y\backslash z)) = (((x\cdot y)\cdot(y\backslash z))/(y\cdot(y\backslash z)))\cdot z \qquad (10)$$

by Axiom 3. We have $(((x\cdot y)\cdot(y\backslash z))/(y\cdot(y\backslash z)))\cdot(y\cdot(y\backslash z)) = (x\cdot y)\cdot(y\backslash z)$ by Axiom 6. Then $(((x\cdot y)\cdot(y\backslash z))/(y\cdot(y\backslash z)))\cdot z = (x\cdot y)\cdot(y\backslash z)$ by (10). Hence we are done by Definition 5. □

Theorem 26. $((x/y)\backslash x)\cdot z = y\cdot z$.

Proof. We have $((y\cdot z)/z)\cdot z = y\cdot z$ by Axiom 6. Then $((((y\cdot z)/z)/(e\backslash y))\cdot(e\backslash y))\cdot z = y\cdot z$ by Axiom 6. Then $((y/(e\backslash y))\cdot(e\backslash y))\cdot z = y\cdot z$ by Axiom 5. Then $(e\cdot(e\backslash y))\cdot z = y\cdot z$ by Proposition 24. Hence we are done by Theorem 19. □

Theorem 27. $x\cdot(((y/x)\backslash y)\backslash z) = z$.

Proof. We have $((y/x)\backslash y)\cdot(((y/x)\backslash y)\backslash z) = z$ by Axiom 4. Hence we are done by Theorem 26. □

Theorem 28. $L((y\backslash x)\backslash e, y\backslash x, y) = x\backslash y$.

Proof. We have $L((y\backslash x)\backslash e, y\backslash x, y) = (y\cdot(y\backslash x))\backslash y$ by Proposition 78. Hence we are done by Axiom 4. □

Theorem 29. $L(y\backslash e, y, x/y) = x\backslash(x/y)$.

Proof. We have $L(y\backslash e, y, x/y) = ((x/y)\cdot y)\backslash(x/y)$ by Proposition 78. Hence we are done by Axiom 6. □

Theorem 30. $L(T(x\backslash e, z), x, y) = T((y\cdot x)\backslash y, z)$.

Proof. We have $L(T(x\backslash e, z), x, y) = T(L(x\backslash e, x, y), z)$ by Axiom 8. Hence we are done by Proposition 78. □

Theorem 31. $R((w\cdot z)\backslash w, x, y) = L(R(z\backslash e, x, y), z, w)$.

Proof. We have $R(L(z\backslash e, z, w), x, y) = L(R(z\backslash e, x, y), z, w)$ by Axiom 10. Hence we are done by Proposition 78. □

Theorem 32. $L((w\cdot z)\backslash w, x, y) = L(L(z\backslash e, x, y), z, w)$.

Proof. We have $L(L(z\backslash e, z, w), x, y) = L(L(z\backslash e, x, y), z, w)$ by Axiom 11. Hence we are done by Proposition 78. □

Theorem 33. $x/L(y\backslash e, y, x) = x\cdot y$.

Proof. We have $x/((x\cdot y)\backslash x) = x\cdot y$ by Proposition 24. Hence we are done by Proposition 78. □

Theorem 34. $L(x, e/x, y) = (y\cdot(e/x))\backslash y$.

Proof. We have $L((e/x)\backslash e, e/x, y) = (y\cdot(e/x))\backslash y$ by Proposition 78. Hence we are done by Proposition 25. □

Theorem 35. $L(x\backslash e, x, e/x) = e/x$.

Proof. We have $e\backslash(e/x) = e/x$ by Proposition 26. Then $((e/x)\cdot x)\backslash(e/x) = e/x$ by Axiom 6. Hence we are done by Proposition 78. □

Theorem 36. $L(x\backslash e, x, x\backslash e) = K(x\backslash e, x)\backslash(x\backslash e)$.

Proof. We have $L(x\backslash e, x, x\backslash e) = ((x\backslash e)\cdot x)\backslash(x\backslash e)$ by Proposition 78. Hence we are done by Proposition 76. □

Theorem 37. $R(e/x, x, x\backslash y) = (x\backslash y)/y$.

Proof. We have $R(e/x, x, x\backslash y) = (x\backslash y)/(x \cdot (x\backslash y))$ by Proposition 79. Hence we are done by Axiom 4. □

Theorem 38. $R(e/(y/x), y/x, x) = x/y$.

Proof. We have $R(e/(y/x), y/x, x) = x/((y/x) \cdot x)$ by Proposition 79. Hence we are done by Axiom 6. □

Theorem 39. $R(T(e/x, z), x, y) = T(y/(x \cdot y), z)$.

Proof. We have $R(T(e/x, z), x, y) = T(R(e/x, x, y), z)$ by Axiom 9. Hence we are done by Proposition 79. □

Theorem 40. $R(w/(z \cdot w), x, y) = R(R(e/z, x, y), z, w)$.

Proof. We have $R(R(e/z, z, w), x, y) = R(R(e/z, x, y), z, w)$ by Axiom 12. Hence we are done by Proposition 79. □

Theorem 41. $R(e/x, x, x\backslash e) = x\backslash e$.

Proof. We have $(x\backslash e)/e = x\backslash e$ by Proposition 27. Then $(x\backslash e)/(x \cdot (x\backslash e)) = x\backslash e$ by Axiom 4. Hence we are done by Proposition 79. □

Theorem 42. $((y \cdot x)\backslash y) \cdot z = L(x\backslash e, x, y) \cdot z$.

Proof. We have $((y/L(x\backslash e, x, y))\backslash y) \cdot z = L(x\backslash e, x, y) \cdot z$ by Theorem 26. Hence we are done by Theorem 33. □

Theorem 43. $L(x\backslash e, x, y) \cdot (((y \cdot x)\backslash y)\backslash z) = z$.

Proof. We have $L(x\backslash e, x, y) \cdot (((y/L(x\backslash e, x, y))\backslash y)\backslash z) = z$ by Theorem 27. Hence we are done by Theorem 33. □

Theorem 44. $L(T(x\backslash e, y), x, e/x) = T(e/x, y)$.

Proof. We have $L(T(x\backslash e, y), x, e/x) = T(L(x\backslash e, x, e/x), y)$ by Axiom 8. Hence we are done by Theorem 35. □

Theorem 45. $R(x, x\backslash e, y)\backslash y = (x\backslash e) \cdot y$.

Proof. We have $R(e/(x\backslash e), x\backslash e, y)\backslash y = (x\backslash e) \cdot y$ by Proposition 80. Hence we are done by Proposition 24. □

Theorem 46. $x\backslash(y \cdot T(y\backslash x, y)) = K(y\backslash x, y)$.

Proof. We have $x\backslash((y\backslash x) \cdot y) = K(y\backslash x, y)$ by Theorem 2. Hence we are done by Proposition 46. □

Theorem 47. $z \cdot T(x, y) = T((z \cdot x)/z, y) \cdot z$.

Proof. We have $z \cdot T(T((z \cdot x)/z, z), y) = T((z \cdot x)/z, y) \cdot z$ by Proposition 50. Hence we are done by Theorem 7. □

Theorem 48. $(y\backslash z) \cdot T((y\backslash z)\backslash z, x) = T(y, x) \cdot (y\backslash z)$.

Proof. We have $(y\backslash z) \cdot T(T(y, y\backslash z), x) = T(y, x) \cdot (y\backslash z)$ by Proposition 50. Hence we are done by Proposition 49. □

Theorem 49. $(y \cdot T(y\backslash x, z))/y = T(x/y, z)$.

Proof. We have $(y \cdot T(T(x/y,z),y))/y = T(x/y,z)$ by Proposition 48. Hence we are done by Proposition 51. □

Theorem 50. $(y \cdot T(x,z))/y = T((y \cdot x)/y, z)$.

Proof. We have $(y \cdot T(T((y \cdot x)/y, y), z))/y = T((y \cdot x)/y, z)$ by Theorem 16. Hence we are done by Theorem 7. □

Theorem 51. $((x/y) \cdot T(y,z))/(x/y) = T(x/(x/y), z)$.

Proof. We have $((x/y) \cdot T(T(x/(x/y), x/y), z))/(x/y) = T(x/(x/y), z)$ by Theorem 16. Hence we are done by Theorem 14. □

Theorem 52. $y \cdot L(z, x\backslash y, x) = x \cdot ((x\backslash y) \cdot z)$.

Proof. We have $(x \cdot (x\backslash y)) \cdot L(z, x\backslash y, x) = x \cdot ((x\backslash y) \cdot z)$ by Proposition 52. Hence we are done by Axiom 4. □

Theorem 53. $x \cdot L(z, y, x/y) = (x/y) \cdot (y \cdot z)$.

Proof. We have $((x/y) \cdot y) \cdot L(z, y, x/y) = (x/y) \cdot (y \cdot z)$ by Proposition 52. Hence we are done by Axiom 6. □

Theorem 54. $z \cdot (y \cdot x) = (z \cdot y) \cdot w \to L(x,y,z) = w$.

Proof. Assume

$$z \cdot (y \cdot x) = (z \cdot y) \cdot w. \tag{11}$$

We have $(z \cdot y) \cdot L(x,y,z) = z \cdot (y \cdot x)$ by Proposition 52. Then $(z \cdot y) \cdot L(x,y,z) = (z \cdot y) \cdot w$ by (11). Hence we are done by Proposition 9. □

Theorem 55. $(x \cdot y) \cdot T(L(z,y,x), w) = x \cdot (y \cdot T(z,w))$.

Proof. We have $(x \cdot y) \cdot L(T(z,w), y, x) = x \cdot (y \cdot T(z,w))$ by Proposition 52. Hence we are done by Axiom 8. □

Theorem 56. $(x \cdot y) \cdot R(L(z,y,x), w, u) = x \cdot (y \cdot R(z,w,u))$.

Proof. We have $(x \cdot y) \cdot L(R(z,w,u), y, x) = x \cdot (y \cdot R(z,w,u))$ by Proposition 52. Hence we are done by Axiom 10. □

Theorem 57. $(x \cdot y) \cdot L(L(z,y,x), w, u) = x \cdot (y \cdot L(z,w,u))$.

Proof. We have $(x \cdot y) \cdot L(L(z,w,u), y, x) = x \cdot (y \cdot L(z,w,u))$ by Proposition 52. Hence we are done by Axiom 11. □

Theorem 58. $(x \cdot y) \cdot L(z, T(x,y), y) = y \cdot (T(x,y) \cdot z)$.

Proof. We have $(y \cdot T(x,y)) \cdot L(z, T(x,y), y) = y \cdot (T(x,y) \cdot z)$ by Proposition 52. Hence we are done by Proposition 46. □

Theorem 59. $K(x\backslash e, x) \cdot L(y, x, x\backslash e) = (x\backslash e) \cdot (x \cdot y)$.

Proof. We have $((x\backslash e) \cdot x) \cdot L(y, x, x\backslash e) = (x\backslash e) \cdot (x \cdot y)$ by Proposition 52. Hence we are done by Proposition 76. □

Theorem 60. $L((y\backslash x)\backslash z, y\backslash x, y) = x\backslash(y \cdot z)$.

Proof. We have $L((y\backslash x)\backslash z, y\backslash x, y) = (y \cdot (y\backslash x))\backslash(y \cdot z)$ by Proposition 53. Hence we are done by Axiom 4. □

Theorem 61. $(y \cdot x)/L(z\backslash x, z, y) = y \cdot z$.

Proof. We have $(y \cdot x)/((y \cdot z)\backslash(y \cdot x)) = y \cdot z$ by Proposition 24. Hence we are done by Proposition 53. □

Theorem 62. $L((x\backslash e)\backslash y, x\backslash e, x) = x \cdot y$.

Proof. We have $e\backslash(x \cdot y) = x \cdot y$ by Proposition 26. Then $(x \cdot (x\backslash e))\backslash(x \cdot y) = x \cdot y$ by Axiom 4. Hence we are done by Proposition 53. □

Theorem 63. $L(x\backslash y, x, e/x) = (e/x) \cdot y$.

Proof. We have $e\backslash((e/x) \cdot y) = (e/x) \cdot y$ by Proposition 26. Then $((e/x) \cdot x)\backslash((e/x) \cdot y) = (e/x) \cdot y$ by Axiom 6. Hence we are done by Proposition 53. □

Theorem 64. $(x \cdot y) \cdot z = w \cdot (y \cdot z) \to R(x, y, z) = w$.

Proof. Assume

$$(x \cdot y) \cdot z = w \cdot (y \cdot z). \tag{12}$$

We have $R(x, y, z) \cdot (y \cdot z) = (x \cdot y) \cdot z$ by Proposition 54. Then $R(x, y, z) \cdot (y \cdot z) = w \cdot (y \cdot z)$ by (12). Hence we are done by Proposition 10. □

Theorem 65. $L(R(x, w, u), y, z) \cdot (w \cdot u) = (L(x, y, z) \cdot w) \cdot u$.

Proof. We have $R(L(x, y, z), w, u) \cdot (w \cdot u) = (L(x, y, z) \cdot w) \cdot u$ by Proposition 54. Hence we are done by Axiom 10. □

Theorem 66. $R(x/y, y, y\backslash z) = (x \cdot (y\backslash z))/z$.

Proof. We have $R(x/y, y, y\backslash z) = (x \cdot (y\backslash z))/(y \cdot (y\backslash z))$ by Proposition 55. Hence we are done by Axiom 4. □

Theorem 67. $R(x/(z/y), z/y, y) = (x \cdot y)/z$.

Proof. We have $R(x/(z/y), z/y, y) = (x \cdot y)/((z/y) \cdot y)$ by Proposition 55. Hence we are done by Axiom 6. □

Theorem 68. $R(x/(e/y), e/y, y) = x \cdot y$.

Proof. We have $(x \cdot y)/e = x \cdot y$ by Proposition 27. Then $(x \cdot y)/((e/y) \cdot y) = x \cdot y$ by Axiom 6. Hence we are done by Proposition 55. □

Theorem 69. $L(x, y\backslash e, y)/((y\backslash e) \cdot x) = y$.

Proof. We have $y \cdot ((y\backslash e) \cdot x) = L(x, y\backslash e, y)$ by Proposition 56. Hence we are done by Proposition 1. □

Theorem 70. $x \cdot K(x\backslash e, x) = L(x, x\backslash e, x)$.

Proof. We have $x \cdot ((x\backslash e) \cdot x) = L(x, x\backslash e, x)$ by Proposition 56. Hence we are done by Proposition 76. □

Theorem 71. $(e/x)\backslash L(y, x, e/x) = x \cdot y$.

Proof. We have $(e/x) \cdot (x \cdot y) = L(y, x, e/x)$ by Proposition 67. Hence we are done by Proposition 2. □

Theorem 72. $(e/y) \cdot (x \cdot y) = L(T(x, y), y, e/y)$.

Proof. We have $(e/y) \cdot (y \cdot T(x, y)) = L(T(x, y), y, e/y)$ by Proposition 67. Hence we are done by Proposition 46. □

Theorem 73. $R(x, y, y\backslash e)/(y\backslash e) = x \cdot y$.

Proof. We have $(x \cdot y) \cdot (y\backslash e) = R(x, y, y\backslash e)$ by Proposition 68. Hence we are done by Proposition 1. □

Theorem 74. $R(x, e/y, y)/y = x \cdot (e/y)$.

Proof. We have $(x \cdot (e/y)) \cdot y = R(x, e/y, y)$ by Proposition 69. Hence we are done by Proposition 1. ☐

Theorem 75. $L(T(y\backslash e, z), y, x/y) = T(x\backslash(x/y), z)$.

Proof. We have $L(T(y\backslash e, z), y, x/y) = T(L(y\backslash e, y, x/y), z)$ by Axiom 8. Hence we are done by Theorem 29. ☐

Theorem 76. $L(x, e/x, x) = K(x, e/x)\backslash x$.

Proof. We have $L(x, e/x, x) = (x \cdot (e/x))\backslash x$ by Theorem 34. Hence we are done by Proposition 77. ☐

Theorem 77. $T(x, x \cdot y) = T(L(x, y, x), y)$.

Proof. We have $L(T(x, y), y, x) = T(L(x, y, x), y)$ by Axiom 8. Hence we are done by Proposition 63. ☐

Theorem 78. $L(T(T(x, y), z), y, x) = T(T(x, x \cdot y), z)$.

Proof. We have $L(T(T(x, y), z), y, x) = T(L(T(x, y), y, x), z)$ by Axiom 8. Hence we are done by Proposition 63. ☐

Theorem 79. $y \cdot T(x, x \cdot y) = L(x, y, x) \cdot y$.

Proof. We have $y \cdot T(L(x, y, x), y) = L(x, y, x) \cdot y$ by Proposition 46. Hence we are done by Theorem 77. ☐

Theorem 80. $(x \cdot T(y, y \cdot x))/x = L(y, x, y)$.

Proof. We have $(x \cdot T(L(y, x, y), x))/x = L(y, x, y)$ by Proposition 48. Hence we are done by Theorem 77. ☐

Theorem 81. $R((e/w) \cdot x, y, z) = L(R(w\backslash x, y, z), w, e/w)$.

Proof. We have $R(L(w\backslash x, w, e/w), y, z) = L(R(w\backslash x, y, z), w, e/w)$ by Axiom 10. Hence we are done by Theorem 63. ☐

Theorem 82. $(e/x)\backslash L(L(y, x, e/x), z, w) = x \cdot L(y, z, w)$.

Proof. We have $(e/x)\backslash L(L(y, z, w), x, e/x) = x \cdot L(y, z, w)$ by Theorem 71. Hence we are done by Axiom 11. ☐

Theorem 83. $L(R(x, w, w\backslash e), y, z)/(w\backslash e) = L(x, y, z) \cdot w$.

Proof. We have $R(L(x, y, z), w, w\backslash e)/(w\backslash e) = L(x, y, z) \cdot w$ by Theorem 73. Hence we are done by Axiom 10. ☐

Theorem 84. $L(R(x, e/w, w), y, z)/w = L(x, y, z) \cdot (e/w)$.

Proof. We have $R(L(x, y, z), e/w, w)/w = L(x, y, z) \cdot (e/w)$ by Theorem 74. Hence we are done by Axiom 10. ☐

Theorem 85. $w \cdot L(w\backslash x, y, z) = L(x/w, y, z) \cdot w$.

Proof. We have $w \cdot L(T(x/w, w), y, z) = L(x/w, y, z) \cdot w$ by Proposition 58. Hence we are done by Proposition 47. ☐

Theorem 86. $w \cdot L(x, y, z) = L((w \cdot x)/w, y, z) \cdot w$.

Proof. We have $w \cdot L(T((w \cdot x)/w, w), y, z) = L((w \cdot x)/w, y, z) \cdot w$ by Proposition 58. Hence we are done by Theorem 7. ☐

Theorem 87. $w \cdot R(x, y, z) = R((w \cdot x)/w, y, z) \cdot w$.

Proof. We have $w \cdot R(T((w \cdot x)/w, w), y, z) = R((w \cdot x)/w, y, z) \cdot w$ by Proposition 59. Hence we are done by Theorem 7. □

Theorem 88. $(y \cdot R(y\backslash x, z, w))/y = R(x/y, z, w)$.

Proof. We have $(y \cdot T(R(x/y, z, w), y))/y = R(x/y, z, w)$ by Proposition 48. Hence we are done by Proposition 60. □

Theorem 89. $T(y/(x \cdot y), x) = R(x\backslash e, x, y)$.

Proof. We have $T(R(e/x, x, y), x) = R(x\backslash e, x, y)$ by Proposition 60. Hence we are done by Proposition 79. □

Theorem 90. $y \cdot R(y\backslash e, y, x) = (x/(y \cdot x)) \cdot y$.

Proof. We have $y \cdot T(x/(y \cdot x), y) = (x/(y \cdot x)) \cdot y$ by Proposition 46. Hence we are done by Theorem 89. □

Theorem 91. $T(y/x, (x\backslash y) \cdot x) = L(x\backslash y, x, x\backslash y)$.

Proof. We have

$$T(y/x, (x\backslash y) \cdot x) \cdot x = x \cdot T(x\backslash y, (x\backslash y) \cdot x) \tag{13}$$

by Proposition 61. We have $L(x\backslash y, x, x\backslash y) \cdot x = x \cdot T(x\backslash y, (x\backslash y) \cdot x)$ by Theorem 79. Hence we are done by (13) and Proposition 8. □

Theorem 92. $T((x \cdot y)/x, y \cdot x) = L(y, x, y)$.

Proof. We have

$$T((x \cdot y)/x, y \cdot x) \cdot x = x \cdot T(y, y \cdot x) \tag{14}$$

by Theorem 47. We have $L(y, x, y) \cdot x = x \cdot T(y, y \cdot x)$ by Theorem 79. Hence we are done by (14) and Proposition 8. □

Theorem 93. $z \cdot ((y \cdot T(x, z))/y) = ((y \cdot x)/y) \cdot z$.

Proof. We have $z \cdot T((y \cdot x)/y, z) = ((y \cdot x)/y) \cdot z$ by Proposition 46. Hence we are done by Theorem 50. □

Theorem 94. $(y \cdot (z\backslash x))/y = T((y \cdot (x/z))/y, z)$.

Proof. We have $(y \cdot T(x/z, z))/y = T((y \cdot (x/z))/y, z)$ by Theorem 50. Hence we are done by Proposition 47. □

Theorem 95. $y\backslash((y/z) \cdot (x \cdot z)) = L(T(x, z), z, y/z)$.

Proof. We have $y\backslash((y/z) \cdot (z \cdot T(x, z))) = L(T(x, z), z, y/z)$ by Theorem 4. Hence we are done by Proposition 46. □

Theorem 96. $L(y, x\backslash e, x)\backslash y = a(x, x\backslash e, y)$.

Proof. We have $e \cdot y = y$ by Axiom 1. Then

$$(x \cdot (x\backslash e)) \cdot y = y \tag{15}$$

by Axiom 4. We have $a(x, x\backslash e, y) = (x \cdot ((x\backslash e) \cdot y))\backslash((x \cdot (x\backslash e)) \cdot y)$ by Definition 1. Then $a(x, x\backslash e, y) = (x \cdot ((x\backslash e) \cdot y))\backslash y$ by (15). Hence we are done by Proposition 56. □

Theorem 97. $L(R(x, w, w\backslash u), y, z) \cdot u = (L(x, y, z) \cdot w) \cdot (w\backslash u)$.

Proof. We have $R(L(x,y,z),w,w\backslash u)\cdot u=(L(x,y,z)\cdot w)\cdot(w\backslash u)$ by Theorem 25. Hence we are done by Axiom 10. □

Theorem 98. $(x\cdot y)\cdot T((x\cdot y)\backslash x,z)=x\cdot(y\cdot T(y\backslash e,z))$.

Proof. We have $(x\cdot y)\cdot L(T(y\backslash e,z),y,x)=x\cdot(y\cdot T(y\backslash e,z))$ by Proposition 52. Hence we are done by Theorem 30. □

Theorem 99. $L(x\backslash e,x,y)\backslash(((y\cdot x)\backslash y)\cdot z)=z$.

Proof. We have $L(x\backslash e,x,y)\cdot z=((y\cdot x)\backslash y)\cdot z$ by Theorem 42. Hence we are done by Proposition 2. □

Theorem 100. $L(x\backslash e,x,y)\backslash z=((y\cdot x)\backslash y)\backslash z$.

Proof. We have $L(x\backslash e,x,y)\cdot(((y\cdot x)\backslash y)\backslash z)=z$ by Theorem 43. Hence we are done by Proposition 2. □

Theorem 101. $L(T(x\backslash e,x),x,e/x)=x\backslash e$.

Proof. We have $T(e/x,x)=x\backslash e$ by Proposition 47. Hence we are done by Theorem 44. □

Theorem 102. $(e/x)\backslash T(e/x,y)=x\cdot T(x\backslash e,y)$.

Proof. We have $(e/x)\backslash L(T(x\backslash e,y),x,e/x)=x\cdot T(x\backslash e,y)$ by Theorem 71. Hence we are done by Theorem 44. □

Theorem 103. $L(T(x\backslash T(y,z),w),x,z)=T((z\cdot x)\backslash(y\cdot z),w)$.

Proof. We have $L(T(x\backslash T(y,z),w),x,z)=T(L(x\backslash T(y,z),x,z),w)$ by Axiom 8. Hence we are done by Proposition 73. □

Theorem 104. $L(T((y\backslash x)\backslash z,w),y\backslash x,y)=T(x\backslash(y\cdot z),w)$.

Proof. We have $L(T((y\backslash x)\backslash z,w),y\backslash x,y)=T(L((y\backslash x)\backslash z,y\backslash x,y),w)$ by Axiom 8. Hence we are done by Theorem 60. □

Theorem 105. $T(R(x/y,y,y\backslash z),z)=z\backslash(x\cdot(y\backslash z))$.

Proof. We have $T((x\cdot(y\backslash z))/z,z)=z\backslash(x\cdot(y\backslash z))$ by Proposition 47. Hence we are done by Theorem 66. □

Theorem 106. $T((x\cdot(y\backslash z))/z,y)=R(y\backslash x,y,y\backslash z)$.

Proof. We have $T(R(x/y,y,y\backslash z),y)=R(y\backslash x,y,y\backslash z)$ by Proposition 60. Hence we are done by Theorem 66. □

Theorem 107. $R(T(x/(z/y),w),z/y,y)=T((x\cdot y)/z,w)$.

Proof. We have $R(T(x/(z/y),w),z/y,y)=T(R(x/(z/y),z/y,y),w)$ by Axiom 9. Hence we are done by Theorem 67. □

Theorem 108. $(e/x)\cdot((x\backslash e)\cdot x)=x\backslash e$.

Proof. We have $L(T(x\backslash e,x),x,e/x)=x\backslash e$ by Theorem 101. Hence we are done by Theorem 72. □

Theorem 109. $(e/x)\cdot K(x\backslash e,x)=x\backslash e$.

Proof. We have $(e/x)\cdot((x\backslash e)\cdot x)=x\backslash e$ by Theorem 108. Hence we are done by Proposition 76. □

Theorem 110. $T(((y\cdot x)\cdot z)/(y\cdot z),y)=R(x,y,z)$.

Proof. We have $T(((y\cdot x)\cdot z)/(y\cdot z),y)=R(y\backslash(y\cdot x),y,z)$ by Proposition 64. Hence we are done by Axiom 3. □

Theorem 111. $(y/x) \cdot (x/y) = (x/y) \cdot R((x/y)\backslash e, x/y, y)$.

Proof. We have $(y/((x/y) \cdot y)) \cdot (x/y) = (x/y) \cdot R((x/y)\backslash e, x/y, y)$ by Theorem 90. Hence we are done by Axiom 6. □

Theorem 112. $R(x, y, x) \cdot (y \cdot x) = (y \cdot x) \cdot T(x, y)$.

Proof. We have $T(((y \cdot x) \cdot x)/(y \cdot x), y) \cdot (y \cdot x) = (y \cdot x) \cdot T(x, y)$ by Theorem 47. Hence we are done by Theorem 110. □

Theorem 113. $T(y, x) = R(T(y, x \cdot y), x, y)$.

Proof. We have

$$(x \cdot y) \cdot T(y, x) = R(y, x, y) \cdot (x \cdot y) \tag{16}$$

by Theorem 112. We have $(x \cdot y) \cdot R(T(y, x \cdot y), x, y) = R(y, x, y) \cdot (x \cdot y)$ by Proposition 59. Hence we are done by (16) and Proposition 7. □

Theorem 114. $(x \cdot y) \cdot T(y, x) = (y \cdot x) \cdot y$.

Proof. We have $R(y, x, y) \cdot (x \cdot y) = (y \cdot x) \cdot y$ by Proposition 54. Hence we are done by Theorem 112. □

Theorem 115. $T(y, x) \cdot (x \cdot y) = (T(y, x \cdot y) \cdot x) \cdot y$.

Proof. We have $R(T(y, x \cdot y), x, y) \cdot (x \cdot y) = (T(y, x \cdot y) \cdot x) \cdot y$ by Proposition 54. Hence we are done by Theorem 113. □

Theorem 116. $K(x, e/x) = T(x, e/x)/x$.

Proof. We have $e \cdot T(x, e/x) = T(x, e/x)$ by Axiom 1. Then

$$T(x, e/x) \cdot (T(x, e/x)\backslash(e \cdot T(x, e/x))) = T(x, e/x) \tag{17}$$

by Axiom 4. We have $T(x, e/x) \cdot (T(x, e/x)\backslash(e \cdot T(x, e/x))) = e \cdot T(x, e/x)$ by Axiom 4. Then $((T(x, e/x) \cdot (T(x, e/x)\backslash(e \cdot T(x, e/x))))/x) \cdot x = e \cdot T(x, e/x)$ by Axiom 6. Then $(T(x, e/x)/x) \cdot x = e \cdot T(x, e/x)$ by (17). Then

$$(T(x, e/x)/x) \cdot x = ((e/x) \cdot x) \cdot T(x, e/x) \tag{18}$$

by Axiom 6. We have $((e/x) \cdot x) \cdot T(x, e/x) = (x \cdot (e/x)) \cdot x$ by Theorem 114. Then

$$(T(x, e/x)/x) \cdot x = (x \cdot (e/x)) \cdot x \tag{19}$$

by (18).

$$\begin{array}{rcl} & K(x, e/x) & \\ = & x \cdot (e/x) & \text{by Proposition 77} \\ = & T(x, e/x)/x & \text{by (19), Proposition 10.} \end{array}$$

Hence we are done. □

Theorem 117. $T(x, e/x) = R(x, e/x, x)$.

Proof. We have

$$((e/x) \cdot x) \cdot T(x, e/x) = (x \cdot (e/x)) \cdot x \tag{20}$$

by Theorem 114. We have $(e/x) \cdot x = e$ by Axiom 6. Then

$$e \cdot T(x, e/x) = (x \cdot (e/x)) \cdot x \tag{21}$$

by (20). We have $e \cdot ((x \cdot (e/x)) \cdot x) = (x \cdot (e/x)) \cdot x$ by Axiom 1. Then $e \cdot T(x, e/x) = e \cdot ((x \cdot (e/x)) \cdot x)$ by (21). Then

$$T(x, e/x) = (x \cdot (e/x)) \cdot x \tag{22}$$

by Proposition 9. We have $(x \cdot (e/x)) \cdot x = R(x, e/x, x)$ by Proposition 69. Hence we are done by (22). □

Theorem 118. $R(T(x, y), e/x, x) = T(T(x, e/x), y)$.

Proof. We have $R(T(x, y), e/x, x) = T(R(x, e/x, x), y)$ by Axiom 9. Hence we are done by Theorem 117. □

Theorem 119. $T(T(y, e/y), x)/y = T(y, x) \cdot (e/y)$.

Proof. We have $R(T(y, x), e/y, y)/y = T(y, x) \cdot (e/y)$ by Theorem 74. Hence we are done by Theorem 118. □

Theorem 120. $L(e/x, x, y) \cdot x = x \cdot ((y \cdot x)\backslash y)$.

Proof. We have $L(e/x, x, y) \cdot x = x \cdot L(x\backslash e, x, y)$ by Theorem 85. Hence we are done by Proposition 78. □

Theorem 121. $(x \cdot ((y \cdot x)\backslash y))/x = L(e/x, x, y)$.

Proof. We have $L(e/x, x, y) \cdot x = x \cdot ((y \cdot x)\backslash y)$ by Theorem 120. Hence we are done by Proposition 1. □

Theorem 122. $(y \cdot L(x, z, w))/y = L((y \cdot x)/y, z, w)$.

Proof. We have $L((y \cdot x)/y, z, w) \cdot y = y \cdot L(x, z, w)$ by Theorem 86. Hence we are done by Proposition 1. □

Theorem 123. $(y \cdot R(x, z, w))/y = R((y \cdot x)/y, z, w)$.

Proof. We have $R((y \cdot x)/y, z, w) \cdot y = y \cdot R(x, z, w)$ by Theorem 87. Hence we are done by Proposition 1. □

Theorem 124. $R((w \cdot x)/w, y, z)\backslash(w \cdot R(x, y, z)) = w$.

Proof. We have $R((w \cdot x)/w, y, z) \cdot w = w \cdot R(x, y, z)$ by Theorem 87. Hence we are done by Proposition 2. □

Theorem 125. $z \cdot ((y \cdot (z\backslash x))/y) = ((y \cdot (x/z))/y) \cdot z$.

Proof. We have $z \cdot ((y \cdot T(x/z, z))/y) = ((y \cdot (x/z))/y) \cdot z$ by Theorem 93. Hence we are done by Proposition 47. □

Theorem 126. $(e/(e/x)) \cdot (x \cdot (e/x)) = x$.

Proof. We have $(e/(e/x)) \cdot (((e/x)\backslash e) \cdot (e/x)) = (e/x)\backslash e$ by Theorem 108. Then

$$(e/(e/x)) \cdot (x \cdot (e/x)) = (e/x)\backslash e \tag{23}$$

by Proposition 25. We have $(e/x)\backslash e = x$ by Proposition 25. Hence we are done by (23). □

Theorem 127. $L((z \cdot (x\backslash e))/z, x, y) = (z \cdot ((y \cdot x)\backslash y))/z$.

Proof. We have $L((z \cdot (x\backslash e))/z, x, y) = (z \cdot L(x\backslash e, x, y))/z$ by Theorem 122. Hence we are done by Proposition 78. $\square$

Theorem 128. $L((w \cdot ((y\backslash x)\backslash z))/w, y\backslash x, y) = (w \cdot (x\backslash (y \cdot z)))/w$.

Proof. We have $L((w \cdot ((y\backslash x)\backslash z))/w, y\backslash x, y) = (w \cdot L((y\backslash x)\backslash z, y\backslash x, y))/w$ by Theorem 122. Hence we are done by Theorem 60. $\square$

Theorem 129. $((y \cdot R(x, z, w))/y) \cdot (z \cdot w) = (((y \cdot x)/y) \cdot z) \cdot w$.

Proof. We have $R((y \cdot x)/y, z, w) \cdot (z \cdot w) = (((y \cdot x)/y) \cdot z) \cdot w$ by Proposition 54. Hence we are done by Theorem 123. $\square$

Theorem 130. $x\backslash ((x \cdot (e/y)) \cdot y) = a(x, e/y, y)$.

Proof. We have $(x \cdot e)\backslash ((x \cdot (e/y)) \cdot y) = a(x, e/y, y)$ by Theorem 1. Hence we are done by Axiom 2. $\square$

Theorem 131. $x\backslash R(x, e/y, y) = a(x, e/y, y)$.

Proof. We have $x\backslash ((x \cdot (e/y)) \cdot y) = a(x, e/y, y)$ by Theorem 130. Hence we are done by Proposition 69. $\square$

Theorem 132. $T(y, x)\backslash T(T(y, e/y), x) = a(T(y, x), e/y, y)$.

Proof. We have $T(y, x)\backslash R(T(y, x), e/y, y) = a(T(y, x), e/y, y)$ by Theorem 131. Hence we are done by Theorem 118. $\square$

Theorem 133. $(x \cdot y) \cdot R((x \cdot y)\backslash x, z, w) = x \cdot (y \cdot R(y\backslash e, z, w))$.

Proof. We have $(x \cdot y) \cdot L(R(y\backslash e, z, w), y, x) = x \cdot (y \cdot R(y\backslash e, z, w))$ by Proposition 52. Hence we are done by Theorem 31. $\square$

Theorem 134. $L(e/y, y, x) = L((x \cdot y)\backslash x, y, e/y)$.

Proof. We have $L(L(y\backslash e, y, e/y), y, x) = L((x \cdot y)\backslash x, y, e/y)$ by Theorem 32. Hence we are done by Theorem 35. $\square$

Theorem 135. $(x \cdot y) \cdot L((x \cdot y)\backslash x, z, w) = x \cdot (y \cdot L(y\backslash e, z, w))$.

Proof. We have $(x \cdot y) \cdot L(L(y\backslash e, z, w), y, x) = x \cdot (y \cdot L(y\backslash e, z, w))$ by Proposition 52. Hence we are done by Theorem 32. $\square$

Theorem 136. $(y \cdot x) \cdot T((y \cdot x)\backslash z, w) = y \cdot (x \cdot T(x\backslash (y\backslash z), w))$.

Proof. We have $(y \cdot x) \cdot T(L(x\backslash (y\backslash z), x, y), w) = y \cdot (x \cdot T(x\backslash (y\backslash z), w))$ by Theorem 55. Hence we are done by Proposition 70. $\square$

Theorem 137. $T((x \cdot w)/(z \cdot w), y) \cdot (z \cdot w) = (T(x/z, y) \cdot z) \cdot w$.

Proof. We have $T(R(x/z, z, w), y) \cdot (z \cdot w) = (T(x/z, y) \cdot z) \cdot w$ by Proposition 65. Hence we are done by Proposition 55. $\square$

Theorem 138. $T(x/(z \cdot w), y) \cdot (z \cdot w) = (T((x/w)/z, y) \cdot z) \cdot w$.

Proof. We have $T(R((x/w)/z, z, w), y) \cdot (z \cdot w) = (T((x/w)/z, y) \cdot z) \cdot w$ by Proposition 65. Hence we are done by Proposition 74. $\square$

Theorem 139. $x \cdot T(x\backslash (x/y), z) = (x/y) \cdot (y \cdot T(y\backslash e, z))$.

Proof. We have $x \cdot L(T(y\backslash e, z), y, x/y) = (x/y) \cdot (y \cdot T(y\backslash e, z))$ by Theorem 53. Hence we are done by Theorem 75. $\square$

Theorem 140. $x\backslash T(x\cdot y,z)=(x\backslash e)\cdot T((x\backslash e)\backslash y,z)$.

Proof. We have $L(T((x\backslash e)\backslash y,z),x\backslash e,x)=T(L((x\backslash e)\backslash y,x\backslash e,x),z)$ by Axiom 8. Then

$$L(T((x\backslash e)\backslash y,z),x\backslash e,x)=T(x\cdot y,z) \tag{24}$$

by Theorem 62. We have $x\backslash L(T((x\backslash e)\backslash y,z),x\backslash e,x)=(x\backslash e)\cdot T((x\backslash e)\backslash y,z)$ by Proposition 57. Hence we are done by (24). □

Theorem 141. $T(x\cdot z,y)/z=T(x/(e/z),y)\cdot(e/z)$.

Proof. We have $R(T(x/(e/z),y),e/z,z)=T(R(x/(e/z),e/z,z),y)$ by Axiom 9. Then

$$R(T(x/(e/z),y),e/z,z)=T(x\cdot z,y) \tag{25}$$

by Theorem 68. We have $R(T(x/(e/z),y),e/z,z)/z=T(x/(e/z),y)\cdot(e/z)$ by Theorem 74. Hence we are done by (25). □

Theorem 142. $R(T(x/y,z),y,y\backslash z)=z\backslash(x\cdot(y\backslash z))$.

Proof. We have $R(T(x/y,z),y,y\backslash z)=T(R(x/y,y,y\backslash z),z)$ by Axiom 9. Hence we are done by Theorem 105. □

Theorem 143. $T(R(z\backslash x,z,z\backslash y),y)=T(y\backslash(x\cdot(z\backslash y)),z)$.

Proof. We have $T(T((x\cdot(z\backslash y))/y,z),y)=T(y\backslash(x\cdot(z\backslash y)),z)$ by Proposition 51. Hence we are done by Theorem 106. □

Theorem 144. $((z\cdot R(e/x,x,y))/z)\cdot(x\cdot y)=(x\cdot y)\cdot((z\cdot((x\cdot y)\backslash y))/z)$.

Proof. We have $((z\cdot(y/(x\cdot y)))/z)\cdot(x\cdot y)=(x\cdot y)\cdot((z\cdot((x\cdot y)\backslash y))/z)$ by Theorem 125. Hence we are done by Proposition 79. □

Theorem 145. $(x\backslash e)\cdot T((x\backslash e)\backslash e,y)=x\backslash T(x,y)$.

Proof. We have $(x\backslash e)\cdot T((x\backslash e)\backslash e,y)=x\backslash T(x\cdot e,y)$ by Theorem 140. Hence we are done by Axiom 2. □

Theorem 146. $y\backslash T(y,x)=T(y,x)\cdot(y\backslash e)$.

Proof. We have $(y\backslash e)\cdot T((y\backslash e)\backslash e,x)=T(y,x)\cdot(y\backslash e)$ by Theorem 48. Hence we are done by Theorem 145. □

Theorem 147. $T(e/(e/y),x)\cdot(e/y)=T(y,x)/y$.

Proof. We have $T(e/(e/y),x)\cdot(e/y)=T(e\cdot y,x)/y$ by Theorem 141. Hence we are done by Axiom 1. □

Theorem 148. $((e/y)\backslash x)\cdot(e/y)=T(x\cdot y,e/y)/y$.

Proof. We have $T(x/(e/y),e/y)\cdot(e/y)=T(x\cdot y,e/y)/y$ by Theorem 141. Hence we are done by Proposition 47. □

Theorem 149. $T(x,y)/x=(e/x)\cdot T((e/x)\backslash e,y)$.

Proof. We have $T(e/(e/x),y)\cdot(e/x)=(e/x)\cdot T((e/x)\backslash e,y)$ by Proposition 61. Hence we are done by Theorem 147. □

Theorem 150. $T(x,y)/x=(e/x)\cdot T(x,y)$.

Proof. We have $(e/x)\cdot T(T(e/(e/x),e/x),y)=T(e/(e/x),y)\cdot(e/x)$ by Proposition 50. Then $(e/x)\cdot T(x,y)=T(e/(e/x),y)\cdot(e/x)$ by Theorem 14. Hence we are done by Theorem 147. □

Theorem 151. $(x/y)\backslash(y\backslash x)=(y\backslash x)\cdot((x/y)\backslash e)$.

Proof. We have $T(x/y, y) \cdot ((x/y)\backslash e) = (x/y)\backslash T(x/y, y)$ by Theorem 146. Then $(y\backslash x) \cdot ((x/y)\backslash e) = (x/y)\backslash T(x/y, y)$ by Proposition 47. Hence we are done by Proposition 47. □

Theorem 152. $T(x, e/y)/y = ((e/y)\backslash(x/y)) \cdot (e/y)$.

Proof. We have $T((x/y) \cdot y, e/y)/y = ((e/y)\backslash(x/y)) \cdot (e/y)$ by Theorem 148. Hence we are done by Axiom 6. □

Theorem 153. $(x\backslash y)/(y/x) = (e/(y/x)) \cdot (x\backslash y)$.

Proof. We have $(e/(y/x)) \cdot T(y/x, x) = T(y/x, x)/(y/x)$ by Theorem 150. Then $(e/(y/x)) \cdot (x\backslash y) = T(y/x, x)/(y/x)$ by Proposition 47. Hence we are done by Proposition 47. □

Theorem 154. $(x/y) \cdot K(y\backslash e, y) = x \cdot T(x\backslash(x/y), y)$.

Proof. We have $T(x\backslash(x/y), y) = L(T(y\backslash e, y), y, x/y)$ by Theorem 75. Then

$$T(x\backslash(x/y), y) = x\backslash((x/y) \cdot ((y\backslash e) \cdot y)) \tag{26}$$

by Theorem 95. We have $x \cdot (x\backslash((x/y) \cdot ((y\backslash e) \cdot y))) = (x/y) \cdot ((y\backslash e) \cdot y)$ by Axiom 4. Then $x \cdot T(x\backslash(x/y), y) = (x/y) \cdot ((y\backslash e) \cdot y)$ by (26). Hence we are done by Proposition 76. □

Theorem 155. $(y \cdot x) \cdot L((y \cdot x)\backslash z, w, u) = y \cdot (x \cdot L(x\backslash(y\backslash z), w, u))$.

Proof. We have $(y \cdot x) \cdot L(L(x\backslash(y\backslash z), x, y), w, u) = y \cdot (x \cdot L(x\backslash(y\backslash z), w, u))$ by Theorem 57. Hence we are done by Proposition 70. □

Theorem 156. $K(x\backslash e, x) \cdot L(x\backslash y, x, x\backslash e) = (x\backslash e) \cdot y$.

Proof. We have $K(x\backslash e, x) \cdot L(x\backslash y, x, x\backslash e) = (x\backslash e) \cdot (x \cdot (x\backslash y))$ by Theorem 59. Hence we are done by Axiom 4. □

Theorem 157. $L((x \cdot u)/(w \cdot u), y, z) \cdot (w \cdot u) = (L(x/w, y, z) \cdot w) \cdot u$.

Proof. We have $L(R(x/w, w, u), y, z) \cdot (w \cdot u) = (L(x/w, y, z) \cdot w) \cdot u$ by Theorem 65. Hence we are done by Proposition 55. □

Theorem 158. $(e/x)\backslash R((e/x) \cdot y, z, w) = x \cdot R(x\backslash y, z, w)$.

Proof. We have $(e/x)\backslash L(R(x\backslash y, z, w), x, e/x) = x \cdot R(x\backslash y, z, w)$ by Theorem 71. Hence we are done by Theorem 81. □

Theorem 159. $(e/x)\backslash L(e/x, y, z) = x \cdot L(x\backslash e, y, z)$.

Proof. We have $(e/x)\backslash L(L(x\backslash e, x, e/x), y, z) = x \cdot L(x\backslash e, y, z)$ by Theorem 82. Hence we are done by Theorem 35. □

Theorem 160. $L(z\backslash e, x, y)/(z\backslash e) = L(e/z, x, y) \cdot z$.

Proof. We have $L(R(e/z, z, z\backslash e), x, y)/(z\backslash e) = L(e/z, x, y) \cdot z$ by Theorem 83. Hence we are done by Theorem 41. □

Theorem 161. $L(x\backslash e, y, z)/(x\backslash e) = x \cdot L(x\backslash e, y, z)$.

Proof. We have $L(e/x, y, z) \cdot x = x \cdot L(x\backslash e, y, z)$ by Theorem 85. Hence we are done by Theorem 160. □

Theorem 162. $L(x\backslash e, x, y)/(x\backslash e) = x \cdot ((y \cdot x)\backslash y)$.

Proof. We have $L(e/x, x, y) \cdot x = x \cdot ((y \cdot x)\backslash y)$ by Theorem 120. Hence we are done by Theorem 160. □

Theorem 163. $(x \cdot ((y \cdot x)\backslash y)) \cdot (x\backslash e) = L(x\backslash e, x, y)$.

Proof. We have $(L(x\backslash e,x,y)/(x\backslash e))\cdot(x\backslash e) = L(x\backslash e,x,y)$ by Axiom 6. Hence we are done by Theorem 162. □

Theorem 164. $L(T(z,e/z),x,y)/z = L(z,x,y)\cdot(e/z)$.

Proof. We have $L(R(z,e/z,z),x,y)/z = L(z,x,y)\cdot(e/z)$ by Theorem 84. Hence we are done by Theorem 117. □

Theorem 165. $L(z,x,y) = (e/z)\backslash(L(z,x,y)/z)$.

Proof. We have $L(T(z,e/z),x,y)/z = L(z,x,y)\cdot(e/z)$ by Theorem 164. Then

$$T(L(z,x,y),e/z)/z = L(z,x,y)\cdot(e/z) \tag{27}$$

by Axiom 8. We have $((e/z)\backslash(L(z,x,y)/z))\cdot(e/z) = T(L(z,x,y),e/z)/z$ by Theorem 152. Hence we are done by (27) and Proposition 8. □

Theorem 166. $(e/z)\cdot L(z,x,y) = L(z,x,y)/z$.

Proof. We have $(e/z)\cdot((e/z)\backslash(L(z,x,y)/z)) = L(z,x,y)/z$ by Axiom 4. Hence we are done by Theorem 165. □

Theorem 167. $T(e/z,L(z,x,y)) = L(z,x,y)\backslash(L(z,x,y)/z)$.

Proof. We have $T(e/z,L(z,x,y)) = L(z,x,y)\backslash((e/z)\cdot L(z,x,y))$ by Definition 3. Hence we are done by Theorem 166. □

Theorem 168. $y\cdot T(y\backslash(x\cdot z),w) = x\cdot((x\backslash y)\cdot T((x\backslash y)\backslash z,w))$.

Proof. We have $y\cdot L(T((x\backslash y)\backslash z,w),x\backslash y,x) = x\cdot((x\backslash y)\cdot T((x\backslash y)\backslash z,w))$ by Theorem 52. Hence we are done by Theorem 104. □

Theorem 169. $T((x\cdot w)/z,y)\cdot z = (T(x/(z/w),y)\cdot(z/w))\cdot w$.

Proof. We have $R(T(x/(z/w),y),z/w,w)\cdot z = (T(x/(z/w),y)\cdot(z/w))\cdot w$ by Theorem 23. Hence we are done by Theorem 107. □

Theorem 170. $(x\cdot y)\cdot((z\cdot((x\cdot y)\backslash x))/z) = x\cdot(y\cdot((z\cdot(y\backslash e))/z))$.

Proof. We have $(x\cdot y)\cdot L((z\cdot(y\backslash e))/z,y,x) = x\cdot(y\cdot((z\cdot(y\backslash e))/z))$ by Proposition 52. Hence we are done by Theorem 127. □

Theorem 171. $R(T(z\backslash x,y),z,z\backslash y) = T(y\backslash(x\cdot(z\backslash y)),z)$.

Proof. We have $R(T(z\backslash x,y),z,z\backslash y) = T(R(z\backslash x,z,z\backslash y),y)$ by Axiom 9. Hence we are done by Theorem 143. □

Theorem 172. $y\cdot(T(y,x)\cdot z) = T(y,x)\cdot(y\cdot z)$.

Proof. We have $(y\cdot z)\cdot L(T(T(y,z),x),z,y) = y\cdot(z\cdot T(T(y,z),x))$ by Proposition 52. Then

$$(y\cdot z)\cdot T(T(y,y\cdot z),x) = y\cdot(z\cdot T(T(y,z),x)) \tag{28}$$

by Theorem 78. We have $(y\cdot z)\cdot T(T(y,y\cdot z),x) = T(y,x)\cdot(y\cdot z)$ by Proposition 50. Then $y\cdot(z\cdot T(T(y,z),x)) = T(y,x)\cdot(y\cdot z)$ by (28). Hence we are done by Proposition 50. □

Theorem 173. $y\backslash(T(y,x)\cdot z) = T(y,x)\cdot(y\backslash z)$.

Proof. We have $T(y,x)\backslash(T(y,x)\cdot z) = z$ by Axiom 3. Then

$$T(y,x)\backslash(y\cdot(y\backslash(T(y,x)\cdot z))) = z \tag{29}$$

by Axiom 4. We have $T(y,x)\cdot(T(y,x)\backslash(y\cdot(y\backslash(T(y,x)\cdot z)))) = y\cdot(y\backslash(T(y,x)\cdot z))$ by Axiom 4. Then $T(y,x)\cdot(y\cdot(y\backslash(T(y,x)\backslash(y\cdot(y\backslash(T(y,x)\cdot z)))))) = y\cdot(y\backslash(T(y,x)\cdot z))$ by Axiom 4. Then

$$T(y,x)\cdot(y\cdot(y\backslash z)) = y\cdot(y\backslash(T(y,x)\cdot z)) \tag{30}$$

by (29). We have $T(y,x)\cdot(y\cdot(y\backslash z)) = y\cdot(T(y,x)\cdot(y\backslash z))$ by Theorem 172. Then $y\cdot(y\backslash(T(y,x)\cdot z)) = y\cdot(T(y,x)\cdot(y\backslash z))$ by (30). Hence we are done by Proposition 9. □

Theorem 174. $y\cdot(T(y,x)\backslash z) = T(y,x)\backslash(y\cdot z)$.

Proof. We have $y\backslash(y\cdot z) = z$ by Axiom 3. Then

$$y\backslash(T(y,x)\cdot(T(y,x)\backslash(y\cdot z))) = z \tag{31}$$

by Axiom 4. We have $y\cdot(y\backslash(T(y,x)\cdot(T(y,x)\backslash(y\cdot z)))) = T(y,x)\cdot(T(y,x)\backslash(y\cdot z))$ by Axiom 4. Then $y\cdot(T(y,x)\cdot(T(y,x)\backslash(y\backslash(T(y,x)\cdot(T(y,x)\backslash(y\cdot z)))))) = T(y,x)\cdot(T(y,x)\backslash(y\cdot z))$ by Axiom 4. Then

$$y\cdot(T(y,x)\cdot(T(y,x)\backslash z)) = T(y,x)\cdot(T(y,x)\backslash(y\cdot z)) \tag{32}$$

by (31). We have $T(y,x)\cdot(y\cdot(T(y,x)\backslash z)) = y\cdot(T(y,x)\cdot(T(y,x)\backslash z))$ by Theorem 172. Hence we are done by (32) and Proposition 7. □

Theorem 175. $y\cdot(T(y,x)\backslash e) = T(y,x)\backslash y$.

Proof. We have $y\backslash(y\cdot e) = e$ by Axiom 3. Then $y\backslash(T(y,x)\cdot(T(y,x)\backslash(y\cdot e))) = e$ by Axiom 4. Then

$$y\backslash(T(y,x)\cdot(T(y,x)\backslash y)) = e \tag{33}$$

by Axiom 2. We have $y\cdot(y\backslash(T(y,x)\cdot(T(y,x)\backslash y))) = T(y,x)\cdot(T(y,x)\backslash y)$ by Axiom 4. Then $y\cdot e = T(y,x)\cdot(T(y,x)\backslash y)$ by (33). Then

$$T(y,x)\cdot(T(y,x)\backslash y) = y\cdot(T(y,x)\cdot(T(y,x)\backslash e)) \tag{34}$$

by Axiom 4. We have $T(y,x)\cdot(y\cdot(T(y,x)\backslash e)) = y\cdot(T(y,x)\cdot(T(y,x)\backslash e))$ by Theorem 172. Hence we are done by (34) and Proposition 7. □

Theorem 176. $T(y,x)\cdot K(y,e/y) = y\cdot(T(y,x)\cdot(e/y))$.

Proof. We have $T(y,x)\cdot(y\cdot(e/y)) = y\cdot(T(y,x)\cdot(e/y))$ by Theorem 172. Hence we are done by Proposition 77. □

Theorem 177. $x\cdot(T(x,y)\backslash y) = T(y,T(x,y))$.

Proof. We have $T(x,y)\backslash(x\cdot y) = T(y,T(x,y))$ by Theorem 13. Hence we are done by Theorem 174. □

Theorem 178. $(y/x)\cdot((x\backslash y)\backslash x) = T(x,x\backslash y)$.

Proof. We have $(y/x)\cdot(T(y/x,x)\backslash x) = T(x,T(y/x,x))$ by Theorem 177. Then $(y/x)\cdot((x\backslash y)\backslash x) = T(x,T(y/x,x))$ by Proposition 47. Hence we are done by Proposition 47. □

Theorem 179. $((x\cdot y)/x)\cdot(y\backslash x) = T(x,y)$.

Proof. We have $((x\cdot y)/x)\cdot((x\backslash(x\cdot y))\backslash x) = T(x,x\backslash(x\cdot y))$ by Theorem 178. Then $((x\cdot y)/x)\cdot(y\backslash x) = T(x,x\backslash(x\cdot y))$ by Axiom 3. Hence we are done by Axiom 3. □

Theorem 180. $(x\backslash y)\backslash(y/x) = (y/x)\cdot((x\backslash y)\backslash e)$.

Proof. We have $(y/x) \cdot (T(y/x, x)\backslash e) = T(y/x, x)\backslash(y/x)$ by Theorem 175. Then $(y/x) \cdot ((x\backslash y)\backslash e) = T(y/x, x)\backslash(y/x)$ by Proposition 47. Hence we are done by Proposition 47. □

Theorem 181. $T(T(y, x \cdot y) \cdot x, y) = T(y, x) \cdot T(x, y)$.

Proof. We have $T(T(y, x \cdot y) \cdot x, y) = y\backslash((T(y, x \cdot y) \cdot x) \cdot y)$ by Definition 3. Then $T(T(y, x \cdot y) \cdot x, y) = y\backslash(T(y, x) \cdot (x \cdot y))$ by Theorem 115. Then $T(y, x) \cdot (y\backslash(x \cdot y)) = T(T(y, x \cdot y) \cdot x, y)$ by Theorem 173. Hence we are done by Definition 3. □

Theorem 182. $(x/y)\backslash(x \cdot T(x\backslash(x/y), z)) = y \cdot T(y\backslash e, z)$.

Proof. We have $(x/y) \cdot (y \cdot T(y\backslash e, z)) = x \cdot T(x\backslash(x/y), z)$ by Theorem 139. Hence we are done by Proposition 2. □

Theorem 183. $(T(e/x, z) \cdot x) \cdot (x\backslash y) = y \cdot T(y\backslash(x\backslash y), z)$.

Proof. We have $R(T(e/x, z), x, x\backslash y) = T(R(e/x, x, x\backslash y), z)$ by Axiom 9. Then

$$R(T(e/x, z), x, x\backslash y) = T((x\backslash y)/y, z) \tag{35}$$

by Theorem 37. We have $R(T(e/x, z), x, x\backslash y) \cdot y = (T(e/x, z) \cdot x) \cdot (x\backslash y)$ by Theorem 25. Then

$$T((x\backslash y)/y, z) \cdot y = (T(e/x, z) \cdot x) \cdot (x\backslash y) \tag{36}$$

by (35). We have $T((x\backslash y)/y, z) \cdot y = y \cdot T(y\backslash(x\backslash y), z)$ by Proposition 61. Hence we are done by (36). □

Theorem 184. $K((x \cdot y)\backslash x, x \cdot y) = y \cdot T(y\backslash e, x \cdot y)$.

Proof. We have $(x \cdot y) \cdot L(T(y\backslash e, x \cdot y), y, x) = L(y\backslash e, y, x) \cdot (x \cdot y)$ by Proposition 58. Then

$$(x \cdot y) \cdot L(T(y\backslash e, x \cdot y), y, x) = ((x \cdot y)\backslash x) \cdot (x \cdot y) \tag{37}$$

by Proposition 78. We have $(x \cdot y) \cdot L(T(y\backslash e, x \cdot y), y, x) = x \cdot (y \cdot T(y\backslash e, x \cdot y))$ by Proposition 52. Then

$$((x \cdot y)\backslash x) \cdot (x \cdot y) = x \cdot (y \cdot T(y\backslash e, x \cdot y)) \tag{38}$$

by (37). We have $x \cdot K((x \cdot y)\backslash x, x \cdot y) = ((x \cdot y)\backslash x) \cdot (x \cdot y)$ by Theorem 17. Then $x \cdot K((x \cdot y)\backslash x, x \cdot y) = x \cdot (y \cdot T(y\backslash e, x \cdot y))$ by (38). Hence we are done by Proposition 9. □

Theorem 185. $K((x \cdot y)\backslash x, x \cdot y)/y = T(e/y, x \cdot y)$.

Proof. We have $(y \cdot T(y\backslash e, x \cdot y))/y = T(e/y, x \cdot y)$ by Theorem 49. Hence we are done by Theorem 184. □

Theorem 186. $((y \cdot x)/y)\backslash x = x \cdot (((y \cdot x)/y)\backslash e)$.

Proof. We have $(y\backslash(y \cdot x)) \cdot (((y \cdot x)/y)\backslash e) = ((y \cdot x)/y)\backslash(y\backslash(y \cdot x))$ by Theorem 151. Then $x \cdot (((y \cdot x)/y)\backslash e) = ((y \cdot x)/y)\backslash(y\backslash(y \cdot x))$ by Axiom 3. Hence we are done by Axiom 3. □

Theorem 187. $(e/x)\backslash R(e/x, y, z) = x \cdot R(x\backslash e, y, z)$.

Proof. We have $(e/x)\backslash R((e/x) \cdot e, y, z) = x \cdot R(x\backslash e, y, z)$ by Theorem 158. Hence we are done by Axiom 2. □

Theorem 188. $(y/(x \cdot y)) \cdot x = (e/x)\backslash R(e/x, x, y)$.

Proof. We have $(y/(x \cdot y)) \cdot x = x \cdot R(x\backslash e, x, y)$ by Theorem 90. Hence we are done by Theorem 187. □

Theorem 189. $(x/(y \cdot x))/((x/(y \cdot x)) \cdot y) = e/y$.

Proof. We have $(e/y)\backslash R(e/y, y, x) = (x/(y \cdot x)) \cdot y$ by Theorem 188. Then

$$(e/y)\backslash(x/(y \cdot x)) = (x/(y \cdot x)) \cdot y \tag{39}$$

by Proposition 79. We have $(x/(y \cdot x))/((e/y)\backslash(x/(y \cdot x))) = e/y$ by Proposition 24. Hence we are done by (39). □

Theorem 190. $(e/z)\backslash L(z\backslash e, x, y) = L(z\backslash e, x, y) \cdot z$.

Proof. We have $(e/z)\backslash L(L(T(z\backslash e, z), z, e/z), x, y) = z \cdot L(T(z\backslash e, z), x, y)$ by Theorem 82. Then

$$(e/z)\backslash L(z\backslash e, x, y) = z \cdot L(T(z\backslash e, z), x, y) \tag{40}$$

by Theorem 101. We have $z \cdot L(T(z\backslash e, z), x, y) = L(z\backslash e, x, y) \cdot z$ by Proposition 58. Hence we are done by (40). □

Theorem 191. $R(e/(e/z), x, y) \cdot (e/z) = R(z, x, y)/z$.

Proof. We have $R(R(e/(e/z), x, y), e/z, z)/z = R(e/(e/z), x, y) \cdot (e/z)$ by Theorem 74. Then $R(R(e/(e/z), e/z, z), x, y)/z = R(e/(e/z), x, y) \cdot (e/z)$ by Axiom 12. Then $R(z/e, x, y)/z = R(e/(e/z), x, y) \cdot (e/z)$ by Theorem 38. Hence we are done by Proposition 27. □

Theorem 192. $R(x, y, z)/x = (e/x) \cdot R(x, y, z)$.

Proof. We have $(e/x) \cdot R(T(e/(e/x), e/x), y, z) = R(e/(e/x), y, z) \cdot (e/x)$ by Proposition 59. Then $(e/x) \cdot R(x, y, z) = R(e/(e/x), y, z) \cdot (e/x)$ by Theorem 14. Hence we are done by Theorem 191. □

Theorem 193. $z \cdot R(z\backslash e, x, y) = R(z\backslash e, x, y)/(z\backslash e)$.

Proof. We have $(e/(z\backslash e)) \cdot R(z\backslash e, x, y) = R(z\backslash e, x, y)/(z\backslash e)$ by Theorem 192. Hence we are done by Proposition 24. □

Theorem 194. $(x \cdot z) \cdot T((x \cdot z)\backslash z, y) = (x \cdot T(x\backslash e, y)) \cdot z$.

Proof. We have $R(T(e/x, y), x, z) \cdot (x \cdot z) = (T(e/x, y) \cdot x) \cdot z$ by Proposition 54. Then

$$T(z/(x \cdot z), y) \cdot (x \cdot z) = (T(e/x, y) \cdot x) \cdot z \tag{41}$$

by Theorem 39. We have $T(z/(x \cdot z), y) \cdot (x \cdot z) = (x \cdot z) \cdot T((x \cdot z)\backslash z, y)$ by Proposition 61. Then $(T(e/x, y) \cdot x) \cdot z = (x \cdot z) \cdot T((x \cdot z)\backslash z, y)$ by (41). Hence we are done by Proposition 61. □

Theorem 195. $y\backslash((x \cdot y)/x) = ((x \cdot y)/x) \cdot (y\backslash e)$.

Proof. We have $((x \cdot y)/x) \cdot ((x\backslash(x \cdot y))\backslash e) = (x\backslash(x \cdot y))\backslash((x \cdot y)/x)$ by Theorem 180. Then $((x \cdot y)/x) \cdot (y\backslash e) = (x\backslash(x \cdot y))\backslash((x \cdot y)/x)$ by Axiom 3. Hence we are done by Axiom 3. □

Theorem 196. $x \cdot T(x\backslash e, y) = K(y\backslash(y/x), y)$.

Proof. We have $(y/x)\backslash(y \cdot T(y\backslash(y/x), y)) = K(y\backslash(y/x), y)$ by Theorem 46. Hence we are done by Theorem 182. □

Theorem 197. $x\backslash K(y\backslash(y/x), y) = T(x\backslash e, y)$.

Proof. We have $x \cdot T(x\backslash e, y) = K(y\backslash(y/x), y)$ by Theorem 196. Hence we are done by Proposition 2. □

Theorem 198. $K(y\backslash(y/(x\backslash e)), y) = x\backslash T(x, y)$.

Proof. We have $(x\backslash e) \cdot T((x\backslash e)\backslash e, y) = x\backslash T(x, y)$ by Theorem 145. Hence we are done by Theorem 196. □

Theorem 199. $K(x\backslash(x/(e/y)), x) = T(y, x)/y$.

Proof. We have $(e/y) \cdot T((e/y)\backslash e, x) = T(y, x)/y$ by Theorem 149. Hence we are done by Theorem 196. □

Theorem 200. $K(x\backslash(x/(((x \cdot y)/x)\backslash e)), x) = ((x \cdot y)/x)\backslash y$.

Proof. We have $K(x\backslash(x/(((x \cdot y)/x)\backslash e)), x) = ((x \cdot y)/x)\backslash T((x \cdot y)/x, x)$ by Theorem 198. Hence we are done by Theorem 7. □

Theorem 201. $K(y\backslash(y/(e/x)), y) \cdot x = T(x, y)$.

Proof. We have $(T(x, y)/x) \cdot x = T(x, y)$ by Axiom 6. Hence we are done by Theorem 199. □

Theorem 202. $K((x/y)\backslash e, x/y) \cdot y = x \cdot T(x\backslash y, x/y)$.

Proof. We have $T(R(e/(x/y), x/y, y), x/y) = R((x/y)\backslash e, x/y, y)$ by Proposition 60. Then

$$T(y/x, x/y) = R((x/y)\backslash e, x/y, y) \tag{42}$$

by Theorem 38. We have $R((x/y)\backslash e, x/y, y) \cdot x = (((x/y)\backslash e) \cdot (x/y)) \cdot y$ by Theorem 23. Then

$$T(y/x, x/y) \cdot x = (((x/y)\backslash e) \cdot (x/y)) \cdot y \tag{43}$$

by (42). We have $T(y/x, x/y) \cdot x = x \cdot T(x\backslash y, x/y)$ by Proposition 61. Then $(((x/y)\backslash e) \cdot (x/y)) \cdot y = x \cdot T(x\backslash y, x/y)$ by (43). Hence we are done by Proposition 76. □

Theorem 203. $T(y, y \cdot T(y\backslash e, x)) = y$.

Proof. We have $(y \cdot y) \cdot T((y \cdot y)\backslash y, x) = y \cdot (y \cdot T(y\backslash e, x))$ by Theorem 98. Then $(y \cdot T(y\backslash e, x)) \cdot y = y \cdot (y \cdot T(y\backslash e, x))$ by Theorem 194. Hence we are done by Theorem 11. □

Theorem 204. $T(x, (x\backslash e) \cdot x) = x$.

Proof. We have $T(x, x \cdot T(x\backslash e, x)) = x$ by Theorem 203. Hence we are done by Proposition 46. □

Theorem 205. $T(y\backslash e, y\backslash T(y, x)) = y\backslash e$.

Proof. We have $T(y\backslash e, (y\backslash e) \cdot T((y\backslash e)\backslash e, x)) = y\backslash e$ by Theorem 203. Hence we are done by Theorem 145. □

Theorem 206. $T(e/y, T(y, x)/y) = e/y$.

Proof. We have $T(e/y, (e/y) \cdot T((e/y)\backslash e, x)) = e/y$ by Theorem 203. Hence we are done by Theorem 149. □

Theorem 207. $R(x, x\backslash e, x) = T(x, x\backslash e)$.

Proof. We have $R(T(x, (x\backslash e) \cdot x), x\backslash e, x) = T(x, x\backslash e)$ by Theorem 113. Hence we are done by Theorem 204. □

Theorem 208. $T(x, x\backslash e)\backslash e = T(x\backslash e, x)$.

Proof. We have

$$R(x, x\backslash e, x)\backslash x = (x\backslash e) \cdot x \tag{44}$$

by Theorem 45.

$$\begin{array}{lcl} & x \cdot (T(x, x\backslash e)\backslash e) & \\ = & T(x, x\backslash e)\backslash x & \text{by Theorem 175} \\ = & (x\backslash e) \cdot x & \text{by (44), Theorem 207} \\ = & x \cdot T(x\backslash e, x) & \text{by Proposition 46.} \end{array}$$

Then $x \cdot (T(x, x\backslash e)\backslash e) = x \cdot T(x\backslash e, x)$. Hence we are done by Proposition 9. □

Theorem 209. $((x\backslash e)\backslash e)\backslash e = T(x\backslash e, x)$.

Proof. We have $T(x, x\backslash e)\backslash e = T(x\backslash e, x)$ by Theorem 208. Hence we are done by Proposition 49. $\square$

Theorem 210. $T(x, e/x) = (x\backslash e)\backslash e$.

Proof. We have $T((e/x)\backslash e, e/x) = (((e/x)\backslash e)\backslash e)\backslash e$ by Theorem 209. Then $T((e/x)\backslash e, e/x) = (x\backslash e)\backslash e$ by Proposition 25. Hence we are done by Proposition 25. $\square$

Theorem 211. $T(x, x\backslash e) = T(x, e/x)$.

Proof. We have $T(x, x\backslash e) = (x\backslash e)\backslash e$ by Proposition 49. Hence we are done by Theorem 210. $\square$

Theorem 212. $((x\backslash e)\backslash e)/x = K(x, e/x)$.

Proof. We have $T(x, e/x)/x = K(x, e/x)$ by Theorem 116. Hence we are done by Theorem 210. $\square$

Theorem 213. $L((z\backslash e)\backslash e, x, y)/z = L(z, x, y) \cdot (e/z)$.

Proof. We have $L(T(z, e/z), x, y)/z = L(z, x, y) \cdot (e/z)$ by Theorem 164. Hence we are done by Theorem 210. $\square$

Theorem 214. $R(x, e/x, x) = T(x, x\backslash e)$.

Proof. We have $R(x, e/x, x) = T(x, e/x)$ by Theorem 117. Hence we are done by Theorem 211. $\square$

Theorem 215. $T(x, x\backslash e)/x = K(x, e/x)$.

Proof. We have $T(x, e/x)/x = K(x, e/x)$ by Theorem 116. Hence we are done by Theorem 211. $\square$

Theorem 216. $T(T(y, y\backslash e), x)/y = T(y, x) \cdot (e/y)$.

Proof. We have $T(T(y, e/y), x)/y = T(y, x) \cdot (e/y)$ by Theorem 119. Hence we are done by Theorem 211. $\square$

Theorem 217. $T(y, x)\backslash T(T(y, y\backslash e), x) = a(T(y, x), e/y, y)$.

Proof. We have $T(y, x)\backslash T(T(y, e/y), x) = a(T(y, x), e/y, y)$ by Theorem 132. Hence we are done by Theorem 211. $\square$

Theorem 218. $x\backslash T(x, x\backslash e) = a(x, e/x, x)$.

Proof. We have $x\backslash R(x, e/x, x) = a(x, e/x, x)$ by Theorem 131. Hence we are done by Theorem 214. $\square$

Theorem 219. $K(x, e/x) = (x\backslash e)\backslash(e/x)$.

Proof. We have $(e/x) \cdot T(x, x\backslash e) = T(x, x\backslash e)/x$ by Theorem 150. Then $(e/x) \cdot ((x\backslash e)\backslash e) = T(x, x\backslash e)/x$ by Proposition 49. Then $((x\backslash e)\backslash e)/x = (e/x) \cdot ((x\backslash e)\backslash e)$ by Proposition 49. Then $(x\backslash e)\backslash(e/x) = ((x\backslash e)\backslash e)/x$ by Theorem 180. Hence we are done by Theorem 212. $\square$

Theorem 220. $x\backslash e = R(e/x, x, e/x)$.

Proof. We have $(e/x)/((x\backslash e)\backslash(e/x)) = x\backslash e$ by Proposition 24. Then

$$(e/x)/K(x, e/x) = x\backslash e \tag{45}$$

by Theorem 219. We have $R(e/x, x, e/x) = (e/x)/(x \cdot (e/x))$ by Proposition 79. Then $R(e/x, x, e/x) = (e/x)/K(x, e/x)$ by Proposition 77. Hence we are done by (45). $\square$

Theorem 221. $R(y\backslash e, y, x) = R(x/(y \cdot x), y, e/y)$.

Proof. We have $R(R(e/y, y, e/y), y, x) = R(x/(y \cdot x), y, e/y)$ by Theorem 40. Hence we are done by Theorem 220. $\square$

Theorem 222. $T(T(y,x),y\backslash e)=T(T(y,x),e/y)$.

Proof. We have $T(T(y,e/y),x)=T(T(y,x),e/y)$ by Axiom 7. Then

$$T((y\backslash e)\backslash e,x)=T(T(y,x),e/y) \tag{46}$$

by Theorem 210. We have $T(T(y,x),y\backslash e)=T((y\backslash e)\backslash e,x)$ by Theorem 18. Hence we are done by (46). □

Theorem 223. $K(y,x)=T(K(y,x),y\backslash e)$.

Proof. We have $T(y\backslash e,y\backslash T(y,x))=y\backslash e$ by Theorem 205. Then

$$T(y\backslash T(y,x),y\backslash e)=y\backslash T(y,x) \tag{47}$$

by Proposition 21. We have $L(T(y\backslash T(y,x),y\backslash e),y,x)=T((x\cdot y)\backslash(y\cdot x),y\backslash e)$ by Theorem 103. Then

$$L(y\backslash T(y,x),y,x)=T((x\cdot y)\backslash(y\cdot x),y\backslash e) \tag{48}$$

by (47). We have $L(y\backslash T(y,x),y,x)=(x\cdot y)\backslash(y\cdot x)$ by Proposition 73. Then

$$T((x\cdot y)\backslash(y\cdot x),y\backslash e)=(x\cdot y)\backslash(y\cdot x) \tag{49}$$

by (48). We have $K(y,x)=(x\cdot y)\backslash(y\cdot x)$ by Definition 2. Then $K(y,x)=T((x\cdot y)\backslash(y\cdot x),y\backslash e)$ by (49). Hence we are done by Definition 2. □

Theorem 224. $T(y\backslash e,K(y,x))=y\backslash e$.

Proof. We have $T(K(y,x),y\backslash e)=K(y,x)$ by Theorem 223. Hence we are done by Proposition 21. □

Theorem 225. $T((z\cdot y)\backslash z,K(y,x))=(z\cdot y)\backslash z$.

Proof. We have $T((z\cdot y)\backslash z,K(y,x))=L(T(y\backslash e,K(y,x)),y,z)$ by Theorem 30. Then

$$T((z\cdot y)\backslash z,K(y,x))=L(y\backslash e,y,z) \tag{50}$$

by Theorem 224. We have $L(y\backslash e,y,z)=(z\cdot y)\backslash z$ by Proposition 78. Hence we are done by (50). □

Theorem 226. $T(K(y,z),(x\cdot y)\backslash x)=K(y,z)$.

Proof. We have $T((x\cdot y)\backslash x,K(y,z))=(x\cdot y)\backslash x$ by Theorem 225. Hence we are done by Proposition 21. □

Theorem 227. $z\cdot(R(z,x,y)\cdot w)=R(z,x,y)\cdot(z\cdot w)$.

Proof. We have $(z\cdot w)\cdot R(T(z,z\cdot w),x,y)=R(z,x,y)\cdot(z\cdot w)$ by Proposition 59. Then

$$(z\cdot w)\cdot R(L(T(z,w),w,z),x,y)=R(z,x,y)\cdot(z\cdot w) \tag{51}$$

by Proposition 63. We have $(z\cdot w)\cdot R(L(T(z,w),w,z),x,y)=z\cdot(w\cdot R(T(z,w),x,y))$ by Theorem 56. Then $R(z,x,y)\cdot(z\cdot w)=z\cdot(w\cdot R(T(z,w),x,y))$ by (51). Hence we are done by Proposition 59. □

Theorem 228. $z\backslash(R(z,x,y)\cdot w)=R(z,x,y)\cdot(z\backslash w)$.

Proof. We have $R(z,x,y)\backslash(R(z,x,y)\cdot w)=w$ by Axiom 3. Then

$$R(z,x,y)\backslash(z\cdot(z\backslash(R(z,x,y)\cdot w)))=w \tag{52}$$

by Axiom 4. We have $R(z,x,y)\cdot(R(z,x,y)\backslash(z\cdot(z\backslash(R(z,x,y)\cdot w))))=z\cdot(z\backslash(R(z,x,y)\cdot w))$ by Axiom 4. Then $R(z,x,y)\cdot(z\cdot(z\backslash(R(z,x,y)\backslash(z\cdot(z\backslash(R(z,x,y)\cdot w))))))=z\cdot(z\backslash(R(z,x,y)\cdot w))$ by Axiom 4. Then

$$R(z,x,y)\cdot(z\cdot(z\backslash w))=z\cdot(z\backslash(R(z,x,y)\cdot w)) \tag{53}$$

by (52). We have $R(z,x,y)\cdot(z\cdot(z\backslash w))=z\cdot(R(z,x,y)\cdot(z\backslash w))$ by Theorem 227. Then $z\cdot(z\backslash(R(z,x,y)\cdot w))=z\cdot(R(z,x,y)\cdot(z\backslash w))$ by (53). Hence we are done by Proposition 9. □

Theorem 229. $x\backslash((x\cdot y)\cdot z) = R(x,y,z)\cdot(x\backslash(y\cdot z))$.

Proof. We have $x\backslash(R(x,y,z)\cdot(y\cdot z)) = R(x,y,z)\cdot(x\backslash(y\cdot z))$ by Theorem 228. Hence we are done by Proposition 54. □

Theorem 230. $R(y,x,y)\cdot T(x,y) = T(y\cdot x,y)$.

Proof. We have $T(y\cdot x,y) = y\backslash((y\cdot x)\cdot y)$ by Definition 3. Then $T(y\cdot x,y) = R(y,x,y)\cdot(y\backslash(x\cdot y))$ by Theorem 229. Hence we are done by Definition 3. □

Theorem 231. $R(y,x,y)\backslash T(y\cdot x,y) = T(x,y)$.

Proof. We have $R(y,x,y)\cdot T(x,y) = T(y\cdot x,y)$ by Theorem 230. Hence we are done by Proposition 2. □

Theorem 232. $((y\cdot(x\cdot z))/y)/z = ((y\cdot(x/(e/z)))/y)\cdot(e/z)$.

Proof. We have $R((y\cdot(x/(e/z)))/y, e/z, z) = (y\cdot R(x/(e/z), e/z, z))/y$ by Theorem 123. Then

$$R((y\cdot(x/(e/z)))/y, e/z, z) = (y\cdot(x\cdot z))/y \tag{54}$$

by Theorem 68. We have $R((y\cdot(x/(e/z)))/y, e/z, z)/z = ((y\cdot(x/(e/z)))/y)\cdot(e/z)$ by Theorem 74. Hence we are done by (54). □

Theorem 233. $x\cdot((z\cdot y)\backslash z) = x\cdot L(y\backslash e, y, z)$.

Proof. We have $(e/x)\backslash L((z\cdot y)\backslash z, x, e/x) = x\cdot((z\cdot y)\backslash z)$ by Theorem 71. Then

$$(e/x)\backslash L(L(y\backslash e, x, e/x), y, z) = x\cdot((z\cdot y)\backslash z) \tag{55}$$

by Theorem 32. We have $(e/x)\backslash L(L(y\backslash e, x, e/x), y, z) = x\cdot L(y\backslash e, y, z)$ by Theorem 82. Hence we are done by (55). □

Theorem 234. $L(x\backslash e, x, y)/((y\cdot x)\backslash y) = e$.

Proof. We have $e\cdot L(x\backslash e, x, y) = L(x\backslash e, x, y)$ by Axiom 1. Then

$$L(x\backslash e, x, y)\cdot(L(x\backslash e, x, y)\backslash(e\cdot L(x\backslash e, x, y))) = L(x\backslash e, x, y) \tag{56}$$

by Axiom 4. We have $L(x\backslash e, x, y)\cdot(L(x\backslash e, x, y)\backslash(e\cdot L(x\backslash e, x, y))) = e\cdot L(x\backslash e, x, y)$ by Axiom 4. Then $((L(x\backslash e, x, y)\cdot(L(x\backslash e, x, y)\backslash(e\cdot L(x\backslash e, x, y))))/((y\cdot x)\backslash y))\cdot((y\cdot x)\backslash y) = e\cdot L(x\backslash e, x, y)$ by Axiom 6. Then

$$(L(x\backslash e, x, y)/((y\cdot x)\backslash y))\cdot((y\cdot x)\backslash y) = e\cdot L(x\backslash e, x, y) \tag{57}$$

by (56). We have $(L(x\backslash e, x, y)/((y\cdot x)\backslash y))\cdot L(x\backslash e, x, y) = (L(x\backslash e, x, y)/((y\cdot x)\backslash y))\cdot((y\cdot x)\backslash y)$ by Theorem 233. Hence we are done by (57) and Proposition 8. □

Theorem 235. $x\cdot(z\cdot T(z\backslash(x\backslash z), y)) = (x\cdot T(x\backslash e, y))\cdot z$.

Proof. We have $(x\cdot z)\cdot T((x\cdot z)\backslash z, y) = (x\cdot T(x\backslash e, y))\cdot z$ by Theorem 194. Hence we are done by Theorem 136. □

Theorem 236. $x\cdot((y\backslash(x\backslash y))\cdot y) = (x\cdot T(x\backslash e, y))\cdot y$.

Proof. We have $x\cdot(y\cdot T(y\backslash(x\backslash y), y)) = (x\cdot T(x\backslash e, y))\cdot y$ by Theorem 235. Hence we are done by Proposition 46. □

Theorem 237. $(T(y/x, w)\cdot x)\cdot z = (x\cdot z)\cdot T((x\cdot z)\backslash(y\cdot z), w)$.

Proof. We have $T((y\cdot z)/(x\cdot z), w)\cdot(x\cdot z) = (x\cdot z)\cdot T((x\cdot z)\backslash(y\cdot z), w)$ by Proposition 61. Hence we are done by Theorem 137. □

Theorem 238. $(x \cdot T(y,z)) \cdot y = (x \cdot y) \cdot T(y,z)$.

Proof. We have $T(((x\cdot y)\cdot y)/(x\cdot y), z)\cdot(x\cdot y) = (x\cdot y)\cdot T(y,z)$ by Theorem 47. Then $(T((x\cdot y)/x, z)\cdot x)\cdot y = (x \cdot y) \cdot T(y,z)$ by Theorem 137. Hence we are done by Theorem 47. □

Theorem 239. $(x \cdot z)/T(z,y) = (x/T(z,y)) \cdot z$.

Proof. We have $(x \cdot z)/z = x$ by Axiom 5. Then

$$(((x \cdot z)/T(z,y)) \cdot T(z,y))/z = x \tag{58}$$

by Axiom 6. We have $((((x \cdot z)/T(z,y)) \cdot T(z,y))/z) \cdot z = ((x \cdot z)/T(z,y)) \cdot T(z,y)$ by Axiom 6. Then $((((((x \cdot z)/T(z,y)) \cdot T(z,y))/z)/T(z,y)) \cdot T(z,y)) \cdot z = ((x \cdot z)/T(z,y)) \cdot T(z,y)$ by Axiom 6. Then

$$((x/T(z,y)) \cdot T(z,y)) \cdot z = ((x \cdot z)/T(z,y)) \cdot T(z,y) \tag{59}$$

by (58). We have $((x/T(z,y)) \cdot T(z,y)) \cdot z = ((x/T(z,y)) \cdot z) \cdot T(z,y)$ by Theorem 238. Then $((x \cdot z)/T(z,y)) \cdot T(z,y) = ((x/T(z,y)) \cdot z) \cdot T(z,y)$ by (59). Hence we are done by Proposition 10. □

Theorem 240. $(x \cdot (z\backslash y)) \cdot (y/z) = (x \cdot (y/z)) \cdot T(y/z, z)$.

Proof. We have $(x \cdot T(y/z, z)) \cdot (y/z) = (x \cdot (y/z)) \cdot T(y/z, z)$ by Theorem 238. Hence we are done by Proposition 47. □

Theorem 241. $(x \cdot (y/z)) \cdot (z\backslash y) = (x \cdot (z\backslash y)) \cdot (y/z)$.

Proof. We have $(x \cdot (y/z)) \cdot T(y/z, z) = (x \cdot (z\backslash y)) \cdot (y/z)$ by Theorem 240. Hence we are done by Proposition 47. □

Theorem 242. $T(e/(e/x), x\backslash e) = x$.

Proof. We have $((e/x) \cdot T(x, x\backslash e))/(e/x) = T(e/(e/x), x\backslash e)$ by Theorem 51. Then

$$((e/x) \cdot ((x\backslash e)\backslash e))/(e/x) = T(e/(e/x), x\backslash e) \tag{60}$$

by Proposition 49. We have $((e/x)\cdot T(x, e/x))/(e/x) = x$ by Proposition 48. Then $((e/x)\cdot((x\backslash e)\backslash e))/(e/x) = x$ by Theorem 210. Hence we are done by (60). □

Theorem 243. $T(y,x) = T(T(e/(e/y), x), y\backslash e)$.

Proof. We have $T(T(e/(e/y), y\backslash e), x) = T(T(e/(e/y), x), y\backslash e)$ by Axiom 7. Hence we are done by Theorem 242. □

Theorem 244. $K(x, x\backslash e) = T(x, x\backslash e)/x$.

Proof. We have $T(e/(e/x), x\backslash e) = (x\backslash e)\backslash((e/(e/x)) \cdot (x\backslash e))$ by Definition 3. Then

$$T(e/(e/x), x\backslash e) = (x\backslash e)\backslash((x\backslash e)/(e/x)) \tag{61}$$

by Theorem 153. We have $K((x\backslash e)\backslash((x\backslash e)/(e/x)), x\backslash e) = T(x, x\backslash e)/x$ by Theorem 199. Then $K(T(e/(e/x), x\backslash e), x\backslash e) = T(x, x\backslash e)/x$ by (61). Hence we are done by Theorem 242. □

Theorem 245. $x \cdot (e/x) = (e/(e/(e/x))) \cdot x$.

Proof. We have $T(e/(e/(e/x)), (e/x)\backslash e) = e/x$ by Theorem 242. Then

$$T(e/(e/(e/x)), x) = e/x \tag{62}$$

by Proposition 25. We have $x \cdot T(e/(e/(e/x)), x) = (e/(e/(e/x))) \cdot x$ by Proposition 46. Hence we are done by (62). □

Theorem 246. $K(x, x\backslash e) \cdot x = T(x, x\backslash e)$.

Proof. We have $(T(x, x\backslash e)/x) \cdot x = T(x, x\backslash e)$ by Axiom 6. Hence we are done by Theorem 244. □

Theorem 247. *$((x\backslash e)\backslash e)/x = K(x, x\backslash e)$.*

Proof. We have $T(x, x\backslash e)/x = K(x, x\backslash e)$ by Theorem 244. Hence we are done by Proposition 49. □

Theorem 248. *$K(x, x\backslash e) = K(x, e/x)$.*

Proof. We have $T(x, x\backslash e)/x = K(x, e/x)$ by Theorem 215. Hence we are done by Theorem 244. □

Theorem 249. *$K(x\backslash e, x) = K(x\backslash e, (x\backslash e)\backslash e)$.*

Proof. We have $K(x\backslash e, e/(x\backslash e)) = K(x\backslash e, (x\backslash e)\backslash e)$ by Theorem 248. Hence we are done by Proposition 24. □

Theorem 250. *$K(x, x\backslash e) = x \cdot (e/x)$.*

Proof. We have $K(x, e/x) = x \cdot (e/x)$ by Proposition 77. Hence we are done by Theorem 248. □

Theorem 251. *$x\backslash K(x, x\backslash e) = e/x$.*

Proof. We have $x\backslash K(x, e/x) = e/x$ by Theorem 22. Hence we are done by Theorem 248. □

Theorem 252. *$(e/(e/x)) \cdot K(x, x\backslash e) = x$.*

Proof. We have $(e/(e/x)) \cdot (x \cdot (e/x)) = x$ by Theorem 126. Then $(e/(e/x)) \cdot K(x, e/x) = x$ by Proposition 77. Hence we are done by Theorem 248. □

Theorem 253. *$K((e/x)\backslash e, e/x) = K(x, x\backslash e)$.*

Proof. We have $K((e/x)\backslash e, e/x) = ((e/x)\backslash e)\cdot(e/x)$ by Proposition 76. Then $K((e/x)\backslash e, e/x) = x\cdot(e/x)$ by Proposition 25. Hence we are done by Theorem 250. □

Theorem 254. *$K(x\backslash e, x) = (x\backslash e)/(e/x)$.*

Proof. We have $(x\backslash e) \cdot T(e/(e/x), x\backslash e) = (e/(e/x)) \cdot (x\backslash e)$ by Proposition 46. Then

$$(x\backslash e) \cdot x = (e/(e/x)) \cdot (x\backslash e) \tag{63}$$

by Theorem 242. We have $(e/(e/x)) \cdot (x\backslash e) = (x\backslash e)/(e/x)$ by Theorem 153. Then

$$(x\backslash e) \cdot x = (x\backslash e)/(e/x) \tag{64}$$

by (63). We have $K(x\backslash e, x) = (x\backslash e) \cdot x$ by Proposition 76. Hence we are done by (64). □

Theorem 255. *$K(x\backslash e, x) \cdot (e/x) = x\backslash e$.*

Proof. We have $((x\backslash e)/(e/x)) \cdot (e/x) = x\backslash e$ by Axiom 6. Hence we are done by Theorem 254. □

Theorem 256. *$K(x\backslash e, x)\backslash(x\backslash e) = e/x$.*

Proof. We have $((x\backslash e)/(e/x))\backslash(x\backslash e) = e/x$ by Proposition 25. Hence we are done by Theorem 254. □

Theorem 257. *$K(x\backslash e, x) = x\backslash(e/(e/x))$.*

Proof. We have $(e/(e/x)) \cdot (((e/x)\backslash e)\backslash e) = ((e/x)\backslash e)\backslash(e/(e/x))$ by Theorem 180. Then $(e/(e/x)) \cdot (x\backslash e) = ((e/x)\backslash e)\backslash(e/(e/x))$ by Proposition 25. Then

$$x\backslash(e/(e/x)) = (e/(e/x)) \cdot (x\backslash e) \tag{65}$$

by Proposition 25. We have $(e/(e/x)) \cdot (x\backslash e) = (x\backslash e)/(e/x)$ by Theorem 153. Then $x\backslash(e/(e/x)) = (x\backslash e)/(e/x)$ by (65). Hence we are done by Theorem 254. □

Theorem 258. $K(e/x, e/(e/x)) = K(x\backslash e, x)$.

Proof. We have $(((e/x)\backslash e)\backslash e)/(e/x) = K(e/x, e/(e/x))$ by Theorem 212. Then $(x\backslash e)/(e/x) = K(e/x, e/(e/x))$ by Proposition 25. Hence we are done by Theorem 254. ☐

Theorem 259. $(x\backslash e) \cdot L(y, e/x, K(x\backslash e, x)) = K(x\backslash e, x) \cdot ((e/x) \cdot y)$.

Proof. We have $(K(x\backslash e, x) \cdot (e/x)) \cdot L(y, e/x, K(x\backslash e, x)) = K(x\backslash e, x) \cdot ((e/x) \cdot y)$ by Proposition 52. Hence we are done by Theorem 255. ☐

Theorem 260. $L((e/x)\backslash y, e/x, K(x\backslash e, x)) = (x\backslash e)\backslash(K(x\backslash e, x) \cdot y)$.

Proof. We have $L((e/x)\backslash y, e/x, K(x\backslash e, x)) = (K(x\backslash e, x) \cdot (e/x))\backslash(K(x\backslash e, x) \cdot y)$ by Proposition 53. Hence we are done by Theorem 255. ☐

Theorem 261. $e/x = L(x\backslash e, x, x\backslash e)$.

Proof. We have $K(x\backslash e, x)\backslash(x\backslash e) = L(x\backslash e, x, x\backslash e)$ by Theorem 36. Hence we are done by Theorem 256. ☐

Theorem 262. $x \cdot K(x\backslash e, x) = e/(e/x)$.

Proof. We have $x \cdot (x\backslash(e/(e/x))) = e/(e/x)$ by Axiom 4. Hence we are done by Theorem 257. ☐

Theorem 263. $e/(e/x) = L(x, x\backslash e, x)$.

Proof. We have $x \cdot K(x\backslash e, x) = L(x, x\backslash e, x)$ by Theorem 70. Hence we are done by Theorem 262. ☐

Theorem 264. $L(T(x, y), x\backslash e, x) = T(e/(e/x), y)$.

Proof. We have $L(T(x, y), x\backslash e, x) = T(L(x, x\backslash e, x), y)$ by Axiom 8. Hence we are done by Theorem 263. ☐

Theorem 265. $x \cdot (e/x) = x/(e/(e/x))$.

Proof. We have $(e/(e/(e/x))) \cdot ((e/x)\backslash e) = ((e/x)\backslash e)/(e/(e/x))$ by Theorem 153. Then $(e/(e/(e/x))) \cdot x = ((e/x)\backslash e)/(e/(e/x))$ by Proposition 25. Then $x/(e/(e/x)) = (e/(e/(e/x))) \cdot x$ by Proposition 25. Hence we are done by Theorem 245. ☐

Theorem 266. $L(x, e/x, x) = e/(e/x)$.

Proof. We have $K(x, e/x) = x \cdot (e/x)$ by Proposition 77. Then

$$K(x, e/x) = x/(e/(e/x)) \tag{66}$$

by Theorem 265. We have $(x/(e/(e/x))) \cdot (e/(e/x)) = x$ by Axiom 6. Then

$$K(x, e/x) \cdot (e/(e/x)) = x \tag{67}$$

by (66). We have $K(x, e/x) \cdot (K(x, e/x)\backslash x) = x$ by Axiom 4. Then $K(x, e/x) \cdot L(x, e/x, x) = x$ by Theorem 76. Then $K(x, e/x) \cdot L(x, e/x, x) = K(x, e/x) \cdot (e/(e/x))$ by (67). Hence we are done by Proposition 9. ☐

Theorem 267. $x = T(x, K(x, e/x))$.

Proof. We have $T(e/(e/x), x\backslash e) = x$ by Theorem 242. Then

$$T(L(x, e/x, x), x\backslash e) = x \tag{68}$$

by Theorem 266. We have $L(T(x, x\backslash e), e/x, x) = T(L(x, e/x, x), x\backslash e)$ by Axiom 8. Then

$$L(T(x, x\backslash e), e/x, x) = x \tag{69}$$

by (68). We have $L(T(x, e/x), e/x, x) = T(x, x \cdot (e/x))$ by Proposition 63. Then $L(T(x, x\backslash e), e/x, x) = T(x, x \cdot (e/x))$ by Theorem 211. Then $L(T(x, x\backslash e), e/x, x) = T(x, K(x, e/x))$ by Proposition 77. Hence we are done by (69). ☐

Theorem 268. $T(x, K(x, x\backslash e)) = x$.

Proof. We have $T(x, K(x, e/x)) = x$ by Theorem 267. Hence we are done by Theorem 248. □

Theorem 269. $K(x, e/x) = a(x, e/x, x)$.

Proof. We have $x\backslash((x \cdot (e/x)) \cdot x) = a(x, e/x, x)$ by Theorem 130. Then

$$x\backslash(K(x, e/x) \cdot x) = a(x, e/x, x) \tag{70}$$

by Proposition 77. We have $T(K(x, e/x), x) = x\backslash(K(x, e/x) \cdot x)$ by Definition 3. Then

$$T(K(x, e/x), x) = a(x, e/x, x) \tag{71}$$

by (70). We have $T(x, K(x, e/x)) = x$ by Theorem 267. Then $T(K(x, e/x), x) = K(x, e/x)$ by Proposition 21. Hence we are done by (71). □

Theorem 270. $x\backslash T(x, x\backslash e) = K(x, e/x)$.

Proof. We have $x\backslash T(x, x\backslash e) = a(x, e/x, x)$ by Theorem 218. Hence we are done by Theorem 269. □

Theorem 271. $x\backslash T(x, x\backslash e) = K(x, x\backslash e)$.

Proof. We have $K(x, e/x) = K(x, x\backslash e)$ by Theorem 248. Hence we are done by Theorem 270. □

Theorem 272. $K(x\backslash e, x) = K(e/x, (e/x)\backslash e)$.

Proof. We have $K(e/x, e/(e/x)) = K(e/x, (e/x)\backslash e)$ by Theorem 248. Hence we are done by Theorem 258.
□

Theorem 273. $T(e/(e/y), x)/((y\backslash e) \cdot T(y, x)) = y$.

Proof. We have $L(T(y, x), y\backslash e, y)/((y\backslash e) \cdot T(y, x)) = y$ by Theorem 69. Hence we are done by Theorem 264. □

Theorem 274. $K(x\backslash e, x) \cdot L(e/x, y, z) = (x\backslash e) \cdot (x \cdot L(x\backslash e, y, z))$.

Proof. We have $L(L(x\backslash e, x, x\backslash e), y, z) = L(L(x\backslash e, y, z), x, x\backslash e)$ by Axiom 11. Then

$$L(e/x, y, z) = L(L(x\backslash e, y, z), x, x\backslash e) \tag{72}$$

by Theorem 261. We have $K(x\backslash e, x) \cdot L(L(x\backslash e, y, z), x, x\backslash e) = (x\backslash e) \cdot (x \cdot L(x\backslash e, y, z))$ by Theorem 59. Hence we are done by (72). □

Theorem 275. $K(y, y\backslash e) \cdot T(y, x) = y \cdot (T(y, x) \cdot (e/y))$.

Proof. We have $K(y, e/y) \cdot T(T(y, K(y, e/y)), x) = T(y, x) \cdot K(y, e/y)$ by Proposition 50. Then

$$K(y, e/y) \cdot T(y, x) = T(y, x) \cdot K(y, e/y) \tag{73}$$

by Theorem 267. We have $T(y, x) \cdot K(y, e/y) = y \cdot (T(y, x) \cdot (e/y))$ by Theorem 176. Then $K(y, e/y) \cdot T(y, x) = y \cdot (T(y, x) \cdot (e/y))$ by (73). Hence we are done by Theorem 248. □

Theorem 276. $z \cdot (L(z, x, y) \cdot w) = L(z, x, y) \cdot (z \cdot w)$.

Proof. We have $(z \cdot w) \cdot L(L(T(z, w), w, z), x, y) = z \cdot (w \cdot L(T(z, w), x, y))$ by Theorem 57. Then

$$(z \cdot w) \cdot L(T(z, z \cdot w), x, y) = z \cdot (w \cdot L(T(z, w), x, y)) \tag{74}$$

by Proposition 63. We have $(z \cdot w) \cdot L(T(z, z \cdot w), x, y) = L(z, x, y) \cdot (z \cdot w)$ by Proposition 58. Then $z \cdot (w \cdot L(T(z, w), x, y)) = L(z, x, y) \cdot (z \cdot w)$ by (74). Hence we are done by Proposition 58. □

Theorem 277. $z \cdot (L(z,x,y)\backslash w) = L(z,x,y)\backslash(z \cdot w)$.

Proof. We have $z\backslash(z \cdot w) = w$ by Axiom 3. Then

$$z\backslash(L(z,x,y) \cdot (L(z,x,y)\backslash(z \cdot w))) = w \tag{75}$$

by Axiom 4. We have $z \cdot (z\backslash(L(z,x,y) \cdot (L(z,x,y)\backslash(z \cdot w)))) = L(z,x,y) \cdot (L(z,x,y)\backslash(z \cdot w))$ by Axiom 4. Then $z \cdot (L(z,x,y) \cdot (L(z,x,y)\backslash(z\backslash(L(z,x,y) \cdot (L(z,x,y)\backslash(z \cdot w)))))) = L(z,x,y) \cdot (L(z,x,y)\backslash(z \cdot w))$ by Axiom 4. Then

$$z \cdot (L(z,x,y) \cdot (L(z,x,y)\backslash w)) = L(z,x,y) \cdot (L(z,x,y)\backslash(z \cdot w)) \tag{76}$$

by (75). We have $L(z,x,y) \cdot (z \cdot (L(z,x,y)\backslash w)) = z \cdot (L(z,x,y) \cdot (L(z,x,y)\backslash w))$ by Theorem 276. Hence we are done by (76) and Proposition 7. □

Theorem 278. $z \cdot (L(z,x,y)\backslash e) = L(z,x,y)\backslash z$.

Proof. We have $z\backslash(z \cdot e) = e$ by Axiom 3. Then $z\backslash(L(z,x,y) \cdot (L(z,x,y)\backslash(z \cdot e))) = e$ by Axiom 4. Then

$$z\backslash(L(z,x,y) \cdot (L(z,x,y)\backslash z)) = e \tag{77}$$

by Axiom 2. We have $z \cdot (z\backslash(L(z,x,y) \cdot (L(z,x,y)\backslash z))) = L(z,x,y) \cdot (L(z,x,y)\backslash z)$ by Axiom 4. Then $z \cdot e = L(z,x,y) \cdot (L(z,x,y)\backslash z)$ by (77). Then

$$L(z,x,y) \cdot (L(z,x,y)\backslash z) = z \cdot (L(z,x,y) \cdot (L(z,x,y)\backslash e)) \tag{78}$$

by Axiom 4. We have $L(z,x,y) \cdot (z \cdot (L(z,x,y)\backslash e)) = z \cdot (L(z,x,y) \cdot (L(z,x,y)\backslash e))$ by Theorem 276. Hence we are done by (78) and Proposition 7. □

Theorem 279. $L(z,x,y) \cdot K(z,z\backslash e) = z \cdot (L(z,x,y) \cdot (e/z))$.

Proof. We have $L(z,x,y) \cdot (z \cdot (e/z)) = z \cdot (L(z,x,y) \cdot (e/z))$ by Theorem 276. Hence we are done by Theorem 250. □

Theorem 280. $x\backslash(L(x,y,z)\backslash x) = L(x,y,z)\backslash e$.

Proof. We have $x \cdot (L(x,y,z)\backslash e) = L(x,y,z)\backslash x$ by Theorem 278. Hence we are done by Proposition 2. □

Theorem 281. $x \cdot (L(x,y,z) \cdot (e/x)) = K(x,x\backslash e) \cdot L(x,y,z)$.

Proof. We have $K(x,x\backslash e) \cdot L(T(x,K(x,x\backslash e)),y,z) = L(x,y,z) \cdot K(x,x\backslash e)$ by Proposition 58. Then $K(x,x\backslash e) \cdot L(x,y,z) = L(x,y,z) \cdot K(x,x\backslash e)$ by Theorem 268. Hence we are done by Theorem 279. □

Theorem 282. $z \cdot T(e/z, L(z,x,y)) = K(z, L(z,x,y)/z)$.

Proof. We have $L(z,x,y)\backslash(z \cdot (L(z,x,y)/z)) = K(z,L(z,x,y)/z)$ by Theorem 3. Then $z \cdot (L(z,x,y)\backslash(L(z,x,y)/z)) = K(z,L(z,x,y)/z)$ by Theorem 277. Hence we are done by Theorem 167. □

Theorem 283. $y \cdot ((y\backslash x) \cdot T((y\backslash x)\backslash e, z)) = x \cdot T(x\backslash y, z)$.

Proof. We have $y \cdot ((y\backslash x) \cdot T((y\backslash x)\backslash e, z)) = x \cdot T(x\backslash(y \cdot e), z)$ by Theorem 168. Hence we are done by Axiom 2. □

Theorem 284. $y \cdot K(z\backslash(z/(y\backslash x)), z) = x \cdot T(x\backslash y, z)$.

Proof. We have $y \cdot ((y\backslash x) \cdot T((y\backslash x)\backslash e, z)) = x \cdot T(x\backslash y, z)$ by Theorem 283. Hence we are done by Theorem 196. □

Theorem 285. $x\backslash(y \cdot T(y\backslash x, z)) = K(z\backslash(z/(x\backslash y)), z)$.

Proof. We have $x \cdot K(z\backslash(z/(x\backslash y)), z) = y \cdot T(y\backslash x, z)$ by Theorem 284. Hence we are done by Proposition 2. □

Theorem 286. $K(z\backslash(z/(x/y)),z)\cdot y = x\cdot T(x\backslash y,z)$.

Proof. We have $(T(e/(x/y),z)\cdot(x/y))\cdot y = T((e\cdot y)/x,z)\cdot x$ by Theorem 169. Then

$$(T(e/(x/y),z)\cdot(x/y))\cdot y = T(y/x,z)\cdot x \tag{79}$$

by Axiom 1. We have $T(y/x,z)\cdot x = x\cdot T(x\backslash y,z)$ by Proposition 61. Then $(T(e/(x/y),z)\cdot(x/y))\cdot y = x\cdot T(x\backslash y,z)$ by (79). Then $((x/y)\cdot T((x/y)\backslash e,z))\cdot y = x\cdot T(x\backslash y,z)$ by Proposition 61. Hence we are done by Theorem 196. □

Theorem 287. $(x\cdot T(x\backslash y,z))/y = K(z\backslash(z/(x/y)),z)$.

Proof. We have $K(z\backslash(z/(x/y)),z)\cdot y = x\cdot T(x\backslash y,z)$ by Theorem 286. Hence we are done by Proposition 1. □

Theorem 288. $K((x\cdot(y/z))\backslash x, x\cdot(y/z)) = (y\cdot T(y\backslash z, x\cdot(y/z)))/z$.

Proof. We have $K((x\cdot(y/z))\backslash((x\cdot(y/z))/(y/z)), x\cdot(y/z)) = (y\cdot T(y\backslash z, x\cdot(y/z)))/z$ by Theorem 287. Hence we are done by Axiom 5. □

Theorem 289. $(x\cdot((y\cdot(x\backslash e))/y))\cdot y = T(y,R(e/x,x,y))$.

Proof. We have $T(y,R(e/x,x,y)) = ((y\cdot R(e/x,x,y))/y)\cdot(R(e/x,x,y)\backslash y)$ by Theorem 179. Then

$$T(y,R(e/x,x,y)) = ((y\cdot R(e/x,x,y))/y)\cdot(x\cdot y) \tag{80}$$

by Proposition 80. We have $((y\cdot R(e/x,x,y))/y)\cdot(x\cdot y) = (((y\cdot(e/x))/y)\cdot x)\cdot y$ by Theorem 129. Then $T(y,R(e/x,x,y)) = (((y\cdot(e/x))/y)\cdot x)\cdot y$ by (80). Hence we are done by Theorem 125. □

Theorem 290. $((y\cdot x)/y)/x = (e/x)\cdot((y\cdot((e/x)\backslash e))/y)$.

Proof. We have $((y\cdot(e/(e/x)))/y)\cdot(e/x) = ((y\cdot(e\cdot x))/y)/x$ by Theorem 232. Then

$$((y\cdot(e/(e/x)))/y)\cdot(e/x) = ((y\cdot x)/y)/x \tag{81}$$

by Axiom 1. We have $((y\cdot(e/(e/x)))/y)\cdot(e/x) = (e/x)\cdot((y\cdot((e/x)\backslash e))/y)$ by Theorem 125. Hence we are done by (81). □

Theorem 291. $(x\cdot w)\cdot T((x\cdot w)\backslash(y\cdot w),z) = (x\cdot T(x\backslash y,z))\cdot w$.

Proof. We have $(x\cdot w)\cdot T((x\cdot w)\backslash(y\cdot w),z) = (T(y/x,z)\cdot x)\cdot w$ by Theorem 237. Hence we are done by Proposition 61. □

Theorem 292. $(x\cdot z)/(y/(y/z)) = (x/(y/(y/z)))\cdot z$.

Proof. We have $(x\cdot z)/z = x$ by Axiom 5. Then

$$(((x\cdot z)/(y/(y/z)))\cdot(y/(y/z)))/z = x \tag{82}$$

by Axiom 6. We have $((((x\cdot z)/(y/(y/z)))\cdot(y/(y/z)))/z)\cdot z = ((x\cdot z)/(y/(y/z)))\cdot(y/(y/z))$ by Axiom 6. Then $((((((x\cdot z)/(y/(y/z)))\cdot(y/(y/z)))/z)/(y/(y/z)))\cdot(y/(y/z)))\cdot z = ((x\cdot z)/(y/(y/z)))\cdot(y/(y/z))$ by Axiom 6. Then

$$((x/(y/(y/z)))\cdot(y/(y/z)))\cdot z = ((x\cdot z)/(y/(y/z)))\cdot(y/(y/z)) \tag{83}$$

by (82). We have $((x/(y/(y/z)))\cdot(y/(y/z)))\cdot T(y/(y/z),y/z) = ((x/(y/(y/z)))\cdot((y/z)\backslash y))\cdot(y/(y/z))$ by Theorem 240. Then $((x/(y/(y/z)))\cdot(y/(y/z)))\cdot z = ((x/(y/(y/z)))\cdot((y/z)\backslash y))\cdot(y/(y/z))$ by Theorem 14. Then $((x/(y/(y/z)))\cdot z)\cdot(y/(y/z)) = ((x/(y/(y/z)))\cdot(y/(y/z)))\cdot z$ by Proposition 25. Then $((x\cdot z)/(y/(y/z)))\cdot(y/(y/z)) = ((x/(y/(y/z)))\cdot z)\cdot(y/(y/z))$ by (83). Hence we are done by Proposition 10. □

Theorem 293. $(T((z/x)/y,w)\cdot y)\cdot x = y\cdot(x\cdot T(x\backslash(y\backslash z),w))$.

Proof. We have $T(z/(y\cdot x),w)\cdot(y\cdot x) = (y\cdot x)\cdot T((y\cdot x)\backslash z,w)$ by Proposition 61. Then

$$(T((z/x)/y,w)\cdot y)\cdot x = (y\cdot x)\cdot T((y\cdot x)\backslash z,w) \tag{84}$$

by Theorem 138. We have $(y\cdot x)\cdot T((y\cdot x)\backslash z,w) = y\cdot(x\cdot T(x\backslash(y\backslash z),w))$ by Theorem 136. Hence we are done by (84). □

Theorem 294. $(x\cdot w)\cdot L((x\cdot w)\backslash w,y,z) = (x\cdot L(x\backslash e,y,z))\cdot w$.

Proof. We have $L(R(e/x,x,w),y,z)\cdot(x\cdot w) = (L(e/x,y,z)\cdot x)\cdot w$ by Theorem 65. Then

$$L(w/(x\cdot w),y,z)\cdot(x\cdot w) = (L(e/x,y,z)\cdot x)\cdot w \tag{85}$$

by Proposition 79. We have $L(w/(x\cdot w),y,z)\cdot(x\cdot w) = (x\cdot w)\cdot L((x\cdot w)\backslash w,y,z)$ by Theorem 85. Then $(L(e/x,y,z)\cdot x)\cdot w = (x\cdot w)\cdot L((x\cdot w)\backslash w,y,z)$ by (85). Hence we are done by Theorem 85. □

Theorem 295. $K(y\backslash(y/T(x,z)),y) = z\backslash((x\cdot T(x\backslash e,y))\cdot z)$.

Proof. We have $K(y\backslash(y/(z\backslash(x\cdot z))),y) = z\backslash((x\cdot z)\cdot T((x\cdot z)\backslash z,y))$ by Theorem 285. Then $K(y\backslash(y/T(x,z)),y) = z\backslash((x\cdot z)\cdot T((x\cdot z)\backslash z,y))$ by Definition 3. Hence we are done by Theorem 194. □

Theorem 296. $T(x\cdot T(x\backslash e,z),y) = K(z\backslash(z/T(x,y)),z)$.

Proof. We have $T(x\cdot T(x\backslash e,z),y) = y\backslash((x\cdot T(x\backslash e,z))\cdot y)$ by Definition 3. Hence we are done by Theorem 295. □

Theorem 297. $(x\cdot R(x\backslash e,y,z))\cdot x = x\cdot(x\cdot R(x\backslash e,y,z))$.

Proof. We have $(x\cdot x)\cdot R((x\cdot x)\backslash x,y,z) = x\cdot(x\cdot R(x\backslash e,y,z))$ by Theorem 133. Hence we are done by Proposition 84. □

Theorem 298. $T(z,z\cdot R(z\backslash e,x,y)) = z$.

Proof. We have $(z\cdot R(z\backslash e,x,y))\cdot z = z\cdot(z\cdot R(z\backslash e,x,y))$ by Theorem 297. Hence we are done by Theorem 11. □

Theorem 299. $T(y,(x/(y\cdot x))\cdot y) = y$.

Proof. We have $T(y,y\cdot R(y\backslash e,y,x)) = y$ by Theorem 298. Hence we are done by Theorem 90. □

Theorem 300. $T(y/(x\cdot y),x\cdot(y/(x\cdot y))) = y/(x\cdot y)$.

Proof. We have $T(y/(x\cdot y),(y/(x\cdot y))\cdot R((y/(x\cdot y))\backslash e,y/(x\cdot y),x\cdot y)) = y/(x\cdot y)$ by Theorem 298. Then $T(y/(x\cdot y),((x\cdot y)/y)\cdot(y/(x\cdot y))) = y/(x\cdot y)$ by Theorem 111. Hence we are done by Axiom 5. □

Theorem 301. $(x\cdot T(x\backslash y,w))\cdot z = x\cdot(z\cdot T(z\backslash(x\backslash(y\cdot z)),w))$.

Proof. We have $(x\cdot z)\cdot T((x\cdot z)\backslash(y\cdot z),w) = x\cdot(z\cdot T(z\backslash(x\backslash(y\cdot z)),w))$ by Theorem 136. Hence we are done by Theorem 291. □

Theorem 302. $(y\cdot T(y\backslash(z/x),w))\cdot x = y\cdot(x\cdot T(x\backslash(y\backslash z),w))$.

Proof. We have $(T((z/x)/y,w)\cdot y)\cdot x = y\cdot(x\cdot T(x\backslash(y\backslash z),w))$ by Theorem 293. Hence we are done by Proposition 61. □

Theorem 303. $(x\cdot L(x\backslash e,y,z))\cdot x = x\cdot(x\cdot L(x\backslash e,y,z))$.

Proof. We have $(x\cdot x)\cdot L((x\cdot x)\backslash x,y,z) = x\cdot(x\cdot L(x\backslash e,y,z))$ by Theorem 135. Hence we are done by Theorem 294. □

Theorem 304. $T(z, z \cdot L(z\backslash e, x, y)) = z$.

Proof. We have $(z \cdot L(z\backslash e, x, y)) \cdot z = z \cdot (z \cdot L(z\backslash e, x, y))$ by Theorem 303. Hence we are done by Theorem 11. □

Theorem 305. $T(y, y \cdot ((x \cdot y)\backslash x)) = y$.

Proof. We have $T(y, y \cdot L(y\backslash e, y, x)) = y$ by Theorem 304. Hence we are done by Proposition 78. □

Theorem 306. $T(x\backslash y, (x\backslash y) \cdot (y\backslash x)) = x\backslash y$.

Proof. We have $T(x\backslash y, (x\backslash y) \cdot L((x\backslash y)\backslash e, x\backslash y, x)) = x\backslash y$ by Theorem 304. Hence we are done by Theorem 28. □

Theorem 307. $T(T(y, x) \cdot ((T(y, x)/y)\backslash(e/y)), T(y, x)) = T(y, x) \cdot ((T(y, x)/y)\backslash(e/y))$.

Proof. We have $T(T(y, x), T(y, x)\cdot(((e/y)\cdot T(y, x))\backslash(e/y))) = T(y, x)$ by Theorem 305. Then $T(T(y, x), T(y, x)\cdot ((T(y, x)/y)\backslash(e/y))) = T(y, x)$ by Theorem 150. Hence we are done by Proposition 21. □

Theorem 308. $T(T(y, z)\backslash e, y\backslash T(y, x)) = T(y, z)\backslash e$.

Proof. We have $T(e, z) = z\backslash(e \cdot z)$ by Definition 3. Then

$$T(e, z) = z\backslash z \tag{86}$$

by Axiom 1. We have $z\backslash z = e$ by Proposition 28. Then

$$T(e, z) = e \tag{87}$$

by (86). We have $T(y, z)\backslash K((y\backslash T(y, x))\backslash((y\backslash T(y, x))/T(y, z)), y\backslash T(y, x)) = T(T(y, z)\backslash e, y\backslash T(y, x))$ by Theorem 197. Then $T(y, z)\backslash T(y \cdot T(y\backslash e, y\backslash T(y, x)), z) = T(T(y, z)\backslash e, y\backslash T(y, x))$ by Theorem 296. Then $T(y, z)\backslash T(y\cdot(y\backslash e), z) = T(T(y, z)\backslash e, y\backslash T(y, x))$ by Theorem 205. Then $T(T(y, z)\backslash e, y\backslash T(y, x)) = T(y, z)\backslash T(e, z)$ by Axiom 4. Hence we are done by (87). □

Theorem 309. $z \cdot T(z\backslash T(x, z), y) = T(z \cdot T(z\backslash x, y), z)$.

Proof. We have $z\cdot(z\cdot T(z\backslash(z\backslash(x\cdot z)), y)) = (z\cdot T(z\backslash x, y))\cdot z$ by Theorem 301. Then $T(z\cdot T(z\backslash x, y), z) = z \cdot T(z\backslash(z\backslash(x \cdot z)), y)$ by Theorem 11. Hence we are done by Definition 3. □

Theorem 310. $T((y\backslash x) \cdot y, y) = (y\backslash T(x, y)) \cdot y$.

Proof. We have $y \cdot T(y\backslash T(x, y), y) = (y\backslash T(x, y)) \cdot y$ by Proposition 46. Then $T(y \cdot T(y\backslash x, y), y) = (y\backslash T(x, y)) \cdot y$ by Theorem 309. Hence we are done by Proposition 46. □

Theorem 311. $T((y\backslash x) \cdot y, y)/y = y\backslash T(x, y)$.

Proof. We have $(y\backslash T(x, y)) \cdot y = T((y\backslash x) \cdot y, y)$ by Theorem 310. Hence we are done by Proposition 1. □

Theorem 312. $y \cdot (T((y\backslash x) \cdot y, y)/y) = T(x, y)$.

Proof. We have $y \cdot (y\backslash T(x, y)) = T(x, y)$ by Axiom 4. Hence we are done by Theorem 311. □

Theorem 313. $T(x, y)/y = y\backslash T(y \cdot (x/y), y)$.

Proof. We have $(x/y) \cdot y = x$ by Axiom 6. Then

$$(y\backslash(y \cdot (x/y))) \cdot y = x \tag{88}$$

by Axiom 3. We have $T((y\backslash(y \cdot (x/y))) \cdot y, y)/y = y\backslash T(y \cdot (x/y), y)$ by Theorem 311. Hence we are done by (88). □

Theorem 314. $y \cdot (T(x,y)/y) = T(y \cdot (x/y), y)$.

Proof. We have $(x/y) \cdot y = x$ by Axiom 6. Then

$$(y \backslash (y \cdot (x/y))) \cdot y = x \tag{89}$$

by Axiom 3. We have $y \cdot (T((y \backslash (y \cdot (x/y))) \cdot y, y)/y) = T(y \cdot (x/y), y)$ by Theorem 312. Hence we are done by (89). ☐

Theorem 315. $((x/y) \cdot K(y \backslash e, y)) \cdot y = x \cdot K(y \backslash e, y)$.

Proof. We have $(x \cdot T(x \backslash (x/y), y)) \cdot y = x \cdot (y \cdot T(y \backslash (x \backslash x), y))$ by Theorem 302. Then $((x/y) \cdot K(y \backslash e, y)) \cdot y = x \cdot (y \cdot T(y \backslash (x \backslash x), y))$ by Theorem 154. Then $((x/y) \cdot K(y \backslash e, y)) \cdot y = x \cdot (y \cdot T(y \backslash e, y))$ by Proposition 28. Then $((x/y) \cdot K(y \backslash e, y)) \cdot y = x \cdot ((y \backslash e) \cdot y)$ by Proposition 46. Hence we are done by Proposition 76. ☐

Theorem 316. $(x \cdot L(x \backslash e, z, w)) \cdot (x \backslash y) = y \cdot L(y \backslash (x \backslash y), z, w)$.

Proof. We have $L((x \backslash y)/y, z, w) \cdot y = y \cdot L(y \backslash (x \backslash y), z, w)$ by Theorem 85. Then

$$L(R(e/x, x, x \backslash y), z, w) \cdot y = y \cdot L(y \backslash (x \backslash y), z, w) \tag{90}$$

by Theorem 37. We have $L(R(e/x, x, x \backslash y), z, w) \cdot y = (L(e/x, z, w) \cdot x) \cdot (x \backslash y)$ by Theorem 97. Then $y \cdot L(y \backslash (x \backslash y), z, w) = (L(e/x, z, w) \cdot x) \cdot (x \backslash y)$ by (90). Hence we are done by Theorem 85. ☐

Theorem 317. $y \cdot L(y \backslash (x \backslash y), x, x \backslash e) = K(x, x \backslash e) \cdot (x \backslash y)$.

Proof. We have $(x \cdot L(x \backslash e, x, x \backslash e)) \cdot (x \backslash y) = y \cdot L(y \backslash (x \backslash y), x, x \backslash e)$ by Theorem 316. Then $(x \cdot (e/x)) \cdot (x \backslash y) = y \cdot L(y \backslash (x \backslash y), x, x \backslash e)$ by Theorem 261. Hence we are done by Theorem 250. ☐

Theorem 318. $y \cdot ((w \cdot (y \backslash (x \cdot z)))/w) = x \cdot ((x \backslash y) \cdot ((w \cdot ((x \backslash y) \backslash z))/w))$.

Proof. We have $y \cdot L((w \cdot ((x \backslash y) \backslash z))/w, x \backslash y, x) = x \cdot ((x \backslash y) \cdot ((w \cdot ((x \backslash y) \backslash z))/w))$ by Theorem 52. Hence we are done by Theorem 128. ☐

Theorem 319. $(e/(x/y)) \backslash (y/x) = (y/x) \cdot (x/y)$.

Proof. We have $(e/(x/y)) \backslash R(e/(x/y), x/y, y) = (x/y) \cdot R((x/y) \backslash e, x/y, y)$ by Theorem 187. Then

$$(e/(x/y)) \backslash (y/x) = (x/y) \cdot R((x/y) \backslash e, x/y, y) \tag{91}$$

by Theorem 38. We have $(x/y) \cdot R((x/y) \backslash e, x/y, y) = (y/x) \cdot (x/y)$ by Theorem 111. Hence we are done by (91). ☐

Theorem 320. $(e/(y/x)) \cdot ((x/y) \cdot (y/x)) = x/y$.

Proof. We have $(e/(y/x)) \cdot ((e/(y/x)) \backslash (x/y)) = x/y$ by Axiom 4. Hence we are done by Theorem 319. ☐

Theorem 321. $(x \backslash y) \cdot (y \backslash x) = (e/(y \backslash x)) \backslash (x \backslash y)$.

Proof. We have $(e/(y \backslash x)) \backslash L((y \backslash x) \backslash e, y \backslash x, y) = L((y \backslash x) \backslash e, y \backslash x, y) \cdot (y \backslash x)$ by Theorem 190. Then $(e/(y \backslash x)) \backslash (x \backslash y) = L((y \backslash x) \backslash e, y \backslash x, y) \cdot (y \backslash x)$ by Theorem 28. Hence we are done by Theorem 28. ☐

Theorem 322. $(e/(y \backslash x)) \cdot ((x \backslash y) \cdot (y \backslash x)) = x \backslash y$.

Proof. We have $(e/(y \backslash x)) \cdot ((e/(y \backslash x)) \backslash (x \backslash y)) = x \backslash y$ by Axiom 4. Hence we are done by Theorem 321. ☐

Theorem 323. $((y \cdot x) \backslash y) \cdot x = (e/x) \backslash ((y \cdot x) \backslash y)$.

Proof. We have $(e/(y\backslash(y\cdot x)))\backslash((y\cdot x)\backslash y) = ((y\cdot x)\backslash y)\cdot(y\backslash(y\cdot x))$ by Theorem 321. Then $(e/x)\backslash((y\cdot x)\backslash y) = ((y\cdot x)\backslash y)\cdot(y\backslash(y\cdot x))$ by Axiom 3. Hence we are done by Axiom 3. □

Theorem 324. $T(y\backslash(K(x,x\backslash e)\cdot(x\backslash y)),x) = y\backslash(x\backslash y)$.

Proof. We have $R(T(x\backslash K(x,x\backslash e),y),x,x\backslash y) = T(y\backslash(K(x,x\backslash e)\cdot(x\backslash y)),x)$ by Theorem 171. Then

$$R(T(e/x,y),x,x\backslash y) = T(y\backslash(K(x,x\backslash e)\cdot(x\backslash y)),x) \quad (92)$$

by Theorem 251. We have $R(T(e/x,y),x,x\backslash y) = y\backslash(e\cdot(x\backslash y))$ by Theorem 142. Then $T(y\backslash(K(x,x\backslash e)\cdot(x\backslash y)),x) = y\backslash(e\cdot(x\backslash y))$ by (92). Hence we are done by Axiom 1. □

Theorem 325. $x\cdot(w\cdot L(w\backslash(x\backslash w),y,z)) = (x\cdot L(x\backslash e,y,z))\cdot w$.

Proof. We have $(x\cdot w)\cdot L((x\cdot w)\backslash w,y,z) = (x\cdot L(x\backslash e,y,z))\cdot w$ by Theorem 294. Hence we are done by Theorem 155. □

Theorem 326. $x\cdot(K(x,x\backslash e)\cdot(x\backslash y)) = (x\cdot(e/x))\cdot y$.

Proof. We have $x\cdot(y\cdot L(y\backslash(x\backslash y),x,x\backslash e)) = (x\cdot L(x\backslash e,x,x\backslash e))\cdot y$ by Theorem 325. Then $x\cdot(y\cdot L(y\backslash(x\backslash y),x,x\backslash e)) = (x\cdot(e/x))\cdot y$ by Theorem 261. Hence we are done by Theorem 317. □

Theorem 327. $x\cdot(K(x,x\backslash e)\cdot(x\backslash y)) = K(x,x\backslash e)\cdot y$.

Proof. We have $x\cdot(K(x,x\backslash e)\cdot(x\backslash y)) = (x\cdot(e/x))\cdot y$ by Theorem 326. Hence we are done by Theorem 250. □

Theorem 328. $K(y,y\backslash e)\cdot(y\backslash T(y,x)) = T(y,x)\cdot(e/y)$.

Proof. We have

$$y\cdot(K(y,y\backslash e)\cdot(y\backslash T(y,x))) = K(y,y\backslash e)\cdot T(y,x) \quad (93)$$

by Theorem 327. We have $y\cdot(T(y,x)\cdot(e/y)) = K(y,y\backslash e)\cdot T(y,x)$ by Theorem 275. Hence we are done by (93) and Proposition 7. □

Theorem 329. $(L(y/x,w,u)\cdot x)\cdot z = (x\cdot z)\cdot L((x\cdot z)\backslash(y\cdot z),w,u)$.

Proof. We have $L((y\cdot z)/(x\cdot z),w,u)\cdot(x\cdot z) = (x\cdot z)\cdot L((x\cdot z)\backslash(y\cdot z),w,u)$ by Theorem 85. Hence we are done by Theorem 157. □

Theorem 330. $(x/y)\backslash(x\cdot((z\cdot(x\backslash(x/y)))/z)) = y\cdot((z\cdot(y\backslash e))/z)$.

Proof. We have $L((z\cdot(y\backslash e))/z,y,x/y) = (z\cdot L(y\backslash e,y,x/y))/z$ by Theorem 122. Then

$$L((z\cdot(y\backslash e))/z,y,x/y) = (z\cdot(x\backslash(x/y)))/z \quad (94)$$

by Theorem 29. We have $x\cdot L((z\cdot(y\backslash e))/z,y,x/y) = (x/y)\cdot(y\cdot((z\cdot(y\backslash e))/z))$ by Theorem 53. Then $x\cdot((z\cdot(x\backslash(x/y)))/z) = (x/y)\cdot(y\cdot((z\cdot(y\backslash e))/z))$ by (94). Hence we are done by Proposition 2. □

Theorem 331. $K(y,y\backslash e)\cdot((e/y)\backslash x) = x\cdot T(x\backslash((e/y)\backslash x),y\backslash e)$.

Proof. We have $(T(e/(e/y),y\backslash e)\cdot(e/y))\cdot((e/y)\backslash x) = x\cdot T(x\backslash((e/y)\backslash x),y\backslash e)$ by Theorem 183. Then $(y\cdot(e/y))\cdot((e/y)\backslash x) = x\cdot T(x\backslash((e/y)\backslash x),y\backslash e)$ by Theorem 242. Hence we are done by Theorem 250. □

Theorem 332. $(x\cdot u)\cdot L((x\cdot u)\backslash(y\cdot u),z,w) = (x\cdot L(x\backslash y,z,w))\cdot u$.

Proof. We have $(x\cdot u)\cdot L((x\cdot u)\backslash(y\cdot u),z,w) = (L(y/x,z,w)\cdot x)\cdot u$ by Theorem 329. Hence we are done by Theorem 85. □

Theorem 333. $x\cdot((y\cdot(x\backslash e))/y) = K(y,(y/x)/y)$.

Proof. We have $(y/x)\backslash(y\cdot((y\cdot(y\backslash(y/x)))/y)) = x\cdot((y\cdot(x\backslash e))/y)$ by Theorem 330. Then

$$(y/x)\backslash(y\cdot((y/x)/y)) = x\cdot((y\cdot(x\backslash e))/y) \tag{95}$$

by Axiom 4. We have $(y/x)\backslash(y\cdot((y/x)/y)) = K(y,(y/x)/y)$ by Theorem 3. Hence we are done by (95). ☐

Theorem 334. $T(x\cdot R(e/x,y,z),x) = x\cdot R(e/x,y,z)$.

Proof. We have $x\cdot(x\cdot R(x\backslash e,y,z)) = (x\cdot R(x\backslash e,y,z))\cdot x$ by Theorem 297. Then

$$T(x\cdot R(x\backslash e,y,z),x) = x\cdot R(x\backslash e,y,z) \tag{96}$$

by Theorem 11. We have $T(x\cdot((x\cdot R(x\backslash e,y,z))/x),x) = x\cdot(T(x\cdot R(x\backslash e,y,z),x)/x)$ by Theorem 314. Then $T(x\cdot((x\cdot R(x\backslash e,y,z))/x),x) = x\cdot((x\cdot R(x\backslash e,y,z))/x)$ by (96). Then $T(x\cdot((x\cdot R(x\backslash e,y,z))/x),x) = x\cdot R(e/x,y,z)$ by Theorem 88. Hence we are done by Theorem 88. ☐

Theorem 335. $T(x\cdot(y/(x\cdot y)),x) = x\cdot(y/(x\cdot y))$.

Proof. We have $T(x\cdot R(e/x,x,y),x) = x\cdot R(e/x,x,y)$ by Theorem 334. Then $T(x\cdot(y/(x\cdot y)),x) = x\cdot R(e/x,x,y)$ by Proposition 79. Hence we are done by Proposition 79. ☐

Theorem 336. $T(y,y\cdot(x/(y\cdot x))) = y$.

Proof. We have $T(y\cdot(x/(y\cdot x)),y) = y\cdot(x/(y\cdot x))$ by Theorem 335. Hence we are done by Proposition 21. ☐

Theorem 337. $T(x/y,(x/y)\cdot(y/x)) = x/y$.

Proof. We have $T(x/y,(x/y)\cdot(y/((x/y)\cdot y))) = x/y$ by Theorem 336. Hence we are done by Axiom 6. ☐

Theorem 338. $(x\backslash e)\cdot(K(x\backslash e,x)\cdot((x\backslash e)\backslash y)) = K(x\backslash e,x)\cdot y$.

Proof. We have $(x\backslash e)\cdot(K(x\backslash e,(x\backslash e)\backslash e)\cdot((x\backslash e)\backslash y)) = K(x\backslash e,(x\backslash e)\backslash e)\cdot y$ by Theorem 327. Then $(x\backslash e)\cdot(K(x\backslash e,x)\cdot((x\backslash e)\backslash y)) = K(x\backslash e,(x\backslash e)\backslash e)\cdot y$ by Theorem 249. Hence we are done by Theorem 249. ☐

Theorem 339. $(x\cdot(y\cdot L(y\backslash e,z,w)))\cdot y = (x\cdot y)\cdot(y\cdot L(y\backslash e,z,w))$.

Proof. We have $((x\cdot y)\cdot y)\cdot L(((x\cdot y)\cdot y)\backslash(x\cdot y),z,w) = (x\cdot y)\cdot(y\cdot L(y\backslash e,z,w))$ by Theorem 135. Then $((x\cdot y)\cdot L((x\cdot y)\backslash x,z,w))\cdot y = (x\cdot y)\cdot(y\cdot L(y\backslash e,z,w))$ by Theorem 332. Hence we are done by Theorem 135. ☐

Theorem 340. $(((x\cdot y)\backslash x)\cdot y)\backslash((x\cdot y)\backslash x) = e/y$.

Proof. We have $K((((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y)))\backslash((((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y)))/(x\backslash(x\cdot y))),((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y))) = x\backslash((x\cdot y)\cdot T((x\cdot y)\backslash x,((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y))))$ by Theorem 285. Then $K((((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y)))\backslash((x\cdot y)\backslash x),((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y))) = x\backslash((x\cdot y)\cdot T((x\cdot y)\backslash x,((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y))))$ by Axiom 5. Then

$$K((((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y)))\backslash((x\cdot y)\backslash x),((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y))) = x\backslash((x\cdot y)\cdot((x\cdot y)\backslash x)) \tag{97}$$

by Theorem 306. We have $x\backslash(x\cdot e) = e$ by Axiom 3. Then $x\backslash((x\cdot y)\cdot((x\cdot y)\backslash(x\cdot e))) = e$ by Axiom 4. Then $x\backslash((x\cdot y)\cdot((x\cdot y)\backslash x)) = e$ by Axiom 2. Then

$$K((((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y)))\backslash((x\cdot y)\backslash x),((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y))) = e \tag{98}$$

by (97). We have $K((((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y)))\backslash((x\cdot y)\backslash x),((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y)))/(x\backslash(x\cdot y)) = T(e/(x\backslash(x\cdot y)),((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y)))$ by Theorem 185. Then

$$e/(x\backslash(x\cdot y)) = T(e/(x\backslash(x\cdot y)),((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y))) \tag{99}$$

by (98). We have $T(e/(x\backslash(x\cdot y)),((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y))) = (((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y)))\backslash((e/(x\backslash(x\cdot y)))\cdot(((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y))))$ by Definition 3. Then $T(e/(x\backslash(x\cdot y)),((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y))) = (((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y)))\backslash((x\cdot y)\backslash x)$ by Theorem 322. Then $e/(x\backslash(x\cdot y)) = (((x\cdot y)\backslash x)\cdot(x\backslash(x\cdot y)))\backslash((x\cdot y)\backslash x)$ by (99). Then $(((x\cdot y)\backslash x)\cdot y)\backslash((x\cdot y)\backslash x) = e/(x\backslash(x\cdot y))$ by Axiom 3. Hence we are done by Axiom 3. ☐

Theorem 341. $((x \cdot y)\backslash x)/(e/y) = ((x \cdot y)\backslash x) \cdot y$.

Proof. We have $((x \cdot y)\backslash x)/((((x \cdot y)\backslash x) \cdot y)\backslash((x \cdot y)\backslash x)) = ((x \cdot y)\backslash x) \cdot y$ by Proposition 24. Hence we are done by Theorem 340. ☐

Theorem 342. $L(y\backslash e, y, (x \cdot y)\backslash x) = e/y$.

Proof. We have $L(y\backslash e, y, (x \cdot y)\backslash x) = (((x \cdot y)\backslash x) \cdot y)\backslash((x \cdot y)\backslash x)$ by Proposition 78. Hence we are done by Theorem 340. ☐

Theorem 343. $(x \cdot z) \cdot ((y \cdot ((x \cdot z)\backslash z))/y) = (x \cdot ((y \cdot (x\backslash e))/y)) \cdot z$.

Proof. We have $((y \cdot R(e/x, x, z))/y) \cdot (x \cdot z) = (((y \cdot (e/x))/y) \cdot x) \cdot z$ by Theorem 129. Then $(x \cdot z) \cdot ((y \cdot ((x \cdot z)\backslash z))/y) = (((y \cdot (e/x))/y) \cdot x) \cdot z$ by Theorem 144. Hence we are done by Theorem 125. ☐

Theorem 344. $T(x \cdot ((z \cdot (x\backslash e))/z), y) = K(z, (z/T(x, y))/z)$.

Proof. We have $y \cdot ((y\backslash(x \cdot y)) \cdot ((z \cdot ((y\backslash(x \cdot y))\backslash e))/z)) = (x \cdot y) \cdot ((z \cdot ((x \cdot y)\backslash(y \cdot e)))/z)$ by Theorem 318. Then $y \cdot K(z, (z/(y\backslash(x \cdot y)))/z) = (x \cdot y) \cdot ((z \cdot ((x \cdot y)\backslash(y \cdot e)))/z)$ by Theorem 333. Then $y \cdot K(z, (z/(y\backslash(x \cdot y)))/z) = (x \cdot y) \cdot ((z \cdot ((x \cdot y)\backslash y))/z)$ by Axiom 2. Then $y \cdot K(z, (z/T(x, y))/z) = (x \cdot y) \cdot ((z \cdot ((x \cdot y)\backslash y))/z)$ by Definition 3. Then $(x \cdot ((z \cdot (x\backslash e))/z)) \cdot y = y \cdot K(z, (z/T(x, y))/z)$ by Theorem 343. Hence we are done by Theorem 11. ☐

Theorem 345. $T(x \cdot ((z \cdot (x\backslash e))/z), y) = T(x, y) \cdot ((z \cdot (T(x, y)\backslash e))/z)$.

Proof. We have $K(z, (z/T(x, y))/z) = T(x, y) \cdot ((z \cdot (T(x, y)\backslash e))/z)$ by Theorem 333. Hence we are done by Theorem 344. ☐

Theorem 346. $L(e/x, T(x, y), e/x) = (T(x, y) \cdot (e/x))/T(x, y)$.

Proof. We have $T((T(x, y) \cdot (e/x))/T(x, y), (e/x) \cdot T(x, y)) = L(e/x, T(x, y), e/x)$ by Theorem 92. Then

$$T((T(x, y) \cdot (e/x))/T(x, y), T(x, y)/x) = L(e/x, T(x, y), e/x) \tag{100}$$

by Theorem 150. We have $(T(x, y) \cdot T(e/x, T(x, y)/x))/T(x, y) = T((T(x, y) \cdot (e/x))/T(x, y), T(x, y)/x)$ by Theorem 50. Then $(T(x, y) \cdot (e/x))/T(x, y) = T((T(x, y) \cdot (e/x))/T(x, y), T(x, y)/x)$ by Theorem 206. Hence we are done by (100). ☐

Theorem 347. $T(x, y/(x \cdot y)) = T(x, R(x\backslash e, x, y))$.

Proof. We have $R(T(y/(x \cdot y), x \cdot (y/(x \cdot y))), x, y/(x \cdot y)) = T(y/(x \cdot y), x)$ by Theorem 113. Then

$$R(y/(x \cdot y), x, y/(x \cdot y)) = T(y/(x \cdot y), x) \tag{101}$$

by Theorem 300. We have $T(y/(x \cdot y), x) = R(x\backslash e, x, y)$ by Theorem 89. Then

$$R(y/(x \cdot y), x, y/(x \cdot y)) = R(x\backslash e, x, y) \tag{102}$$

by (101). We have $R(y/(x \cdot y), x, y/(x \cdot y))\backslash T((y/(x \cdot y)) \cdot x, y/(x \cdot y)) = T(x, y/(x \cdot y))$ by Theorem 231. Then

$$R(x\backslash e, x, y)\backslash T((y/(x \cdot y)) \cdot x, y/(x \cdot y)) = T(x, y/(x \cdot y)) \tag{103}$$

by (102). We have $T(y/(x \cdot y), (y/(x \cdot y)) \cdot ((x \cdot y)/y)) = y/(x \cdot y)$ by Theorem 337. Then $T(y/(x \cdot y), (y/(x \cdot y)) \cdot x) = y/(x \cdot y)$ by Axiom 5. Then $T((y/(x \cdot y)) \cdot x, y/(x \cdot y)) = (y/(x \cdot y)) \cdot x$ by Proposition 21. Then

$$R(x\backslash e, x, y)\backslash((y/(x \cdot y)) \cdot x) = T(x, y/(x \cdot y)) \tag{104}$$

by (103). We have $T(x, R(x\backslash e, x, y)) = R(x\backslash e, x, y)\backslash(x \cdot R(x\backslash e, x, y))$ by Definition 3. Then $T(x, R(x\backslash e, x, y)) = R(x\backslash e, x, y)\backslash((y/(x \cdot y)) \cdot x)$ by Theorem 90. Hence we are done by (104). ☐

Theorem 348. $((y\backslash e) \cdot (y \cdot L(y\backslash e, y, x))) \cdot y = (y \cdot ((x \cdot y)\backslash x)) \cdot K(y\backslash e, y)$.

Proof. We have $T(e/y, (y\backslash e)\cdot y) = L(y\backslash e, y, y\backslash e)$ by Theorem 91. Then $T(e/y, K(y\backslash e, y)) = L(y\backslash e, y, y\backslash e)$ by Proposition 76. Then

$$T(e/y, K(y\backslash e, y)) = e/y \tag{105}$$

by Theorem 261. We have $K(y\backslash e, y)\cdot L(T(e/y, K(y\backslash e, y)), y, x) = L(e/y, y, x)\cdot K(y\backslash e, y)$ by Proposition 58. Then $K(y\backslash e, y) \cdot L(e/y, y, x) = L(e/y, y, x) \cdot K(y\backslash e, y)$ by (105). Then

$$L(e/y, y, x) \cdot K(y\backslash e, y) = (y\backslash e) \cdot (y \cdot L(y\backslash e, y, x)) \tag{106}$$

by Theorem 274. We have $(((y\cdot((x\cdot y)\backslash x))/y)\cdot K(y\backslash e, y))\cdot y = (y\cdot((x\cdot y)\backslash x))\cdot K(y\backslash e, y)$ by Theorem 315. Then $(L(e/y, y, x) \cdot K(y\backslash e, y)) \cdot y = (y \cdot ((x \cdot y)\backslash x)) \cdot K(y\backslash e, y)$ by Theorem 121. Hence we are done by (106). ☐

Theorem 349. $T(x \cdot R(x\backslash e, z, w), y) = T(x, y) \cdot R(T(x, y)\backslash e, z, w)$.

Proof. We have $(x\cdot y)\cdot L(R(T(x, y)\backslash e, z, w), T(x, y), y) = y\cdot(T(x, y)\cdot R(T(x, y)\backslash e, z, w))$ by Theorem 58. Then $(x \cdot y) \cdot R((y \cdot T(x, y))\backslash y, z, w) = y \cdot (T(x, y) \cdot R(T(x, y)\backslash e, z, w))$ by Theorem 31. Then

$$(x \cdot y) \cdot R((x \cdot y)\backslash y, z, w) = y \cdot (T(x, y) \cdot R(T(x, y)\backslash e, z, w)) \tag{107}$$

by Proposition 46. We have $(x \cdot y) \cdot R((x \cdot y)\backslash y, z, w) = (x \cdot R(x\backslash e, z, w)) \cdot y$ by Proposition 84. Then $y \cdot (T(x, y) \cdot R(T(x, y)\backslash e, z, w)) = (x \cdot R(x\backslash e, z, w)) \cdot y$ by (107). Hence we are done by Theorem 11. ☐

Theorem 350. $w \cdot (T(x, w) \cdot L(T(x, w)\backslash e, y, z)) = (x \cdot L(x\backslash e, y, z)) \cdot w$.

Proof. We have $(x\cdot w)\cdot L(L(T(x, w)\backslash e, y, z), T(x, w), w) = w\cdot(T(x, w)\cdot L(T(x, w)\backslash e, y, z))$ by Theorem 58. Then $(x \cdot w) \cdot L((w \cdot T(x, w))\backslash w, y, z) = w \cdot (T(x, w) \cdot L(T(x, w)\backslash e, y, z))$ by Theorem 32. Then

$$(x \cdot w) \cdot L((x \cdot w)\backslash w, y, z) = w \cdot (T(x, w) \cdot L(T(x, w)\backslash e, y, z)) \tag{108}$$

by Proposition 46. We have $(x \cdot w) \cdot L((x \cdot w)\backslash w, y, z) = (x \cdot L(x\backslash e, y, z)) \cdot w$ by Theorem 294. Hence we are done by (108). ☐

Theorem 351. $T(x \cdot L(x\backslash e, z, w), y) = T(x, y) \cdot L(T(x, y)\backslash e, z, w)$.

Proof. We have $y \cdot (T(x, y) \cdot L(T(x, y)\backslash e, z, w)) = (x \cdot L(x\backslash e, z, w)) \cdot y$ by Theorem 350. Hence we are done by Theorem 11. ☐

Theorem 352. $e/((y\backslash x)/x) = (y \cdot ((y\backslash x)/x))\backslash y$.

Proof. We have $T(y, y \cdot ((y\backslash x)/(y \cdot (y\backslash x)))) = y$ by Theorem 336. Then

$$T(y, y \cdot ((y\backslash x)/x)) = y \tag{109}$$

by Axiom 4. We have $T(T(y, y \cdot ((y\backslash x)/x)), y\backslash x) = T((y\backslash x)\backslash x, y \cdot ((y\backslash x)/x))$ by Theorem 18. Then

$$T(y, y\backslash x) = T((y\backslash x)\backslash x, y \cdot ((y\backslash x)/x)) \tag{110}$$

by (109). We have $K((y \cdot ((y\backslash x)/x))\backslash y, y \cdot ((y\backslash x)/x)) = ((y\backslash x) \cdot T((y\backslash x)\backslash x, y \cdot ((y\backslash x)/x)))/x$ by Theorem 288. Then $K((y \cdot ((y\backslash x)/x))\backslash y, y \cdot ((y\backslash x)/x)) = ((y\backslash x) \cdot T(y, y\backslash x))/x$ by (110). Then

$$K((y \cdot ((y\backslash x)/x))\backslash y, y \cdot ((y\backslash x)/x)) = x/x \tag{111}$$

by Theorem 8. We have $x/x = e$ by Proposition 29. Then

$$K((y \cdot ((y\backslash x)/x))\backslash y, y \cdot ((y\backslash x)/x)) = e \tag{112}$$

by (111). We have $K((y \cdot ((y\backslash x)/x))\backslash y, y \cdot ((y\backslash x)/x))/((y\backslash x)/x) = T(e/((y\backslash x)/x), y \cdot ((y\backslash x)/x))$ by Theorem 185. Then

$$e/((y\backslash x)/x) = T(e/((y\backslash x)/x), y \cdot ((y\backslash x)/x)) \tag{113}$$

by (112). We have $x/(y\backslash x) = y$ by Proposition 24. Then $(e/((y\backslash x)/x)) \cdot ((x/(y\backslash x)) \cdot ((y\backslash x)/x)) = y$ by Theorem 320. Then

$$(e/((y\backslash x)/x)) \cdot (y \cdot ((y\backslash x)/x)) = y \tag{114}$$

by Proposition 24. We have $T(e/((y\backslash x)/x), y \cdot ((y\backslash x)/x)) = (y \cdot ((y\backslash x)/x))\backslash((e/((y\backslash x)/x)) \cdot (y \cdot ((y\backslash x)/x)))$ by Definition 3. Then $T(e/((y\backslash x)/x), y \cdot ((y\backslash x)/x)) = (y \cdot ((y\backslash x)/x))\backslash y$ by (114). Hence we are done by (113). ☐

Theorem 353. $((y/x) \cdot (x/y))\backslash(y/x) = e/(x/y)$.

Proof. We have $((y/x) \cdot (((y/x)\backslash y)/y))\backslash(y/x) = e/(((y/x)\backslash y)/y)$ by Theorem 352. Then $((y/x) \cdot (x/y))\backslash(y/x) = e/(((y/x)\backslash y)/y)$ by Proposition 25. Hence we are done by Proposition 25. ☐

Theorem 354. $((x/y) \cdot (y/x)) \cdot (e/(y/x)) = x/y$.

Proof. We have $((x/y) \cdot (y/x)) \cdot (((x/y) \cdot (y/x))\backslash(x/y)) = x/y$ by Axiom 4. Hence we are done by Theorem 353. ☐

Theorem 355. $K(e/(x/y), (y/x) \cdot (x/y)) = e$.

Proof. We have $T(T(y/x, (y/x) \cdot (x/y)), x) = T(x\backslash y, (y/x) \cdot (x/y))$ by Proposition 51. Then

$$T(y/x, x) = T(x\backslash y, (y/x) \cdot (x/y)) \tag{115}$$

by Theorem 337. We have $T(y/x, x) = x\backslash y$ by Proposition 47. Then

$$T(x\backslash y, (y/x) \cdot (x/y)) = x\backslash y \tag{116}$$

by (115). We have $K(((y/x)\cdot(x/y))\backslash(y/x), (y/x)\cdot(x/y)) = (x\cdot T(x\backslash y, (y/x)\cdot(x/y)))/y$ by Theorem 288. Then $K(((y/x) \cdot (x/y))\backslash(y/x), (y/x) \cdot (x/y)) = (x \cdot (x\backslash y))/y$ by (116). Then

$$K(((y/x) \cdot (x/y))\backslash(y/x), (y/x) \cdot (x/y)) = y/y \tag{117}$$

by Axiom 4. We have $y/y = e$ by Proposition 29. Then $K(((y/x) \cdot (x/y))\backslash(y/x), (y/x) \cdot (x/y)) = e$ by (117). Hence we are done by Theorem 353. ☐

Theorem 356. $K(e/y, (x/(y \cdot x)) \cdot y) = e$.

Proof. We have $K(e/((y \cdot x)/x), (x/(y \cdot x)) \cdot ((y \cdot x)/x)) = e$ by Theorem 355. Then $K(e/y, (x/(y \cdot x)) \cdot ((y \cdot x)/x)) = e$ by Axiom 5. Hence we are done by Axiom 5. ☐

Theorem 357. $e/x = L(x\backslash e, x, y/(x \cdot y))$.

Proof. We have $K(e/x, (y/(x \cdot y)) \cdot x) = e$ by Theorem 356. Then

$$T(e/x, (y/(x \cdot y)) \cdot x) = e/x \tag{118}$$

by Proposition 23. We have

$$T((y/(x \cdot y))/((y/(x \cdot y)) \cdot x), (y/(x \cdot y)) \cdot x) = ((y/(x \cdot y)) \cdot x)\backslash(y/(x \cdot y)) \tag{119}$$

by Proposition 47.

$$\begin{aligned} & L(x\backslash e, x, y/(x \cdot y)) \\ = \quad & ((y/(x \cdot y)) \cdot x)\backslash(y/(x \cdot y)) && \text{by Proposition 78} \\ = \quad & T(e/x, (y/(x \cdot y)) \cdot x) && \text{by (119), Theorem 189.} \end{aligned}$$

Then $L(x\backslash e, x, y/(x \cdot y)) = T(e/x, (y/(x \cdot y)) \cdot x)$. Hence we are done by (118). ☐

Theorem 358. $R(x\backslash e, x, y) \cdot K(x, x\backslash e) = y/(x \cdot y)$.

Proof. We have $((y/(x \cdot y)) \cdot x) \cdot (((y/(x \cdot y)) \cdot x)\backslash(y/(x \cdot y))) = y/(x \cdot y)$ by Axiom 4. Then $((y/(x \cdot y)) \cdot x) \cdot L(x\backslash e, x, y/(x \cdot y)) = y/(x \cdot y)$ by Proposition 78. Then

$$((y/(x \cdot y)) \cdot x) \cdot (e/x) = y/(x \cdot y) \tag{120}$$

by Theorem 357. We have

$$R(y/(x \cdot y), x, e/x)\backslash(((y/(x \cdot y)) \cdot x) \cdot (e/x)) = x \cdot (e/x) \tag{121}$$

by Theorem 6.

$$\begin{aligned} & K(x, x\backslash e) \\ = \quad & x \cdot (e/x) && \text{by Theorem 250} \\ = \quad & R(x\backslash e, x, y)\backslash(((y/(x \cdot y)) \cdot x) \cdot (e/x)) && \text{by (121), Theorem 221.} \end{aligned}$$

Then $K(x, x\backslash e) = R(x\backslash e, x, y)\backslash(((y/(x \cdot y)) \cdot x) \cdot (e/x))$. Then

$$R(x\backslash e, x, y)\backslash(y/(x \cdot y)) = K(x, x\backslash e) \tag{122}$$

by (120). We have $R(x\backslash e, x, y) \cdot (R(x\backslash e, x, y)\backslash(y/(x \cdot y))) = y/(x \cdot y)$ by Axiom 4. Hence we are done by (122). □

Theorem 359. $T(e/(e/y), R(e/y, y, x)) = y$.

Proof. We have $L(y\backslash e, y, x/(y \cdot x)) = e/y$ by Theorem 357. Then

$$L(y\backslash e, y, R(e/y, y, x)) = e/y \tag{123}$$

by Proposition 79. We have $R(e/y, y, x)/L(y\backslash e, y, R(e/y, y, x)) = R(e/y, y, x) \cdot y$ by Theorem 33. Then

$$R(e/y, y, x)/(e/y) = R(e/y, y, x) \cdot y \tag{124}$$

by (123). We have $(e/(e/y)) \cdot R(e/y, y, x) = R(e/y, y, x)/(e/y)$ by Theorem 192. Then $R(e/y, y, x) \cdot y = (e/(e/y)) \cdot R(e/y, y, x)$ by (124). Hence we are done by Theorem 11. □

Theorem 360. $T(e/(e/y), x/(y \cdot x)) = y$.

Proof. We have $T(e/(e/y), R(e/y, y, x)) = y$ by Theorem 359. Hence we are done by Proposition 79. □

Theorem 361. $T(x, x\backslash e) = T(x, R(e/x, x, y))$.

Proof. We have $T(T(e/(e/x), R(e/x, x, y)), x\backslash e) = T(x, R(e/x, x, y))$ by Theorem 243. Hence we are done by Theorem 359. □

Theorem 362. $T(x, x\backslash e) = T(x, y/(x \cdot y))$.

Proof. We have $T(T(e/(e/x), y/(x \cdot y)), x\backslash e) = T(x, y/(x \cdot y))$ by Theorem 243. Hence we are done by Theorem 360. □

Theorem 363. $T(y, (y\backslash x)/x) = T(y, y\backslash e)$.

Proof. We have $T(y, (y\backslash x)/(y \cdot (y\backslash x))) = T(y, y\backslash e)$ by Theorem 362. Hence we are done by Axiom 4. □

Theorem 364. $T(x, (y \cdot x)\backslash y)/x = K(x, (y \cdot x)\backslash y)$.

Proof. We have $T(x,(y\cdot x)\backslash y)=K(((y\cdot x)\backslash y)\backslash(((y\cdot x)\backslash y)/(e/x)),(y\cdot x)\backslash y)\cdot x$ by Theorem 201. Then $T(x,(y\cdot x)\backslash y)=K(((y\cdot x)\backslash y)\backslash(((y\cdot x)\backslash y)\cdot x),(y\cdot x)\backslash y)\cdot x$ by Theorem 341. Then $K(x,(y\cdot x)\backslash y)\cdot x=T(x,(y\cdot x)\backslash y)$ by Axiom 3. Hence we are done by Proposition 1. ☐

Theorem 365. $K(x,y)=((e/y)\cdot K(x,y))/(e/y)$.

Proof. We have $((e/y)\cdot(e/y))\cdot((x\cdot(((e/y)\cdot(e/y))\backslash(e/y)))/x)=(e/y)\cdot((e/y)\cdot((x\cdot((e/y)\backslash e))/x))$ by Theorem 170. Then $((e/y)\cdot((x\cdot((e/y)\backslash e))/x))\cdot(e/y)=(e/y)\cdot((e/y)\cdot((x\cdot((e/y)\backslash e))/x))$ by Theorem 343. Then $(((x\cdot y)/x)/y)\cdot(e/y)=(e/y)\cdot((e/y)\cdot((x\cdot((e/y)\backslash e))/x))$ by Theorem 290. Then $(e/y)\cdot(((x\cdot y)/x)/y)=(((x\cdot y)/x)/y)\cdot(e/y)$ by Theorem 290. Then

$$((e/y)\cdot(((x\cdot y)/x)/y))/(e/y)=((x\cdot y)/x)/y \tag{125}$$

by Proposition 1. We have $R(((e/y)\cdot(((x\cdot y)/x)/y))/(e/y),y,x)=((e/y)\cdot R(((x\cdot y)/x)/y,y,x))/(e/y)$ by Theorem 123. Then $R(((e/y)\cdot(((x\cdot y)/x)/y))/(e/y),y,x)=((e/y)\cdot((x\cdot y)/(y\cdot x)))/(e/y)$ by Proposition 74. Then

$$R(((x\cdot y)/x)/y,y,x)=((e/y)\cdot((x\cdot y)/(y\cdot x)))/(e/y) \tag{126}$$

by (125). We have $R(((x\cdot y)/x)/y,y,x)=(x\cdot y)/(y\cdot x)$ by Proposition 74. Then

$$((e/y)\cdot((x\cdot y)/(y\cdot x)))/(e/y)=(x\cdot y)/(y\cdot x) \tag{127}$$

by (126). We have $T(((e/y)\cdot((x\cdot y)/(y\cdot x)))/(e/y),y\cdot x)=((e/y)\cdot((y\cdot x)\backslash(x\cdot y)))/(e/y)$ by Theorem 94. Then

$$T((x\cdot y)/(y\cdot x),y\cdot x)=((e/y)\cdot((y\cdot x)\backslash(x\cdot y)))/(e/y) \tag{128}$$

by (127). We have $T((x\cdot y)/(y\cdot x),y\cdot x)=(y\cdot x)\backslash(x\cdot y)$ by Proposition 47. Then

$$((e/y)\cdot((y\cdot x)\backslash(x\cdot y)))/(e/y)=(y\cdot x)\backslash(x\cdot y) \tag{129}$$

by (128). We have $K(x,y)=(y\cdot x)\backslash(x\cdot y)$ by Definition 2. Then $K(x,y)=((e/y)\cdot((y\cdot x)\backslash(x\cdot y)))/(e/y)$ by (129). Hence we are done by Definition 2. ☐

Theorem 366. $T(K(x,y),e/y)=K(x,y)$.

Proof. We have $T(((e/y)\cdot K(x,y))/(e/y),e/y)=K(x,y)$ by Theorem 7. Hence we are done by Theorem 365. ☐

Theorem 367. $T(e/y,K(x,y))=e/y$.

Proof. We have $T(K(x,y),e/y)=K(x,y)$ by Theorem 366. Hence we are done by Proposition 21. ☐

Theorem 368. $K(y,K(x,y\backslash e))=e$.

Proof. We have $T(e/(y\backslash e),K(x,y\backslash e))=e/(y\backslash e)$ by Theorem 367. Then $K(e/(y\backslash e),K(x,y\backslash e))=e$ by Proposition 22. Hence we are done by Proposition 24. ☐

Theorem 369. $T(y,K(x,y\backslash e))=y$.

Proof. We have $K(y,K(x,y\backslash e))=e$ by Theorem 368. Hence we are done by Proposition 23. ☐

Theorem 370. $T(z,x)=T(T(z,x),K(y,z\backslash e))$.

Proof. We have $T(T(z,K(y,z\backslash e)),x)=T(T(z,x),K(y,z\backslash e))$ by Axiom 7. Hence we are done by Theorem 369. ☐

Theorem 371. $T(K(y,z\backslash e),T(z,x))=K(y,z\backslash e)$.

Proof. We have $T(T(z,x),K(y,z\backslash e))=T(z,x)$ by Theorem 370. Hence we are done by Proposition 21. ☐

Theorem 372. $T(T(y,z), x\backslash T(x, y\backslash e)) = T(y,z)$.

Proof. We have $T(T(y,z), K((y\backslash e)\backslash((y\backslash e)/(x\backslash e)), y\backslash e)) = T(y,z)$ by Theorem 370. Hence we are done by Theorem 198. ☐

Theorem 373. $(z/(y\cdot z))\cdot K(x,y) = K(x,y)\cdot(z/(y\cdot z))$.

Proof. We have $((e/y)\cdot K(x,y))/K(x,y) = e/y$ by Axiom 5. Then $((((e/y)\cdot K(x,y))/(e/y))\cdot(e/y))/K(x,y) = e/y$ by Axiom 6. Then

$$(K(x,y)\cdot(e/y))/K(x,y) = e/y \tag{130}$$

by Theorem 365. We have $R((K(x,y)\cdot(e/y))/K(x,y), y, z)\backslash(K(x,y)\cdot R(e/y, y, z)) = K(x,y)$ by Theorem 124. Then

$$R(e/y, y, z)\backslash(K(x,y)\cdot R(e/y, y, z)) = K(x,y) \tag{131}$$

by (130). We have $T(K(x,y), R(e/y, y, z)) = R(e/y, y, z)\backslash(K(x,y)\cdot R(e/y, y, z))$ by Definition 3. Then $T(K(x,y), R(e/y, y, z)) = K(x,y)$ by (131). Then

$$T(K(x,y), z/(y\cdot z)) = K(x,y) \tag{132}$$

by Proposition 79. We have $(z/(y\cdot z))\cdot T(K(x,y), z/(y\cdot z)) = K(x,y)\cdot(z/(y\cdot z))$ by Proposition 46. Hence we are done by (132). ☐

Theorem 374. $(x\backslash e)/(x\backslash T(x,y)) = T(x,y)\backslash e$.

Proof. We have $T(x\backslash e, (x\backslash e)\cdot T((x\backslash e)\backslash e, y)) = x\backslash e$ by Theorem 203. Then

$$T(x\backslash e, T(x,y)\cdot(x\backslash e)) = x\backslash e \tag{133}$$

by Theorem 48. We have $R(T(x\backslash e, T(x,y)\cdot(x\backslash e)), T(x,y), x\backslash e) = T(x\backslash e, T(x,y))$ by Theorem 113. Then

$$R(x\backslash e, T(x,y), x\backslash e) = T(x\backslash e, T(x,y)) \tag{134}$$

by (133). We have $((x\backslash e)/(T(x,y)\cdot(x\backslash e)))\cdot T(x,y) = T(x,y)\cdot R(T(x,y)\backslash e, T(x,y), x\backslash e)$ by Theorem 90. Then

$$((x\backslash e)/(x\backslash T(x,y)))\cdot T(x,y) = T(x,y)\cdot R(T(x,y)\backslash e, T(x,y), x\backslash e) \tag{135}$$

by Theorem 146. We have $T(x,y)\cdot R(T(x,y)\backslash e, T(x,y), x\backslash e) = T(x\cdot R(x\backslash e, T(x,y), x\backslash e), y)$ by Theorem 349. Then $((x\backslash e)/(x\backslash T(x,y)))\cdot T(x,y) = T(x\cdot R(x\backslash e, T(x,y), x\backslash e), y)$ by (135). Then

$$T(x\cdot T(x\backslash e, T(x,y)), y) = ((x\backslash e)/(x\backslash T(x,y)))\cdot T(x,y) \tag{136}$$

by (134). We have $T(x\cdot T(x\backslash e, T(x,y)), y) = K(T(x,y)\backslash(T(x,y)/T(x,y)), T(x,y))$ by Theorem 296. Then

$$T(x\cdot T(x\backslash e, T(x,y)), y) = K(T(x,y)\backslash e, T(x,y)) \tag{137}$$

by Proposition 29. We have $K(T(x,y)\backslash e, T(x,y)) = (T(x,y)\backslash e)\cdot T(x,y)$ by Proposition 76. Then $T(x\cdot T(x\backslash e, T(x,y)), y) = (T(x,y)\backslash e)\cdot T(x,y)$ by (137). Hence we are done by (136) and Proposition 8. ☐

Theorem 375. $(x\backslash T(x,y))\backslash(x\backslash e) = T(x,y)\backslash e$.

Proof. We have $T((x\backslash e)/(x\backslash T(x,y)), x\backslash T(x,y)) = (x\backslash T(x,y))\backslash(x\backslash e)$ by Proposition 47. Then

$$T(T(x,y)\backslash e, x\backslash T(x,y)) = (x\backslash T(x,y))\backslash(x\backslash e) \tag{138}$$

by Theorem 374. We have $T(T(x,y)\backslash e, x\backslash T(x,y)) = T(x,y)\backslash e$ by Theorem 308. Hence we are done by (138). ☐

Theorem 376. $(x\cdot T(x\backslash e, y))\backslash x = T(e/x, y)\backslash e$.

Proof. We have $((e/x)\backslash T(e/x,y))\backslash((e/x)\backslash e) = T(e/x,y)\backslash e$ by Theorem 375. Then $(x\cdot T(x\backslash e,y))\backslash((e/x)\backslash e) = T(e/x,y)\backslash e$ by Theorem 102. Hence we are done by Proposition 25. ☐

Theorem 377. $x/(T(e/x,y)\backslash e) = x \cdot T(x\backslash e,y)$.

Proof. We have $x/((x \cdot T(x\backslash e,y))\backslash x) = x \cdot T(x\backslash e,y)$ by Proposition 24. Hence we are done by Theorem 376. ☐

Theorem 378. $L(x\backslash e,x,y) = (x\backslash e)\cdot(x\cdot L(x\backslash e,x,y))$.

Proof. We have

$$(e/x)\cdot(K(e/x,(e/x)\backslash e)\cdot((e/x)\backslash L(e/x,x,y))) = K(e/x,(e/x)\backslash e)\cdot L(e/x,x,y) \tag{139}$$

by Theorem 327. We have $(e/x)\cdot(L(e/x,x,y)\cdot(e/(e/x))) = K(e/x,(e/x)\backslash e)\cdot L(e/x,x,y)$ by Theorem 281. Then

$$K(e/x,(e/x)\backslash e)\cdot((e/x)\backslash L(e/x,x,y)) = L(e/x,x,y)\cdot(e/(e/x)) \tag{140}$$

by (139) and Proposition 7. We have $L(((e/x)\backslash e)\backslash e,x,y)/(e/x) = L(e/x,x,y)\cdot(e/(e/x))$ by Theorem 213. Then $L(x\backslash e,x,y)/(e/x) = L(e/x,x,y)\cdot(e/(e/x))$ by Proposition 25. Then $K(e/x,(e/x)\backslash e)\cdot((e/x)\backslash L(e/x,x,y)) = L(x\backslash e,x,y)/(e/x)$ by (140). Then $K(x\backslash e,x)\cdot((e/x)\backslash L(e/x,x,y)) = L(x\backslash e,x,y)/(e/x)$ by Theorem 272. Then

$$K(x\backslash e,x)\cdot(x\cdot L(x\backslash e,x,y)) = L(x\backslash e,x,y)/(e/x) \tag{141}$$

by Theorem 159. We have $((x\backslash e)\cdot(x\cdot L(x\backslash e,x,y)))\cdot x = ((x\backslash e)\cdot x)\cdot(x\cdot L(x\backslash e,x,y))$ by Theorem 339. Then $(x\cdot((y\cdot x)\backslash y))\cdot K(x\backslash e,x) = ((x\backslash e)\cdot x)\cdot(x\cdot L(x\backslash e,x,y))$ by Theorem 348. Then $K(x\backslash e,x)\cdot(x\cdot L(x\backslash e,x,y)) = (x\cdot((y\cdot x)\backslash y))\cdot K(x\backslash e,x)$ by Proposition 76. Then

$$L(x\backslash e,x,y)/(e/x) = (x\cdot((y\cdot x)\backslash y))\cdot K(x\backslash e,x) \tag{142}$$

by (141). We have $(((y\cdot x)\backslash y)\cdot x)\cdot((((y\cdot x)\backslash y)\cdot x)\backslash((y\cdot x)\backslash y)) = (y\cdot x)\backslash y$ by Axiom 4. Then

$$(((y\cdot x)\backslash y)\cdot x)\cdot(e/x) = (y\cdot x)\backslash y \tag{143}$$

by Theorem 340.

$$\begin{array}{lcl} & L(x\backslash e,x,y) & \\ = & (y\cdot x)\backslash y & \text{by Proposition 78} \\ = & (L(x\backslash e,x,y)\cdot x)\cdot(e/x) & \text{by (143), Proposition 78.} \end{array}$$

Then $L(x\backslash e,x,y) = (L(x\backslash e,x,y)\cdot x)\cdot(e/x)$. Then $L(x\backslash e,x,y)/(e/x) = L(x\backslash e,x,y)\cdot x$ by Proposition 1. Then

$$(x\cdot((y\cdot x)\backslash y))\cdot K(x\backslash e,x) = L(x\backslash e,x,y)\cdot x \tag{144}$$

by (142). We have $((x\backslash e)\cdot(x\cdot L(x\backslash e,x,y)))\cdot x = (x\cdot((y\cdot x)\backslash y))\cdot K(x\backslash e,x)$ by Theorem 348. Hence we are done by (144) and Proposition 8. ☐

Theorem 379. $(y\cdot x)\backslash y = (x\backslash e)\cdot(x\cdot((y\cdot x)\backslash y))$.

Proof. We have $L(x\backslash e,x,y) = (y\cdot x)\backslash y$ by Proposition 78. Then $(x\backslash e)\cdot(x\cdot L(x\backslash e,x,y)) = (y\cdot x)\backslash y$ by Theorem 378. Hence we are done by Proposition 78. ☐

Theorem 380. $(x\backslash e)\backslash((y\cdot x)\backslash y) = x\cdot((y\cdot x)\backslash y)$.

Proof. We have $(x\backslash e)\cdot(x\cdot((y\cdot x)\backslash y)) = (y\cdot x)\backslash y$ by Theorem 379. Hence we are done by Proposition 2. ☐

Theorem 381. $e = K(x\backslash e, x\cdot((y\cdot x)\backslash y))$.

Proof. We have $((y\cdot x)\backslash y)\backslash(((y\cdot x)\backslash y)\cdot e) = e$ by Axiom 3. Then $((y\cdot x)\backslash y)\backslash((x\backslash e)\cdot((x\backslash e)\backslash(((y\cdot x)\backslash y)\cdot e))) = e$ by Axiom 4. Then

$$((y\cdot x)\backslash y)\backslash((x\backslash e)\cdot((x\backslash e)\backslash((y\cdot x)\backslash y))) = e \tag{145}$$

by Axiom 2. We have $((y\cdot x)\backslash y)\cdot(((y\cdot x)\backslash y)\backslash((x\backslash e)\cdot((x\backslash e)\backslash((y\cdot x)\backslash y)))) = (x\backslash e)\cdot((x\backslash e)\backslash((y\cdot x)\backslash y))$ by Axiom 4. Then $((y\cdot x)\backslash y)\cdot e = (x\backslash e)\cdot((x\backslash e)\backslash((y\cdot x)\backslash y))$ by (145). Then

$$(x\backslash e)\cdot(x\cdot((y\cdot x)\backslash y)) = ((y\cdot x)\backslash y)\cdot e \tag{146}$$

by Theorem 380. We have $L(x\backslash e, x, y)/(x\backslash e) = x\cdot((y\cdot x)\backslash y)$ by Theorem 162. Then

$$((y\cdot x)\backslash y)/(x\backslash e) = x\cdot((y\cdot x)\backslash y) \tag{147}$$

by Proposition 78. We have $(((y\cdot x)\backslash y)/(x\backslash e))\cdot(x\backslash e) = (y\cdot x)\backslash y$ by Axiom 6. Then

$$(x\cdot((y\cdot x)\backslash y))\cdot(x\backslash e) = (y\cdot x)\backslash y \tag{148}$$

by (147). We have $((x\cdot((y\cdot x)\backslash y))\cdot(x\backslash e))\cdot K(x\backslash e, x\cdot((y\cdot x)\backslash y)) = (x\backslash e)\cdot(x\cdot((y\cdot x)\backslash y))$ by Proposition 82. Then $((y\cdot x)\backslash y)\cdot K(x\backslash e, x\cdot((y\cdot x)\backslash y)) = (x\backslash e)\cdot(x\cdot((y\cdot x)\backslash y))$ by (148). Hence we are done by (146) and Proposition 7. □

Theorem 382. $e = (y\backslash e)\cdot T(y, x\backslash(x/y))$.

Proof. We have $(y\backslash e)\cdot T(e/(y\backslash e), L(y\backslash e, y, x/y)) = K(y\backslash e, L(y\backslash e, y, x/y)/(y\backslash e))$ by Theorem 282. Then $(y\backslash e)\cdot T(y, L(y\backslash e, y, x/y)) = K(y\backslash e, L(y\backslash e, y, x/y)/(y\backslash e))$ by Proposition 24. Then $K(y\backslash e, y\cdot L(y\backslash e, y, x/y)) = (y\backslash e)\cdot T(y, L(y\backslash e, y, x/y))$ by Theorem 161. Then $K(y\backslash e, y\cdot L(y\backslash e, y, x/y)) = (y\backslash e)\cdot T(y, x\backslash(x/y))$ by Theorem 29. Then

$$K(y\backslash e, y\cdot(x\backslash(x/y))) = (y\backslash e)\cdot T(y, x\backslash(x/y)) \tag{149}$$

by Theorem 29. We have $K(y\backslash e, y\cdot(((x/y)\cdot y)\backslash(x/y))) = e$ by Theorem 381. Then $K(y\backslash e, y\cdot(x\backslash(x/y))) = e$ by Axiom 6. Hence we are done by (149). □

Theorem 383. $T(y, y\backslash e) = T(y, x\backslash(x/y))$.

Proof. We have $(y\backslash e)\cdot T(y, x\backslash(x/y)) = e$ by Theorem 382. Hence we are done by Theorem 20. □

Theorem 384. $(y\backslash e)\backslash e = T(y, x\backslash(x/y))$.

Proof. We have $(y\backslash e)\cdot T(y, x\backslash(x/y)) = e$ by Theorem 382. Hence we are done by Proposition 2. □

Theorem 385. $T(y, (x\cdot y)\backslash x) = T(y, y\backslash e)$.

Proof. We have $T(y, (x\cdot y)\backslash((x\cdot y)/y)) = T(y, y\backslash e)$ by Theorem 383. Hence we are done by Axiom 5. □

Theorem 386. $T(y, (x\cdot y)\backslash x) = (y\backslash e)\backslash e$.

Proof. We have $T(y, (x\cdot y)\backslash((x\cdot y)/y)) = (y\backslash e)\backslash e$ by Theorem 384. Hence we are done by Axiom 5. □

Theorem 387. $T(x\backslash y, y\backslash x) = ((x\backslash y)\backslash e)\backslash e$.

Proof. We have $T(x\backslash y, y\backslash(y/(x\backslash y))) = ((x\backslash y)\backslash e)\backslash e$ by Theorem 384. Hence we are done by Proposition 24. □

Theorem 388. $T(y, L(y\backslash e, y, x)) = T(y, y\backslash e)$.

Proof. We have $T(y, (x\cdot y)\backslash x) = T(y, y\backslash e)$ by Theorem 385. Hence we are done by Proposition 78. □

Theorem 389. $T(y, L(y\backslash e, y, x)) = (y\backslash e)\backslash e$.

Proof. We have $T(y, (x\cdot y)\backslash x) = (y\backslash e)\backslash e$ by Theorem 386. Hence we are done by Proposition 78. □

Theorem 390. $x = T(e/(e/x), (y \cdot x)\backslash y)$.

Proof. We have $K(x\backslash e, x \cdot ((y \cdot x)\backslash y)) = ((x \cdot ((y \cdot x)\backslash y)) \cdot (x\backslash e))\backslash((x\backslash e) \cdot (x \cdot ((y \cdot x)\backslash y)))$ by Definition 2. Then

$$K(x\backslash e, x \cdot ((y \cdot x)\backslash y)) = L(x\backslash e, x, y)\backslash((x\backslash e) \cdot (x \cdot ((y \cdot x)\backslash y))) \tag{150}$$

by Theorem 163. We have $L(x\backslash e, x, y)\backslash((x\backslash e) \cdot (x \cdot ((y \cdot x)\backslash y))) = (x\backslash e) \cdot (L(x\backslash e, x, y)\backslash(x \cdot ((y \cdot x)\backslash y)))$ by Theorem 277. Then

$$K(x\backslash e, x \cdot ((y \cdot x)\backslash y)) = (x\backslash e) \cdot (L(x\backslash e, x, y)\backslash(x \cdot ((y \cdot x)\backslash y))) \tag{151}$$

by (150). We have $L(x\backslash e, x, y)\backslash(((y \cdot x)\backslash y) \cdot T(x, (y \cdot x)\backslash y)) = T(x, (y \cdot x)\backslash y)$ by Theorem 99. Then $L(x\backslash e, x, y)\backslash(x \cdot ((y \cdot x)\backslash y)) = T(x, (y \cdot x)\backslash y)$ by Proposition 46. Then

$$(x\backslash e) \cdot T(x, (y \cdot x)\backslash y) = K(x\backslash e, x \cdot ((y \cdot x)\backslash y)) \tag{152}$$

by (151). We have $T(e/(e/x), (y \cdot x)\backslash y)/((x\backslash e) \cdot T(x, (y \cdot x)\backslash y)) = x$ by Theorem 273. Then $T(e/(e/x), (y \cdot x)\backslash y)/K(x\backslash e, x \cdot ((y \cdot x)\backslash y)) = x$ by (152). Then

$$T(e/(e/x), (y \cdot x)\backslash y)/e = x \tag{153}$$

by Theorem 381. We have $T(e/(e/x), (y \cdot x)\backslash y)/e = T(e/(e/x), (y \cdot x)\backslash y)$ by Proposition 27. Hence we are done by (153). ☐

Theorem 391. $T(x\backslash(x/y), y) = ((x\backslash(x/y))\backslash e)\backslash e$.

Proof. We have $T(x\backslash(x/y), (x/y)\backslash x) = ((x\backslash(x/y))\backslash e)\backslash e$ by Theorem 387. Hence we are done by Proposition 25. ☐

Theorem 392. $K(y, (x \cdot y)\backslash x) = K(y, y\backslash e)$.

Proof. We have $T(y, (x \cdot y)\backslash x)/y = K(y, (x \cdot y)\backslash x)$ by Theorem 364. Then

$$((y\backslash e)\backslash e)/y = K(y, (x \cdot y)\backslash x) \tag{154}$$

by Theorem 386. We have $((y\backslash e)\backslash e)/y = K(y, y\backslash e)$ by Theorem 247. Hence we are done by (154). ☐

Theorem 393. $e/(e/y) = L(y, (x \cdot y)\backslash x, y)$.

Proof. We have $(((x \cdot y)\backslash x) \cdot T(e/(e/y), (x \cdot y)\backslash x))/((x \cdot y)\backslash x) = e/(e/y)$ by Proposition 48. Then

$$(((x \cdot y)\backslash x) \cdot y)/((x \cdot y)\backslash x) = e/(e/y) \tag{155}$$

by Theorem 390. We have $(((x \cdot y)\backslash x) \cdot T(y, y \cdot ((x \cdot y)\backslash x)))/((x \cdot y)\backslash x) = L(y, (x \cdot y)\backslash x, y)$ by Theorem 80. Then $(((x \cdot y)\backslash x) \cdot y)/((x \cdot y)\backslash x) = L(y, (x \cdot y)\backslash x, y)$ by Theorem 305. Hence we are done by (155). ☐

Theorem 394. $L(x\backslash y, y\backslash x, x\backslash y) = e/(e/(x\backslash y))$.

Proof. We have $L(x\backslash y, (x \cdot (x\backslash y))\backslash x, x\backslash y) = e/(e/(x\backslash y))$ by Theorem 393. Hence we are done by Axiom 4. ☐

Theorem 395. $e/(e/((y \cdot x)\backslash y)) = L(e/x, x, y)$.

Proof. We have $L((y \cdot x)\backslash y, y\backslash(y \cdot x), (y \cdot x)\backslash y) = e/(e/((y \cdot x)\backslash y))$ by Theorem 394. Then

$$L((y \cdot x)\backslash y, x, (y \cdot x)\backslash y) = e/(e/((y \cdot x)\backslash y)) \tag{156}$$

by Axiom 3. We have $L(L(x\backslash e, x, (y \cdot x)\backslash y), x, y) = L((y \cdot x)\backslash y, x, (y \cdot x)\backslash y)$ by Theorem 32. Then $L(e/x, x, y) = L((y \cdot x)\backslash y, x, (y \cdot x)\backslash y)$ by Theorem 342. Hence we are done by (156). ☐

Theorem 396. $L(e/y, y, x/y) = e/(e/(x\backslash(x/y)))$.

Proof. We have $L(e/y, y, x/y) = e/(e/(((x/y)\cdot y)\backslash(x/y)))$ by Theorem 395. Hence we are done by Axiom 6. ☐

Theorem 397. $K(T(x, R(x\backslash e, y, z)), R(x\backslash e, y, z)) = x\backslash T(x, R(x\backslash e, y, z))$.

Proof. We have $K(R(x\backslash e, y, z)\backslash(R(x\backslash e, y, z)/(x\backslash e)), R(x\backslash e, y, z)) = x\backslash T(x, R(x\backslash e, y, z))$ by Theorem 198. Then $K(R(x\backslash e, y, z)\backslash(x\cdot R(x\backslash e, y, z)), R(x\backslash e, y, z)) = x\backslash T(x, R(x\backslash e, y, z))$ by Theorem 193. Hence we are done by Definition 3. ☐

Theorem 398. $a(T(y, x), e/y, y) = y\cdot L(y\backslash e, T(y, x), e/y)$.

Proof. We have $((T(y, x)\cdot(e/y))/T(y, x))\cdot((e/y)\backslash e) = (e/y)\backslash((T(y, x)\cdot(e/y))/T(y, x))$ by Theorem 195. Then

$$((T(y, x)\cdot(e/y))/T(y, x))\cdot y = (e/y)\backslash((T(y, x)\cdot(e/y))/T(y, x)) \tag{157}$$

by Proposition 25. We have $((T(y, x)\cdot(e/y))/T(y, x))\cdot y = y\cdot((T(y, x)\cdot(y\backslash e))/T(y, x))$ by Theorem 125. Then

$$(e/y)\backslash((T(y, x)\cdot(e/y))/T(y, x)) = y\cdot((T(y, x)\cdot(y\backslash e))/T(y, x)) \tag{158}$$

by (157). We have $(y\cdot((T(y, x)\cdot(y\backslash e))/T(y, x)))\cdot T(y, x) = T(T(y, x), R(e/y, y, T(y, x)))$ by Theorem 289. Then $T(T(y, x), R(e/y, y, T(y, x)))/T(y, x) = y\cdot((T(y, x)\cdot(y\backslash e))/T(y, x))$ by Proposition 1. Then $(e/y)\backslash((T(y, x)\cdot(e/y))/T(y, x)) = T(T(y, x), R(e/y, y, T(y, x)))/T(y, x)$ by (158). Then

$$(e/y)\backslash((T(T(y, y\backslash e), x)/y)/T(y, x)) = T(T(y, x), R(e/y, y, T(y, x)))/T(y, x) \tag{159}$$

by Theorem 216. We have $T(T(y, R(e/y, y, T(y, x))), x) = T(T(y, x), R(e/y, y, T(y, x)))$ by Axiom 7. Then $T(T(y, y\backslash e), x) = T(T(y, x), R(e/y, y, T(y, x)))$ by Theorem 361. Then

$$(e/y)\backslash((T(T(y, y\backslash e), x)/y)/T(y, x)) = T(T(y, y\backslash e), x)/T(y, x) \tag{160}$$

by (159). We have $T(y, x)\backslash T(T(y, e/y), x) = a(T(y, x), e/y, y)$ by Theorem 132. Then $T(y, x)\backslash T(T(y, x), e/y) = a(T(y, x), e/y, y)$ by Axiom 7. Then

$$T(y, x)\backslash T(T(y, x), y\backslash e) = a(T(y, x), e/y, y) \tag{161}$$

by Theorem 222. We have $T(T(y, x), T(y, x)\backslash T(T(y, x), y\backslash e)) = T(y, x)$ by Theorem 372. Then

$$T(T(y, x), a(T(y, x), e/y, y)) = T(y, x) \tag{162}$$

by (161). We have $T(T(y, y\backslash e), x)/T(T(y, x), T(y, x)\backslash T(T(y, y\backslash e), x)) = T(y, x)\backslash T(T(y, y\backslash e), x)$ by Theorem 15. Then

$$T(T(y, y\backslash e), x)/T(T(y, x), a(T(y, x), e/y, y)) = T(y, x)\backslash T(T(y, y\backslash e), x) \tag{163}$$

by Theorem 217. We have $T(y, x)\backslash T(T(y, y\backslash e), x) = a(T(y, x), e/y, y)$ by Theorem 217. Then $T(T(y, y\backslash e), x)/T(T(y, x), a(T(y, x), e/y, y)) = a(T(y, x), e/y, y)$ by (163). Then $T(T(y, y\backslash e), x)/T(y, x) = a(T(y, x), e/y, y)$ by (162). Then $(e/y)\backslash((T(T(y, y\backslash e), x)/y)/T(y, x)) = a(T(y, x), e/y, y)$ by (160). Then $(e/y)\backslash((T(y, x)\cdot(e/y))/T(y, x)) = a(T(y, x), e/y, y)$ by Theorem 216. Then

$$(e/y)\backslash L(e/y, T(y, x), e/y) = a(T(y, x), e/y, y) \tag{164}$$

by Theorem 346. We have $(e/y)\backslash L(e/y, T(y, x), e/y) = y\cdot L(y\backslash e, T(y, x), e/y)$ by Theorem 159. Hence we are done by (164). ☐

Theorem 399. $y\cdot((y\backslash T(y, x))/T(y, x)) = a(T(y, x), e/y, y)$.

Proof. We have $T((T(y,x)\cdot(e/y))/T(y,x),y) = (T(y,x)\cdot(y\backslash e))/T(y,x)$ by Theorem 94. Then

$$T(L(e/y,T(y,x),e/y),y) = (T(y,x)\cdot(y\backslash e))/T(y,x) \tag{165}$$

by Theorem 346. We have $T(L(e/y,T(y,x),e/y),y) = L(y\backslash e,T(y,x),e/y)$ by Proposition 72. Then $(T(y,x)\cdot(y\backslash e))/T(y,x) = L(y\backslash e,T(y,x),e/y)$ by (165). Then

$$(y\backslash T(y,x))/T(y,x) = L(y\backslash e,T(y,x),e/y) \tag{166}$$

by Theorem 146. We have $y\cdot L(y\backslash e,T(y,x),e/y) = a(T(y,x),e/y,y)$ by Theorem 398. Hence we are done by (166). □

Theorem 400. $(y\backslash e)\cdot T(y,x) = K(y\backslash e,y)\cdot(T(y,x)/y)$.

Proof. We have $((y\backslash e)/(e/y))\cdot((e/y)\cdot T((e/y)\backslash e,x)) = (y\backslash e)\cdot T((y\backslash e)\backslash((y\backslash e)/(e/y)),x)$ by Theorem 139. Then $K(y\backslash e,y)\cdot((e/y)\cdot T((e/y)\backslash e,x)) = (y\backslash e)\cdot T((y\backslash e)\backslash((y\backslash e)/(e/y)),x)$ by Theorem 254. Then $K(y\backslash e,y)\cdot(T(y,x)/y) = (y\backslash e)\cdot T((y\backslash e)\backslash((y\backslash e)/(e/y)),x)$ by Theorem 149. Then $(y\backslash e)\cdot T((y\backslash e)\backslash K(y\backslash e,y),x) = K(y\backslash e,y)\cdot(T(y,x)/y)$ by Theorem 254. Hence we are done by Theorem 21. □

Theorem 401. $T(e/(e/y),x)\backslash T(y,x) = K(T(y,x),e/y)$.

Proof. We have $K((e/y)\backslash((e/y)/((((e/y)\cdot T(y,x))/(e/y))\backslash e)),e/y) = (((e/y)\cdot T(y,x))/(e/y))\backslash T(y,x)$ by Theorem 200. Then $K((e/y)\backslash((e/y)/(T(e/(e/y),x)\backslash e)),e/y) = (((e/y)\cdot T(y,x))/(e/y))\backslash T(y,x)$ by Theorem 51. Then $K((e/y)\backslash((e/y)\cdot T((e/y)\backslash e,x)),e/y) = (((e/y)\cdot T(y,x))/(e/y))\backslash T(y,x)$ by Theorem 377. Then

$$K((e/y)\backslash(T(y,x)/y),e/y) = (((e/y)\cdot T(y,x))/(e/y))\backslash T(y,x) \tag{167}$$

by Theorem 149. We have $(e/y)\cdot T(y,x) = T(y,x)/y$ by Theorem 150. Then $(e/y)\backslash(T(y,x)/y) = T(y,x)$ by Proposition 2. Then $(((e/y)\cdot T(y,x))/(e/y))\backslash T(y,x) = K(T(y,x),e/y)$ by (167). Hence we are done by Theorem 51. □

Theorem 402. $K(x/y,y/x) = K(x/y,(x/y)\backslash e)$.

Proof. We have $T(T(x/y,(y/((x/y)\cdot y))\cdot(x/y))\cdot(y/((x/y)\cdot y)),x/y) = T(x/y,y/((x/y)\cdot y))\cdot T(y/((x/y)\cdot y),x/y)$ by Theorem 181. Then $T((x/y)\cdot(y/((x/y)\cdot y)),x/y) = T(x/y,y/((x/y)\cdot y))\cdot T(y/((x/y)\cdot y),x/y)$ by Theorem 299. Then $T(x/y,y/((x/y)\cdot y))\cdot R((x/y)\backslash e,x/y,y) = T((x/y)\cdot(y/((x/y)\cdot y)),x/y)$ by Theorem 89. Then

$$T(x/y,y/((x/y)\cdot y))\cdot R((x/y)\backslash e,x/y,y) = (x/y)\cdot(y/((x/y)\cdot y)) \tag{168}$$

by Theorem 335. We have $((x/y)\cdot R((x/y)\backslash e,x/y,y))\backslash(T(x/y,R((x/y)\backslash e,x/y,y))\cdot R((x/y)\backslash e,x/y,y)) = K(T(x/y,R((x/y)\backslash e,x/y,y)),R((x/y)\backslash e,x/y,y))$ by Theorem 12. Then $((x/y)\cdot R((x/y)\backslash e,x/y,y))\backslash(T(x/y,y/((x/y)\cdot y))\cdot R((x/y)\backslash e,x/y,y)) = K(T(x/y,R((x/y)\backslash e,x/y,y)),R((x/y)\backslash e,x/y,y))$ by Theorem 347. Then

$$((x/y)\cdot R((x/y)\backslash e,x/y,y))\backslash((x/y)\cdot(y/((x/y)\cdot y))) = K(T(x/y,R((x/y)\backslash e,x/y,y)),R((x/y)\backslash e,x/y,y)) \tag{169}$$

by (168). We have $K(x/y,y/((x/y)\cdot y)) = ((y/((x/y)\cdot y))\cdot(x/y))\backslash((x/y)\cdot(y/((x/y)\cdot y)))$ by Definition 2. Then $K(x/y,y/((x/y)\cdot y)) = ((x/y)\cdot R((x/y)\backslash e,x/y,y))\backslash((x/y)\cdot(y/((x/y)\cdot y)))$ by Theorem 90. Then

$$K(T(x/y,R((x/y)\backslash e,x/y,y)),R((x/y)\backslash e,x/y,y)) = K(x/y,y/((x/y)\cdot y)) \tag{170}$$

by (169). We have $K(T(x/y,R((x/y)\backslash e,x/y,y)),R((x/y)\backslash e,x/y,y)) = (x/y)\backslash T(x/y,R((x/y)\backslash e,x/y,y))$ by Theorem 397. Then $K(T(x/y,R((x/y)\backslash e,x/y,y)),R((x/y)\backslash e,x/y,y)) = (x/y)\backslash T(x/y,y/((x/y)\cdot y))$ by Theorem 347. Then

$$K(x/y,y/((x/y)\cdot y)) = (x/y)\backslash T(x/y,y/((x/y)\cdot y)) \tag{171}$$

by (170). We have $(x/y)\backslash T(x/y,(x/y)\backslash e) = K(x/y,(x/y)\backslash e)$ by Theorem 271. Then $(x/y)\backslash T(x/y,y/((x/y)\cdot y)) = K(x/y,(x/y)\backslash e)$ by Theorem 362. Then $K(x/y,y/((x/y)\cdot y)) = K(x/y,(x/y)\backslash e)$ by (171). Hence we are done by Axiom 6. □

Theorem 403. $K(x/(y\backslash x),(y\backslash x)/x)=K(y,y\backslash e)$.

Proof. We have $K(x/(y\backslash x),(x/(y\backslash x))\backslash e)=K(x/(y\backslash x),(y\backslash x)/x)$ by Theorem 402. Then $K(y,(x/(y\backslash x))\backslash e)=K(x/(y\backslash x),(y\backslash x)/x)$ by Proposition 24. Hence we are done by Proposition 24. □

Theorem 404. $((y\cdot x)\backslash y)\cdot K(x,x\backslash e)=L(e/x,x,y)$.

Proof. We have $((e/x)\cdot T((e/x)\backslash e,(y\cdot x)\backslash y))\cdot((y\cdot x)\backslash y)=(e/x)\cdot((((y\cdot x)\backslash y)\backslash((e/x)\backslash((y\cdot x)\backslash y)))\cdot((y\cdot x)\backslash y))$ by Theorem 236. Then $((e/x)\cdot T((e/x)\backslash e,(y\cdot x)\backslash y))\cdot((y\cdot x)\backslash y)=(e/x)\cdot((((y\cdot x)\backslash y)\backslash(((y\cdot x)\backslash y)\cdot x))\cdot((y\cdot x)\backslash y))$ by Theorem 323. Then $(e/x)\cdot((((y\cdot x)\backslash y)\backslash(((y\cdot x)\backslash y)\cdot x))\cdot((y\cdot x)\backslash y))=(T(x,(y\cdot x)\backslash y)/x)\cdot((y\cdot x)\backslash y)$ by Theorem 149. Then

$$(e/x)\cdot(x\cdot((y\cdot x)\backslash y))=(T(x,(y\cdot x)\backslash y)/x)\cdot((y\cdot x)\backslash y) \tag{172}$$

by Axiom 3. We have $(e/x)\cdot(x\cdot((y\cdot x)\backslash y))=L((y\cdot x)\backslash y,x,e/x)$ by Proposition 67. Then $(e/x)\cdot(x\cdot((y\cdot x)\backslash y))=L(e/x,x,y)$ by Theorem 134. Then $(T(x,(y\cdot x)\backslash y)/x)\cdot((y\cdot x)\backslash y)=L(e/x,x,y)$ by (172). Then $K(x,(y\cdot x)\backslash y)\cdot((y\cdot x)\backslash y)=L(e/x,x,y)$ by Theorem 364. Then

$$K(x,x\backslash e)\cdot((y\cdot x)\backslash y)=L(e/x,x,y) \tag{173}$$

by Theorem 392. We have $((y\cdot x)\backslash y)\cdot T(K(x,x\backslash e),(y\cdot x)\backslash y)=K(x,x\backslash e)\cdot((y\cdot x)\backslash y)$ by Proposition 46. Then $((y\cdot x)\backslash y)\cdot T(K(x,x\backslash e),(y\cdot x)\backslash y)=L(e/x,x,y)$ by (173). Hence we are done by Theorem 226. □

Theorem 405. $(y\cdot x)\cdot K(x,x\backslash e)=y\cdot T(x,L(x\backslash e,x,y))$.

Proof. We have $L(e/x,x,y)=e/(e/((y\cdot x)\backslash y))$ by Theorem 395. Then

$$L(e/x,x,y)=e/(e/L(x\backslash e,x,y)) \tag{174}$$

by Proposition 78. We have $L(x\backslash e,x,y)\backslash(e/(e/L(x\backslash e,x,y)))=((y\cdot x)\backslash y)\backslash(e/(e/L(x\backslash e,x,y)))$ by Theorem 100. Then $K(L(x\backslash e,x,y)\backslash e,L(x\backslash e,x,y))=((y\cdot x)\backslash y)\backslash(e/(e/L(x\backslash e,x,y)))$ by Theorem 257. Then

$$K(L(x\backslash e,x,y)\backslash e,L(x\backslash e,x,y))=((y\cdot x)\backslash y)\backslash L(e/x,x,y) \tag{175}$$

by (174). We have $((y\cdot x)\backslash y)\cdot K(x,x\backslash e)=L(e/x,x,y)$ by Theorem 404. Then $((y\cdot x)\backslash y)\backslash L(e/x,x,y)=K(x,x\backslash e)$ by Proposition 2. Then

$$K(L(x\backslash e,x,y)\backslash e,L(x\backslash e,x,y))=K(x,x\backslash e) \tag{176}$$

by (175). We have $y\cdot T(y\backslash(y\cdot x),L(x\backslash e,x,y))=(y\cdot x)\cdot K(L(x\backslash e,x,y)\backslash(L(x\backslash e,x,y)/((y\cdot x)\backslash y)),L(x\backslash e,x,y))$ by Theorem 284. Then $y\cdot T(x,L(x\backslash e,x,y))=(y\cdot x)\cdot K(L(x\backslash e,x,y)\backslash(L(x\backslash e,x,y)/((y\cdot x)\backslash y)),L(x\backslash e,x,y))$ by Axiom 3. Then $(y\cdot x)\cdot K(L(x\backslash e,x,y)\backslash e,L(x\backslash e,x,y))=y\cdot T(x,L(x\backslash e,x,y))$ by Theorem 234. Hence we are done by (176). □

Theorem 406. $x\cdot T(y,y\backslash e)=(x\cdot y)\cdot K(y,y\backslash e)$.

Proof. We have $x\cdot T(y,L(y\backslash e,y,x))=(x\cdot y)\cdot K(y,y\backslash e)$ by Theorem 405. Hence we are done by Theorem 388. □

Theorem 407. $x\cdot((y\backslash e)\backslash e)=(x\cdot y)\cdot K(y,y\backslash e)$.

Proof. We have $x\cdot T(y,L(y\backslash e,y,x))=(x\cdot y)\cdot K(y,y\backslash e)$ by Theorem 405. Hence we are done by Theorem 389. □

Theorem 408. $(x\cdot y)\backslash(x\cdot((y\backslash e)\backslash e))=K(y,y\backslash e)$.

Proof. We have $(x\cdot y)\cdot K(y,y\backslash e)=x\cdot((y\backslash e)\backslash e)$ by Theorem 407. Hence we are done by Proposition 2. □

Theorem 409. $R(y,K(x,x\backslash e),x)=y$.

Proof. We have $(y \cdot K(x, x\backslash e))/K(x, x\backslash e) = y$ by Axiom 5. Then

$$(((y \cdot K(x, x\backslash e))/T(x, x\backslash e)) \cdot T(x, x\backslash e))/K(x, x\backslash e) = y \tag{177}$$

by Axiom 6. We have $((((y \cdot K(x, x\backslash e))/T(x, x\backslash e)) \cdot T(x, x\backslash e))/K(x, x\backslash e)) \cdot K(x, x\backslash e) = ((y \cdot K(x, x\backslash e))/T(x, x\backslash e)) \cdot T(x, x\backslash e)$ by Axiom 6. Then $((((((y \cdot K(x, x\backslash e))/T(x, x\backslash e)) \cdot T(x, x\backslash e))/K(x, x\backslash e))/x) \cdot x) \cdot K(x, x\backslash e) = ((y \cdot K(x, x\backslash e))/T(x, x\backslash e)) \cdot T(x, x\backslash e)$ by Axiom 6. Then

$$((y/x) \cdot x) \cdot K(x, x\backslash e) = ((y \cdot K(x, x\backslash e))/T(x, x\backslash e)) \cdot T(x, x\backslash e) \tag{178}$$

by (177). We have $((y/x) \cdot x) \cdot K(x, x\backslash e) = (y/x) \cdot T(x, x\backslash e)$ by Theorem 406. Then $((y \cdot K(x, x\backslash e))/T(x, x\backslash e)) \cdot T(x, x\backslash e) = (y/x) \cdot T(x, x\backslash e)$ by (178). Then

$$(y \cdot K(x, x\backslash e))/T(x, x\backslash e) = y/x \tag{179}$$

by Proposition 10. We have $R(y, K(x, x\backslash e), x) = ((y \cdot K(x, x\backslash e)) \cdot x)/(K(x, x\backslash e) \cdot x)$ by Definition 5. Then

$$R(y, K(x, x\backslash e), x) = ((y \cdot K(x, x\backslash e)) \cdot x)/T(x, x\backslash e) \tag{180}$$

by Theorem 246. We have $((y \cdot K(x, x\backslash e)) \cdot x)/T(x, x\backslash e) = ((y \cdot K(x, x\backslash e))/T(x, x\backslash e)) \cdot x$ by Theorem 239. Then $R(y, K(x, x\backslash e), x) = ((y \cdot K(x, x\backslash e))/T(x, x\backslash e)) \cdot x$ by (180). Then

$$(y/x) \cdot x = R(y, K(x, x\backslash e), x) \tag{181}$$

by (179). We have $(y/x) \cdot x = y$ by Axiom 6. Hence we are done by (181). □

Theorem 410. $R(y, K(x\backslash e, x), e/x) = y$.

Proof. We have $R(y, K(e/x, (e/x)\backslash e), e/x) = y$ by Theorem 409. Hence we are done by Theorem 272. □

Theorem 411. $x \cdot (y\backslash e) = (x \cdot K(y\backslash e, y)) \cdot (e/y)$.

Proof. We have $R(x, K(y\backslash e, y), e/y) \cdot (K(y\backslash e, y) \cdot (e/y)) = (x \cdot K(y\backslash e, y)) \cdot (e/y)$ by Proposition 54. Then $R(x, K(y\backslash e, y), e/y) \cdot (y\backslash e) = (x \cdot K(y\backslash e, y)) \cdot (e/y)$ by Theorem 255. Hence we are done by Theorem 410. □

Theorem 412. $(x \cdot (e/y)) \cdot K(y\backslash e, y) = x \cdot (y\backslash e)$.

Proof. We have

$$((x \cdot (e/y)) \cdot K(y\backslash e, y)) \cdot (e/y) = (x \cdot (e/y)) \cdot (y\backslash e) \tag{182}$$

by Theorem 411. We have $(x \cdot (y\backslash e)) \cdot (e/y) = (x \cdot (e/y)) \cdot (y\backslash e)$ by Theorem 241. Hence we are done by (182) and Proposition 8. □

Theorem 413. $R(y, e/x, K(x\backslash e, x)) = y$.

Proof. We have

$$(y \cdot (e/x)) \cdot K(x\backslash e, x) = y \cdot (x\backslash e) \tag{183}$$

by Theorem 412. We have $(e/x) \cdot K(x\backslash e, x) = x\backslash e$ by Theorem 109. Then $(y \cdot (e/x)) \cdot K(x\backslash e, x) = y \cdot ((e/x) \cdot K(x\backslash e, x))$ by (183). Hence we are done by Theorem 64. □

Theorem 414. $R(y, e/(e/x), K(x, x\backslash e)) = y$.

Proof. We have $R(y, e/(e/x), K((e/x)\backslash e, e/x)) = y$ by Theorem 413. Hence we are done by Theorem 253. □

Theorem 415. $x \cdot y = (x \cdot (e/(e/y))) \cdot K(y, y\backslash e)$.

Proof. We have $R(x, e/(e/y), K(y, y\backslash e))\cdot((e/(e/y))\cdot K(y, y\backslash e)) = (x\cdot(e/(e/y)))\cdot K(y, y\backslash e)$ by Proposition 54. Then $R(x, e/(e/y), K(y, y\backslash e)) \cdot y = (x \cdot (e/(e/y))) \cdot K(y, y\backslash e)$ by Theorem 252. Hence we are done by Theorem 414. □

Theorem 416. $x \cdot K(y, y\backslash e) = (x/(e/(e/y))) \cdot y$.

Proof. We have $((x/(e/(e/y))) \cdot (e/(e/y))) \cdot K(y, y\backslash e) = (x/(e/(e/y))) \cdot y$ by Theorem 415. Hence we are done by Axiom 6. □

Theorem 417. $(x/y) \cdot K(y, y\backslash e) = x/(e/(e/y))$.

Proof. We have $((x/y) \cdot y)/(e/(e/y)) = ((x/y)/(e/(e/y))) \cdot y$ by Theorem 292. Then $x/(e/(e/y)) = ((x/y)/(e/(e/y))) \cdot y$ by Axiom 6. Hence we are done by Theorem 416. □

Theorem 418. $x\backslash(x/(e/(e/y))) = e/(e/(x\backslash(x/y)))$.

Proof. We have $x\backslash((x/y) \cdot (y \cdot (e/y))) = L(e/y, y, x/y)$ by Theorem 4. Then $x\backslash((x/y) \cdot (y \cdot (e/y))) = e/(e/(x\backslash(x/y)))$ by Theorem 396. Then $x\backslash((x/y) \cdot K(y, y\backslash e)) = e/(e/(x\backslash(x/y)))$ by Theorem 250. Hence we are done by Theorem 417. □

Theorem 419. $(x\backslash(x/(e/y)))\backslash e = e/(x\backslash(x/(y\backslash e)))$.

Proof. We have $(e/(e/(x\backslash(x/(y\backslash e)))))\backslash e = e/(x\backslash(x/(y\backslash e)))$ by Proposition 25. Then $(x\backslash(x/(e/(e/(y\backslash e)))))\backslash e = e/(x\backslash(x/(y\backslash e)))$ by Theorem 418. Hence we are done by Proposition 24. □

Theorem 420. $((x\backslash(x/y))\backslash e)\backslash e = x\backslash(x/((y\backslash e)\backslash e))$.

Proof. We have $e/(x\backslash(x/((y\backslash e)\backslash e))) = (x\backslash(x/(e/(y\backslash e))))\backslash e$ by Theorem 419. Then

$$e/(x\backslash(x/((y\backslash e)\backslash e))) = (x\backslash(x/y))\backslash e \tag{184}$$

by Proposition 24. We have $(e/(x\backslash(x/((y\backslash e)\backslash e))))\backslash e = x\backslash(x/((y\backslash e)\backslash e))$ by Proposition 25. Hence we are done by (184). □

Theorem 421. $T(x\backslash(x/y), y) = x\backslash(x/((y\backslash e)\backslash e))$.

Proof. We have $T(x\backslash(x/y), y) = ((x\backslash(x/y))\backslash e)\backslash e$ by Theorem 391. Hence we are done by Theorem 420. □

Theorem 422. $x \cdot T(x\backslash(x/y), y) = x/((y\backslash e)\backslash e)$.

Proof. We have $x \cdot (x\backslash(x/((y\backslash e)\backslash e))) = x/((y\backslash e)\backslash e)$ by Axiom 4. Hence we are done by Theorem 421. □

Theorem 423. $(x/y) \cdot K(y\backslash e, y) = x/((y\backslash e)\backslash e)$.

Proof. We have $(x/y) \cdot K(y\backslash e, y) = x \cdot T(x\backslash(x/y), y)$ by Theorem 154. Hence we are done by Theorem 422. □

Theorem 424. $(x/((y\backslash e)\backslash e)) \cdot y = x \cdot K(y\backslash e, y)$.

Proof. We have $((x/y) \cdot K(y\backslash e, y)) \cdot y = x \cdot K(y\backslash e, y)$ by Theorem 315. Hence we are done by Theorem 423. □

Theorem 425. $x \cdot K(y\backslash e, y) = x/K(y, y\backslash e)$.

Proof. We have $x/(((x/((y\backslash e)\backslash e)) \cdot y)\backslash((x/((y\backslash e)\backslash e)) \cdot ((y\backslash e)\backslash e))) = (x/((y\backslash e)\backslash e)) \cdot y$ by Theorem 24. Then $x/K(y, y\backslash e) = (x/((y\backslash e)\backslash e)) \cdot y$ by Theorem 408. Hence we are done by Theorem 424. □

Theorem 426. $(y \cdot K(x\backslash e, x)) \cdot K(x, x\backslash e) = y$.

Proof. We have

$$((y \cdot K(x\backslash e,x)) \cdot K(x,x\backslash e))/K(x,x\backslash e) = ((y \cdot K(x\backslash e,x)) \cdot K(x,x\backslash e)) \cdot K(x\backslash e,x) \quad (185)$$

by Theorem 425. We have $((y \cdot K(x\backslash e,x)) \cdot K(x,x\backslash e))/K(x,x\backslash e) = y \cdot K(x\backslash e,x)$ by Axiom 5. Then $((y \cdot K(x\backslash e,x)) \cdot K(x,x\backslash e)) \cdot K(x\backslash e,x) = y \cdot K(x\backslash e,x)$ by (185). Hence we are done by Proposition 10. □

Theorem 427. $(y/(x \cdot y)) \cdot K(x\backslash e,x) = R(x\backslash e,x,y)$.

Proof. We have

$$((y/(x \cdot y)) \cdot K(x\backslash e,x)) \cdot K(x,x\backslash e) = y/(x \cdot y) \quad (186)$$

by Theorem 426. We have $R(x\backslash e,x,y) \cdot K(x,x\backslash e) = y/(x \cdot y)$ by Theorem 358. Hence we are done by (186) and Proposition 8. □

Theorem 428. $K(y\backslash e,y) = K(x/(y \cdot x),y)$.

Proof. We have $((y \cdot x)/x) \cdot x = y \cdot x$ by Axiom 6. Then $((((y \cdot x)/x)/(e/(x/(y \cdot x)))) \cdot (e/(x/(y \cdot x)))) \cdot x = y \cdot x$ by Axiom 6. Then $((y/(e/(x/(y \cdot x)))) \cdot (e/(x/(y \cdot x)))) \cdot x = y \cdot x$ by Axiom 5. Then

$$(y \cdot x)/x = (y/(e/(x/(y \cdot x)))) \cdot (e/(x/(y \cdot x))) \quad (187)$$

by Proposition 1. We have $(((y \cdot x)/x) \cdot (x/(y \cdot x))) \cdot (e/(x/(y \cdot x))) = (y \cdot x)/x$ by Theorem 354. Then $((y \cdot x)/x) \cdot (x/(y \cdot x)) = y/(e/(x/(y \cdot x)))$ by (187) and Proposition 8. Then

$$y \cdot (x/(y \cdot x)) = y/(e/(x/(y \cdot x))) \quad (188)$$

by Axiom 5. We have $K(y\backslash(y/(e/(x/(y \cdot x)))),y) = T(x/(y \cdot x),y)/(x/(y \cdot x))$ by Theorem 199. Then $K(y\backslash(y/(e/(x/(y \cdot x)))),y) = R(y\backslash e,y,x)/(x/(y \cdot x))$ by Theorem 89. Then $K(y\backslash(y \cdot (x/(y \cdot x))),y) = R(y\backslash e,y,x)/(x/(y \cdot x))$ by (188). Then

$$K(x/(y \cdot x),y) = R(y\backslash e,y,x)/(x/(y \cdot x)) \quad (189)$$

by Axiom 3. We have $(R(y\backslash e,y,x)/(x/(y \cdot x))) \cdot (x/(y \cdot x)) = R(y\backslash e,y,x)$ by Axiom 6. Then $K(x/(y \cdot x),y) \cdot (x/(y \cdot x)) = R(y\backslash e,y,x)$ by (189). Then

$$(x/(y \cdot x)) \cdot K(x/(y \cdot x),y) = R(y\backslash e,y,x) \quad (190)$$

by Theorem 373. We have $(x/(y \cdot x)) \cdot K(y\backslash e,y) = R(y\backslash e,y,x)$ by Theorem 427. Hence we are done by (190) and Proposition 7. □

Theorem 429. $K(y/x,x/y) = K((x/y)\backslash e,x/y)$.

Proof. We have $K(y/((x/y) \cdot y),x/y) = K((x/y)\backslash e,x/y)$ by Theorem 428. Hence we are done by Axiom 6. □

Theorem 430. $K(y/x,x/y) \cdot y = x \cdot T(x\backslash y,x/y)$.

Proof. We have $K((x/y)\backslash e,x/y) \cdot y = x \cdot T(x\backslash y,x/y)$ by Theorem 202. Hence we are done by Theorem 429. □

Theorem 431. $T(x,x\backslash e) \cdot (x\backslash y) = K(x,x\backslash e) \cdot y$.

Proof. We have $(x\backslash y) \cdot T((x\backslash y)\backslash y,(x\backslash y)/y) = T(x,(x\backslash y)/y) \cdot (x\backslash y)$ by Theorem 48. Then $K(y/(x\backslash y),(x\backslash y)/y) \cdot y = T(x,(x\backslash y)/y) \cdot (x\backslash y)$ by Theorem 430. Then $T(x,(x\backslash y)/y) \cdot (x\backslash y) = K(x,x\backslash e) \cdot y$ by Theorem 403. Hence we are done by Theorem 363. □

Theorem 432. $L(y,x,K(x,x\backslash e)) = y$.

Proof. We have

$$T(x, x\backslash e) \cdot (x\backslash(x \cdot y)) = K(x, x\backslash e) \cdot (x \cdot y) \tag{191}$$

by Theorem 431. We have $K(x, x\backslash e) \cdot x = T(x, x\backslash e)$ by Theorem 246. Then $(K(x, x\backslash e) \cdot x) \cdot (x\backslash(x \cdot y)) = K(x, x\backslash e) \cdot (x \cdot y)$ by (191). Then

$$L(y, x, K(x, x\backslash e)) = x\backslash(x \cdot y) \tag{192}$$

by Theorem 54. We have $x\backslash(x \cdot y) = y$ by Axiom 3. Hence we are done by (192). □

Theorem 433. *$L(y, e/x, K(x\backslash e, x)) = y$.*

Proof. We have $L(y, e/x, K(e/x, (e/x)\backslash e)) = y$ by Theorem 432. Hence we are done by Theorem 272. □

Theorem 434. *$L(x\backslash y, x, x\backslash e) = (e/x) \cdot y$.*

Proof.

$$\begin{aligned} & K(x\backslash e, x) \cdot L(x\backslash L(y, e/x, K(x\backslash e, x)), x, x\backslash e) \\ = \quad & (x\backslash e) \cdot L(y, e/x, K(x\backslash e, x)) && \text{by Theorem 156} \\ = \quad & K(x\backslash e, x) \cdot ((e/x) \cdot y) && \text{by Theorem 259.} \end{aligned}$$

Then $K(x\backslash e, x) \cdot L(x\backslash L(y, e/x, K(x\backslash e, x)), x, x\backslash e) = K(x\backslash e, x) \cdot ((e/x) \cdot y)$. Then $L(x\backslash L(y, e/x, K(x\backslash e, x)), x, x\backslash e) = (e/x) \cdot y$ by Proposition 9. Hence we are done by Theorem 433. □

Theorem 435. *$(e/x)\backslash y = (x\backslash e)\backslash(K(x\backslash e, x) \cdot y)$.*

Proof. We have $L((e/x)\backslash y, e/x, K(x\backslash e, x)) = (x\backslash e)\backslash(K(x\backslash e, x) \cdot y)$ by Theorem 260. Hence we are done by Theorem 433. □

Theorem 436. *$(e/x)\backslash y = K(x\backslash e, x) \cdot ((x\backslash e)\backslash y)$.*

Proof. We have $(x\backslash e) \cdot (K(x\backslash e, x) \cdot ((x\backslash e)\backslash y)) = K(x\backslash e, x) \cdot y$ by Theorem 338. Then

$$(x\backslash e)\backslash(K(x\backslash e, x) \cdot y) = K(x\backslash e, x) \cdot ((x\backslash e)\backslash y) \tag{193}$$

by Proposition 2. We have $L((e/x)\backslash y, e/x, K(x\backslash e, x)) = (x\backslash e)\backslash(K(x\backslash e, x) \cdot y)$ by Theorem 260. Then $L((e/x)\backslash y, e/x, K(x\backslash e, x)) = K(x\backslash e, x) \cdot ((x\backslash e)\backslash y)$ by (193). Hence we are done by Theorem 433. □

Theorem 437. *$((y\backslash e) \cdot x)/((e/y) \cdot x) = K(y\backslash e, y)$.*

Proof. We have $K(y\backslash e, y) = (y\backslash e) \cdot y$ by Proposition 76. Then $K(y\backslash e, y) = ((y\backslash e) \cdot x)/L(y\backslash x, y, y\backslash e)$ by Theorem 61. Hence we are done by Theorem 434. □

Theorem 438. *$K(x\backslash e, x) \cdot y = (e/x)\backslash((x\backslash e) \cdot y)$.*

Proof. We have $K(x\backslash e, x) \cdot ((x\backslash e)\backslash((x\backslash e) \cdot y)) = (e/x)\backslash((x\backslash e) \cdot y)$ by Theorem 436. Hence we are done by Axiom 3. □

Theorem 439. *$T(y\backslash((e/x)\backslash y), x\backslash e) = y\backslash((x\backslash e)\backslash y)$.*

Proof. We have $T(y\backslash(K(x\backslash e, (x\backslash e)\backslash e) \cdot ((x\backslash e)\backslash y)), x\backslash e) = y\backslash((x\backslash e)\backslash y)$ by Theorem 324. Then $T(y\backslash(K(x\backslash e, x) \cdot ((x\backslash e)\backslash y)), x\backslash e) = y\backslash((x\backslash e)\backslash y)$ by Theorem 249. Hence we are done by Theorem 436. □

Theorem 440. *$K(x, x\backslash e) \cdot ((e/x)\backslash y) = (x\backslash e)\backslash y$.*

Proof. We have $y \cdot (y\backslash((x\backslash e)\backslash y)) = (x\backslash e)\backslash y$ by Axiom 4. Then

$$y \cdot T(y\backslash((e/x)\backslash y), x\backslash e) = (x\backslash e)\backslash y \tag{194}$$

by Theorem 439. We have $K(x, x\backslash e) \cdot ((e/x)\backslash y) = y \cdot T(y\backslash((e/x)\backslash y), x\backslash e)$ by Theorem 331. Hence we are done by (194). □

Theorem 441. $(e/x)\backslash((x\backslash e)\cdot(K(x,x\backslash e)\cdot y))=y$.

Proof. We have

$$K(x,x\backslash e)\cdot((e/x)\backslash((x\backslash e)\cdot(K(x,x\backslash e)\cdot y)))=(x\backslash e)\backslash((x\backslash e)\cdot(K(x,x\backslash e)\cdot y)) \tag{195}$$

by Theorem 440. We have $K(x,x\backslash e)\cdot y=(x\backslash e)\backslash((x\backslash e)\cdot(K(x,x\backslash e)\cdot y))$ by Axiom 3. Hence we are done by (195) and Proposition 7. □

Theorem 442. $y=K(x,x\backslash e)\cdot(K(x\backslash e,x)\cdot y)$.

Proof. We have

$$K(x\backslash e,x)\cdot y=(e/x)\backslash((x\backslash e)\cdot(K(x,x\backslash e)\cdot(K(x\backslash e,x)\cdot y))) \tag{196}$$

by Theorem 441. We have $K(x\backslash e,x)\cdot(K(x,x\backslash e)\cdot(K(x\backslash e,x)\cdot y))=(e/x)\backslash((x\backslash e)\cdot(K(x,x\backslash e)\cdot(K(x\backslash e,x)\cdot y)))$ by Theorem 438. Hence we are done by (196) and Proposition 7. □

Theorem 443. $K(T(y,x),e/y)=K(T(y,x),y\backslash e)$.

Proof. We have

$$(y\backslash e)\cdot y=K(y\backslash e,y) \tag{197}$$

by Proposition 76. We have $(y\backslash e)\cdot T(K(y\backslash e,y)/(y\backslash e),y\backslash e)=K(y\backslash e,y)$ by Theorem 10. Then

$$y=T(K(y\backslash e,y)/(y\backslash e),y\backslash e) \tag{198}$$

by (197) and Proposition 7. We have $(y\backslash e)\cdot T(T(K(y\backslash e,y)/(y\backslash e),y\backslash e),x)=T(K(y\backslash e,y)/(y\backslash e),x)\cdot(y\backslash e)$ by Proposition 50. Then

$$(y\backslash e)\cdot T(y,x)=T(K(y\backslash e,y)/(y\backslash e),x)\cdot(y\backslash e) \tag{199}$$

by (198). We have $((y\backslash e)\cdot K(T(y,x),e/y))\cdot T(((y\backslash e)\cdot K(T(y,x),e/y))\backslash(K(y\backslash e,y)\cdot K(T(y,x),e/y)),x)=(y\backslash e)\cdot(K(T(y,x),e/y)\cdot T(K(T(y,x),e/y)\backslash((y\backslash e)\backslash(K(y\backslash e,y)\cdot K(T(y,x),e/y))),x))$ by Theorem 136. Then $(T(K(y\backslash e,y)/(y\backslash e),x)\cdot(y\backslash e))\cdot K(T(y,x),e/y)=(y\backslash e)\cdot(K(T(y,x),e/y)\cdot T(K(T(y,x),e/y)\backslash((y\backslash e)\backslash(K(y\backslash e,y)\cdot K(T(y,x),e/y))),x))$ by Theorem 237. Then $(y\backslash e)\cdot(K(T(y,x),e/y)\cdot T(K(T(y,x),e/y)\backslash((y\backslash e)\backslash(K(y\backslash e,y)\cdot K(T(y,x),e/y))),x))=((y\backslash e)\cdot T(y,x))\cdot K(T(y,x),e/y)$ by (199). Then

$$(y\backslash e)\cdot(K(T(y,x),e/y)\cdot T(K(T(y,x),e/y)\backslash((e/y)\backslash K(T(y,x),e/y)),x))=((y\backslash e)\cdot T(y,x))\cdot K(T(y,x),e/y) \tag{200}$$

by Theorem 435. We have $((y\backslash e)\cdot(K(T(y,x),e/y)\cdot T(K(T(y,x),e/y)\backslash((e/y)\backslash K(T(y,x),e/y)),x)))/((e/y)\cdot(K(T(y,x),e/y)\cdot T(K(T(y,x),e/y)\backslash((e/y)\backslash K(T(y,x),e/y)),x)))=K(y\backslash e,y)$ by Theorem 437. Then $((y\backslash e)\cdot(K(T(y,x),e/y)\cdot T(K(T(y,x),e/y)\backslash((e/y)\backslash K(T(y,x),e/y)),x)))/(((e/y)\cdot T((e/y)\backslash e,x))\cdot K(T(y,x),e/y))=K(y\backslash e,y)$ by Theorem 235. Then $((y\backslash e)\cdot(K(T(y,x),e/y)\cdot T(K(T(y,x),e/y)\backslash((e/y)\backslash K(T(y,x),e/y)),x)))/((T(y,x)/y)\cdot K(T(y,x),e/y))=K(y\backslash e,y)$ by Theorem 149. Then

$$(((y\backslash e)\cdot T(y,x))\cdot K(T(y,x),e/y))/((T(y,x)/y)\cdot K(T(y,x),e/y))=K(y\backslash e,y) \tag{201}$$

by (200). We have $((((y\backslash e)\cdot T(y,x))\cdot K(T(y,x),e/y))/((T(y,x)/y)\cdot K(T(y,x),e/y)))\cdot((T(y,x)/y)\cdot K(T(y,x),e/y))=((y\backslash e)\cdot T(y,x))\cdot K(T(y,x),e/y)$ by Axiom 6. Then

$$K(y\backslash e,y)\cdot((T(y,x)/y)\cdot K(T(y,x),e/y))=((y\backslash e)\cdot T(y,x))\cdot K(T(y,x),e/y) \tag{202}$$

by (201). We have $L(K(T(y,x),e/y),T(y,x)/y,K(y\backslash e,y))=(K(y\backslash e,y)\cdot(T(y,x)/y))\backslash(K(y\backslash e,y)\cdot((T(y,x)/y)\cdot K(T(y,x),e/y)))$ by Definition 4. Then $L(K(T(y,x),e/y),T(y,x)/y,K(y\backslash e,y))=((y\backslash e)\cdot T(y,x))\backslash(K(y\backslash e,y)\cdot((T(y,x)/y)\cdot K(T(y,x),e/y)))$ by Theorem 400. Then

$$((y\backslash e)\cdot T(y,x))\backslash(((y\backslash e)\cdot T(y,x))\cdot K(T(y,x),e/y))=L(K(T(y,x),e/y),T(y,x)/y,K(y\backslash e,y)) \tag{203}$$

by (202). We have $((y\backslash e)\cdot T(y,x))\backslash(((y\backslash e)\cdot T(y,x))\cdot K(T(y,x),e/y)) = K(T(y,x),e/y)$ by Axiom 3. Then

$$L(K(T(y,x),e/y),T(y,x)/y,K(y\backslash e,y)) = K(T(y,x),e/y) \tag{204}$$

by (203). We have $K(T(y,x),e/y) = ((e/y)\cdot T(y,x))\backslash(T(y,x)\cdot(e/y))$ by Definition 2. Then

$$K(T(y,x),e/y) = (T(y,x)/y)\backslash(T(y,x)\cdot(e/y)) \tag{205}$$

by Theorem 150. We have $L((T(y,x)/y)\backslash(T(y,x)\cdot(e/y)),T(y,x)/y,K(y\backslash e,y)) = (K(y\backslash e,y)\cdot(T(y,x)/y))\backslash(K(y\backslash e,y)\cdot(T(y,x)\cdot(e/y)))$ by Proposition 53. Then $L(K(T(y,x),e/y),T(y,x)/y,K(y\backslash e,y)) = (K(y\backslash e,y)\cdot(T(y,x)/y))\backslash(K(y\backslash e,y)\cdot(T(y,x)\cdot(e/y)))$ by (205). Then

$$((y\backslash e)\cdot T(y,x))\backslash(K(y\backslash e,y)\cdot(T(y,x)\cdot(e/y))) = L(K(T(y,x),e/y),T(y,x)/y,K(y\backslash e,y)) \tag{206}$$

by Theorem 400. We have

$$K(y,y\backslash e)\cdot(K(y\backslash e,y)\cdot(T(y,x)\cdot(e/y))) = T(y,x)\cdot(e/y) \tag{207}$$

by Theorem 442. We have $K(y,y\backslash e)\cdot(y\backslash T(y,x)) = T(y,x)\cdot(e/y)$ by Theorem 328. Then $K(y\backslash e,y)\cdot(T(y,x)\cdot(e/y)) = y\backslash T(y,x)$ by (207) and Proposition 7. Then

$$((y\backslash e)\cdot T(y,x))\backslash(y\backslash T(y,x)) = L(K(T(y,x),e/y),T(y,x)/y,K(y\backslash e,y)) \tag{208}$$

by (206). We have $K(T(y,x),y\backslash e) = ((y\backslash e)\cdot T(y,x))\backslash(T(y,x)\cdot(y\backslash e))$ by Definition 2. Then $K(T(y,x),y\backslash e) = ((y\backslash e)\cdot T(y,x))\backslash(y\backslash T(y,x))$ by Theorem 146. Then $L(K(T(y,x),e/y),T(y,x)/y,K(y\backslash e,y)) = K(T(y,x),y\backslash e)$ by (208). Hence we are done by (204). ☐

Theorem 444. $T(e/(e/y),x)\backslash T(y,x) = K(T(y,x),y\backslash e)$.

Proof. We have $T(e/(e/y),x)\backslash T(y,x) = K(T(y,x),e/y)$ by Theorem 401. Hence we are done by Theorem 443. ☐

Theorem 445. $T(a(T(y,x),e/y,y),x) = K(T(y,x),y\backslash e)$.

Proof. We have

$$T(T(y,x)\cdot((T(y,x)/y)\backslash(e/y)),T(y,x)) = T(y,x)\cdot((T(y,x)/y)\backslash(e/y)) \tag{209}$$

by Theorem 307. Then $T(T(y,x)\cdot((T(y,x)/y)\backslash(e/y)),T(y,x)) = T(y,x)\cdot(K(x\backslash(x/(e/y)),x)\backslash(e/y))$ by Theorem 199. Then

$$T(y,x)\cdot(K(x\backslash(x/(e/y)),x)\backslash(e/y)) = T(y,x)\cdot((T(y,x)/y)\backslash(e/y)) \tag{210}$$

by (209). We have $((e/y)\cdot T((e/y)\backslash e,x))\backslash(e/y) = T(e/(e/y),x)\backslash e$ by Theorem 376. Then $K(x\backslash(x/(e/y)),x)\backslash(e/y) = T(e/(e/y),x)\backslash e$ by Theorem 196. Then

$$T(y,x)\cdot(T(e/(e/y),x)\backslash e) = T(y,x)\cdot((T(y,x)/y)\backslash(e/y)) \tag{211}$$

by (210). We have $T(y,x)\cdot((((e/y)\cdot T(y,x))/(e/y))\backslash e) = (((e/y)\cdot T(y,x))/(e/y))\backslash T(y,x)$ by Theorem 186. Then $T(y,x)\cdot(T(e/(e/y),x)\backslash e) = (((e/y)\cdot T(y,x))/(e/y))\backslash T(y,x)$ by Theorem 51. Then $T(e/(e/y),x)\backslash T(y,x) = T(y,x)\cdot(T(e/(e/y),x)\backslash e)$ by Theorem 51. Then $T(y,x)\cdot((T(y,x)/y)\backslash(e/y)) = T(e/(e/y),x)\backslash T(y,x)$ by (211). Then

$$T(y,x)\cdot((T(y,x)/y)\backslash(e/y)) = K(T(y,x),y\backslash e) \tag{212}$$

by Theorem 444. We have $L(T(y,x)\backslash e,T(y,x),e/y) = ((e/y)\cdot T(y,x))\backslash((e/y)\cdot e)$ by Proposition 53. Then

$$L(T(y,x)\backslash e,T(y,x),e/y) = (T(y,x)/y)\backslash((e/y)\cdot e) \tag{213}$$

by Theorem 150. We have $T(y,x)\cdot L(T(y,x)\backslash e,T(y,x),e/y) = T(y\cdot L(y\backslash e,T(y,x),e/y),x)$ by Theorem 351. Then $T(y,x)\cdot((T(y,x)/y)\backslash((e/y)\cdot e)) = T(y\cdot L(y\backslash e,T(y,x),e/y),x)$ by (213). Then $T(y,x)\cdot((T(y,x)/y)\backslash(e/y)) = T(y\cdot L(y\backslash e,T(y,x),e/y),x)$ by Axiom 2. Then $T(y\cdot L(y\backslash e,T(y,x),e/y),x) = K(T(y,x),y\backslash e)$ by (212). Hence we are done by Theorem 398. ☐

Theorem 446. $T(e/(e/x),y)\backslash e = e/T(x,y)$.

Proof. We have $L(T(x,y),x\backslash e,x)\backslash T(x,y) = a(x,x\backslash e,T(x,y))$ by Theorem 96. Then $T(e/(e/x),y)\backslash T(x,y) = a(x,x\backslash e,T(x,y))$ by Theorem 264. Then

$$a(x,x\backslash e,T(x,y)) = K(T(x,y),e/x) \tag{214}$$

by Theorem 401. We have $T(x,y)\backslash(L(T(x,y),x\backslash e,x)\backslash T(x,y)) = L(T(x,y),x\backslash e,x)\backslash e$ by Theorem 280. Then $T(x,y)\backslash a(x,x\backslash e,T(x,y)) = L(T(x,y),x\backslash e,x)\backslash e$ by Theorem 96. Then $L(T(x,y),x\backslash e,x)\backslash e = T(x,y)\backslash K(T(x,y),e/x)$ by (214). Then $T(x,y)\backslash K(T(x,y),e/x) = T(e/(e/x),y)\backslash e$ by Theorem 264. Then

$$T(x,y)\backslash K(T(x,y),x\backslash e) = T(e/(e/x),y)\backslash e \tag{215}$$

by Theorem 443. We have $T(x,y)\backslash T(T(x,y)\cdot((T(x,y)\cdot(T(x,y)\backslash e))/T(x,y)),T(x,y)) = T(T(x,y)\cdot(T(x,y)\backslash e),T(x,y))/T(x,y)$ by Theorem 313. Then $T(x,y)\backslash T(T(x\cdot((T(x,y)\cdot(x\backslash e))/T(x,y)),y),T(x,y)) = T(T(x,y)\cdot(T(x,y)\backslash e),T(x,y))/T(x,y)$ by Theorem 345. Then $T(x,y)\backslash T(T(x\cdot((x\backslash T(x,y))/T(x,y)),y),T(x,y)) = T(T(x,y)\cdot(T(x,y)\backslash e),T(x,y))/T(x,y)$ by Theorem 146. Then $T(x,y)\backslash T(T(a(T(x,y),e/x,x),y),T(x,y)) = T(T(x,y)\cdot(T(x,y)\backslash e),T(x,y))/T(x,y)$ by Theorem 399. Then $T(x,y)\backslash T(K(T(x,y),x\backslash e),T(x,y)) = T(T(x,y)\cdot(T(x,y)\backslash e),T(x,y))/T(x,y)$ by Theorem 445. Then $T(T(x,y)\cdot(T(x,y)\backslash e),T(x,y))/T(x,y) = T(x,y)\backslash K(T(x,y),x\backslash e)$ by Theorem 371. Then $T(T(x,y)\cdot(T(x,y)\backslash e),T(x,y))/T(x,y) = T(e/(e/x),y)\backslash e$ by (215). Then

$$T(e,T(x,y))/T(x,y) = T(e/(e/x),y)\backslash e \tag{216}$$

by Axiom 4. We have $T(e,T(x,y)) = T(x,y)\backslash(e\cdot T(x,y))$ by Definition 3. Then

$$T(e,T(x,y)) = T(x,y)\backslash T(x,y) \tag{217}$$

by Axiom 1. We have $T(x,y)\backslash T(x,y) = e$ by Proposition 28. Then $T(e,T(x,y)) = e$ by (217). Hence we are done by (216). ☐

The following was first conjectured in [9].

Theorem 447. $e/(e/T(x,y)) = T(e/(e/x),y)$.

Proof. We have $e/(T(e/(e/x),y)\backslash e) = T(e/(e/x),y)$ by Proposition 24. Hence we are done by Theorem 446. ☐

Theorem 448. $K(y/x,x) = (x\cdot(y/x))\backslash y$.

Proof. We have $K(y/x,x) = (x\cdot(y/x))\backslash((y/x)\cdot x)$ by Definition 2. Hence we are done by Axiom 6. ☐

Theorem 449. $L(z,x\backslash y,x) = y\backslash(x\cdot((x\backslash y)\cdot z))$.

Proof. We have $L(z,x\backslash y,x) = (x\cdot(x\backslash y))\backslash(x\cdot((x\backslash y)\cdot z))$ by Definition 4. Hence we are done by Axiom 4. ☐

Theorem 450. $T(x,x) = x$.

Proof. We have $x\backslash(x\cdot x) = x$ by Axiom 3. Hence we are done by Definition 3. ☐

Theorem 451. $T(y,x) = T(T(y,x),y)$.

Proof. We have $T(T(y,y),x) = T(T(y,x),y)$ by Axiom 7. Hence we are done by Theorem 450. ☐

Theorem 452. $(y\cdot x)/K(y,x) = x\cdot y$.

Proof. We have $(y\cdot x)/((x\cdot y)\backslash(y\cdot x)) = x\cdot y$ by Proposition 24. Hence we are done by Definition 2. ☐

Theorem 453. $(y\cdot(z\cdot x))/L(x,z,y) = y\cdot z$.

Proof. We have $(y \cdot (z \cdot x))/((y \cdot z)\backslash(y \cdot (z \cdot x))) = y \cdot z$ by Proposition 24. Hence we are done by Definition 4. □

Theorem 454. $y = x\backslash(y \cdot T(x, y))$.

Proof. We have $x \cdot y = y \cdot T(x, y)$ by Proposition 46. Hence we are done by Proposition 2. □

Theorem 455. $K(T(y, x), y) = e$.

Proof. We have $T(T(y, x), y) = T(y, x)$ by Theorem 451. Hence we are done by Proposition 22. □

Theorem 456. $(R(x, y, z) \cdot w)/R(T(x, w), y, z) = w$.

Proof. We have $(R(x, y, z) \cdot w)/T(R(x, y, z), w) = w$ by Theorem 5. Hence we are done by Axiom 9. □

Theorem 457. $K(x, (y \cdot x)/y) = e$.

Proof. We have $K(T((y \cdot x)/y, y), (y \cdot x)/y) = e$ by Theorem 455. Hence we are done by Theorem 7. □

Theorem 458. $T(y, (x \cdot y)/x) = y$.

Proof. We have $K(y, (x \cdot y)/x) = e$ by Theorem 457. Hence we are done by Proposition 23. □

Theorem 459. $x \cdot K(y, x/y) = y \cdot (x/y)$.

Proof. We have $((x/y) \cdot y) \cdot K(y, x/y) = y \cdot (x/y)$ by Proposition 82. Hence we are done by Axiom 6. □

Theorem 460. $x \cdot y = (y \cdot x) \cdot z \to K(x, y) = z$.

Proof. Assume

$$x \cdot y = (y \cdot x) \cdot z. \tag{218}$$

We have $(y \cdot x) \cdot K(x, y) = x \cdot y$ by Proposition 82. Then $(y \cdot x) \cdot K(x, y) = (y \cdot x) \cdot z$ by (218). Hence we are done by Proposition 9. □

Theorem 461. $R(y, x, K(x, y)) = (x \cdot y)/(x \cdot K(x, y))$.

Proof. We have $R(y, x, K(x, y)) = ((y \cdot x) \cdot K(x, y))/(x \cdot K(x, y))$ by Definition 5. Hence we are done by Proposition 82. □

Theorem 462. $x/K(y, y\backslash x) = (y\backslash x) \cdot y$.

Proof. We have $(y \cdot (y\backslash x))/K(y, y\backslash x) = (y\backslash x) \cdot y$ by Theorem 452. Hence we are done by Axiom 4. □

Theorem 463. $T(x\backslash y, T(x, x\backslash y)) = T(x, x\backslash y)\backslash y$.

Proof. We have $T(x\backslash y, T(x, x\backslash y)) = T(x, x\backslash y)\backslash((x\backslash y) \cdot T(x, x\backslash y))$ by Definition 3. Hence we are done by Theorem 8. □

Theorem 464. $((x\backslash y)\backslash y) \cdot z = T(x, x\backslash y) \cdot z$.

Proof. We have $((y/T(x, x\backslash y))\backslash y) \cdot z = T(x, x\backslash y) \cdot z$ by Theorem 26. Hence we are done by Theorem 15. □

Theorem 465. $K(x, e/x)\backslash e = K(e/x, x)$.

Proof. We have $(x \cdot (e/x))\backslash e = K(e/x, x)$ by Theorem 448. Hence we are done by Proposition 77. □

Theorem 466. $z \cdot T(z\backslash(x \cdot z), y) = T(x, y) \cdot z$.

Proof. We have $z \cdot T(T(x, z), y) = T(x, y) \cdot z$ by Proposition 50. Hence we are done by Definition 3. □

Theorem 467. $L(T(x\backslash z, w), x, y) = T((y \cdot x)\backslash(y \cdot z), w)$.

Proof. We have $L(T(x\backslash z, w), x, y) = T(L(x\backslash z, x, y), w)$ by Axiom 8. Hence we are done by Proposition 53. ☐

Theorem 468. $R(R(x, w, u), y, z) \cdot (w \cdot u) = (R(x, y, z) \cdot w) \cdot u$.

Proof. We have $R(R(x, y, z), w, u) \cdot (w \cdot u) = (R(x, y, z) \cdot w) \cdot u$ by Proposition 54. Hence we are done by Axiom 12. ☐

Theorem 469. $R(x, y\backslash e, y) \cdot K(y\backslash e, y) = (x \cdot (y\backslash e)) \cdot y$.

Proof. We have $R(x, y\backslash e, y) \cdot ((y\backslash e) \cdot y) = (x \cdot (y\backslash e)) \cdot y$ by Proposition 54. Hence we are done by Proposition 76. ☐

Theorem 470. $R(T(x/y, w), y, z) = T((x \cdot z)/(y \cdot z), w)$.

Proof. We have $R(T(x/y, w), y, z) = T(R(x/y, y, z), w)$ by Axiom 9. Hence we are done by Proposition 55. ☐

Theorem 471. $T(R(y/x, x, z), x \cdot z) = (x \cdot z)\backslash(y \cdot z)$.

Proof. We have $T((y \cdot z)/(x \cdot z), x \cdot z) = (x \cdot z)\backslash(y \cdot z)$ by Proposition 47. Hence we are done by Proposition 55. ☐

Theorem 472. $(x \cdot y)/L(z, y/z, x) = x \cdot (y/z)$.

Proof. We have $(x \cdot ((y/z) \cdot z))/L(z, y/z, x) = x \cdot (y/z)$ by Theorem 453. Hence we are done by Axiom 6. ☐

Theorem 473. $R(T(x/y, z), y, y\backslash e) = T(x \cdot (y\backslash e), z)$.

Proof. We have $R(T(x/y, z), y, y\backslash e) = T(R(x/y, y, y\backslash e), z)$ by Axiom 9. Hence we are done by Proposition 71. ☐

Theorem 474. $x\backslash R(L(y, x\backslash e, x), z, w) = (x\backslash e) \cdot R(y, z, w)$.

Proof. We have $x\backslash L(R(y, z, w), x\backslash e, x) = (x\backslash e) \cdot R(y, z, w)$ by Proposition 57. Hence we are done by Axiom 10. ☐

Theorem 475. $(y \cdot L(y\backslash x, z, w))/y = L(x/y, z, w)$.

Proof. We have $(y \cdot T(L(x/y, z, w), y))/y = L(x/y, z, w)$ by Proposition 48. Hence we are done by Proposition 72. ☐

Theorem 476. $T(x/(y \cdot z), y) = R(y\backslash(x/z), y, z)$.

Proof. We have $T(R((x/z)/y, y, z), y) = R(y\backslash(x/z), y, z)$ by Proposition 60. Hence we are done by Proposition 74. ☐

Theorem 477. $T((x\backslash y)/y, x) = R(x\backslash e, x, x\backslash y)$.

Proof. We have $T(R(e/x, x, x\backslash y), x) = R(x\backslash e, x, x\backslash y)$ by Proposition 60. Hence we are done by Theorem 37. ☐

Theorem 478. $K(x, (y\backslash z)\backslash z) = (T(y, y\backslash z) \cdot x)\backslash(x \cdot ((y\backslash z)\backslash z))$.

Proof. We have $K(x, (y\backslash z)\backslash z) = (((y\backslash z)\backslash z) \cdot x)\backslash(x \cdot ((y\backslash z)\backslash z))$ by Definition 2. Hence we are done by Theorem 464. ☐

Theorem 479. $(z/y) \cdot (y \cdot T(x, z)) = L(x, y, z/y) \cdot z$.

Proof. We have $z \cdot L(T(x, z), y, z/y) = L(x, y, z/y) \cdot z$ by Proposition 58. Hence we are done by Theorem 53. ☐

Theorem 480. $x\backslash((x\backslash e)\backslash e) = K((x\backslash e)\backslash e, x\backslash e)$.

Proof. We have $((x \cdot K((x\backslash e)\backslash e, x\backslash e))/K((x\backslash e)\backslash e, x\backslash e)) \cdot K((x\backslash e)\backslash e, x\backslash e) = x \cdot K((x\backslash e)\backslash e, x\backslash e)$ by Axiom 6. Then $((((x \cdot K((x\backslash e)\backslash e, x\backslash e))/K((x\backslash e)\backslash e, x\backslash e))/K((x\backslash e)\backslash e, x\backslash e)) \cdot K((x\backslash e)\backslash e, x\backslash e)) \cdot K((x\backslash e)\backslash e, x\backslash e) = x \cdot K((x\backslash e)\backslash e, x\backslash e)$ by Axiom 6. Then

$$((x/K((x\backslash e)\backslash e, x\backslash e)) \cdot K((x\backslash e)\backslash e, x\backslash e)) \cdot K((x\backslash e)\backslash e, x\backslash e) = x \cdot K((x\backslash e)\backslash e, x\backslash e) \qquad (219)$$

by Axiom 5. We have $(e/(x\backslash e)) \cdot (x\backslash e) = e$ by Axiom 6. Then $(((e/(x\backslash e))/K((x\backslash e)\backslash e, x\backslash e)) \cdot K((x\backslash e)\backslash e, x\backslash e)) \cdot (x\backslash e) = e$ by Axiom 6. Then

$$((x/K((x\backslash e)\backslash e, x\backslash e)) \cdot K((x\backslash e)\backslash e, x\backslash e)) \cdot (x\backslash e) = e \qquad (220)$$

by Proposition 24.

$$\begin{array}{rcl} & & (x \cdot K((x\backslash e)\backslash e, x\backslash e))/K((x\backslash e)\backslash e, x\backslash e) \\ & = & (x/K((x\backslash e)\backslash e, x\backslash e)) \cdot K((x\backslash e)\backslash e, x\backslash e) \quad \text{by (219), Proposition 1} \\ & = & e/(x\backslash e) \quad \text{by (220), Proposition 1.} \end{array}$$

Then

$$(x \cdot K((x\backslash e)\backslash e, x\backslash e))/K((x\backslash e)\backslash e, x\backslash e) = e/(x\backslash e). \qquad (221)$$

We have $x\backslash(x \cdot K((x\backslash e)\backslash e, x\backslash e)) = K((x\backslash e)\backslash e, x\backslash e)$ by Axiom 3. Then $x\backslash(((x \cdot K((x\backslash e)\backslash e, x\backslash e))/K((x\backslash e)\backslash e, x\backslash e)) \cdot K((x\backslash e)\backslash e, x\backslash e)) = K((x\backslash e)\backslash e, x\backslash e)$ by Axiom 6. Then $x\backslash((e/(x\backslash e)) \cdot K((x\backslash e)\backslash e, x\backslash e)) = K((x\backslash e)\backslash e, x\backslash e)$ by (221). Hence we are done by Theorem 109. □

Theorem 481. $T(R(z\backslash x, z, y), z \cdot y) = T((z \cdot y)\backslash(x \cdot y), z)$.

Proof. We have $T(T((x \cdot y)/(z \cdot y), z), z \cdot y) = T((z \cdot y)\backslash(x \cdot y), z)$ by Proposition 51. Hence we are done by Proposition 64. □

Theorem 482. $z \cdot R(x, z, y) = (((z \cdot x) \cdot y)/(z \cdot y)) \cdot z$.

Proof. We have $z \cdot T(((z \cdot x) \cdot y)/(z \cdot y), z) = (((z \cdot x) \cdot y)/(z \cdot y)) \cdot z$ by Proposition 46. Hence we are done by Theorem 110. □

Theorem 483. $(y\backslash z) \cdot ((x \cdot ((y\backslash z)\backslash z))/x) = ((x \cdot y)/x) \cdot (y\backslash z)$.

Proof. We have $(y\backslash z) \cdot ((x \cdot T(y, y\backslash z))/x) = ((x \cdot y)/x) \cdot (y\backslash z)$ by Theorem 93. Hence we are done by Proposition 49. □

Theorem 484. $T(x\backslash e, y)/(x\backslash e) = x \cdot T(x\backslash e, y)$.

Proof. We have $R(T(e/x, y), x, x\backslash e) = T(R(e/x, x, x\backslash e), y)$ by Axiom 9. Then

$$R(T(e/x, y), x, x\backslash e) = T(x\backslash e, y) \qquad (222)$$

by Theorem 41. We have $R(T(e/x, y), x, x\backslash e)/(x\backslash e) = T(e/x, y) \cdot x$ by Theorem 73. Then

$$T(x\backslash e, y)/(x\backslash e) = T(e/x, y) \cdot x \qquad (223)$$

by (222). We have $T(e/x, y) \cdot x = x \cdot T(x\backslash e, y)$ by Proposition 61. Hence we are done by (223). □

Theorem 485. $R((z \cdot (e/x))/z, x, y) = (z \cdot (y/(x \cdot y)))/z$.

Proof. We have $R((z \cdot (e/x))/z, x, y) = (z \cdot R(e/x, x, y))/z$ by Theorem 123. Hence we are done by Proposition 79. □

Theorem 486. $(x \cdot y) \cdot T((x \cdot y)\backslash(x \cdot z), w) = x \cdot (y \cdot T(y\backslash z, w))$.

Proof. We have $(x \cdot y) \cdot T(L(y\backslash z, y, x), w) = x \cdot (y \cdot T(y\backslash z, w))$ by Theorem 55. Hence we are done by Proposition 53. $\square$

Theorem 487. $T((z \cdot w)\backslash(w \cdot x), y) = L(T(T(z, w)\backslash x, y), T(z, w), w)$.

Proof. We have $T((w \cdot T(z, w))\backslash(w \cdot x), y) = L(T(T(z, w)\backslash x, y), T(z, w), w)$ by Theorem 467. Hence we are done by Proposition 46. $\square$

Theorem 488. $R(T(y/x, x \cdot z), x, z) = (x \cdot z)\backslash(y \cdot z)$.

Proof. We have $T((y \cdot z)/(x \cdot z), x \cdot z) = (x \cdot z)\backslash(y \cdot z)$ by Proposition 47. Hence we are done by Theorem 470. $\square$

Theorem 489. $(z \cdot w) \cdot ((y \cdot ((z \cdot w)\backslash(x \cdot w)))/y) = ((y \cdot R(x/z, z, w))/y) \cdot (z \cdot w)$.

Proof. We have $(z \cdot w) \cdot ((y \cdot T(R(x/z, z, w), z \cdot w))/y) = ((y \cdot R(x/z, z, w))/y) \cdot (z \cdot w)$ by Theorem 93. Hence we are done by Theorem 471. $\square$

Theorem 490. $T(x \cdot (z\backslash e), y)/(z\backslash e) = T(x/z, y) \cdot z$.

Proof. We have $R(T(x/z, y), z, z\backslash e)/(z\backslash e) = T(x/z, y) \cdot z$ by Theorem 73. Hence we are done by Theorem 473. $\square$

Theorem 491. $(z/x)\backslash(L(y, x, z/x) \cdot z) = x \cdot T(y, z)$.

Proof. We have $(z/x) \cdot (x \cdot T(y, z)) = L(y, x, z/x) \cdot z$ by Theorem 479. Hence we are done by Proposition 2. $\square$

Theorem 492. $((x \cdot T(y, x \cdot y)) \cdot y)/(x \cdot y) = y$.

Proof. We have $x \cdot T(y, x) = y \cdot x$ by Proposition 46. Then

$$x \cdot R(T(y, x \cdot y), x, y) = y \cdot x \tag{224}$$

by Theorem 113. We have $(((x \cdot T(y, x \cdot y)) \cdot y)/(x \cdot y)) \cdot x = x \cdot R(T(y, x \cdot y), x, y)$ by Theorem 482. Hence we are done by (224) and Proposition 8. $\square$

Theorem 493. $T((y/x) \cdot T(x, y), x) = y$.

Proof. We have $((((y/x) \cdot T(x, (y/x) \cdot x)) \cdot x)/((y/x) \cdot x)) \cdot ((y/x) \cdot x) = ((y/x) \cdot T(x, (y/x) \cdot x)) \cdot x$ by Axiom 6. Then $x \cdot ((y/x) \cdot x) = ((y/x) \cdot T(x, (y/x) \cdot x)) \cdot x$ by Theorem 492. Then

$$T((y/x) \cdot T(x, (y/x) \cdot x), x) = (y/x) \cdot x \tag{225}$$

by Theorem 11. We have $(y/x) \cdot x = y$ by Axiom 6. Then $T((y/x) \cdot T(x, (y/x) \cdot x), x) = y$ by (225). Hence we are done by Axiom 6. $\square$

Theorem 494. $T(x \cdot T(x\backslash y, y), x\backslash y) = y$.

Proof. We have $T((y/(x\backslash y)) \cdot T(x\backslash y, y), x\backslash y) = y$ by Theorem 493. Hence we are done by Proposition 24. $\square$

Theorem 495. $((x\backslash y) \cdot y)/(x\backslash y) = x \cdot T(x\backslash y, y)$.

Proof. We have $((x\backslash y) \cdot T(x \cdot T(x\backslash y, y), x\backslash y))/(x\backslash y) = x \cdot T(x\backslash y, y)$ by Proposition 48. Hence we are done by Theorem 494. $\square$

Theorem 496. $(x\backslash e) \cdot T((x\backslash e)\backslash(x\backslash y), z) = x\backslash T(y, z)$.

Proof. We have $(x\backslash e) \cdot T((x\backslash e)\backslash(x\backslash y), z) = x\backslash T(x \cdot (x\backslash y), z)$ by Theorem 140. Hence we are done by Axiom 4. $\square$

Theorem 497. $T(y \cdot (x\backslash e), z)/(x\backslash e) = x \cdot T(x\backslash y, z)$.

Proof. We have $T(y/x, z) \cdot x = x \cdot T(x\backslash y, z)$ by Proposition 61. Hence we are done by Theorem 490. ☐

Theorem 498. $T(y \cdot x, z)/x = (e/x) \cdot T((e/x)\backslash y, z)$.

Proof. We have $T(y/(e/x), z) \cdot (e/x) = (e/x) \cdot T((e/x)\backslash y, z)$ by Proposition 61. Hence we are done by Theorem 141. ☐

Theorem 499. $T(((e/x) \cdot y) \cdot x, z)/x = (e/x) \cdot T(y, z)$.

Proof. We have $T(((e/x) \cdot y)/(e/x), z) \cdot (e/x) = (e/x) \cdot T(y, z)$ by Theorem 47. Hence we are done by Theorem 141. ☐

Theorem 500. $(T(x, y)/x)/(e/x) = T(e/(e/x), y)$.

Proof. We have $T(e/(e/x), y) \cdot (e/x) = T(x, y)/x$ by Theorem 147. Hence we are done by Proposition 1. ☐

Theorem 501. $T(e/y, T(y, x)) = T(y, x)\backslash(T(y, x)/y)$.

Proof. We have $T(e/y, T(y, x)) = T(y, x)\backslash((e/y) \cdot T(y, x))$ by Definition 3. Hence we are done by Theorem 150. ☐

Theorem 502. $((y\backslash x)/(x/y))/(e/(x/y)) = T(e/(e/(x/y)), y)$.

Proof. We have $(T(x/y, y)/(x/y))/(e/(x/y)) = T(e/(e/(x/y)), y)$ by Theorem 500. Hence we are done by Proposition 47. ☐

Theorem 503. $L(x\backslash e, x, T(x, y)/x) = T(e/x, T(x, y))$.

Proof. We have $L(x\backslash e, x, T(x, y)/x) = T(x, y)\backslash(T(x, y)/x)$ by Theorem 29. Hence we are done by Theorem 501. ☐

Theorem 504. $K(y\backslash e, y) \cdot (y\backslash x) = x \cdot T(x\backslash(y\backslash x), y)$.

Proof. We have $R(y\backslash e, y, y\backslash x) \cdot x = ((y\backslash e) \cdot y) \cdot (y\backslash x)$ by Theorem 25. Then

$$T((y\backslash x)/x, y) \cdot x = ((y\backslash e) \cdot y) \cdot (y\backslash x) \tag{226}$$

by Theorem 477. We have $T((y\backslash x)/x, y) \cdot x = x \cdot T(x\backslash(y\backslash x), y)$ by Proposition 61. Then $((y\backslash e) \cdot y) \cdot (y\backslash x) = x \cdot T(x\backslash(y\backslash x), y)$ by (226). Hence we are done by Proposition 76. ☐

Theorem 505. $(y\backslash T(x, z))/(y\backslash e) = T((y\backslash x)/(y\backslash e), z)$.

Proof. We have $((y\backslash e) \cdot T((y\backslash e)\backslash(y\backslash x), z))/(y\backslash e) = T((y\backslash x)/(y\backslash e), z)$ by Theorem 49. Hence we are done by Theorem 496. ☐

Theorem 506. $(y \cdot T(y\backslash x, z)) \cdot (y\backslash e) = T(x \cdot (y\backslash e), z)$.

Proof. We have $(T(x \cdot (y\backslash e), z)/(y\backslash e)) \cdot (y\backslash e) = T(x \cdot (y\backslash e), z)$ by Axiom 6. Hence we are done by Theorem 497. ☐

Theorem 507. $((e/y) \cdot T(x, z)) \cdot y = T(((e/y) \cdot x) \cdot y, z)$.

Proof. We have $(T(((e/y) \cdot x) \cdot y, z)/y) \cdot y = T(((e/y) \cdot x) \cdot y, z)$ by Axiom 6. Hence we are done by Theorem 499. ☐

Theorem 508. $T(x \cdot y, x)/y = ((x \cdot y) \cdot x)/(x \cdot y)$.

Proof. We have $(x \cdot y) \cdot T((y \cdot x)/y, x \cdot y) = ((y \cdot x)/y) \cdot (x \cdot y)$ by Proposition 46. Then

$$(x \cdot y) \cdot L(x, y, x) = ((y \cdot x)/y) \cdot (x \cdot y) \tag{227}$$

by Theorem 92. We have $L(((x \cdot y) \cdot x)/(x \cdot y), y, x) \cdot (x \cdot y) = (x \cdot y) \cdot L(x, y, x)$ by Theorem 86. Then

$$(y \cdot x)/y = L(((x \cdot y) \cdot x)/(x \cdot y), y, x) \tag{228}$$

by (227) and Proposition 8. We have $((y \cdot x)/y) \cdot y = y \cdot x$ by Axiom 6. Then

$$L(((x \cdot y) \cdot x)/(x \cdot y), y, x) \cdot y = y \cdot x \tag{229}$$

by (228). We have $y \cdot L(T(((x \cdot y) \cdot x)/(x \cdot y), y), y, x) = L(((x \cdot y) \cdot x)/(x \cdot y), y, x) \cdot y$ by Proposition 58. Then $y \cdot ((x \cdot y) \backslash (x \cdot ((((x \cdot y) \cdot x)/(x \cdot y)) \cdot y))) = L(((x \cdot y) \cdot x)/(x \cdot y), y, x) \cdot y$ by Proposition 62. Then

$$x = (x \cdot y) \backslash (x \cdot ((((x \cdot y) \cdot x)/(x \cdot y)) \cdot y)) \tag{230}$$

by (229) and Proposition 7. We have $(x \cdot y) \cdot ((x \cdot y) \backslash (x \cdot ((((x \cdot y) \cdot x)/(x \cdot y)) \cdot y))) = x \cdot ((((x \cdot y) \cdot x)/(x \cdot y)) \cdot y)$ by Axiom 4. Then $(x \cdot y) \cdot x = x \cdot ((((x \cdot y) \cdot x)/(x \cdot y)) \cdot y)$ by (230). Then $T(x \cdot y, x) = (((x \cdot y) \cdot x)/(x \cdot y)) \cdot y$ by Theorem 11. Hence we are done by Proposition 1. □

Theorem 509. $T(T(y \cdot x, y)/x, y \cdot x) = y$.

Proof. We have $T(((y \cdot x) \cdot y)/(y \cdot x), y \cdot x) = y$ by Theorem 7. Hence we are done by Theorem 508. □

Theorem 510. $T(T(x, x/y)/y, x) = x/y$.

Proof. We have $T(T((x/y) \cdot y, x/y)/y, (x/y) \cdot y) = x/y$ by Theorem 509. Then $T(T(x, x/y)/y, (x/y) \cdot y) = x/y$ by Axiom 6. Hence we are done by Axiom 6. □

Theorem 511. $(x \cdot (x/y))/x = T(x, x/y)/y$.

Proof. We have $(x \cdot T(T(x, x/y)/y, x))/x = T(x, x/y)/y$ by Proposition 48. Hence we are done by Theorem 510. □

Theorem 512. $((x \cdot (x/y))/x) \cdot y = T(x, x/y)$.

Proof. We have $(T(x, x/y)/y) \cdot y = T(x, x/y)$ by Axiom 6. Hence we are done by Theorem 511. □

Theorem 513. $T(x \backslash T(y, y/x), y) = x \backslash y$.

Proof. We have $T(T(T(y, y/x)/x, y), x) = T(x \backslash T(y, y/x), y)$ by Proposition 51. Then

$$T(y/x, x) = T(x \backslash T(y, y/x), y) \tag{231}$$

by Theorem 510. We have $T(y/x, x) = x \backslash y$ by Proposition 47. Hence we are done by (231). □

Theorem 514. $T((x \backslash y) \backslash T(y, x), y) = (x \backslash y) \backslash y$.

Proof. We have $T((x \backslash y) \backslash T(y, y/(x \backslash y)), y) = (x \backslash y) \backslash y$ by Theorem 513. Hence we are done by Proposition 24. □

Theorem 515. $T(y, y/x) = x \cdot ((y \cdot (x \backslash y))/y)$.

Proof. We have $((y \cdot (y/x))/y) \cdot x = x \cdot ((y \cdot (x \backslash y))/y)$ by Theorem 125. Hence we are done by Theorem 512. □

Theorem 516. $(x \cdot y) \cdot R((x \cdot y) \backslash (x \cdot z), w, u) = x \cdot (y \cdot R(y \backslash z, w, u))$.

Proof. We have $(x \cdot y) \cdot R(L(y \backslash z, y, x), w, u) = x \cdot (y \cdot R(y \backslash z, w, u))$ by Theorem 56. Hence we are done by Proposition 53. □

Theorem 517. $R((x \cdot u)/(w \cdot u), y, z) \cdot (w \cdot u) = (R(x/w, y, z) \cdot w) \cdot u$.

Proof. We have $R(R(x/w, w, u), y, z) \cdot (w \cdot u) = (R(x/w, y, z) \cdot w) \cdot u$ by Theorem 468. Hence we are done by Proposition 55. ☐

Theorem 518. $R(T(z\backslash x, z \cdot y), z, y) = T((z \cdot y)\backslash(x \cdot y), z)$.

Proof. We have $R(T(z\backslash x, z \cdot y), z, y) = T(R(z\backslash x, z, y), z \cdot y)$ by Axiom 9. Hence we are done by Theorem 481. ☐

Theorem 519. $R(T(x, z \cdot y), z, y) = T((z \cdot y)\backslash((z \cdot x) \cdot y), z)$.

Proof. We have $R(T(z\backslash(z \cdot x), z \cdot y), z, y) = T((z \cdot y)\backslash((z \cdot x) \cdot y), z)$ by Theorem 518. Hence we are done by Axiom 3. ☐

Theorem 520. $T(z, x) \cdot (y \cdot z) = z \cdot (T(z, x) \cdot T(y, z))$.

Proof. We have $T(z, x) \cdot (z \cdot T(y, z)) = z \cdot (T(z, x) \cdot T(y, z))$ by Theorem 172. Hence we are done by Proposition 46. ☐

Theorem 521. $(y\backslash x) \cdot ((x/y) \cdot z) = (x/y) \cdot (T(x/y, y) \cdot z)$.

Proof. We have $T(x/y, y) \cdot ((x/y) \cdot z) = (x/y) \cdot (T(x/y, y) \cdot z)$ by Theorem 172. Hence we are done by Proposition 47. ☐

Theorem 522. $x\backslash z = T(x, y) \cdot (x\backslash(T(x, y)\backslash z))$.

Proof. We have $x\backslash(T(x, y) \cdot (T(x, y)\backslash z)) = T(x, y) \cdot (x\backslash(T(x, y)\backslash z))$ by Theorem 173. Hence we are done by Axiom 4. ☐

Theorem 523. $T(y, x)\backslash K(y, e/y) = y \cdot (T(y, x)\backslash(e/y))$.

Proof. We have $T(y, x)\backslash(y \cdot (e/y)) = y \cdot (T(y, x)\backslash(e/y))$ by Theorem 174. Hence we are done by Proposition 77. ☐

Theorem 524. $y \cdot (T(y, x)\backslash(T(y, x)/y)) = K(y, T(y, x)/y)$.

Proof. We have $T(y, x)\backslash(y \cdot (T(y, x)/y)) = K(y, T(y, x)/y)$ by Theorem 3. Hence we are done by Theorem 174. ☐

Theorem 525. $T(T(y, x), y \cdot (T(y, x)\backslash z)) = (y \cdot (T(y, x)\backslash z))\backslash(y \cdot z)$.

Proof. We have $T(y, x) \cdot (y \cdot (T(y, x)\backslash z)) = y \cdot (T(y, x) \cdot (T(y, x)\backslash z))$ by Theorem 172. Then

$$T(y, x) \cdot (y \cdot (T(y, x)\backslash z)) = y \cdot z \tag{232}$$

by Axiom 4. We have $T(T(y, x), y \cdot (T(y, x)\backslash z)) = (y \cdot (T(y, x)\backslash z))\backslash(T(y, x) \cdot (y \cdot (T(y, x)\backslash z)))$ by Definition 3. Hence we are done by (232). ☐

Theorem 526. $T(x, y)\backslash(x\backslash z) = x\backslash(T(x, y)\backslash z)$.

Proof. We have $T(x, y) \cdot (x\backslash(T(x, y)\backslash z)) = x\backslash z$ by Theorem 522. Hence we are done by Proposition 2. ☐

Theorem 527. $T(x, y) \cdot T(T(x, y)\backslash x, x) = L(x, T(x, y)\backslash x, T(x, y))$.

Proof. We have

$$x \cdot (T(x, y) \cdot T(T(x, y)\backslash x, x)) = T(x, y) \cdot ((T(x, y)\backslash x) \cdot x) \tag{233}$$

by Theorem 520. We have $x \cdot L(x, T(x, y)\backslash x, T(x, y)) = T(x, y) \cdot ((T(x, y)\backslash x) \cdot x)$ by Theorem 52. Hence we are done by (233) and Proposition 7. ☐

Theorem 528. $T(y,z) \cdot (T(y,x)\backslash e) = T(y,x)\backslash T(y,z)$.

Proof. We have $y \cdot (T(y,x)\backslash e) = T(y,x)\backslash y$ by Theorem 175. Then

$$(T(y,x)\backslash y)/(T(y,x)\backslash e) = y \tag{234}$$

by Proposition 1. We have $(T(y,x)\backslash T(y,z))/(T(y,x)\backslash e) = T((T(y,x)\backslash y)/(T(y,x)\backslash e), z)$ by Theorem 505. Then

$$(T(y,x)\backslash T(y,z))/(T(y,x)\backslash e) = T(y,z) \tag{235}$$

by (234). We have $((T(y,x)\backslash T(y,z))/(T(y,x)\backslash e)) \cdot (T(y,x)\backslash e) = T(y,x)\backslash T(y,z)$ by Axiom 6. Hence we are done by (235). □

Theorem 529. $y \cdot T(e/y, T(y,x)) = K(y, T(y,x)/y)$.

Proof. We have $y \cdot (T(y,x)\backslash(T(y,x)/y)) = K(y, T(y,x)/y)$ by Theorem 524. Hence we are done by Theorem 501. □

Theorem 530. $((z\backslash(y/x)) \cdot z) \cdot x = (z \cdot x) \cdot T((z \cdot x)\backslash y, z)$.

Proof. We have $R(z\backslash(y/x), z, x) \cdot (z \cdot x) = ((z\backslash(y/x)) \cdot z) \cdot x$ by Proposition 54. Then

$$T(y/(z \cdot x), z) \cdot (z \cdot x) = ((z\backslash(y/x)) \cdot z) \cdot x \tag{236}$$

by Theorem 476. We have $T(y/(z \cdot x), z) \cdot (z \cdot x) = (z \cdot x) \cdot T((z \cdot x)\backslash y, z)$ by Proposition 61. Hence we are done by (236). □

Theorem 531. $(x\backslash e) \cdot R((x\backslash e)\backslash e, y, z) = x\backslash R(x, y, z)$.

Proof. We have $x\backslash R(L((x\backslash e)\backslash e, x\backslash e, x), y, z) = (x\backslash e) \cdot R((x\backslash e)\backslash e, y, z)$ by Theorem 474. Then $x\backslash R(e\backslash x, y, z) = (x\backslash e) \cdot R((x\backslash e)\backslash e, y, z)$ by Theorem 28. Hence we are done by Proposition 26. □

Theorem 532. $(z\backslash R(z, x, y))/R(T(z, z\backslash e), x, y) = z\backslash e$.

Proof. We have $(R(z, x, y) \cdot (z\backslash e))/R(T(z, z\backslash e), x, y) = z\backslash e$ by Theorem 456. Hence we are done by Proposition 89. □

Theorem 533. $(R(z, x, y)/z)/R(z, x, y) = e/z$.

Proof. We have $(e/z) \cdot R(z, x, y) = R(z, x, y)/z$ by Theorem 192. Hence we are done by Proposition 1. □

Theorem 534. $L(R(z, x, y)\backslash w, R(z, x, y), e/z) = (R(z, x, y)/z)\backslash((e/z) \cdot w)$.

Proof. We have $L(R(z, x, y)\backslash w, R(z, x, y), e/z) = ((e/z) \cdot R(z, x, y))\backslash((e/z) \cdot w)$ by Proposition 53. Hence we are done by Theorem 192. □

Theorem 535. $(x/y) \cdot ((y\backslash x) \cdot z) = (y\backslash x) \cdot ((x/y) \cdot z)$.

Proof. We have $(x/y) \cdot (T(x/y, y) \cdot z) = (y\backslash x) \cdot ((x/y) \cdot z)$ by Theorem 521. Hence we are done by Proposition 47. □

Theorem 536. $(x\backslash y)\backslash((y/x) \cdot z) = (y/x) \cdot ((x\backslash y)\backslash z)$.

Proof. We have $(y/x)\backslash((y/x) \cdot z) = z$ by Axiom 3. Then

$$(y/x)\backslash((x\backslash y) \cdot ((x\backslash y)\backslash((y/x) \cdot z))) = z \tag{237}$$

by Axiom 4. We have $(y/x) \cdot ((y/x)\backslash((x\backslash y) \cdot ((x\backslash y)\backslash((y/x) \cdot z)))) = (x\backslash y) \cdot ((x\backslash y)\backslash((y/x) \cdot z))$ by Axiom 4. Then $(y/x) \cdot ((x\backslash y) \cdot ((x\backslash y)\backslash((y/x)\backslash((x\backslash y) \cdot ((x\backslash y)\backslash((y/x) \cdot z)))))) = (x\backslash y) \cdot ((x\backslash y)\backslash((y/x) \cdot z))$ by Axiom 4. Then

$$(y/x) \cdot ((x\backslash y) \cdot ((x\backslash y)\backslash z)) = (x\backslash y) \cdot ((x\backslash y)\backslash((y/x) \cdot z)) \tag{238}$$

by (237). We have $(y/x) \cdot ((x\backslash y) \cdot ((x\backslash y)\backslash z)) = (x\backslash y) \cdot ((y/x) \cdot ((x\backslash y)\backslash z))$ by Theorem 535. Then $(x\backslash y) \cdot ((x\backslash y)\backslash((y/x) \cdot z)) = (x\backslash y) \cdot ((y/x) \cdot ((x\backslash y)\backslash z))$ by (238). Hence we are done by Proposition 9. □

Theorem 537. $(x\backslash y)\backslash K(y\backslash x, y) = T((x\backslash y)\backslash e, y)$.

Proof. We have $(x\backslash y)\backslash K(y\backslash(y/(x\backslash y)), y) = T((x\backslash y)\backslash e, y)$ by Theorem 197. Hence we are done by Proposition 24. ☐

Theorem 538. $((y \cdot x)\backslash y)\backslash K(x, y) = T(((y \cdot x)\backslash y)\backslash e, y)$.

Proof. We have $((y \cdot x)\backslash y)\backslash K(y\backslash(y \cdot x), y) = T(((y \cdot x)\backslash y)\backslash e, y)$ by Theorem 537. Hence we are done by Axiom 3. ☐

Theorem 539. $((y \cdot x)\backslash y) \cdot T(((y \cdot x)\backslash y)\backslash e, y) = K(x, y)$.

Proof. We have $((y \cdot x)\backslash y) \cdot (((y \cdot x)\backslash y)\backslash K(x, y)) = K(x, y)$ by Axiom 4. Hence we are done by Theorem 538. ☐

Theorem 540. $T(L(x\backslash e, x, y)\backslash e, y) = ((y \cdot x)\backslash y)\backslash K(x, y)$.

Proof. We have $T(((y \cdot x)\backslash y)\backslash e, y) = ((y \cdot x)\backslash y)\backslash K(x, y)$ by Theorem 538. Hence we are done by Proposition 78. ☐

Theorem 541. $T(y, y \cdot K(x, y)) = y$.

Proof. We have $T(y, (((y\cdot((y\cdot x)\backslash y))\backslash y)\cdot y)/((y\cdot((y\cdot x)\backslash y))\backslash y)) = y$ by Theorem 458. Then $T(y, (y\cdot((y\cdot x)\backslash y))\cdot T((y\cdot((y\cdot x)\backslash y))\backslash y, y)) = y$ by Theorem 495. Then $T(y, (y\cdot((y\cdot x)\backslash y))\cdot L(T(((y\cdot x)\backslash y)\backslash e, y), (y\cdot x)\backslash y, y)) = y$ by Theorem 30. Then $T(y, y \cdot (((y \cdot x)\backslash y) \cdot T(((y \cdot x)\backslash y)\backslash e, y))) = y$ by Proposition 52. Hence we are done by Theorem 539. ☐

Theorem 542. $L(x\backslash e, x, y) \cdot T(L(x\backslash e, x, y)\backslash e, y) = K(x, y)$.

Proof. We have $L(x\backslash e, x, y) \cdot (((y \cdot x)\backslash y)\backslash K(x, y)) = K(x, y)$ by Theorem 43. Hence we are done by Theorem 540. ☐

Theorem 543. $(e/y)\backslash((y\backslash x)/x) = ((y\backslash x)/x) \cdot y$.

Proof. We have $((y\backslash x)/(y\cdot(y\backslash x)))/(((y\backslash x)/(y\cdot(y\backslash x)))\cdot y) = e/y$ by Theorem 189. Then $((y\backslash x)/x)/(((y\backslash x)/(y\cdot(y\backslash x))) \cdot y) = e/y$ by Axiom 4. Then

$$((y\backslash x)/x)/(((y\backslash x)/x) \cdot y) = e/y \tag{239}$$

by Axiom 4. We have $(((y\backslash x)/x)/(((y\backslash x)/x) \cdot y))\backslash((y\backslash x)/x) = ((y\backslash x)/x) \cdot y$ by Proposition 25. Hence we are done by (239). ☐

Theorem 544. $R(x, e/x, x) = (x\backslash e)\backslash e$.

Proof. We have $R(x, e/x, x) = T(x, e/x)$ by Theorem 117. Hence we are done by Theorem 210. ☐

Theorem 545. $R(z, x, y)\backslash R(T(z, z\backslash e), x, y) = a(R(z, x, y), e/z, z)$.

Proof. We have $R(R(z, e/z, z), x, y) = R(R(z, x, y), e/z, z)$ by Axiom 12. Then $R(T(z, e/z), x, y) = R(R(z, x, y), e/z, z)$ by Theorem 117. Then

$$R(R(z, x, y), e/z, z) = R(T(z, z\backslash e), x, y) \tag{240}$$

by Theorem 211. We have $R(z, x, y)\backslash R(R(z, x, y), e/z, z) = a(R(z, x, y), e/z, z)$ by Theorem 131. Hence we are done by (240). ☐

Theorem 546. $(x\backslash e) \cdot ((y \cdot ((x\backslash e)\backslash e))/y) = x\backslash((y \cdot x)/y)$.

Proof. We have $((y \cdot x)/y) \cdot (x\backslash e) = x\backslash((y \cdot x)/y)$ by Theorem 195. Hence we are done by Theorem 483. ☐

Theorem 547. $y \cdot (z \cdot T(z\backslash(y\backslash x), y)) = ((y\backslash(x/z)) \cdot y) \cdot z$.

Proof. We have $(y \cdot z) \cdot T((y \cdot z)\backslash x, y) = ((y\backslash(x/z)) \cdot y) \cdot z$ by Theorem 530. Hence we are done by Theorem 136. □

Theorem 548. $x \cdot (K(x\backslash e, x) \cdot (x\backslash y)) = K(x\backslash e, x) \cdot y$.

Proof. We have $x \cdot (y \cdot T(y\backslash(x\backslash y), x)) = ((x\backslash(y/y)) \cdot x) \cdot y$ by Theorem 547. Then $x \cdot (K(x\backslash e, x) \cdot (x\backslash y)) = ((x\backslash(y/y)) \cdot x) \cdot y$ by Theorem 504. Then $((x\backslash e) \cdot x) \cdot y = x \cdot (K(x\backslash e, x) \cdot (x\backslash y))$ by Proposition 29. Hence we are done by Proposition 76. □

Theorem 549. $x \cdot (K(x\backslash e, x) \cdot y) = K(x\backslash e, x) \cdot (x \cdot y)$.

Proof. We have $x \cdot (K(x\backslash e, x) \cdot (x\backslash(x \cdot y))) = K(x\backslash e, x) \cdot (x \cdot y)$ by Theorem 548. Hence we are done by Axiom 3. □

Theorem 550. $y/T(y, x) = (e/T(y, x)) \cdot y$.

Proof. We have $e \cdot y = y$ by Axiom 1. Then

$$y \cdot (y\backslash(e \cdot y)) = y \tag{241}$$

by Axiom 4. We have $y \cdot (y\backslash(e \cdot y)) = e \cdot y$ by Axiom 4. Then $((y \cdot (y\backslash(e \cdot y)))/T(y, x)) \cdot T(y, x) = e \cdot y$ by Axiom 6. Then $(y/T(y, x)) \cdot T(y, x) = e \cdot y$ by (241). Then

$$(y/T(y, x)) \cdot T(y, x) = ((e/T(y, x)) \cdot T(y, x)) \cdot y \tag{242}$$

by Axiom 6. We have $((e/T(y, x)) \cdot T(y, x)) \cdot y = ((e/T(y, x)) \cdot y) \cdot T(y, x)$ by Theorem 238. Then $(y/T(y, x)) \cdot T(y, x) = ((e/T(y, x)) \cdot y) \cdot T(y, x)$ by (242). Hence we are done by Proposition 10. □

Theorem 551. $T(e/T(x, y), x) = x\backslash(x/T(x, y))$.

Proof. We have $T(e/T(x, y), x) = x\backslash((e/T(x, y)) \cdot x)$ by Definition 3. Hence we are done by Theorem 550. □

Theorem 552. $K(x, x\backslash e) \cdot x = (x\backslash e)\backslash e$.

Proof. We have $(x \cdot (e/x)) \cdot x = R(x, e/x, x)$ by Proposition 69. Then $K(x, e/x) \cdot x = R(x, e/x, x)$ by Proposition 77. Then $K(x, e/x) \cdot x = (x\backslash e)\backslash e$ by Theorem 544. Hence we are done by Theorem 248. □

Theorem 553. $(x\backslash e) \cdot K(x, x\backslash e) = e/x$.

Proof. We have $(x\backslash e) \cdot ((x\backslash e)\backslash(e/x)) = e/x$ by Axiom 4. Then $(x\backslash e) \cdot K(x, e/x) = e/x$ by Theorem 219. Hence we are done by Theorem 248. □

Theorem 554. $L(x\backslash y, x, K(x, x\backslash e)) = T(x, x\backslash e)\backslash(K(x, x\backslash e) \cdot y)$.

Proof. We have $L(x\backslash y, x, K(x, x\backslash e)) = (K(x, x\backslash e) \cdot x)\backslash(K(x, x\backslash e) \cdot y)$ by Proposition 53. Hence we are done by Theorem 246. □

Theorem 555. $K(x, x\backslash e) = (e/(e/(e/x))) \cdot x$.

Proof. We have $K(x, x\backslash e) = x \cdot (e/x)$ by Theorem 250. Hence we are done by Theorem 245. □

Theorem 556. $K(x, x\backslash e) = x/(e/(e/x))$.

Proof. We have $K(x, x\backslash e) = x \cdot (e/x)$ by Theorem 250. Hence we are done by Theorem 265. □

Theorem 557. $K(x, x\backslash e) \cdot (e/(e/x)) = x$.

Proof. We have $(x/(e/(e/x))) \cdot (e/(e/x)) = x$ by Axiom 6. Hence we are done by Theorem 556. □

Theorem 558. $K(x, x\backslash e)\backslash x = e/(e/x)$.

Proof. We have $(x/(e/(e/x)))\backslash x = e/(e/x)$ by Proposition 25. Hence we are done by Theorem 556. □

Theorem 559. $K(x, e/x) = x\backslash((x\backslash e)\backslash e)$.

Proof. We have $x\backslash R(x, e/x, x) = a(x, e/x, x)$ by Theorem 131. Then $x\backslash((x\backslash e)\backslash e) = a(x, e/x, x)$ by Theorem 544. Hence we are done by Theorem 269. □

Theorem 560. $K((x\backslash e)\backslash e, x\backslash e) = K(x, e/x)$.

Proof. We have $K((x\backslash e)\backslash e, x\backslash e) = x\backslash((x\backslash e)\backslash e)$ by Theorem 480. Hence we are done by Theorem 559. □

Theorem 561. $K((x\backslash e)\backslash e, x\backslash e) = K(x, x\backslash e)$.

Proof. We have $K(x, e/x) = K(x, x\backslash e)$ by Theorem 248. Then

$$x\backslash((x\backslash e)\backslash e) = K(x, x\backslash e) \tag{243}$$

by Theorem 559. We have $K((x\backslash e)\backslash e, x\backslash e) = x\backslash((x\backslash e)\backslash e)$ by Theorem 480. Hence we are done by (243). □

Theorem 562. $x \cdot K(x, x\backslash e) = T(x, x\backslash e)$.

Proof. We have $x \cdot (x\backslash T(x, x\backslash e)) = T(x, x\backslash e)$ by Axiom 4. Then $x \cdot K(x, e/x) = T(x, x\backslash e)$ by Theorem 270. Hence we are done by Theorem 248. □

Theorem 563. $K(y, y\backslash e) \cdot ((e/y)\backslash x) = x \cdot T(x\backslash((e/y)\backslash x), e/y)$.

Proof. We have $K((e/y)\backslash e, e/y) \cdot ((e/y)\backslash x) = x \cdot T(x\backslash((e/y)\backslash x), e/y)$ by Theorem 504. Hence we are done by Theorem 253. □

Theorem 564. $K(e/x, x) = K(x\backslash e, x)$.

Proof. We have $K(e/x, (e/x)\backslash e) = K(x\backslash e, x)$ by Theorem 272. Hence we are done by Proposition 25. □

Theorem 565. $K(x, x\backslash e)\backslash e = K(x\backslash e, x)$.

Proof. We have $K(x, e/x)\backslash e = K(e/x, x)$ by Theorem 465. Then $K(x, x\backslash e)\backslash e = K(e/x, x)$ by Theorem 248. Hence we are done by Theorem 564. □

Theorem 566. $K(e/(e/x), e/x) = K(x, x\backslash e)$.

Proof. We have $K((e/x)\backslash e, e/x) = K(x, x\backslash e)$ by Theorem 253. Hence we are done by Theorem 564. □

Theorem 567. $K(x\backslash e, x) \cdot T(e/x, y) = (x\backslash e) \cdot (x \cdot T(x\backslash e, y))$.

Proof. We have $L(T(x\backslash e, y), x, x\backslash e) = T(L(x\backslash e, x, x\backslash e), y)$ by Axiom 8. Then

$$L(T(x\backslash e, y), x, x\backslash e) = T(e/x, y) \tag{244}$$

by Theorem 261. We have $K(x\backslash e, x) \cdot L(T(x\backslash e, y), x, x\backslash e) = (x\backslash e) \cdot (x \cdot T(x\backslash e, y))$ by Theorem 59. Hence we are done by (244). □

Theorem 568. $x\backslash(y \cdot T(y\backslash x, z)) = (x\backslash y) \cdot T((x\backslash y)\backslash e, z)$.

Proof. We have $x \cdot ((x\backslash y) \cdot T((x\backslash y)\backslash e, z)) = y \cdot T(y\backslash x, z)$ by Theorem 283. Hence we are done by Proposition 2. □

Theorem 569. $T(y, y \cdot (T(y, x)\backslash y)) = y$.

Proof. We have $T(y, y \cdot K(y\backslash(y/(T(y,x)\backslash(x\backslash y))), y)) = y$ by Theorem 541. Then $T(y, y \cdot (T(y,x)\backslash((x\backslash y) \cdot T((x\backslash y)\backslash T(y,x), y)))) = y$ by Theorem 285. Then $T(y, y \cdot (T(y,x)\backslash((x\backslash y) \cdot ((x\backslash y)\backslash y)))) = y$ by Theorem 514. Hence we are done by Axiom 4. ☐

Theorem 570. $(x \cdot (T(x,y)\backslash x))\backslash(x \cdot x) = T(x,y)$.

Proof. We have $T(T(x, x \cdot (T(x,y)\backslash x)), y) = T(T(x,y), x \cdot (T(x,y)\backslash x))$ by Axiom 7. Then $T(x,y) = T(T(x,y), x \cdot (T(x,y)\backslash x))$ by Theorem 569. Hence we are done by Theorem 525. ☐

Theorem 571. $(y \cdot y)/T(y,x) = y \cdot (T(y,x)\backslash y)$.

Proof. We have $(y \cdot y)/((y \cdot (T(y,x)\backslash y))\backslash(y \cdot y)) = y \cdot (T(y,x)\backslash y)$ by Proposition 24. Hence we are done by Theorem 570. ☐

Theorem 572. $y \cdot (T(y,x)\backslash y) = (y/T(y,x)) \cdot y$.

Proof. We have $(y \cdot y)/T(y,x) = (y/T(y,x)) \cdot y$ by Theorem 239. Hence we are done by Theorem 571. ☐

Theorem 573. $T(x/y, z)\backslash(y\backslash x) = (y\backslash x) \cdot (T(x/y, z)\backslash e)$.

Proof. We have $T(x/y, y) \cdot (T(x/y, z)\backslash e) = T(x/y, z)\backslash T(x/y, y)$ by Theorem 528. Then $(y\backslash x) \cdot (T(x/y, z)\backslash e) = T(x/y, z)\backslash T(x/y, y)$ by Proposition 47. Hence we are done by Proposition 47. ☐

Theorem 574. $((x\backslash y)\backslash y) \cdot (x \cdot z) = x \cdot (((x\backslash y)\backslash y) \cdot z)$.

Proof. We have $(y/(x\backslash y)) \cdot (((x\backslash y)\backslash y) \cdot z) = ((x\backslash y)\backslash y) \cdot ((y/(x\backslash y)) \cdot z)$ by Theorem 535. Then $x \cdot (((x\backslash y)\backslash y) \cdot z) = ((x\backslash y)\backslash y) \cdot ((y/(x\backslash y)) \cdot z)$ by Proposition 24. Hence we are done by Proposition 24. ☐

Theorem 575. $T(x\backslash T(x,z), y) = K(z\backslash(z/T(x\backslash e, y)), z)$.

Proof. We have $T((x\backslash e) \cdot T((x\backslash e)\backslash e, z), y) = K(z\backslash(z/T(x\backslash e, y)), z)$ by Theorem 296. Hence we are done by Theorem 145. ☐

Theorem 576. $T(x \cdot T(x\backslash e, z), y) = T(x,y) \cdot T(T(x,y)\backslash e, z)$.

Proof. We have $K(z\backslash(z/T(x,y)), z) = T(x,y) \cdot T(T(x,y)\backslash e, z)$ by Theorem 196. Hence we are done by Theorem 296. ☐

Theorem 577. $K(x\backslash(x/T(x,y)), x) = T(K(x\backslash e, x), y)$.

Proof. We have $T(x \cdot T(x\backslash e, x), y) = K(x\backslash(x/T(x,y)), x)$ by Theorem 296. Then $T((x\backslash e) \cdot x, y) = K(x\backslash(x/T(x,y)), x)$ by Proposition 46. Hence we are done by Proposition 76. ☐

Theorem 578. $T(z\backslash e, z\backslash R(z,x,y)) = z\backslash e$.

Proof. We have $T(z\backslash e, (z\backslash e) \cdot R((z\backslash e)\backslash e, x, y)) = z\backslash e$ by Theorem 298. Hence we are done by Theorem 531. ☐

Theorem 579. $(R(z,x,y) \cdot (e/z))/R(z,x,y) = L(e/z, R(z,x,y), e/z)$.

Proof. We have $T(e/z, (e/z) \cdot R((e/z)\backslash e, x, y)) = e/z$ by Theorem 298. Then

$$T(e/z, (e/z) \cdot R(z,x,y)) = e/z \qquad (245)$$

by Proposition 25. We have $(R(z,x,y) \cdot T(e/z, (e/z) \cdot R(z,x,y)))/R(z,x,y) = L(e/z, R(z,x,y), e/z)$ by Theorem 80. Hence we are done by (245). ☐

Theorem 580. $R(x\backslash e, R(x,y,z), x\backslash e) = T(x\backslash e, R(x,y,z))$.

Proof. We have $T(x\backslash e, (x\backslash e)\cdot R((x\backslash e)\backslash e, y, z)) = x\backslash e$ by Theorem 298. Then

$$T(x\backslash e, R(x, y, z)\cdot (x\backslash e)) = x\backslash e \tag{246}$$

by Proposition 88. We have $R(T(x\backslash e, R(x, y, z)\cdot(x\backslash e)), R(x, y, z), x\backslash e) = T(x\backslash e, R(x, y, z))$ by Theorem 113. Hence we are done by (246). □

Theorem 581. $T(y\backslash T(y, x), y) = T(y\backslash T(y, x), T(y, y\backslash e))$.

Proof. We have $T(y\backslash e, T(y, y\backslash e)) = T(y, y\backslash e)\backslash e$ by Theorem 463. Then

$$T(y\backslash e, T(y, y\backslash e)) = T(y\backslash e, y) \tag{247}$$

by Theorem 208. We have $T(y\backslash T(y, x), T(y, y\backslash e)) = K(x\backslash(x/T(y\backslash e, T(y, y\backslash e))), x)$ by Theorem 575. Then

$$T(y\backslash T(y, x), T(y, y\backslash e)) = K(x\backslash(x/T(y\backslash e, y)), x) \tag{248}$$

by (247). We have $T(y\backslash T(y, x), y) = K(x\backslash(x/T(y\backslash e, y)), x)$ by Theorem 575. Hence we are done by (248). □

Theorem 582. $K(y\backslash T(y, x), T(y, z)\backslash e) = e$.

Proof. We have $T(T(y, z)\backslash e, y\backslash T(y, x)) = T(y, z)\backslash e$ by Theorem 308. Then $T(y\backslash T(y, x), T(y, z)\backslash e) = y\backslash T(y, x)$ by Proposition 21. Hence we are done by Proposition 22. □

Theorem 583. $T(x\cdot y, y)/y = y\backslash T(y\cdot x, y)$.

Proof. We have $T((y\backslash(y\cdot x))\cdot y, y)/y = y\backslash T(y\cdot x, y)$ by Theorem 311. Hence we are done by Axiom 3. □

Theorem 584. $y\cdot(T(x\cdot y, y)/y) = T(y\cdot x, y)$.

Proof. We have $y\cdot(y\backslash T(y\cdot x, y)) = T(y\cdot x, y)$ by Axiom 4. Hence we are done by Theorem 583. □

Theorem 585. $T(K(y, x), T(y, y\backslash e)) = T(K(y, x), y)$.

Proof. We have $L(T(y\backslash T(y, x), T(y, y\backslash e)), y, x) = T((x\cdot y)\backslash(y\cdot x), T(y, y\backslash e))$ by Theorem 103. Then

$$L(T(y\backslash T(y, x), y), y, x) = T((x\cdot y)\backslash(y\cdot x), T(y, y\backslash e)) \tag{249}$$

by Theorem 581. We have $L(T(y\backslash T(y, x), y), y, x) = T((x\cdot y)\backslash(y\cdot x), y)$ by Theorem 103. Then $T((x\cdot y)\backslash(y\cdot x), T(y, y\backslash e)) = T((x\cdot y)\backslash(y\cdot x), y)$ by (249). Then $T((x\cdot y)\backslash(y\cdot x), T(y, y\backslash e)) = T(K(y, x), y)$ by Definition 2. Hence we are done by Definition 2. □

Theorem 586. $x\backslash(K(x, x\backslash e)\cdot y) = K(x, x\backslash e)\cdot(x\backslash y)$.

Proof. We have $x\cdot(K(x, x\backslash e)\cdot(x\backslash y)) = K(x, x\backslash e)\cdot y$ by Theorem 327. Hence we are done by Proposition 2. □

Theorem 587. $L(x, y, K(y, y\backslash e)) = L(x, K(y, y\backslash e), y)$.

Proof. We have

$$(K(y, y\backslash e)\cdot y)\cdot L(x, y, K(y, y\backslash e)) = K(y, y\backslash e)\cdot(y\cdot x) \tag{250}$$

by Proposition 52. We have

$$y\cdot(K(y, y\backslash e)\cdot(y\backslash(y\cdot x))) = K(y, y\backslash e)\cdot(y\cdot x) \tag{251}$$

by Theorem 327.

$$
\begin{array}{lll}
& T(y, y\backslash e) \cdot L(x, y, K(y, y\backslash e)) & \\
= & K(y, y\backslash e) \cdot (y \cdot x) & \text{by (250), Theorem 246} \\
= & y \cdot (K(y, y\backslash e) \cdot x) & \text{by (251), Axiom 3.}
\end{array}
$$

Then

$$T(y, y\backslash e) \cdot L(x, y, K(y, y\backslash e)) = y \cdot (K(y, y\backslash e) \cdot x). \tag{252}$$

We have $(y \cdot K(y, y\backslash e)) \cdot L(x, K(y, y\backslash e), y) = y \cdot (K(y, y\backslash e) \cdot x)$ by Proposition 52. Then $T(y, y\backslash e) \cdot L(x, K(y, y\backslash e), y) = y \cdot (K(y, y\backslash e) \cdot x)$ by Theorem 562. Hence we are done by (252) and Proposition 7. □

Theorem 588. $L(x, K(y\backslash e, y), e/y) = L(x, e/y, K(e/y, (e/y)\backslash e))$.

Proof. We have $L(x, K(e/y, (e/y)\backslash e), e/y) = L(x, e/y, K(e/y, (e/y)\backslash e))$ by Theorem 587. Hence we are done by Theorem 272. □

Theorem 589. $(x\backslash((y \cdot x)/y)) \cdot y = T(y, R(x, x\backslash e, y))$.

Proof. We have $T(y, R(x, x\backslash e, y)) = ((y \cdot R(x, x\backslash e, y))/y) \cdot (R(x, x\backslash e, y)\backslash y)$ by Theorem 179. Then

$$T(y, R(x, x\backslash e, y)) = ((y \cdot R(x, x\backslash e, y))/y) \cdot ((x\backslash e) \cdot y) \tag{253}$$

by Theorem 45. We have $((y \cdot R(x, x\backslash e, y))/y) \cdot ((x\backslash e) \cdot y) = (((y \cdot x)/y) \cdot (x\backslash e)) \cdot y$ by Theorem 129. Then $T(y, R(x, x\backslash e, y)) = (((y \cdot x)/y) \cdot (x\backslash e)) \cdot y$ by (253). Hence we are done by Theorem 195. □

Theorem 590. $T(y, R(x, x\backslash e, y))/y = x\backslash((y \cdot x)/y)$.

Proof. We have $(x\backslash((y \cdot x)/y)) \cdot y = T(y, R(x, x\backslash e, y))$ by Theorem 589. Hence we are done by Proposition 1. □

Theorem 591. $(T(x, y)\backslash x) \cdot y = T(y, R(T(x, y), T(x, y)\backslash e, y))$.

Proof. We have $(T(x, y)\backslash((y \cdot T(x, y))/y)) \cdot y = T(y, R(T(x, y), T(x, y)\backslash e, y))$ by Theorem 589. Hence we are done by Proposition 48. □

Theorem 592. $K(T(x, x\backslash e), T(x, x\backslash e)\backslash e) = K(x, x\backslash e)$.

Proof. We have $K((((x\backslash e)\backslash e)\backslash e)\backslash e, ((x\backslash e)\backslash e)\backslash e) = K((x\backslash e)\backslash e, e/((x\backslash e)\backslash e))$ by Theorem 560. Then $K(T((x\backslash e)\backslash e, x\backslash e), ((x\backslash e)\backslash e)\backslash e) = K((x\backslash e)\backslash e, e/((x\backslash e)\backslash e))$ by Theorem 209. Then $K(T((x\backslash e)\backslash e, x\backslash e), ((x\backslash e)\backslash e)\backslash e) = K((x\backslash e)\backslash e, x\backslash e)$ by Proposition 24. Then

$$K(T((x\backslash e)\backslash e, x\backslash e), T(x\backslash e, (x\backslash e)\backslash e)) = K((x\backslash e)\backslash e, x\backslash e) \tag{254}$$

by Proposition 49. We have $K((x\backslash e)\backslash e, x\backslash e) = K(x, x\backslash e)$ by Theorem 561. Then

$$K(T((x\backslash e)\backslash e, x\backslash e), T(x\backslash e, (x\backslash e)\backslash e)) = K(x, x\backslash e) \tag{255}$$

by (254). We have $T((x\backslash e)\backslash e, x\backslash e) = (((x\backslash e)\backslash e)\backslash e)\backslash e$ by Theorem 209. Then $T((x\backslash e)\backslash e, x\backslash e) = T(x\backslash e, x)\backslash e$ by Theorem 209. Then $K(T(x\backslash e, x)\backslash e, T(x\backslash e, (x\backslash e)\backslash e)) = K(x, x\backslash e)$ by (255). Then $K(T(x\backslash e, x)\backslash e, ((x\backslash e)\backslash e)\backslash e) = K(x, x\backslash e)$ by Proposition 49. Then

$$K(T(x\backslash e, x)\backslash e, T(x\backslash e, x)) = K(x, x\backslash e) \tag{256}$$

by Theorem 209. We have $e/(T(x, x\backslash e)\backslash e) = T(x, x\backslash e)$ by Proposition 24. Then

$$e/T(x\backslash e, x) = T(x, x\backslash e) \tag{257}$$

by Theorem 208. We have $K(e/T(x\backslash e, x), T(x\backslash e, x)) = K(T(x\backslash e, x)\backslash e, T(x\backslash e, x))$ by Theorem 564. Then $K(T(x, x\backslash e), T(x\backslash e, x)) = K(T(x\backslash e, x)\backslash e, T(x\backslash e, x))$ by (257). Then $K(x, x\backslash e) = K(T(x, x\backslash e), T(x\backslash e, x))$ by (256). Hence we are done by Theorem 208. □

Theorem 593. $T(y,z)\backslash((y\backslash x)\backslash x) = ((y\backslash x)\backslash x)\cdot(T(y,z)\backslash e)$.

Proof. We have $((y\backslash x)\backslash x)\cdot(T(x/(y\backslash x),z)\backslash e) = T(x/(y\backslash x),z)\backslash((y\backslash x)\backslash x)$ by Theorem 573. Then $((y\backslash x)\backslash x)\cdot(T(y,z)\backslash e) = T(x/(y\backslash x),z)\backslash((y\backslash x)\backslash x)$ by Proposition 24. Hence we are done by Proposition 24. □

Theorem 594. $(x\cdot u)\cdot R((x\cdot u)\backslash(y\cdot u),z,w) = (x\cdot R(x\backslash y,z,w))\cdot u$.

Proof. We have $R((y\cdot u)/(x\cdot u),z,w)\cdot(x\cdot u) = (x\cdot u)\cdot R((x\cdot u)\backslash(y\cdot u),z,w)$ by Proposition 83. Then $(R(y/x,z,w)\cdot x)\cdot u = (x\cdot u)\cdot R((x\cdot u)\backslash(y\cdot u),z,w)$ by Theorem 517. Hence we are done by Proposition 83. □

Theorem 595. $((x/z)\cdot L((x/z)\backslash y,w,u))\cdot z = x\cdot L(x\backslash(y\cdot z),w,u)$.

Proof. We have $R(L(y/(x/z),w,u),x/z,z)\cdot x = (L(y/(x/z),w,u)\cdot(x/z))\cdot z$ by Theorem 23. Then $L(R(y/(x/z),x/z,z),w,u)\cdot x = (L(y/(x/z),w,u)\cdot(x/z))\cdot z$ by Axiom 10. Then

$$L((y\cdot z)/x,w,u)\cdot x = (L(y/(x/z),w,u)\cdot(x/z))\cdot z \tag{258}$$

by Theorem 67. We have $L((y\cdot z)/x,w,u)\cdot x = x\cdot L(x\backslash(y\cdot z),w,u)$ by Theorem 85. Then $(L(y/(x/z),w,u)\cdot(x/z))\cdot z = x\cdot L(x\backslash(y\cdot z),w,u)$ by (258). Hence we are done by Theorem 85. □

Theorem 596. $K(y,(y/(x\backslash e))/y) = x\backslash((y\cdot x)/y)$.

Proof. We have $(x\backslash e)\cdot((y\cdot((x\backslash e)\backslash e))/y) = x\backslash((y\cdot x)/y)$ by Theorem 546. Hence we are done by Theorem 333. □

Theorem 597. $(x\cdot L(x\backslash(z\cdot y),w,u))/y = (x/y)\cdot L((x/y)\backslash z,w,u)$.

Proof. We have $((x/y)\cdot L((x/y)\backslash z,w,u))\cdot y = x\cdot L(x\backslash(z\cdot y),w,u)$ by Theorem 595. Hence we are done by Proposition 1. □

Theorem 598. $(x\cdot L(x\backslash y,z,w))/y = (x/y)\cdot L((x/y)\backslash e,z,w)$.

Proof. We have $(x\cdot L(x\backslash(e\cdot y),z,w))/y = (x/y)\cdot L((x/y)\backslash e,z,w)$ by Theorem 597. Hence we are done by Axiom 1. □

Theorem 599. $(x\cdot R(x\backslash y,z,w))/y = (x/y)\cdot R((x/y)\backslash e,z,w)$.

Proof. We have $R(R(e/(x/y),z,w),x/y,y)\cdot x = (R(e/(x/y),z,w)\cdot(x/y))\cdot y$ by Theorem 23. Then $R(y/((x/y)\cdot y),z,w)\cdot x = (R(e/(x/y),z,w)\cdot(x/y))\cdot y$ by Theorem 40. Then

$$(R(e/(x/y),z,w)\cdot(x/y))\cdot y = R(y/x,z,w)\cdot x \tag{259}$$

by Axiom 6. We have $R(y/x,z,w)\cdot x = x\cdot R(x\backslash y,z,w)$ by Proposition 83. Then $(R(e/(x/y),z,w)\cdot(x/y))\cdot y = x\cdot R(x\backslash y,z,w)$ by (259). Then $((x/y)\cdot R((x/y)\backslash e,z,w))\cdot y = x\cdot R(x\backslash y,z,w)$ by Proposition 83. Hence we are done by Proposition 1. □

Theorem 600. $(x\backslash e)\backslash(K(x,x\backslash e)\cdot y) = K(x,x\backslash e)\cdot((x\backslash e)\backslash y)$.

Proof. We have $(x\backslash e)\cdot(K((x\backslash e)\backslash e,x\backslash e)\cdot((x\backslash e)\backslash y)) = K((x\backslash e)\backslash e,x\backslash e)\cdot y$ by Theorem 548. Then $(x\backslash e)\cdot(K(x,x\backslash e)\cdot((x\backslash e)\backslash y)) = K((x\backslash e)\backslash e,x\backslash e)\cdot y$ by Theorem 561. Then $(x\backslash e)\cdot(K(x,x\backslash e)\cdot((x\backslash e)\backslash y)) = K(x,x\backslash e)\cdot y$ by Theorem 561. Hence we are done by Proposition 2. □

Theorem 601. $(x\cdot K(y,y\backslash e))\cdot y = (x\cdot y)\cdot K(y,y\backslash e)$.

Proof. We have $(x\cdot(y\cdot L(y\backslash e,y,e/y)))\cdot y = (x\cdot y)\cdot(y\cdot L(y\backslash e,y,e/y))$ by Theorem 339. Then $(x\cdot(y\cdot(e/y)))\cdot y = (x\cdot y)\cdot(y\cdot L(y\backslash e,y,e/y))$ by Theorem 35. Then $(x\cdot y)\cdot(y\cdot L(y\backslash e,y,e/y)) = (x\cdot K(y,y\backslash e))\cdot y$ by Theorem 250. Then $(x\cdot y)\cdot(y\cdot(e/y)) = (x\cdot K(y,y\backslash e))\cdot y$ by Theorem 35. Hence we are done by Theorem 250. □

Theorem 602. $(x \cdot K(y, y\backslash e))\backslash((x \cdot y) \cdot K(y, y\backslash e)) = y$.

Proof. We have $(x \cdot K(y, y\backslash e)) \cdot y = (x \cdot y) \cdot K(y, y\backslash e)$ by Theorem 601. Hence we are done by Proposition 2. □

Theorem 603. $L(y\backslash e, y, x\backslash(x/y)) = e/y$.

Proof. We have $L(y\backslash e, y, ((x/y) \cdot y)\backslash(x/y)) = e/y$ by Theorem 342. Hence we are done by Axiom 6. □

Theorem 604. $(x\backslash(x/y))/(e/y) = (x\backslash(x/y)) \cdot y$.

Proof. We have $(x\backslash(x/y))/L(y\backslash e, y, x\backslash(x/y)) = (x\backslash(x/y)) \cdot y$ by Theorem 33. Hence we are done by Theorem 603. □

Theorem 605. $T(e/(e/(x/y)), y/x) = x/y$.

Proof. We have $T(e/(e/(x/y)), R(e/(x/y), x/y, y)) = x/y$ by Theorem 359. Hence we are done by Theorem 38. □

Theorem 606. $((x\backslash y)/y) \cdot T(x, x\backslash e) = x \cdot ((x\backslash y)/y)$.

Proof. We have $((x\backslash y)/y) \cdot T(x, (x\backslash y)/y) = x \cdot ((x\backslash y)/y)$ by Proposition 46. Hence we are done by Theorem 363. □

Theorem 607. $T(y\backslash e, K(x, y)) = y\backslash e$.

Proof. We have $T(T(e/y, K(x, y)), y) = T(y\backslash e, K(x, y))$ by Proposition 51. Then

$$T(e/y, y) = T(y\backslash e, K(x, y)) \tag{260}$$

by Theorem 367. We have $T(e/y, y) = y\backslash e$ by Proposition 47. Hence we are done by (260). □

Theorem 608. $(z \cdot w) \cdot ((y \cdot ((z \cdot w)\backslash(x \cdot w)))/y) = (((y \cdot (x/z))/y) \cdot z) \cdot w$.

Proof. We have $((y \cdot R(x/z, z, w))/y) \cdot (z \cdot w) = (((y \cdot (x/z))/y) \cdot z) \cdot w$ by Theorem 129. Hence we are done by Theorem 489. □

Theorem 609. $L(y, T(y, x)\backslash y, T(y, x)) = L(y, T(y, x)\backslash y, y)$.

Proof. We have $T(((T(y, x)\backslash y) \cdot y)/(T(y, x)\backslash y), y \cdot (T(y, x)\backslash y)) = L(y, T(y, x)\backslash y, y)$ by Theorem 92. Then $T(((T(y, x)\backslash y) \cdot y)/(T(y, x)\backslash y), (y \cdot y)/T(y, x)) = L(y, T(y, x)\backslash y, y)$ by Theorem 571. Then $T(T(y, x) \cdot T(T(y, x)\backslash y, y), (y \cdot y)/T(y, x)) = L(y, T(y, x)\backslash y, y)$ by Theorem 495. Then $T(L(y, T(y, x)\backslash y, T(y, x)), (y \cdot y)/T(y, x)) = L(y, T(y, x)\backslash y, y)$ by Theorem 527. Then

$$T(L(y, T(y, x)\backslash y, T(y, x)), y \cdot (T(y, x)\backslash y)) = L(y, T(y, x)\backslash y, y) \tag{261}$$

by Theorem 571. We have $L(T(y, y \cdot (T(y, x)\backslash y)), T(y, x)\backslash y, T(y, x)) = T(L(y, T(y, x)\backslash y, T(y, x)), y \cdot (T(y, x)\backslash y))$ by Axiom 8. Then $L(T(y, y \cdot (T(y, x)\backslash y)), T(y, x)\backslash y, T(y, x)) = L(y, T(y, x)\backslash y, y)$ by (261). Hence we are done by Theorem 569. □

Theorem 610. $((y\backslash x) \cdot y) \cdot (e/y) = T(x \cdot (e/y), y)$.

Proof. We have $y \cdot (T(((e/y) \cdot ((e/y)\backslash(y\backslash(x \cdot (e/y))))) \cdot y, y)/y) = T(y \cdot ((e/y) \cdot ((e/y)\backslash(y\backslash(x \cdot (e/y))))), y)$ by Theorem 584. Then

$$y \cdot ((e/y) \cdot T((e/y)\backslash(y\backslash(x \cdot (e/y))), y)) = T(y \cdot ((e/y) \cdot ((e/y)\backslash(y\backslash(x \cdot (e/y))))), y) \tag{262}$$

by Theorem 499. We have $y \cdot ((e/y) \cdot T((e/y)\backslash(y\backslash(x \cdot (e/y))), y)) = (y \cdot T(y\backslash x, y)) \cdot (e/y)$ by Theorem 301. Then

$$T(y \cdot ((e/y) \cdot ((e/y)\backslash(y\backslash(x \cdot (e/y))))), y) = (y \cdot T(y\backslash x, y)) \cdot (e/y) \tag{263}$$

by (262). We have $y \cdot (y\backslash(x \cdot (e/y))) = x \cdot (e/y)$ by Axiom 4. Then $y \cdot ((e/y) \cdot ((e/y)\backslash(y\backslash(x \cdot (e/y))))) = x \cdot (e/y)$ by Axiom 4. Then $T(x \cdot (e/y), y) = (y \cdot T(y\backslash x, y)) \cdot (e/y)$ by (263). Hence we are done by Proposition 46. □

Theorem 611. $(x \cdot y) \cdot (e/y) = T((y \cdot x) \cdot (e/y), y)$.

Proof. We have $((y\backslash(y \cdot x)) \cdot y) \cdot (e/y) = T((y \cdot x) \cdot (e/y), y)$ by Theorem 610. Hence we are done by Axiom 3. ☐

Theorem 612. $K(x\backslash y, y\backslash x) = K(x\backslash y, (x\backslash y)\backslash e)$.

Proof. We have $K(x\backslash y, (x \cdot (x\backslash y))\backslash x) = K(x\backslash y, (x\backslash y)\backslash e)$ by Theorem 392. Hence we are done by Axiom 4. ☐

Theorem 613. $(x\backslash y)\backslash K(x\backslash y, y\backslash x) = e/(x\backslash y)$.

Proof. We have $(x\backslash y)\backslash K(x\backslash y, (x\backslash y)\backslash e) = e/(x\backslash y)$ by Theorem 251. Hence we are done by Theorem 612. ☐

Theorem 614. $e/(e/(x\backslash y)) = L(e/(y\backslash x), y\backslash x, y)$.

Proof. We have $(y\backslash x) \cdot T(x\backslash y, (x\backslash y) \cdot (y\backslash x)) = L(x\backslash y, y\backslash x, x\backslash y) \cdot (y\backslash x)$ by Theorem 79. Then

$$(y\backslash x) \cdot (x\backslash y) = L(x\backslash y, y\backslash x, x\backslash y) \cdot (y\backslash x) \tag{264}$$

by Theorem 306. We have $L(e/(y\backslash x), y\backslash x, y) \cdot (y\backslash x) = (y\backslash x) \cdot L((y\backslash x)\backslash e, y\backslash x, y)$ by Theorem 85. Then $L(e/(y\backslash x), y\backslash x, y) \cdot (y\backslash x) = (y\backslash x) \cdot (x\backslash y)$ by Theorem 28. Then $L(x\backslash y, y\backslash x, x\backslash y) = L(e/(y\backslash x), y\backslash x, y)$ by (264) and Proposition 8. Hence we are done by Theorem 394. ☐

Theorem 615. $x\backslash(y \cdot K(y\backslash x, x\backslash y)) = e/(e/(x\backslash y))$.

Proof. We have $x\backslash(y \cdot ((y\backslash x) \cdot (e/(y\backslash x)))) = L(e/(y\backslash x), y\backslash x, y)$ by Theorem 449. Then $x\backslash(y \cdot K(y\backslash x, (y\backslash x)\backslash e)) = L(e/(y\backslash x), y\backslash x, y)$ by Theorem 250. Then $x\backslash(y \cdot K(y\backslash x, (y\backslash x)\backslash e)) = e/(e/(x\backslash y))$ by Theorem 614. Hence we are done by Theorem 612. ☐

Theorem 616. $L(x, x\backslash e, y) = e/(e/((y \cdot (x\backslash e))\backslash y))$.

Proof. We have $L(e/(x\backslash e), x\backslash e, y) = e/(e/((y \cdot (x\backslash e))\backslash y))$ by Theorem 395. Hence we are done by Proposition 24. ☐

Theorem 617. $L(e/x, x, y)\backslash e = e/((y \cdot x)\backslash y)$.

Proof. We have $(e/(e/((y \cdot x)\backslash y)))\backslash e = e/((y \cdot x)\backslash y)$ by Proposition 25. Hence we are done by Theorem 395. ☐

Theorem 618. $L(L(y, x, x\backslash e), K(x\backslash e, x), e/x) = (e/x) \cdot (x \cdot y)$.

Proof. We have $L(L(y, x, x\backslash e), K(x\backslash e, x), e/x) = ((e/x) \cdot K(x\backslash e, x))\backslash((e/x) \cdot (K(x\backslash e, x) \cdot L(y, x, x\backslash e)))$ by Definition 4. Then

$$L(L(y, x, x\backslash e), K(x\backslash e, x), e/x) = (x\backslash e)\backslash((e/x) \cdot (K(x\backslash e, x) \cdot L(y, x, x\backslash e))) \tag{265}$$

by Theorem 109. We have $(x\backslash e)\backslash((e/x) \cdot (K(x\backslash e, x) \cdot L(y, x, x\backslash e))) = (e/x) \cdot ((x\backslash e)\backslash(K(x\backslash e, x) \cdot L(y, x, x\backslash e)))$ by Theorem 536. Then $L(L(y, x, x\backslash e), K(x\backslash e, x), e/x) = (e/x) \cdot ((x\backslash e)\backslash(K(x\backslash e, x) \cdot L(y, x, x\backslash e)))$ by (265). Then $(e/x) \cdot ((x\backslash e)\backslash((x\backslash e) \cdot (x \cdot y))) = L(L(y, x, x\backslash e), K(x\backslash e, x), e/x)$ by Theorem 59. Hence we are done by Axiom 3. ☐

Theorem 619. $T(y\backslash T(y, z), K(x, y)) = y\backslash T(y, z)$.

Proof. We have $T(y\backslash e, K(x, y)) \cdot T(T(y\backslash e, K(x, y))\backslash e, z) = T((y\backslash e) \cdot T((y\backslash e)\backslash e, z), K(x, y))$ by Theorem 576. Then $(y\backslash e) \cdot T(T(y\backslash e, K(x, y))\backslash e, z) = T((y\backslash e) \cdot T((y\backslash e)\backslash e, z), K(x, y))$ by Theorem 607. Then

$$T((y\backslash e) \cdot T((y\backslash e)\backslash e, z), K(x, y)) = (y\backslash e) \cdot T((y\backslash e)\backslash e, z) \tag{266}$$

by Theorem 607. We have $(y\backslash e) \cdot T((y\backslash e)\backslash e, z) = y\backslash T(y, z)$ by Theorem 145. Then $T((y\backslash e) \cdot T((y\backslash e)\backslash e, z), K(x, y)) = y\backslash T(y, z)$ by (266). Hence we are done by Theorem 145. ☐

Theorem 620. $K(y,y\backslash e)\cdot T(K(y,y\backslash e)\backslash x,y)=T(x,y)$.

Proof. We have $(T((x/(e/y))/y,y)\cdot y)\cdot(e/y)=T(x/(y\cdot(e/y)),y)\cdot(y\cdot(e/y))$ by Theorem 138. Then $(T((x/(e/y))/y,y)\cdot y)\cdot(e/y)=T(x/(y\cdot(e/y)),y)\cdot K(y,y\backslash e)$ by Theorem 250. Then

$$(T((x/(e/y))/y,y)\cdot y)\cdot(e/y)=T(x/K(y,y\backslash e),y)\cdot K(y,y\backslash e) \tag{267}$$

by Theorem 250. We have $T(x/K(y,y\backslash e),y)\cdot K(y,y\backslash e)=K(y,y\backslash e)\cdot T(K(y,y\backslash e)\backslash x,y)$ by Proposition 61. Then

$$(T((x/(e/y))/y,y)\cdot y)\cdot(e/y)=K(y,y\backslash e)\cdot T(K(y,y\backslash e)\backslash x,y) \tag{268}$$

by (267). We have $(T((x/(e/y))/y,y)\cdot y)\cdot(e/y)=y\cdot((e/y)\cdot T((e/y)\backslash(y\backslash x),y))$ by Theorem 293. Then $K(y,y\backslash e)\cdot T(K(y,y\backslash e)\backslash x,y)=y\cdot((e/y)\cdot T((e/y)\backslash(y\backslash x),y))$ by (268). Then

$$K(y,y\backslash e)\cdot T(K(y,y\backslash e)\backslash x,y)=y\cdot(T((y\backslash x)\cdot y,y)/y) \tag{269}$$

by Theorem 498. We have $y\cdot(T((y\backslash x)\cdot y,y)/y)=T(x,y)$ by Theorem 312. Hence we are done by (269). ☐

Theorem 621. $T(x\cdot K(y,y\backslash e),y)=T(x,y)\cdot K(y,y\backslash e)$.

Proof. We have $K(y,y\backslash e)\cdot T(K(y,y\backslash e)\backslash(x\cdot K(y,y\backslash e)),y)=T(x,y)\cdot K(y,y\backslash e)$ by Theorem 466. Hence we are done by Theorem 620. ☐

Theorem 622. $y\cdot((y\backslash x)\cdot K(y,y\backslash e))=x\cdot K(y,y\backslash e)$.

Proof. We have $T((x/y)\cdot K(y,y\backslash e),y)=y\backslash(((x/y)\cdot K(y,y\backslash e))\cdot y)$ by Definition 3. Then $T((x/y)\cdot K(y,y\backslash e),y)=y\backslash(((x/y)\cdot y)\cdot K(y,y\backslash e))$ by Theorem 601. Then

$$y\backslash(x\cdot K(y,y\backslash e))=T((x/y)\cdot K(y,y\backslash e),y) \tag{270}$$

by Axiom 6. We have $T((x/y)\cdot K(y,y\backslash e),y)=T(x/y,y)\cdot K(y,y\backslash e)$ by Theorem 621. Then $T((x/y)\cdot K(y,y\backslash e),y)=(y\backslash x)\cdot K(y,y\backslash e)$ by Proposition 47. Then

$$(y\backslash x)\cdot K(y,y\backslash e)=y\backslash(x\cdot K(y,y\backslash e)) \tag{271}$$

by (270). We have $y\cdot(y\backslash(x\cdot K(y,y\backslash e)))=x\cdot K(y,y\backslash e)$ by Axiom 4. Hence we are done by (271). ☐

Theorem 623. $y\cdot(x\cdot K(y,y\backslash e))=(y\cdot x)\cdot K(y,y\backslash e)$.

Proof. We have $y\cdot((y\backslash(y\cdot x))\cdot K(y,y\backslash e))=(y\cdot x)\cdot K(y,y\backslash e)$ by Theorem 622. Hence we are done by Axiom 3. ☐

Theorem 624. $R(y,x,K(y,y\backslash e))=y$.

Proof. We have $(y\cdot x)\cdot K(y,y\backslash e)=y\cdot(x\cdot K(y,y\backslash e))$ by Theorem 623. Hence we are done by Theorem 64. ☐

Theorem 625. $L(K(y,y\backslash e),x,y)=K(y,y\backslash e)$.

Proof. We have $(y\cdot x)\cdot K(y,y\backslash e)=y\cdot(x\cdot K(y,y\backslash e))$ by Theorem 623. Hence we are done by Theorem 54. ☐

Theorem 626. $T(x,y\cdot K(x,x\backslash e))=T(x,y)$.

Proof. We have $R(T(x,y\cdot K(x,x\backslash e)),y,K(x,x\backslash e))=T(R(x,y,K(x,x\backslash e)),y\cdot K(x,x\backslash e))$ by Axiom 9. Then

$$R(T(x,y\cdot K(x,x\backslash e)),y,K(x,x\backslash e))=T(x,y\cdot K(x,x\backslash e)) \tag{272}$$

by Theorem 624. We have $T((y\cdot K(x,x\backslash e))\backslash((y\cdot x)\cdot K(x,x\backslash e)),y)=R(T(x,y\cdot K(x,x\backslash e)),y,K(x,x\backslash e))$ by Theorem 519. Then $T(x,y)=R(T(x,y\cdot K(x,x\backslash e)),y,K(x,x\backslash e))$ by Theorem 602. Hence we are done by (272). ☐

Theorem 627. $T(K(y\backslash e,y)\cdot x,y)=K(y\backslash e,y)\cdot T(x,y)$.

Proof. We have

$$T(((e/K(y,y\backslash e))\cdot x)\cdot K(y,y\backslash e),y)=T((e/K(y,y\backslash e))\cdot x,y)\cdot K(y,y\backslash e) \tag{273}$$

by Theorem 621. We have $T(((e/K(y,y\backslash e))\cdot x)\cdot K(y,y\backslash e),y)=((e/K(y,y\backslash e))\cdot T(x,y))\cdot K(y,y\backslash e)$ by Theorem 507. Then $((e/K(y,y\backslash e))\cdot T(x,y))\cdot K(y,y\backslash e)=T((e/K(y,y\backslash e))\cdot x,y)\cdot K(y,y\backslash e)$ by (273). Then $(e/K(y,y\backslash e))\cdot T(x,y)=T((e/K(y,y\backslash e))\cdot x,y)$ by Proposition 10. Then $T(((y\backslash e)\cdot y)\cdot x,y)=(e/K(y,y\backslash e))\cdot T(x,y)$ by Theorem 462. Then $(e/K(y,y\backslash e))\cdot T(x,y)=T(K(y\backslash e,y)\cdot x,y)$ by Proposition 76. Then $((y\backslash e)\cdot y)\cdot T(x,y)=T(K(y\backslash e,y)\cdot x,y)$ by Theorem 462. Hence we are done by Proposition 76. ☐

Theorem 628. $K(y\backslash e,y)\cdot(x\cdot y)=(K(y\backslash e,y)\cdot x)\cdot y$.

Proof. We have $y\cdot(K(y\backslash e,y)\cdot(y\backslash(x\cdot y)))=K(y\backslash e,y)\cdot(x\cdot y)$ by Theorem 548. Then $y\cdot(K(y\backslash e,y)\cdot T(x,y))=K(y\backslash e,y)\cdot(x\cdot y)$ by Definition 3. Then

$$y\cdot T(K(y\backslash e,y)\cdot x,y)=K(y\backslash e,y)\cdot(x\cdot y) \tag{274}$$

by Theorem 627. We have $y\cdot T(K(y\backslash e,y)\cdot x,y)=(K(y\backslash e,y)\cdot x)\cdot y$ by Proposition 46. Hence we are done by (274). ☐

Theorem 629. $L(y,x,K(y\backslash e,y))=y$.

Proof. We have $(K(y\backslash e,y)\cdot x)\cdot y=K(y\backslash e,y)\cdot(x\cdot y)$ by Theorem 628. Hence we are done by Theorem 54. ☐

Theorem 630. $T(x,y)=T(x,K(x\backslash e,x)\cdot y)$.

Proof. We have $(K(x\backslash e,x)\cdot y)\backslash(K(x\backslash e,x)\cdot(x\cdot y))=L(T(x,y),y,K(x\backslash e,x))$ by Proposition 62. Then

$$(K(x\backslash e,x)\cdot y)\backslash(x\cdot(K(x\backslash e,x)\cdot y))=L(T(x,y),y,K(x\backslash e,x)) \tag{275}$$

by Theorem 549. We have $T(x,K(x\backslash e,x)\cdot y)=(K(x\backslash e,x)\cdot y)\backslash(x\cdot(K(x\backslash e,x)\cdot y))$ by Definition 3. Then

$$T(x,K(x\backslash e,x)\cdot y)=L(T(x,y),y,K(x\backslash e,x)) \tag{276}$$

by (275). We have $L(T(x,y),y,K(x\backslash e,x))=T(L(x,y,K(x\backslash e,x)),y)$ by Axiom 8. Then $L(T(x,y),y,K(x\backslash e,x))=T(x,y)$ by Theorem 629. Hence we are done by (276). ☐

Theorem 631. $T(K(x\backslash e,x),K(x\backslash e,x)\backslash y)=T(K(x\backslash e,x),y)$.

Proof. We have $T(x,K(x\backslash e,x)\cdot(K(x\backslash e,x)\backslash y))=T(x,K(x\backslash e,x)\backslash y)$ by Theorem 630. Then

$$T(x,y)=T(x,K(x\backslash e,x)\backslash y) \tag{277}$$

by Axiom 4. We have $K(x\backslash(x/T(x,K(x\backslash e,x)\backslash y)),x)=T(K(x\backslash e,x),K(x\backslash e,x)\backslash y)$ by Theorem 577. Then

$$K(x\backslash(x/T(x,y)),x)=T(K(x\backslash e,x),K(x\backslash e,x)\backslash y) \tag{278}$$

by (277). We have $K(x\backslash(x/T(x,y)),x)=T(K(x\backslash e,x),y)$ by Theorem 577. Hence we are done by (278). ☐

Theorem 632. $T(x,K(y,y\backslash e))=(K(y,y\backslash e)\backslash x)\cdot K(y,y\backslash e)$.

Proof. We have $T(K((e/y)\backslash e, e/y), K((e/y)\backslash e, e/y)\backslash x) = (K((e/y)\backslash e, e/y)\backslash x)\backslash x$ by Proposition 49. Then $T(K((e/y)\backslash e, e/y), x) = (K((e/y)\backslash e, e/y)\backslash x)\backslash x$ by Theorem 631. Then $(K(y, y\backslash e)\backslash x)\backslash x = T(K((e/y)\backslash e, e/y), x)$ by Theorem 253. Then

$$T(K(y, y\backslash e), x) = (K(y, y\backslash e)\backslash x)\backslash x \tag{279}$$

by Theorem 253. We have $(K(y, y\backslash e)\backslash x) \cdot ((x \cdot ((K(y, y\backslash e)\backslash x)\backslash x))/x) = T(x, x/(K(y, y\backslash e)\backslash x))$ by Theorem 515. Then $(K(y, y\backslash e)\backslash x) \cdot ((x \cdot T(K(y, y\backslash e), x))/x) = T(x, x/(K(y, y\backslash e)\backslash x))$ by (279). Then $(K(y, y\backslash e)\backslash x) \cdot K(y, y\backslash e) = T(x, x/(K(y, y\backslash e)\backslash x))$ by Proposition 48. Hence we are done by Proposition 24. □

Theorem 633. $x \cdot K(y, y\backslash e) = T(K(y, y\backslash e) \cdot x, K(y, y\backslash e))$.

Proof. We have $(K(y, y\backslash e)\backslash(K(y, y\backslash e) \cdot x)) \cdot K(y, y\backslash e) = T(K(y, y\backslash e) \cdot x, K(y, y\backslash e))$ by Theorem 632. Hence we are done by Axiom 3. □

Theorem 634. $(T(y, z) \cdot x) \cdot K(x, y) = x \cdot (K(x, y) \cdot T(K(x, y)\backslash y, z))$.

Proof. We have $T((x \cdot y)/(x \cdot K(x, y)), z) \cdot (x \cdot K(x, y)) = (x \cdot K(x, y)) \cdot T((x \cdot K(x, y))\backslash(x \cdot y), z)$ by Proposition 61. Then

$$T(R(y, x, K(x, y)), z) \cdot (x \cdot K(x, y)) = (x \cdot K(x, y)) \cdot T((x \cdot K(x, y))\backslash(x \cdot y), z) \tag{280}$$

by Theorem 461. We have $T(R(y, x, K(x, y)), z) \cdot (x \cdot K(x, y)) = (T(y, z) \cdot x) \cdot K(x, y)$ by Proposition 65. Then

$$(x \cdot K(x, y)) \cdot T((x \cdot K(x, y))\backslash(x \cdot y), z) = (T(y, z) \cdot x) \cdot K(x, y) \tag{281}$$

by (280). We have $(x \cdot K(x, y)) \cdot T((x \cdot K(x, y))\backslash(x \cdot y), z) = x \cdot (K(x, y) \cdot T(K(x, y)\backslash y, z))$ by Theorem 486. Hence we are done by (281). □

Theorem 635. $L(x \cdot T(x\backslash e, w), y, z) = K(w\backslash(w/L(x, y, z)), w)$.

Proof. We have $(z \cdot (y \cdot x)) \cdot T((z \cdot (y \cdot x))\backslash(z \cdot y), w) = (z \cdot y) \cdot K(w\backslash(w/((z \cdot y)\backslash(z \cdot (y \cdot x)))), w)$ by Theorem 284. Then

$$(z \cdot (y \cdot x)) \cdot T((z \cdot (y \cdot x))\backslash(z \cdot y), w) = (z \cdot y) \cdot K(w\backslash(w/L(x, y, z)), w) \tag{282}$$

by Definition 4. We have $(z \cdot (y \cdot x)) \cdot T((z \cdot (y \cdot x))\backslash(z \cdot y), w) = z \cdot ((y \cdot x) \cdot T((y \cdot x)\backslash y, w))$ by Theorem 486. Then $(z \cdot y) \cdot K(w\backslash(w/L(x, y, z)), w) = z \cdot ((y \cdot x) \cdot T((y \cdot x)\backslash y, w))$ by (282). Then $(z \cdot y) \cdot K(w\backslash(w/L(x, y, z)), w) = z \cdot (y \cdot (x \cdot T(x\backslash e, w)))$ by Theorem 98. Hence we are done by Theorem 54. □

Theorem 636. $R(x \cdot T(x\backslash e, w), y, z) = K(w\backslash(w/R(x, y, z)), w)$.

Proof. We have $K(w\backslash(w/(((x \cdot y) \cdot z)/(y \cdot z))), w) = (((x \cdot y) \cdot z) \cdot T(((x \cdot y) \cdot z)\backslash(y \cdot z), w))/(y \cdot z)$ by Theorem 287. Then $K(w\backslash(w/R(x, y, z)), w) = (((x \cdot y) \cdot z) \cdot T(((x \cdot y) \cdot z)\backslash(y \cdot z), w))/(y \cdot z)$ by Definition 5. Then

$$K(w\backslash(w/R(x, y, z)), w) = (((x \cdot y) \cdot T((x \cdot y)\backslash y, w)) \cdot z)/(y \cdot z) \tag{283}$$

by Theorem 291.

$$\begin{aligned} & R(x \cdot T(x\backslash e, w), y, z) \\ = \;& (((x \cdot T(x\backslash e, w)) \cdot y) \cdot z)/(y \cdot z) && \text{by Definition 5} \\ = \;& K(w\backslash(w/R(x, y, z)), w) && \text{by (283), Theorem 194.} \end{aligned}$$

Hence we are done. □

Theorem 637. $R(x\cdot T(x\backslash e, R(x,y,z)), y, z) = K(R(x,y,z)\backslash e, R(x,y,z))$.

Proof. We have $R(x\cdot T(x\backslash e, R(x,y,z)), y, z) = K(R(x,y,z)\backslash(R(x,y,z)/R(x,y,z)), R(x,y,z))$ by Theorem 636. Hence we are done by Proposition 29. □

Theorem 638. $(x\backslash T(x,y))\backslash e = (x\backslash e)\backslash(T(x,y)\backslash e)$.

Proof. We have $T(e/(x\backslash e), y)\cdot(x\backslash e) = (x\backslash e)\cdot T((x\backslash e)\backslash e, y)$ by Proposition 61. Then

$$T(e/(x\backslash e), y)\cdot(x\backslash e) = x\backslash T(x,y) \tag{284}$$

by Theorem 145. We have $(T(e/(x\backslash e), y)\cdot(x\backslash e))\cdot((x\backslash e)\backslash((x\backslash e)/(x\backslash T(x,y)))) = ((x\backslash e)/(x\backslash T(x,y)))\cdot T(((x\backslash e)/(x\backslash T(x,y)))\backslash((x\backslash e)\backslash((x\backslash e)/(x\backslash T(x,y)))), y)$ by Theorem 183. Then

$$(x\backslash T(x,y))\cdot((x\backslash e)\backslash((x\backslash e)/(x\backslash T(x,y)))) = ((x\backslash e)/(x\backslash T(x,y)))\cdot T(((x\backslash e)/(x\backslash T(x,y)))\backslash((x\backslash e)\backslash((x\backslash e)/(x\backslash T(x,y)))), y) \tag{285}$$

by (284). We have $(x\backslash e)\cdot(((x\backslash e)/(x\backslash T(x,y)))\cdot T(((x\backslash e)/(x\backslash T(x,y)))\backslash((x\backslash e)\backslash((x\backslash e)/(x\backslash T(x,y)))), y)) = ((x\backslash e)\cdot T((x\backslash e)\backslash e, y))\cdot((x\backslash e)/(x\backslash T(x,y)))$ by Theorem 235. Then $(x\backslash e)\cdot((x\backslash T(x,y))\cdot((x\backslash e)\backslash((x\backslash e)/(x\backslash T(x,y))))) = ((x\backslash e)\cdot T((x\backslash e)\backslash e, y))\cdot((x\backslash e)/(x\backslash T(x,y)))$ by (285). Then

$$(x\backslash e)\cdot((x\backslash T(x,y))\cdot((x\backslash e)\backslash((x\backslash e)/(x\backslash T(x,y))))) = (x\backslash T(x,y))\cdot((x\backslash e)/(x\backslash T(x,y))) \tag{286}$$

by Theorem 145. We have $(x\backslash e)\cdot K(x\backslash T(x,y), (x\backslash e)/(x\backslash T(x,y))) = (x\backslash T(x,y))\cdot((x\backslash e)/(x\backslash T(x,y)))$ by Theorem 459. Then $(x\backslash T(x,y))\cdot((x\backslash e)\backslash((x\backslash e)/(x\backslash T(x,y)))) = K(x\backslash T(x,y), (x\backslash e)/(x\backslash T(x,y)))$ by (286) and Proposition 7. Then

$$(x\backslash T(x,y))\cdot((x\backslash e)\backslash(T(x,y)\backslash e)) = K(x\backslash T(x,y), (x\backslash e)/(x\backslash T(x,y))) \tag{287}$$

by Theorem 374. We have $K(x\backslash T(x,y), T(x,y)\backslash e) = e$ by Theorem 582. Then $K(x\backslash T(x,y), (x\backslash e)/(x\backslash T(x,y))) = e$ by Theorem 374. Then $(x\backslash T(x,y))\cdot((x\backslash e)\backslash(T(x,y)\backslash e)) = e$ by (287). Hence we are done by Proposition 2. □

Theorem 639. $K(x,y) = (((y\cdot x)\backslash y)\backslash K(x,y))/(L(x\backslash e, x, y)\backslash e)$.

Proof. We have

$$T(L(x\backslash e, x, y)\backslash e, y)/(L(x\backslash e, x, y)\backslash e) = L(x\backslash e, x, y)\cdot T(L(x\backslash e, x, y)\backslash e, y) \tag{288}$$

by Theorem 484.

$$\begin{aligned} & K(x,y) \\ = \quad & L(x\backslash e, x, y)\cdot T(L(x\backslash e, x, y)\backslash e, y) && \text{by Theorem 542} \\ = \quad & (((y\cdot x)\backslash y)\backslash K(x,y))/(L(x\backslash e, x, y)\backslash e) && \text{by (288), Theorem 540.} \end{aligned}$$

Hence we are done. □

Theorem 640. $(K(y/x, x/y)\cdot x)/y = e/(e/(x/y))$.

Proof. We have $R(((y/x)\cdot(e/(y/x)))/(y/x), y/x, x) = (((y/x)\cdot(e/(y/x)))\cdot x)/y$ by Theorem 67. Then $((y/x)\cdot(x/((y/x)\cdot x)))/(y/x) = (((y/x)\cdot(e/(y/x)))\cdot x)/y$ by Theorem 485. Then $((y/x)\cdot(x/y))/(y/x) = (((y/x)\cdot(e/(y/x)))\cdot x)/y$ by Axiom 6. Then $(K(y/x, (y/x)\backslash e)\cdot x)/y = ((y/x)\cdot(x/y))/(y/x)$ by Theorem 250. Then

$$(K(y/x, x/y)\cdot x)/y = ((y/x)\cdot(x/y))/(y/x) \tag{289}$$

by Theorem 402. We have $((y/x)\cdot T(e/(e/(x/y)), y/x))/(y/x) = e/(e/(x/y))$ by Proposition 48. Then $((y/x)\cdot(x/y))/(y/x) = e/(e/(x/y))$ by Theorem 605. Hence we are done by (289). □

Theorem 641. $R(x\cdot R(x\backslash e, w, u), y, z) = R(x,y,z)\cdot R(R(x,y,z)\backslash e, w, u)$.

Proof. We have $(((x \cdot y) \cdot z)/(y \cdot z)) \cdot R((((x \cdot y) \cdot z)/(y \cdot z))\backslash e, w, u) = (((x \cdot y) \cdot z) \cdot R(((x \cdot y) \cdot z)\backslash(y \cdot z), w, u))/(y \cdot z)$ by Theorem 599. Then $R(x, y, z) \cdot R((((x \cdot y) \cdot z)/(y \cdot z))\backslash e, w, u) = (((x \cdot y) \cdot z) \cdot R(((x \cdot y) \cdot z)\backslash(y \cdot z), w, u))/(y \cdot z)$ by Definition 5. Then $(((x \cdot y) \cdot z) \cdot R(((x \cdot y) \cdot z)\backslash(y \cdot z), w, u))/(y \cdot z) = R(x, y, z) \cdot R(R(x, y, z)\backslash e, w, u)$ by Definition 5. Then $(((x \cdot y) \cdot R((x \cdot y)\backslash y, w, u)) \cdot z)/(y \cdot z) = R(x, y, z) \cdot R(R(x, y, z)\backslash e, w, u)$ by Theorem 594. Then

$$(((x \cdot R(x\backslash e, w, u)) \cdot y) \cdot z)/(y \cdot z) = R(x, y, z) \cdot R(R(x, y, z)\backslash e, w, u) \tag{290}$$

by Proposition 84. We have $R(x \cdot R(x\backslash e, w, u), y, z) = (((x \cdot R(x\backslash e, w, u)) \cdot y) \cdot z)/(y \cdot z)$ by Definition 5. Hence we are done by (290). □

Theorem 642. *$R(x, z, w) \cdot ((y \cdot (R(x, z, w)\backslash e))/y) = R(x \cdot ((y \cdot (x\backslash e))/y), z, w)$.*

Proof. We have $(R(x, z, w) \cdot (z \cdot w)) \cdot ((y \cdot ((R(x, z, w) \cdot (z \cdot w))\backslash(z \cdot w)))/y) = (R(x, z, w) \cdot ((y \cdot (R(x, z, w)\backslash e))/y)) \cdot (z \cdot w)$ by Theorem 343. Then $((x \cdot z) \cdot w) \cdot ((y \cdot ((R(x, z, w) \cdot (z \cdot w))\backslash(z \cdot w)))/y) = (R(x, z, w) \cdot ((y \cdot (R(x, z, w)\backslash e))/y)) \cdot (z \cdot w)$ by Proposition 54. Then

$$((x \cdot z) \cdot w) \cdot ((y \cdot (((x \cdot z) \cdot w)\backslash(z \cdot w)))/y) = (R(x, z, w) \cdot ((y \cdot (R(x, z, w)\backslash e))/y)) \cdot (z \cdot w) \tag{291}$$

by Proposition 54. We have $((x \cdot z) \cdot w) \cdot ((y \cdot (((x \cdot z) \cdot w)\backslash(z \cdot w)))/y) = (((y \cdot (z/(x \cdot z)))/y) \cdot (x \cdot z)) \cdot w$ by Theorem 608. Then $((x \cdot z) \cdot w) \cdot ((y \cdot (((x \cdot z) \cdot w)\backslash(z \cdot w)))/y) = ((x \cdot z) \cdot ((y \cdot ((x \cdot z)\backslash z))/y)) \cdot w$ by Theorem 125. Then $(R(x, z, w) \cdot ((y \cdot (R(x, z, w)\backslash e))/y)) \cdot (z \cdot w) = ((x \cdot z) \cdot ((y \cdot ((x \cdot z)\backslash z))/y)) \cdot w$ by (291). Then $(R(x, z, w) \cdot ((y \cdot (R(x, z, w)\backslash e))/y)) \cdot (z \cdot w) = ((x \cdot ((y \cdot (x\backslash e))/y)) \cdot z) \cdot w$ by Theorem 343. Hence we are done by Theorem 64. □

Theorem 643. *$x = L(x, K(x, x\backslash e), y)$.*

Proof. We have $(y \cdot K(x, x\backslash e))\backslash((y \cdot x) \cdot K(x, x\backslash e)) = x$ by Theorem 602. Then

$$(y \cdot K(x, x\backslash e))\backslash(y \cdot T(x, x\backslash e)) = x \tag{292}$$

by Theorem 406. We have $L(x, K(x, x\backslash e), y) = (y \cdot K(x, x\backslash e))\backslash(y \cdot (K(x, x\backslash e) \cdot x))$ by Definition 4. Then $L(x, K(x, x\backslash e), y) = (y \cdot K(x, x\backslash e))\backslash(y \cdot T(x, x\backslash e))$ by Theorem 246. Hence we are done by (292). □

Theorem 644. *$L(e/y, K(y\backslash e, y), x) = e/y$.*

Proof. We have $L(e/y, K(e/y, (e/y)\backslash e), x) = e/y$ by Theorem 643. Hence we are done by Theorem 272. □

Theorem 645. *$x/(e/(e/y)) = (x \cdot K(y, y\backslash e))/y$.*

Proof. We have $R(x/(e/(e/y)), e/(e/y), K(y, y\backslash e)) = (x \cdot K(y, y\backslash e))/((e/(e/y)) \cdot K(y, y\backslash e))$ by Proposition 55. Then $R(x/(e/(e/y)), e/(e/y), K(y, y\backslash e)) = (x \cdot K(y, y\backslash e))/y$ by Theorem 252. Hence we are done by Theorem 414. □

Theorem 646. *$x/(e/(e/y)) = x \cdot (e/(e/(x\backslash(x/y))))$.*

Proof. We have $x \cdot L(e/y, y, x/y) = (x/y) \cdot (y \cdot (e/y))$ by Theorem 53. Then $x \cdot (e/(e/(x\backslash(x/y)))) = (x/y) \cdot (y \cdot (e/y))$ by Theorem 396. Then $(x/y) \cdot K(y, y\backslash e) = x \cdot (e/(e/(x\backslash(x/y))))$ by Theorem 250. Hence we are done by Theorem 417. □

Theorem 647. *$y \cdot (e/(e/(y\backslash x))) = y/(e/(e/(x\backslash y)))$.*

Proof. We have $y \cdot (e/(e/(y\backslash(y/(x\backslash y))))) = y/(e/(e/(x\backslash y)))$ by Theorem 646. Hence we are done by Proposition 24. □

Theorem 648. *$x/L(y, y\backslash e, x) = x \cdot (e/y)$.*

Proof. We have $x/(e/(e/((x\cdot(y\backslash e))\backslash x))) = x\cdot(e/(e/(x\backslash(x\cdot(y\backslash e)))))$ by Theorem 647. Then $x/(e/(e/((x\cdot(y\backslash e))\backslash x))) = x \cdot (e/(e/(y\backslash e)))$ by Axiom 3. Then $x/L(y, y\backslash e, x) = x \cdot (e/(e/(y\backslash e)))$ by Theorem 616. Hence we are done by Proposition 24. □

Theorem 649. $(y \cdot (e/x))\backslash y = L(x, x\backslash e, y)$.

Proof. We have $(y/L(x, x\backslash e, y))\backslash y = L(x, x\backslash e, y)$ by Proposition 25. Hence we are done by Theorem 648. □

Theorem 650. $K(x\backslash(x/L(y, y\backslash e, x)), z) = K(e/y, z)$.

Proof. We have

$$(z \cdot (x\backslash(x \cdot (e/y))))\backslash((z\backslash(z \cdot (x\backslash(x \cdot (e/y))))) \cdot z) = K(z\backslash(z \cdot (x\backslash(x \cdot (e/y)))), z) \tag{293}$$

by Theorem 2. Then $(z \cdot (x\backslash(x \cdot (e/y))))\backslash((z\backslash(z \cdot (x\backslash(x \cdot (e/y))))) \cdot z) = K(x\backslash(x \cdot (e/y)), z)$ by Axiom 3. Then

$$K(x\backslash(x \cdot (e/y)), z) = K(z\backslash(z \cdot (x\backslash(x \cdot (e/y)))), z) \tag{294}$$

by (293). We have $x\backslash(x \cdot (e/y)) = e/y$ by Axiom 3. Then $z\backslash(z \cdot (x\backslash(x \cdot (e/y)))) = e/y$ by Axiom 3. Then $K(e/y, z) = K(x\backslash(x \cdot (e/y)), z)$ by (294). Hence we are done by Theorem 648. □

Theorem 651. $L(x, e/x, y) = L(x, x\backslash e, y)$.

Proof. We have $L(x, e/x, y) = (y \cdot (e/x))\backslash y$ by Theorem 34. Hence we are done by Theorem 649. □

Theorem 652. $L(x\backslash e, x, y) = L(x\backslash e, (x\backslash e)\backslash e, y)$.

Proof. We have $L(x\backslash e, e/(x\backslash e), y) = L(x\backslash e, (x\backslash e)\backslash e, y)$ by Theorem 651. Hence we are done by Proposition 24. □

Theorem 653. $L(x\backslash e, T(x, x\backslash e), y) = L(x\backslash e, x, y)$.

Proof. We have $L(x\backslash e, (x\backslash e)\backslash e, y) = L(x\backslash e, x, y)$ by Theorem 652. Hence we are done by Proposition 49. □

Theorem 654. $L(x \cdot T(x\backslash e, y), x\backslash e, y) = K(e/x, y)$.

Proof. We have $L(x\cdot T(x\backslash e, y), x\backslash e, y) = K(y\backslash(y/L(x, x\backslash e, y)), y)$ by Theorem 635. Hence we are done by Theorem 650. □

Theorem 655. $x \cdot y = (x \cdot ((y\backslash e)\backslash e)) \cdot K(y\backslash e, y)$.

Proof. We have $((x \cdot ((y\backslash e)\backslash e))/((y\backslash e)\backslash e)) \cdot y = (x \cdot ((y\backslash e)\backslash e)) \cdot K(y\backslash e, y)$ by Theorem 424. Hence we are done by Axiom 5. □

Theorem 656. $x/K(y\backslash e, y) = x \cdot K(y, y\backslash e)$.

Proof. We have $((x \cdot K(y, y\backslash e))/K(y, y\backslash e)) \cdot K(y, y\backslash e) = x \cdot K(y, y\backslash e)$ by Axiom 6. Then $((((x \cdot K(y, y\backslash e))/K(y, y\backslash e))/K(y\backslash e, y))\cdot K(y\backslash e, y))\cdot K(y, y\backslash e) = x\cdot K(y, y\backslash e)$ by Axiom 6. Then $((x/K(y\backslash e, y))\cdot K(y\backslash e, y)) \cdot K(y, y\backslash e) = x \cdot K(y, y\backslash e)$ by Axiom 5. Then

$$(x \cdot K(y, y\backslash e))/K(y, y\backslash e) = (x/K(y\backslash e, y)) \cdot K(y\backslash e, y) \tag{295}$$

by Proposition 1. We have $(x\cdot K(y, y\backslash e))/K(y, y\backslash e) = (x\cdot K(y, y\backslash e))\cdot K(y\backslash e, y)$ by Theorem 425. Then $(x/K(y\backslash e, y))\cdot K(y\backslash e, y) = (x\cdot K(y, y\backslash e))\cdot K(y\backslash e, y)$ by (295). Hence we are done by Proposition 10. □

Theorem 657. $(x \cdot K(y, y\backslash e))\backslash x = K(y\backslash e, y)$.

Proof. We have $(x/K(y\backslash e, y))\backslash x = K(y\backslash e, y)$ by Proposition 25. Hence we are done by Theorem 656. □

Theorem 658. $T((K(x\backslash e,x)\cdot y)\cdot K(x,x\backslash e),K(x\backslash e,x))=y$.

Proof. We have $T((K(x\backslash e,x)\cdot y)/K(x\backslash e,x),K(x\backslash e,x))=y$ by Theorem 7. Hence we are done by Theorem 656. ☐

Theorem 659. $T(y,K(x,x\backslash e))\cdot K(x\backslash e,x)=K(x,x\backslash e)\backslash y$.

Proof. We have

$$(T(y,K(x,x\backslash e))\cdot K(x\backslash e,x))\cdot K(x,x\backslash e)=T(y,K(x,x\backslash e)) \tag{296}$$

by Theorem 426. We have $(K(x,x\backslash e)\backslash y)\cdot K(x,x\backslash e)=T(y,K(x,x\backslash e))$ by Theorem 632. Hence we are done by (296) and Proposition 8. ☐

Theorem 660. $T(y,x)=T(y,x\cdot K(y\backslash e,y))$.

Proof. We have $T(y,(x\cdot K(y\backslash e,y))\cdot K(y,y\backslash e))=T(y,x\cdot K(y\backslash e,y))$ by Theorem 626. Hence we are done by Theorem 426. ☐

Theorem 661. $(x\cdot y)\backslash(x\cdot(e/(e/y)))=K(y\backslash e,y)$.

Proof. We have $((x\cdot(e/(e/y)))\cdot K(y,y\backslash e))\backslash(x\cdot(e/(e/y)))=K(y\backslash e,y)$ by Theorem 657. Hence we are done by Theorem 415. ☐

Theorem 662. $(x\cdot(y\backslash e))\cdot((y\backslash e)\backslash e)=R(x,y\backslash e,y)$.

Proof. We have

$$((x\cdot(y\backslash e))\cdot((y\backslash e)\backslash e))\cdot K(y\backslash e,y)=(x\cdot(y\backslash e))\cdot y \tag{297}$$

by Theorem 655. We have $R(x,y\backslash e,y)\cdot K(y\backslash e,y)=(x\cdot(y\backslash e))\cdot y$ by Theorem 469. Hence we are done by (297) and Proposition 8. ☐

Theorem 663. $L(K(y\backslash e,y),y,x)=K(y\backslash e,y)$.

Proof. We have $L(y\backslash(e/(e/y)),y,x)=(x\cdot y)\backslash(x\cdot(e/(e/y)))$ by Proposition 53. Then $L(K(y\backslash e,y),y,x)=(x\cdot y)\backslash(x\cdot(e/(e/y)))$ by Theorem 257. Hence we are done by Theorem 661. ☐

Theorem 664. $K(y\backslash e,y)=K((x\cdot y)\backslash x,x\backslash(x\cdot y))$.

Proof. We have $x\cdot(x\backslash((x\cdot y)\cdot K((x\cdot y)\backslash x,x\backslash(x\cdot y))))=(x\cdot y)\cdot K((x\cdot y)\backslash x,x\backslash(x\cdot y))$ by Axiom 4. Then

$$x\cdot(y\cdot(y\backslash(x\backslash((x\cdot y)\cdot K((x\cdot y)\backslash x,x\backslash(x\cdot y))))))=(x\cdot y)\cdot K((x\cdot y)\backslash x,x\backslash(x\cdot y)) \tag{298}$$

by Axiom 4. We have $(x\cdot y)\cdot L(y\backslash(x\backslash((x\cdot y)\cdot K((x\cdot y)\backslash x,x\backslash(x\cdot y)))),y,x)=x\cdot(y\cdot(y\backslash(x\backslash((x\cdot y)\cdot K((x\cdot y)\backslash x,x\backslash(x\cdot y))))))$ by Proposition 52. Then $L(y\backslash(x\backslash((x\cdot y)\cdot K((x\cdot y)\backslash x,x\backslash(x\cdot y)))),y,x)=K((x\cdot y)\backslash x,x\backslash(x\cdot y))$ by (298) and Proposition 7. Then $L(y\backslash(e/(e/(x\backslash(x\cdot y)))),y,x)=K((x\cdot y)\backslash x,x\backslash(x\cdot y))$ by Theorem 615. Then $L(y\backslash(e/(e/y)),y,x)=K((x\cdot y)\backslash x,x\backslash(x\cdot y))$ by Axiom 3. Then $L(K(y\backslash e,y),y,x)=K((x\cdot y)\backslash x,x\backslash(x\cdot y))$ by Theorem 257. Hence we are done by Theorem 663. ☐

Theorem 665. $K((x\cdot y)\backslash x,y)=K(y\backslash e,y)$.

Proof. We have $K((x\cdot y)\backslash x,x\backslash(x\cdot y))=K(y\backslash e,y)$ by Theorem 664. Hence we are done by Axiom 3. ☐

Theorem 666. $K(y\backslash x,x\backslash y)=K((x\backslash y)\backslash e,x\backslash y)$.

Proof. We have $K((x\cdot(x\backslash y))\backslash x,x\backslash y)=K((x\backslash y)\backslash e,x\backslash y)$ by Theorem 665. Hence we are done by Axiom 4. ☐

Theorem 667. $K(x/(y\cdot x),(x/(y\cdot x))\backslash e)=K(y\backslash e,y)$.

Proof. We have $K(x/(y\cdot x),(x/(y\cdot x))\backslash e)=K(x/(y\cdot x),(y\cdot x)/x)$ by Theorem 402. Then $K(x/(y\cdot x),(x/(y\cdot x))\backslash e)=K(x/(y\cdot x),y)$ by Axiom 5. Hence we are done by Theorem 428. ☐

Theorem 668. $(y \cdot x) \cdot K(y\backslash e, y) = y \cdot (x \cdot K(y\backslash e, y))$.

Proof. We have $(y \cdot x)/L(K(y, y\backslash e), x/K(y, y\backslash e), y) = y \cdot (x/K(y, y\backslash e))$ by Theorem 472. Then $(y \cdot x)/K(y, y\backslash e) = y \cdot (x/K(y, y\backslash e))$ by Theorem 625. Then $(y \cdot x) \cdot K(y\backslash e, y) = y \cdot (x/K(y, y\backslash e))$ by Theorem 425. Hence we are done by Theorem 425. ☐

Theorem 669. $R(y, x, K(y\backslash e, y)) = y$.

Proof. We have $(y \cdot x) \cdot K(y\backslash e, y) = y \cdot (x \cdot K(y\backslash e, y))$ by Theorem 668. Hence we are done by Theorem 64. ☐

Theorem 670. $((y/(x \cdot y))\backslash e)\backslash e = R(x\backslash e, x, y)$.

Proof. We have $K(y/(x \cdot y), (y/(x \cdot y))\backslash e) \cdot (y/(x \cdot y)) = ((y/(x \cdot y))\backslash e)\backslash e$ by Theorem 552. Then

$$K(x\backslash e, x) \cdot (y/(x \cdot y)) = ((y/(x \cdot y))\backslash e)\backslash e \tag{299}$$

by Theorem 667. We have $K(x\backslash e, x) \cdot (y/(x \cdot y)) = (y/(x \cdot y)) \cdot K(x\backslash e, x)$ by Theorem 373. Then $K(x\backslash e, x) \cdot (y/(x \cdot y)) = R(x\backslash e, x, y)$ by Theorem 427. Hence we are done by (299). ☐

Theorem 671. $R(x\backslash e, x, y) = (R(e/x, x, y)\backslash e)\backslash e$.

Proof. We have $R(x\backslash e, x, y) = ((y/(x \cdot y))\backslash e)\backslash e$ by Theorem 670. Hence we are done by Proposition 79. ☐

Theorem 672. $K(e/(e/y), x) = (y \cdot K(y, x))/y$.

Proof. We have $L((e/y) \cdot T((e/y)\backslash e, x), (e/y)\backslash e, x) = K(e/(e/y), x)$ by Theorem 654. Then $L((e/y) \cdot T((e/y)\backslash e, x), y, x) = K(e/(e/y), x)$ by Proposition 25. Then

$$L(T(y, x)/y, y, x) = K(e/(e/y), x) \tag{300}$$

by Theorem 149. We have $L((x\backslash(y \cdot x))/y, y, x) = (y \cdot ((x \cdot y)\backslash(y \cdot x)))/y$ by Proposition 75. Then $L((x\backslash(y \cdot x))/y, y, x) = (y \cdot K(y, x))/y$ by Definition 2. Then $L(T(y, x)/y, y, x) = (y \cdot K(y, x))/y$ by Definition 3. Hence we are done by (300). ☐

Theorem 673. $T(K(e/(e/x), y), x) = K(x, y)$.

Proof. We have $T((x \cdot K(x, y))/x, x) = K(x, y)$ by Theorem 7. Hence we are done by Theorem 672. ☐

Theorem 674. $K((y\backslash e)\backslash e, x) = T(K(y, x), y)$.

Proof. We have $T(K(e/(e/T(y, y\backslash e)), x), T(y, y\backslash e)) = K(T(y, y\backslash e), x)$ by Theorem 673. Then $T(K(e/(y\backslash e), x), T(y, y\backslash e)) = K(T(y, y\backslash e), x)$ by Theorem 15. Then

$$T(K(y, x), T(y, y\backslash e)) = K(T(y, y\backslash e), x) \tag{301}$$

by Proposition 24. We have $T(K(y, x), T(y, y\backslash e)) = T(K(y, x), y)$ by Theorem 585. Then $K(T(y, y\backslash e), x) = T(K(y, x), y)$ by (301). Hence we are done by Proposition 49. ☐

Theorem 675. $(((y \cdot x)\backslash y)\backslash e)\backslash(((y \cdot x)\backslash y)\backslash K(x, y)) = K((x\backslash e)\backslash e, y)$.

Proof. We have $L(e/T(x, x\backslash e), T(x, x\backslash e), y)\backslash e = e/((y \cdot T(x, x\backslash e))\backslash y)$ by Theorem 617. Then

$$L(e/T(x, x\backslash e), T(x, x\backslash e), y)\backslash e = e/L(T(x, x\backslash e)\backslash e, T(x, x\backslash e), y) \tag{302}$$

by Proposition 78. We have $(e/L(T(x, x\backslash e)\backslash e, T(x, x\backslash e), y))\backslash e = L(T(x, x\backslash e)\backslash e, T(x, x\backslash e), y)$ by Proposition 25. Then $(L(e/T(x, x\backslash e), T(x, x\backslash e), y)\backslash e)\backslash e = L(T(x, x\backslash e)\backslash e, T(x, x\backslash e), y)$ by (302). Then $(L(x\backslash e, T(x, x\backslash e), y)\backslash e)\backslash e = L(T(x, x\backslash e)\backslash e, T(x, x\backslash e), y)$ by Theorem 15. Then $L(T(x\backslash e, x), T(x, x\backslash e), y) = (L(x\backslash e, T(x, x\backslash e), y)\backslash e)\backslash e$ by Theorem 208. Then

$$L(T(x\backslash e, x), T(x, x\backslash e), y) = (L(x\backslash e, x, y)\backslash e)\backslash e \tag{303}$$

by Theorem 653. We have $y/L(T(x,x\backslash e)\backslash e, T(x,x\backslash e), y) = y \cdot T(x,x\backslash e)$ by Theorem 33. Then $y/L(T(x\backslash e,x), T(x,x\backslash e), y) = y \cdot T(x,x\backslash e)$ by Theorem 208. Then

$$y/((L(x\backslash e,x,y)\backslash e)\backslash e) = y \cdot T(x,x\backslash e) \tag{304}$$

by (303). We have $T(L(x\backslash e,x,y) \cdot T(L(x\backslash e,x,y)\backslash e, y), L(x\backslash e,x,y)\backslash e) = K(y\backslash(y/T(L(x\backslash e,x,y), L(x\backslash e,x,y)\backslash e)), y)$ by Theorem 296. Then $T(K(x,y), L(x\backslash e,x,y)\backslash e) = K(y\backslash(y/T(L(x\backslash e,x,y), L(x\backslash e,x,y)\backslash e)), y)$ by Theorem 542. Then $K(y\backslash(y/((L(x\backslash e,x,y)\backslash e)\backslash e)), y) = T(K(x,y), L(x\backslash e,x,y)\backslash e)$ by Proposition 49. Then $T(K(x,y), L(x\backslash e,x,y)\backslash e) = K(y\backslash(y \cdot T(x,x\backslash e)), y)$ by (304). Then $T(K(x,y), L(x\backslash e,x,y)\backslash e) = K(T(x,x\backslash e), y)$ by Axiom 3. Then $T(K(x,y), L(x\backslash e,x,y)\backslash e) = K((x\backslash e)\backslash e, y)$ by Proposition 49. Then

$$T(K(x,y), ((y \cdot x)\backslash y)\backslash e) = K((x\backslash e)\backslash e, y) \tag{305}$$

by Proposition 78. We have $(((y \cdot x)\backslash y)\backslash K(x,y))/(L(x\backslash e,x,y)\backslash e) = K(x,y)$ by Theorem 639. Then

$$(((y \cdot x)\backslash y)\backslash K(x,y))/(((y \cdot x)\backslash y)\backslash e) = K(x,y) \tag{306}$$

by Proposition 78. We have $T((((y \cdot x)\backslash y)\backslash K(x,y))/(((y \cdot x)\backslash y)\backslash e), ((y \cdot x)\backslash y)\backslash e) = (((y \cdot x)\backslash y)\backslash e)\backslash(((y \cdot x)\backslash y)\backslash K(x,y))$ by Proposition 47. Then $T(K(x,y), ((y \cdot x)\backslash y)\backslash e) = (((y \cdot x)\backslash y)\backslash e)\backslash(((y \cdot x)\backslash y)\backslash K(x,y))$ by (306). Hence we are done by (305). □

Theorem 676. $T(x,x\backslash e) \cdot y = K(x,x\backslash e) \cdot (x \cdot y)$.

Proof. We have $T(x,x\backslash e) \cdot (x\backslash(x \cdot y)) = K(x,x\backslash e) \cdot (x \cdot y)$ by Theorem 431. Hence we are done by Axiom 3. □

Theorem 677. $((x\backslash e)\backslash e) \cdot (x\backslash y) = K(x,x\backslash e) \cdot y$.

Proof. We have $T(x,x\backslash e) \cdot (x\backslash y) = K(x,x\backslash e) \cdot y$ by Theorem 431. Hence we are done by Proposition 49. □

Theorem 678. $L(y, e/x, K(e/x, x)) = y$.

Proof. We have $L(y, e/x, K(e/x, (e/x)\backslash e)) = y$ by Theorem 432. Hence we are done by Proposition 25. □

Theorem 679. $((x\backslash e)\backslash e) \cdot y = K(x,x\backslash e) \cdot (x \cdot y)$.

Proof. We have $(K(x,x\backslash e) \cdot x) \cdot L(y, x, K(x,x\backslash e)) = K(x,x\backslash e) \cdot (x \cdot y)$ by Proposition 52. Then $((x\backslash e)\backslash e) \cdot L(y, x, K(x,x\backslash e)) = K(x,x\backslash e) \cdot (x \cdot y)$ by Theorem 552. Hence we are done by Theorem 432. □

Theorem 680. $x\backslash y = K(x,x\backslash e) \cdot (((x\backslash e)\backslash e)\backslash y)$.

Proof. We have $T(x,x\backslash e)\backslash(K(T(x,x\backslash e), T(x,x\backslash e)\backslash e) \cdot y) = K(T(x,x\backslash e), T(x,x\backslash e)\backslash e) \cdot (T(x,x\backslash e)\backslash y)$ by Theorem 586. Then

$$T(x,x\backslash e)\backslash(K(x,x\backslash e) \cdot y) = K(T(x,x\backslash e), T(x,x\backslash e)\backslash e) \cdot (T(x,x\backslash e)\backslash y) \tag{307}$$

by Theorem 592. We have $T(x,x\backslash e)\backslash(K(x,x\backslash e) \cdot y) = L(x\backslash y, x, K(x,x\backslash e))$ by Theorem 554. Then $K(T(x,x\backslash e), T(x,x\backslash e)\backslash e) \cdot (T(x,x\backslash e)\backslash y) = L(x\backslash y, x, K(x,x\backslash e))$ by (307). Then $K(x,x\backslash e) \cdot (T(x,x\backslash e)\backslash y) = L(x\backslash y, x, K(x,x\backslash e))$ by Theorem 592. Then $K(x,x\backslash e) \cdot (((x\backslash e)\backslash e)\backslash y) = L(x\backslash y, x, K(x,x\backslash e))$ by Proposition 49. Hence we are done by Theorem 432. □

Theorem 681. $L(y, e/(e/x), K(x,x\backslash e)) = y$.

Proof. We have $L(y, e/(e/x), K(e/(e/x), e/x)) = y$ by Theorem 678. Hence we are done by Theorem 566. □

Theorem 682. $(x\backslash e) \cdot y = K(x\backslash e, x) \cdot ((e/x) \cdot y)$.

Proof. We have $(x\backslash e) \cdot L(y, e/x, K(x\backslash e, x)) = K(x\backslash e, x) \cdot ((e/x) \cdot y)$ by Theorem 259. Hence we are done by Theorem 433. □

Theorem 683. $L(x, y, y\backslash e) = L(x, y, e/y)$.

Proof. We have $(e/y) \cdot (y \cdot x) = L(x, y, e/y)$ by Proposition 67. Then

$$L(L(x, y, y\backslash e), K(y\backslash e, y), e/y) = L(x, y, e/y) \tag{308}$$

by Theorem 618. We have $L(L(x, y, y\backslash e), e/y, K(e/y, (e/y)\backslash e)) = L(L(x, y, y\backslash e), K(y\backslash e, y), e/y)$ by Theorem 588. Then $L(L(x, y, y\backslash e), e/y, K(y\backslash e, y)) = L(L(x, y, y\backslash e), K(y\backslash e, y), e/y)$ by Theorem 272. Then $L(x, y, e/y) = L(L(x, y, y\backslash e), e/y, K(y\backslash e, y))$ by (308). Hence we are done by Theorem 433. □

Theorem 684. $x \cdot y = (e/(e/x)) \cdot (K(x, x\backslash e) \cdot y)$.

Proof. We have $L(y, K(e/(e/x), (e/(e/x))\backslash e), e/(e/x)) = L(y, e/(e/x), K(e/(e/x), (e/(e/x))\backslash e))$ by Theorem 587. Then $L(y, K(e/(e/x), e/x), e/(e/x)) = L(y, e/(e/x), K(e/(e/x), (e/(e/x))\backslash e))$ by Proposition 25. Then $L(y, e/(e/x), K(e/(e/x), (e/(e/x))\backslash e)) = L(y, K(x, x\backslash e), e/(e/x))$ by Theorem 566. Then

$$y = L(y, K(x, x\backslash e), e/(e/x)) \tag{309}$$

by Theorem 432. We have $((e/(e/x)) \cdot K(x, x\backslash e)) \cdot L(y, K(x, x\backslash e), e/(e/x)) = (e/(e/x)) \cdot (K(x, x\backslash e) \cdot y)$ by Proposition 52. Then $x \cdot L(y, K(x, x\backslash e), e/(e/x)) = (e/(e/x)) \cdot (K(x, x\backslash e) \cdot y)$ by Theorem 252. Hence we are done by (309). □

Theorem 685. $x \cdot y = K(x, x\backslash e) \cdot ((e/(e/x)) \cdot y)$.

Proof. We have $(K(x, x\backslash e) \cdot (e/(e/x))) \cdot L(y, e/(e/x), K(x, x\backslash e)) = K(x, x\backslash e) \cdot ((e/(e/x)) \cdot y)$ by Proposition 52. Then $x \cdot L(y, e/(e/x), K(x, x\backslash e)) = K(x, x\backslash e) \cdot ((e/(e/x)) \cdot y)$ by Theorem 557. Hence we are done by Theorem 681. □

Theorem 686. $(e/(e/x))\backslash y = x\backslash(K(x, x\backslash e) \cdot y)$.

Proof. We have $L((e/(e/x))\backslash y, e/(e/x), K(x, x\backslash e)) = (K(x, x\backslash e) \cdot (e/(e/x)))\backslash(K(x, x\backslash e) \cdot y)$ by Proposition 53. Then $L((e/(e/x))\backslash y, e/(e/x), K(x, x\backslash e)) = x\backslash(K(x, x\backslash e) \cdot y)$ by Theorem 557. Hence we are done by Theorem 681. □

Theorem 687. $K(x, x\backslash e) = K((y/(x \cdot y))\backslash e, y/(x \cdot y))$.

Proof. We have $y \cdot T(y\backslash(x \cdot y), y/(x \cdot y)) = T(x, y/(x \cdot y)) \cdot y$ by Theorem 466. Then $K((y/(x \cdot y))\backslash e, y/(x \cdot y)) \cdot (x \cdot y) = T(x, y/(x \cdot y)) \cdot y$ by Theorem 202. Then

$$K((y/(x \cdot y))\backslash e, y/(x \cdot y)) \cdot (x \cdot y) = T(x, x\backslash e) \cdot y \tag{310}$$

by Theorem 362. We have $K(x, x\backslash e) \cdot (x \cdot y) = T(x, x\backslash e) \cdot y$ by Theorem 676. Hence we are done by (310) and Proposition 8. □

Theorem 688. $K(y, y\backslash e) \cdot (T(y, x)\backslash e) = y \cdot (T(y, x)\backslash(e/y))$.

Proof. We have $((y\backslash e)\backslash e)\backslash(((y\backslash e)\backslash e) \cdot (T(y, x)\backslash e)) = T(y, x)\backslash e$ by Axiom 3. Then

$$((y\backslash e)\backslash e)\backslash(y \cdot (y\backslash(((y\backslash e)\backslash e) \cdot (T(y, x)\backslash e)))) = T(y, x)\backslash e \tag{311}$$

by Axiom 4. We have $((y\backslash e)\backslash e) \cdot (((y\backslash e)\backslash e)\backslash(y \cdot (y\backslash(((y\backslash e)\backslash e) \cdot (T(y, x)\backslash e))))) = y \cdot (y\backslash(((y\backslash e)\backslash e) \cdot (T(y, x)\backslash e)))$ by Axiom 4. Then $((y\backslash e)\backslash e) \cdot (y \cdot (y\backslash(((y\backslash e)\backslash e)\backslash(y \cdot (y\backslash(((y\backslash e)\backslash e) \cdot (T(y, x)\backslash e))))))) = y \cdot (y\backslash(((y\backslash e)\backslash e) \cdot (T(y, x)\backslash e)))$ by Axiom 4. Then

$$((y\backslash e)\backslash e) \cdot (y \cdot (y\backslash(T(y, x)\backslash e))) = y \cdot (y\backslash(((y\backslash e)\backslash e) \cdot (T(y, x)\backslash e))) \tag{312}$$

by (311). We have $((y\backslash e)\backslash e) \cdot (y \cdot (y\backslash(T(y,x)\backslash e))) = y \cdot (((y\backslash e)\backslash e) \cdot (y\backslash(T(y,x)\backslash e)))$ by Theorem 574. Then $y \cdot (y\backslash(((y\backslash e)\backslash e) \cdot (T(y,x)\backslash e))) = y \cdot (((y\backslash e)\backslash e) \cdot (y\backslash(T(y,x)\backslash e)))$ by (312). Then $y\backslash(((y\backslash e)\backslash e) \cdot (T(y,x)\backslash e)) = ((y\backslash e)\backslash e) \cdot (y\backslash(T(y,x)\backslash e))$ by Proposition 9. Then

$$y\backslash(T(y,x)\backslash((y\backslash e)\backslash e)) = ((y\backslash e)\backslash e) \cdot (y\backslash(T(y,x)\backslash e)) \tag{313}$$

by Theorem 593. We have $T(y,x)\backslash(y\backslash((y\backslash e)\backslash e)) = y\backslash(T(y,x)\backslash((y\backslash e)\backslash e))$ by Theorem 526. Then

$$T(y,x)\backslash K(y,e/y) = y\backslash(T(y,x)\backslash((y\backslash e)\backslash e)) \tag{314}$$

by Theorem 559. We have $T(y,x)\backslash K(y,e/y) = y \cdot (T(y,x)\backslash(e/y))$ by Theorem 523. Then $y\backslash(T(y,x)\backslash((y\backslash e)\backslash e)) = y \cdot (T(y,x)\backslash(e/y))$ by (314). Then $((y\backslash e)\backslash e) \cdot (y\backslash(T(y,x)\backslash e)) = y \cdot (T(y,x)\backslash(e/y))$ by (313). Hence we are done by Theorem 677. □

Theorem 689. $K(x,x\backslash e)\backslash(x \cdot y) = (e/(e/x)) \cdot y$.

Proof. We have $K(x,x\backslash e) \cdot ((e/(e/x)) \cdot y) = x \cdot y$ by Theorem 685. Hence we are done by Proposition 2. □

Theorem 690. $(e/(e/x))\backslash y = K(x,x\backslash e) \cdot (x\backslash y)$.

Proof. We have $x\backslash(K(x,x\backslash e) \cdot y) = K(x,x\backslash e) \cdot (x\backslash y)$ by Theorem 586. Hence we are done by Theorem 686. □

Theorem 691. $(e/(e/y)) \cdot (x \cdot K(y,y\backslash e)) = T(y \cdot x, K(y,y\backslash e))$.

Proof. We have $T(y \cdot x, K(y,y\backslash e)) = K(y,y\backslash e)\backslash((y \cdot x) \cdot K(y,y\backslash e))$ by Definition 3. Then $T(y \cdot x, K(y,y\backslash e)) = K(y,y\backslash e)\backslash(y \cdot (x \cdot K(y,y\backslash e)))$ by Theorem 623. Hence we are done by Theorem 689. □

Theorem 692. $(e/x)\backslash(K(x,x\backslash e) \cdot y) = (x\backslash e)\backslash y$.

Proof. We have

$$K(x,x\backslash e) \cdot ((e/x)\backslash(K(x,x\backslash e) \cdot y)) = (x\backslash e)\backslash(K(x,x\backslash e) \cdot y) \tag{315}$$

by Theorem 440. We have $K(x,x\backslash e) \cdot ((x\backslash e)\backslash y) = (x\backslash e)\backslash(K(x,x\backslash e) \cdot y)$ by Theorem 600. Hence we are done by (315) and Proposition 7. □

Theorem 693. $y = K(x\backslash e,x) \cdot (K(x,x\backslash e) \cdot y)$.

Proof. We have $(e/x)\backslash((x\backslash e) \cdot (K(x,x\backslash e) \cdot y)) = K(x\backslash e,x) \cdot (K(x,x\backslash e) \cdot y)$ by Theorem 438. Hence we are done by Theorem 441. □

Theorem 694. $K(x,x\backslash e)\backslash y = K(x\backslash e,x) \cdot y$.

Proof. We have $K(x,x\backslash e) \cdot (K(x\backslash e,x) \cdot y) = y$ by Theorem 442. Hence we are done by Proposition 2. □

Theorem 695. $K(x\backslash e,x) \cdot ((e/(e/x))\backslash y) = x\backslash y$.

Proof. We have

$$K(x,x\backslash e) \cdot (K(x\backslash e,x) \cdot ((e/(e/x))\backslash y)) = (e/(e/x))\backslash y \tag{316}$$

by Theorem 442. We have $K(x,x\backslash e) \cdot (x\backslash y) = (e/(e/x))\backslash y$ by Theorem 690. Hence we are done by (316) and Proposition 7. □

Theorem 696. $K(x,x\backslash e) \cdot y = K(x\backslash e,x)\backslash y$.

Proof. We have $K(x,x\backslash e) \cdot (K(x\backslash e,x) \cdot (K(x\backslash e,x)\backslash y)) = K(x\backslash e,x)\backslash y$ by Theorem 442. Hence we are done by Axiom 4. □

Theorem 697. $K(x,x\backslash e) \cdot ((x\backslash e) \cdot y) = (e/x) \cdot y$.

Proof. We have $K(x, x\backslash e) \cdot (K(x\backslash e, x) \cdot ((e/x) \cdot y)) = (e/x) \cdot y$ by Theorem 442. Hence we are done by Theorem 682. □

Theorem 698. $T(x, y) = T(x, K(x, x\backslash e) \cdot y)$.

Proof. We have $T(x, K(x\backslash e, x) \cdot (K(x, x\backslash e) \cdot y)) = T(x, K(x, x\backslash e) \cdot y)$ by Theorem 630. Hence we are done by Theorem 693. □

Theorem 699. $T(y, K(x, x\backslash e)) \cdot K(x\backslash e, x) = K(x\backslash e, x) \cdot y$.

Proof. We have $T(y, K(x, x\backslash e)) \cdot K(x\backslash e, x) = K(x, x\backslash e)\backslash y$ by Theorem 659. Hence we are done by Theorem 694. □

Theorem 700. $T((e/y) \cdot x, K(y, y\backslash e)) = ((y\backslash e) \cdot x) \cdot K(y, y\backslash e)$.

Proof. We have $T(K(y, y\backslash e) \cdot ((y\backslash e) \cdot x), K(y, y\backslash e)) = ((y\backslash e) \cdot x) \cdot K(y, y\backslash e)$ by Theorem 633. Hence we are done by Theorem 697. □

Theorem 701. $T(T(y, K(x, x\backslash e)), K(x\backslash e, x)) = y$.

Proof. We have $K(x\backslash e, x) \cdot y = T(y, K(x, x\backslash e)) \cdot K(x\backslash e, x)$ by Theorem 699. Hence we are done by Theorem 11. □

Theorem 702. $R(x\backslash e, x, y)\backslash y = ((x\backslash e)\backslash e) \cdot y$.

Proof. We have

$$K(y/(x \cdot y), (y/(x \cdot y))\backslash e) \cdot ((((y/(x \cdot y))\backslash e)\backslash e)\backslash y) = (y/(x \cdot y))\backslash y \tag{317}$$

by Theorem 680. We have $K(y/(x\cdot y), (y/(x\cdot y))\backslash e)\cdot(K(y/(x\cdot y), (y/(x\cdot y))\backslash e)\backslash(x\cdot y)) = (y/(x\cdot y))\backslash y$ by Theorem 19. Then $(((y/(x\cdot y))\backslash e)\backslash e)\backslash y = K(y/(x\cdot y), (y/(x\cdot y))\backslash e)\backslash(x\cdot y)$ by (317) and Proposition 7. Then $K(x\backslash e, x)\backslash(x \cdot y) = (((y/(x \cdot y))\backslash e)\backslash e)\backslash y$ by Theorem 667. Then

$$K(x\backslash e, x)\backslash(x \cdot y) = R(x\backslash e, x, y)\backslash y \tag{318}$$

by Theorem 670. We have $K(x, x\backslash e) \cdot (x \cdot y) = ((x\backslash e)\backslash e) \cdot y$ by Theorem 679. Then $K(x\backslash e, x)\backslash(x \cdot y) = ((x\backslash e)\backslash e) \cdot y$ by Theorem 696. Hence we are done by (318). □

Theorem 703. $y/(((x\backslash e)\backslash e) \cdot y) = R(x\backslash e, x, y)$.

Proof. We have $y/(R(x\backslash e, x, y)\backslash y) = R(x\backslash e, x, y)$ by Proposition 24. Hence we are done by Theorem 702. □

Theorem 704. $R(x\backslash e, (x\backslash e)\backslash e, y) = R(x\backslash e, x, y)$.

Proof. We have $R(x\backslash e, (x\backslash e)\backslash e, y) = y/(((x\backslash e)\backslash e) \cdot y)$ by Proposition 66. Hence we are done by Theorem 703. □

Theorem 705. $y \cdot T(x, (y\backslash e)\backslash e) = T(x \cdot y, K(y, y\backslash e))$.

Proof. We have $(((y\backslash e)\backslash e)/y)\backslash(L(x, y, ((y\backslash e)\backslash e)/y) \cdot ((y\backslash e)\backslash e)) = y \cdot T(x, (y\backslash e)\backslash e)$ by Theorem 491. Then $K(y, y\backslash e)\backslash(L(x, y, ((y\backslash e)\backslash e)/y)\cdot((y\backslash e)\backslash e)) = y\cdot T(x, (y\backslash e)\backslash e)$ by Theorem 247. Then $K(y, y\backslash e)\backslash(L(x, y, K(y, y\backslash e))\cdot ((y\backslash e)\backslash e)) = y \cdot T(x, (y\backslash e)\backslash e)$ by Theorem 247. Then

$$K(y, y\backslash e)\backslash(x \cdot ((y\backslash e)\backslash e)) = y \cdot T(x, (y\backslash e)\backslash e) \tag{319}$$

by Theorem 432. We have $T(x \cdot y, K(y, y\backslash e)) = K(y, y\backslash e)\backslash((x \cdot y) \cdot K(y, y\backslash e))$ by Definition 3. Then $T(x \cdot y, K(y, y\backslash e)) = K(y, y\backslash e)\backslash(x \cdot ((y\backslash e)\backslash e))$ by Theorem 407. Hence we are done by (319). □

Theorem 706. $T(x, (y\backslash e)\backslash e) = T(T(x, y), K(y, y\backslash e))$.

Proof. We have $(e/(e/y))\cdot(K(y,y\backslash e)\cdot T(T(x,y),K(y,y\backslash e))) = y\cdot T(T(x,y),K(y,y\backslash e))$ by Theorem 684. Then $(e/(e/y))\cdot(T(x,y)\cdot K(y,y\backslash e)) = y\cdot T(T(x,y),K(y,y\backslash e))$ by Proposition 46. Then

$$T(y\cdot T(x,y),K(y,y\backslash e)) = y\cdot T(T(x,y),K(y,y\backslash e)) \tag{320}$$

by Theorem 691. We have $T(((y\cdot T(x,y))/y)\cdot y,K(y,y\backslash e)) = y\cdot T((y\cdot T(x,y))/y,(y\backslash e)\backslash e)$ by Theorem 705. Then $T(y\cdot T(x,y),K(y,y\backslash e)) = y\cdot T((y\cdot T(x,y))/y,(y\backslash e)\backslash e)$ by Axiom 6. Then $y\cdot T((y\cdot T(x,y))/y,(y\backslash e)\backslash e) = y\cdot T(T(x,y),K(y,y\backslash e))$ by (320). Then $T((y\cdot T(x,y))/y,(y\backslash e)\backslash e) = T(T(x,y),K(y,y\backslash e))$ by Proposition 9. Hence we are done by Proposition 48. □

Theorem 707. $T(T(x,y),K(y,y\backslash e)) = T(x,T(y,y\backslash e))$.

Proof. We have $T(T(x,y),K(y,y\backslash e)) = T(x,(y\backslash e)\backslash e)$ by Theorem 706. Hence we are done by Proposition 49. □

Theorem 708. $T(T(x,(y\backslash e)\backslash e),K(y\backslash e,y)) = T(x,y)$.

Proof. We have $T(T(T(x,y),K(y,y\backslash e)),K(y\backslash e,y)) = T(x,y)$ by Theorem 701. Hence we are done by Theorem 706. □

Theorem 709. $(x\backslash e)\backslash((x\backslash y)/y) = x\cdot((x\backslash y)/y)$.

Proof. We have $K(x,x\backslash e)\cdot((e/x)\backslash((x\backslash y)/y)) = ((x\backslash y)/y)\cdot T(((x\backslash y)/y)\backslash((e/x)\backslash((x\backslash y)/y)),e/x)$ by Theorem 563. Then $K(x,x\backslash e)\cdot(((x\backslash y)/y)\cdot x) = ((x\backslash y)/y)\cdot T(((x\backslash y)/y)\backslash((e/x)\backslash((x\backslash y)/y)),e/x)$ by Theorem 543. Then $((x\backslash y)/y)\cdot T(((x\backslash y)/y)\backslash(((x\backslash y)/y)\cdot x),e/x) = K(x,x\backslash e)\cdot(((x\backslash y)/y)\cdot x)$ by Theorem 543. Then $((x\backslash y)/y)\cdot T(x,e/x) = K(x,x\backslash e)\cdot(((x\backslash y)/y)\cdot x)$ by Axiom 3. Then

$$K(x,x\backslash e)\cdot(((x\backslash y)/y)\cdot x) = ((x\backslash y)/y)\cdot T(x,x\backslash e) \tag{321}$$

by Theorem 211. We have $((x\backslash y)/y)\cdot T(x,x\backslash e) = x\cdot((x\backslash y)/y)$ by Theorem 606. Then

$$K(x,x\backslash e)\cdot(((x\backslash y)/y)\cdot x) = x\cdot((x\backslash y)/y) \tag{322}$$

by (321). We have $K(x,x\backslash e)\cdot((e/x)\backslash((x\backslash y)/y)) = (x\backslash e)\backslash((x\backslash y)/y)$ by Theorem 440. Then $K(x,x\backslash e)\cdot(((x\backslash y)/y)\cdot x) = (x\backslash e)\backslash((x\backslash y)/y)$ by Theorem 543. Hence we are done by (322). □

Theorem 710. $T(e/(e/(y/(x\cdot y))),x\cdot y) = (x\cdot y)\backslash(K(x,x\backslash e)\cdot y)$.

Proof. We have $K((y/(x\cdot y))\backslash e,y/(x\cdot y)) = K((x\cdot y)/y,y/(x\cdot y))$ by Theorem 429. Then

$$K(x,x\backslash e) = K((x\cdot y)/y,y/(x\cdot y)) \tag{323}$$

by Theorem 687. We have $(K((x\cdot y)/y,y/(x\cdot y))\cdot y)/(x\cdot y) = e/(e/(y/(x\cdot y)))$ by Theorem 640. Then $(K((x\cdot y)/y,y/(x\cdot y))\cdot y)/(x\cdot y) = e/(e/R(e/x,x,y))$ by Proposition 79. Then

$$(K(x,x\backslash e)\cdot y)/(x\cdot y) = e/(e/R(e/x,x,y)) \tag{324}$$

by (323). We have $R(e/(e/(e/x)),x,y) = (((e/(e/(e/x)))\cdot x)\cdot y)/(x\cdot y)$ by Definition 5. Then $R(e/(e/(e/x)),x,y) = (K(x,x\backslash e)\cdot y)/(x\cdot y)$ by Theorem 555. Then

$$e/(e/R(e/x,x,y)) = R(e/(e/(e/x)),x,y) \tag{325}$$

by (324). We have

$$(((x\cdot y)\backslash y)/(y/(x\cdot y)))/(e/(y/(x\cdot y))) = T(e/(e/(y/(x\cdot y))),x\cdot y) \tag{326}$$

by Theorem 502. Then $(((x\cdot y)\backslash y)/(y/(x\cdot y)))/(e/(y/(x\cdot y))) = T(e/(e/R(e/x,x,y)),x\cdot y)$ by Proposition 79. Then $T(e/(e/R(e/x,x,y)),x\cdot y) = T(e/(e/(y/(x\cdot y))),x\cdot y)$ by (326). Then

$$T(R(e/(e/(e/x)),x,y),x\cdot y) = T(e/(e/(y/(x\cdot y))),x\cdot y) \tag{327}$$

by (325). We have $R(T(e/(e/(e/x)), x\cdot y), x, y) = T(R(e/(e/(e/x)), x, y), x\cdot y)$ by Axiom 9. Then

$$R(T(e/(e/(e/x)), x\cdot y), x, y) = T(e/(e/(y/(x\cdot y))), x\cdot y) \tag{328}$$

by (327). We have $R(T((e\cdot K(x, x\backslash e))/x, x\cdot y), x, y) = (x\cdot y)\backslash((e\cdot K(x, x\backslash e))\cdot y)$ by Theorem 488. Then $R(T(e/(e/(e/x)), x\cdot y), x, y) = (x\cdot y)\backslash((e\cdot K(x, x\backslash e))\cdot y)$ by Theorem 645. Then $T(e/(e/(y/(x\cdot y))), x\cdot y) = (x\cdot y)\backslash((e\cdot K(x, x\backslash e))\cdot y)$ by (328). Hence we are done by Axiom 1. □

Theorem 711. $K(T(x, y), x\backslash e) = K(T(x, y), T(x, y)\backslash e)$.

Proof. We have $T(x, y)\cdot((T(x, y)\cdot(T(x, y)\backslash e))/T(x, y)) = T(x\cdot((T(x, y)\cdot(x\backslash e))/T(x, y)), y)$ by Theorem 345. Then

$$T(x, y)\cdot(e/T(x, y)) = T(x\cdot((T(x, y)\cdot(x\backslash e))/T(x, y)), y) \tag{329}$$

by Axiom 4. We have $K(T(x, y), T(x, y)\backslash e) = T(x, y)\cdot(e/T(x, y))$ by Theorem 250. Then $K(T(x, y), T(x, y)\backslash e) = T(x\cdot((T(x, y)\cdot(x\backslash e))/T(x, y)), y)$ by (329). Then $T(x\cdot((x\backslash T(x, y))/T(x, y)), y) = K(T(x, y), T(x, y)\backslash e)$ by Theorem 146. Then $T(a(T(x, y), e/x, x), y) = K(T(x, y), T(x, y)\backslash e)$ by Theorem 399. Hence we are done by Theorem 445. □

Theorem 712. $T(e/(e/x), y)\backslash T(x, y) = K(T(x, y), T(x, y)\backslash e)$.

Proof. We have $T(e/(e/x), y)\backslash T(x, y) = K(T(x, y), x\backslash e)$ by Theorem 444. Hence we are done by Theorem 711. □

Theorem 713. $e/(e/(y\backslash x)) = T(e/(e/(x/y)), y)$.

Proof. We have $e/(e/T(x/y, y)) = T(e/(e/(x/y)), y)$ by Theorem 447. Hence we are done by Proposition 47.

□

Theorem 714. $(x\cdot y)\cdot(e/(e/((x\cdot y)\backslash y))) = K(x, x\backslash e)\cdot y$.

Proof. We have $T(e/(e/(y/(x\cdot y))), x\cdot y) = (x\cdot y)\backslash(K(x, x\backslash e)\cdot y)$ by Theorem 710. Then

$$e/(e/((x\cdot y)\backslash y)) = (x\cdot y)\backslash(K(x, x\backslash e)\cdot y) \tag{330}$$

by Theorem 713. We have $(x\cdot y)\cdot((x\cdot y)\backslash(K(x, x\backslash e)\cdot y)) = K(x, x\backslash e)\cdot y$ by Axiom 4. Hence we are done by (330). □

Theorem 715. $K(x\backslash e, x)\cdot(e/T(x, y)) = x\backslash(x/T(x, y))$.

Proof. We have $((y\cdot(e/(e/T(x, y))))/y)\backslash((e/T(x, y))\cdot T((y\cdot(e/(e/T(x, y))))/y, e/T(x, y))) = e/T(x, y)$ by Theorem 454. Then $((y\cdot(e/(e/T(x, y))))/y)\backslash((e/T(x, y))\cdot((y\cdot((e/T(x, y))\backslash e))/y)) = e/T(x, y)$ by Theorem 94. Then $((y\cdot(e/(e/T(x, y))))/y)\backslash((e/T(x, y))\cdot((y\cdot T(x, y))/y)) = e/T(x, y)$ by Proposition 25. Then $((y\cdot(e/(e/T(x, y))))/y)\backslash((e/T(x, y))\cdot x) = e/T(x, y)$ by Proposition 48. Then $((y\cdot(e/(e/T(x, y))))/y)\backslash(x/T(x, y)) = e/T(x, y)$ by Theorem 550. Then $((y\cdot T(e/(e/x), y))/y)\backslash(x/T(x, y)) = e/T(x, y)$ by Theorem 447. Then

$$(e/(e/x))\backslash(x/T(x, y)) = e/T(x, y) \tag{331}$$

by Proposition 48. We have $K(x\backslash e, x)\cdot((e/(e/x))\backslash(x/T(x, y))) = x\backslash(x/T(x, y))$ by Theorem 695. Hence we are done by (331). □

Theorem 716. $K(x\backslash e, x) = T(K(x\backslash e, x), y)$.

Proof. We have $T(e/T(x, y), x)\cdot T(x, y) = T(x, y)\cdot T(T(x, y)\backslash e, x)$ by Proposition 61. Then $(x\backslash(x/T(x, y)))\cdot T(x, y) = T(x, y)\cdot T(T(x, y)\backslash e, x)$ by Theorem 551. Then $T(x\cdot T(x\backslash e, x), y) = (x\backslash(x/T(x, y)))\cdot T(x, y)$ by Theorem 576. Then $(x\backslash(x/T(x, y)))\cdot T(x, y) = T((x\backslash e)\cdot x, y)$ by Proposition 46. Then

$$(x\backslash(x/T(x, y)))\cdot T(x, y) = T(K(x\backslash e, x), y) \tag{332}$$

by Proposition 76. We have $(x\backslash(x/T(x,y)))/(e/T(x,y)) = (x\backslash(x/T(x,y))) \cdot T(x,y)$ by Theorem 604. Then $T(e/T(x,y),x)/(e/T(x,y)) = (x\backslash(x/T(x,y))) \cdot T(x,y)$ by Theorem 551. Then

$$T(e/T(x,y),x)/(e/T(x,y)) = T(K(x\backslash e,x),y) \tag{333}$$

by (332). We have $K(x\backslash(x/(e/(e/T(x,y)))),x) = T(e/T(x,y),x)/(e/T(x,y))$ by Theorem 199. Then

$$K(x\backslash(x/(e/(e/T(x,y)))),x) = T(K(x\backslash e,x),y) \tag{334}$$

by (333). We have $T(e/T(x,y),x) = K(x\backslash(x/(e/(e/T(x,y)))),x) \cdot (e/T(x,y))$ by Theorem 201. Then $x\backslash(x/T(x,y)) = K(x\backslash(x/(e/(e/T(x,y)))),x) \cdot (e/T(x,y))$ by Theorem 551. Then

$$T(K(x\backslash e,x),y) \cdot (e/T(x,y)) = x\backslash(x/T(x,y)) \tag{335}$$

by (334). We have $K(x\backslash e,x) \cdot (e/T(x,y)) = x\backslash(x/T(x,y))$ by Theorem 715. Hence we are done by (335) and Proposition 8. □

Theorem 717. $T(y,K(x\backslash e,x)) = y$.

Proof. We have $T(K(x\backslash e,x),y) = K(x\backslash e,x)$ by Theorem 716. Hence we are done by Proposition 21. □

Theorem 718. $T(y,K(x,x\backslash e)) = y$.

Proof. We have $T(y,K((e/x)\backslash e,e/x)) = y$ by Theorem 717. Hence we are done by Theorem 253. □

Theorem 719. $T(z,K(y\backslash x,x\backslash y)) = z$.

Proof. We have $T(z,K((x\backslash y)\backslash e,x\backslash y)) = z$ by Theorem 717. Hence we are done by Theorem 666. □

Theorem 720. $(K(x\backslash e,x) \cdot y) \cdot K(x,x\backslash e) = y$.

Proof. We have $T((K(x\backslash e,x) \cdot y) \cdot K(x,x\backslash e),K(x\backslash e,x)) = y$ by Theorem 658. Hence we are done by Theorem 717. □

Theorem 721. $T(x,(y\backslash e)\backslash e) = T(x,y)$.

Proof. We have $T(T(x,(y\backslash e)\backslash e),K(y\backslash e,y)) = T(x,y)$ by Theorem 708. Hence we are done by Theorem 717. □

Theorem 722. $T(x,T(y,y\backslash e)) = T(x,y)$.

Proof. We have $T(T(T(x,y),K(y,y\backslash e)),K(y\backslash e,y)) = T(x,y)$ by Theorem 701. Then $T(T(x,T(y,y\backslash e)),K(y\backslash e,y)) = T(x,y)$ by Theorem 707. Hence we are done by Theorem 717. □

Theorem 723. $T(K(y,y\backslash e),x) = K(y,y\backslash e)$.

Proof. We have $T(K((e/y)\backslash e,e/y),x) = K((e/y)\backslash e,e/y)$ by Theorem 716. Then $K(K((e/y)\backslash e,e/y),x) = e$ by Proposition 22. Then $K(K(y,y\backslash e),x) = e$ by Theorem 253. Hence we are done by Proposition 23. □

Theorem 724. $z \cdot K(y\backslash x,x\backslash y) = K(y\backslash x,x\backslash y) \cdot z$.

Proof. We have

$$T((K(y\backslash x,x\backslash y) \cdot z) \cdot (e/K(y\backslash x,x\backslash y)),K(y\backslash x,x\backslash y)) = (K(y\backslash x,x\backslash y) \cdot z) \cdot (e/K(y\backslash x,x\backslash y)) \tag{336}$$

by Theorem 719. We have $T((K(y\backslash x,x\backslash y) \cdot z) \cdot (e/K(y\backslash x,x\backslash y)),K(y\backslash x,x\backslash y)) = (z \cdot K(y\backslash x,x\backslash y)) \cdot (e/K(y\backslash x,x\backslash y))$ by Theorem 611. Then $(z \cdot K(y\backslash x,x\backslash y)) \cdot (e/K(y\backslash x,x\backslash y)) = (K(y\backslash x,x\backslash y) \cdot z) \cdot (e/K(y\backslash x,x\backslash y))$ by (336). Hence we are done by Proposition 10. □

Theorem 725. $T(x, R(y\backslash e, y, z)) = T(x, R(e/y, y, z))$.

Proof. We have $T(x, (R(e/y, y, z)\backslash e)\backslash e) = T(x, R(e/y, y, z))$ by Theorem 721. Hence we are done by Theorem 671. ☐

Theorem 726. $T(y, y\backslash e) \cdot T(x, y) = x \cdot T(y, y\backslash e)$.

Proof. We have $T(y, y\backslash e) \cdot T(x, T(y, y\backslash e)) = x \cdot T(y, y\backslash e)$ by Proposition 46. Hence we are done by Theorem 722. ☐

Theorem 727. $T(x, R(e/y, y, z)) = T(x, R(y\backslash e, (y\backslash e)\backslash e, z))$.

Proof. We have $T(x, R(e/y, y, z)) = T(x, R(y\backslash e, y, z))$ by Theorem 725. Hence we are done by Theorem 704. ☐

Theorem 728. $T(x\backslash e, y) = (x\backslash e) \cdot (x \cdot T(x\backslash e, y))$.

Proof. We have $R(x\backslash e, y, K((x\backslash e)\backslash e, x\backslash e)) = x\backslash e$ by Theorem 669. Then

$$R(x\backslash e, y, K(x, x\backslash e)) = x\backslash e \tag{337}$$

by Theorem 561. We have $R(x\backslash e, y, K(x, x\backslash e)) \cdot (y \cdot K(x, x\backslash e)) = ((x\backslash e) \cdot y) \cdot K(x, x\backslash e)$ by Proposition 54. Then $(x\backslash e) \cdot (y \cdot K(x, x\backslash e)) = ((x\backslash e) \cdot y) \cdot K(x, x\backslash e)$ by (337). Then

$$T((e/x) \cdot y, K(x, x\backslash e)) = (x\backslash e) \cdot (y \cdot K(x, x\backslash e)) \tag{338}$$

by Theorem 700. We have $T(x\backslash e, y \cdot K(x, x\backslash e)) = (y \cdot K(x, x\backslash e))\backslash((x\backslash e) \cdot (y \cdot K(x, x\backslash e)))$ by Definition 3. Then

$$T(x\backslash e, y \cdot K(x, x\backslash e)) = (y \cdot K(x, x\backslash e))\backslash T((e/x) \cdot y, K(x, x\backslash e)) \tag{339}$$

by (338). We have $K(x, x\backslash e) \cdot (K(x\backslash e, x) \cdot (y \cdot K(x, x\backslash e))) = y \cdot K(x, x\backslash e)$ by Theorem 442. Then

$$T(y, K(x, x\backslash e)) = K(x\backslash e, x) \cdot (y \cdot K(x, x\backslash e)) \tag{340}$$

by Theorem 11. We have $T(x\backslash e, K(x\backslash e, (x\backslash e)\backslash e) \cdot (y \cdot K(x, x\backslash e))) = T(x\backslash e, y \cdot K(x, x\backslash e))$ by Theorem 698. Then $T(x\backslash e, K(x\backslash e, x) \cdot (y \cdot K(x, x\backslash e))) = T(x\backslash e, y \cdot K(x, x\backslash e))$ by Theorem 249. Then $T(x\backslash e, T(y, K(x, x\backslash e))) = T(x\backslash e, y \cdot K(x, x\backslash e))$ by (340). Then

$$T(x\backslash e, T(y, K(x, x\backslash e))) = (y \cdot K(x, x\backslash e))\backslash T((e/x) \cdot y, K(x, x\backslash e)) \tag{341}$$

by (339). We have $T(x\backslash e, K((x\backslash e)\backslash e, x\backslash e) \cdot T(y, K((x\backslash e)\backslash e, x\backslash e))) = T(x\backslash e, T(y, K((x\backslash e)\backslash e, x\backslash e)))$ by Theorem 630. Then $T(x\backslash e, y \cdot K((x\backslash e)\backslash e, x\backslash e)) = T(x\backslash e, T(y, K((x\backslash e)\backslash e, x\backslash e)))$ by Proposition 46. Then $T(x\backslash e, T(y, K((x\backslash e)\backslash e, x\backslash e))) = T(x\backslash e, y)$ by Theorem 660. Then $T(x\backslash e, T(y, K(x, x\backslash e))) = T(x\backslash e, y)$ by Theorem 561. Then $(y \cdot K(x, x\backslash e))\backslash T((e/x) \cdot y, K(x, x\backslash e)) = T(x\backslash e, y)$ by (341). Then

$$(y \cdot K(x, x\backslash e))\backslash((e/x) \cdot y) = T(x\backslash e, y) \tag{342}$$

by Theorem 718. We have $(y/K(x\backslash e, x))\backslash(L(e/x, K(x\backslash e, x), y/K(x\backslash e, x)) \cdot y) = K(x\backslash e, x) \cdot T(e/x, y)$ by Theorem 491. Then $(y/K(x\backslash e, x))\backslash((e/x) \cdot y) = K(x\backslash e, x) \cdot T(e/x, y)$ by Theorem 644. Then

$$(y \cdot K(x, x\backslash e))\backslash((e/x) \cdot y) = K(x\backslash e, x) \cdot T(e/x, y) \tag{343}$$

by Theorem 656. We have $K(x\backslash e, x) \cdot T(e/x, y) = (x\backslash e) \cdot (x \cdot T(x\backslash e, y))$ by Theorem 567. Then $(y \cdot K(x, x\backslash e))\backslash((e/x) \cdot y) = (x\backslash e) \cdot (x \cdot T(x\backslash e, y))$ by (343). Hence we are done by (342). ☐

Theorem 729. $(x\backslash e)\backslash T(x\backslash e, y) = x \cdot T(x\backslash e, y)$.

Proof. We have $(x\backslash e) \cdot (x \cdot T(x\backslash e, y)) = T(x\backslash e, y)$ by Theorem 728. Hence we are done by Proposition 2. ☐

Theorem 730. $K(y\backslash(y/x), y) = (x\backslash e)\backslash T(x\backslash e, y)$.

Proof. We have $K(y\backslash(y/x), y) = x \cdot T(x\backslash e, y)$ by Theorem 196. Hence we are done by Theorem 729. $\square$

Theorem 731. $K(y\backslash x, x\backslash y) \cdot y = x \cdot (e/(e/(x\backslash y)))$.

Proof. We have $x \cdot (x\backslash(y \cdot K(y\backslash x, x\backslash y))) = y \cdot K(y\backslash x, x\backslash y)$ by Axiom 4. Then $x \cdot (e/(e/(x\backslash y))) = y \cdot K(y\backslash x, x\backslash y)$ by Theorem 615. Hence we are done by Theorem 724. $\square$

Theorem 732. $K(x\backslash e, y) = K(e/x, y)$.

Proof. We have $K(y\backslash(y/((x\backslash e)\backslash e)), y) = (x\backslash e)\backslash T(x\backslash e, y)$ by Theorem 198. Then

$$K(y\backslash(y/((x\backslash e)\backslash e)), y) = K(y\backslash(y/x), y) \tag{344}$$

by Theorem 730. We have

$$L((x\backslash e)\backslash T(x\backslash e, y), x\backslash e, y) = (y \cdot (x\backslash e))\backslash((x\backslash e) \cdot y) \tag{345}$$

by Proposition 73.

$$\begin{array}{lll} & K(x\backslash e, y) & \\ = & (y \cdot (x\backslash e))\backslash((x\backslash e) \cdot y) & \text{by Definition 2} \\ = & L(K(y\backslash(y/((x\backslash e)\backslash e)), y), x\backslash e, y) & \text{by (345), Theorem 198.} \end{array}$$

Then $K(x\backslash e, y) = L(K(y\backslash(y/((x\backslash e)\backslash e)), y), x\backslash e, y)$. Then $L(K(y\backslash(y/x), y), x\backslash e, y) = K(x\backslash e, y)$ by (344). Then

$$L(x \cdot T(x\backslash e, y), x\backslash e, y) = K(x\backslash e, y) \tag{346}$$

by Theorem 196. We have $L(x \cdot T(x\backslash e, y), x\backslash e, y) = K(e/x, y)$ by Theorem 654. Hence we are done by (346). $\square$

Theorem 733. $K(x, y) = K((x\backslash e)\backslash e, y)$.

Proof. We have $K(e/(x\backslash e), y) = K((x\backslash e)\backslash e, y)$ by Theorem 732. Hence we are done by Proposition 24. $\square$

Theorem 734. $T(K(x, y), x) = K(x, y)$.

Proof. We have $T(K(x, y), x) = K((x\backslash e)\backslash e, y)$ by Theorem 674. Hence we are done by Theorem 733. $\square$

Theorem 735. $(((y \cdot x)\backslash y)\backslash e)\backslash(((y \cdot x)\backslash y)\backslash K(x, y)) = K(x, y)$.

Proof. We have $(((y \cdot x)\backslash y)\backslash e)\backslash(((y \cdot x)\backslash y)\backslash K(x, y)) = K((x\backslash e)\backslash e, y)$ by Theorem 675. Hence we are done by Theorem 733. $\square$

Theorem 736. $T(y, K(y, x)) = y$.

Proof. We have $T(K(y, x), y) = K(y, x)$ by Theorem 734. Hence we are done by Proposition 21. $\square$

Theorem 737. $K(x\backslash((y \cdot x)/y), y) = e$.

Proof. We have $T(K(y, (y/(x\backslash e))/y), y) = K(y, (y/(x\backslash e))/y)$ by Theorem 734. Then $K(K(y, (y/(x\backslash e))/y), y) = e$ by Proposition 22. Hence we are done by Theorem 596. $\square$

Theorem 738. $T(T(y, x)\backslash y, x) = T(y, x)\backslash y$.

Proof. We have $K(T(y, x)\backslash((x \cdot T(y, x))/x), x) = e$ by Theorem 737. Then $K(T(y, x)\backslash y, x) = e$ by Proposition 48. Hence we are done by Proposition 23. $\square$

Theorem 739. $T(y, T(x, y)\backslash x) = y$.

Proof. We have $T(y, K(y, (y/(T(x, y)\backslash e))/y)) = y$ by Theorem 736. Then $T(y, T(x, y)\backslash((y \cdot T(x, y))/y)) = y$ by Theorem 596. Hence we are done by Proposition 48. □

Theorem 740. $(T(y, x)\backslash y) \cdot x = x \cdot (T(y, x)\backslash y)$.

Proof. We have $(T(y, x)\backslash y) \cdot T(x, T(y, x)\backslash y) = x \cdot (T(y, x)\backslash y)$ by Proposition 46. Hence we are done by Theorem 739. □

Theorem 741. $K(x\backslash T(x, y), T(z, y)\backslash z) = e$.

Proof. We have $T(y\backslash T(y, R(T(z, y), T(z, y)\backslash e, y)), K(y\backslash(y/(x\backslash e)), y)) = y\backslash T(y, R(T(z, y), T(z, y)\backslash e, y))$ by Theorem 619. Then $T(K(y\backslash(y/(x\backslash e)), y), y\backslash T(y, R(T(z, y), T(z, y)\backslash e, y))) = K(y\backslash(y/(x\backslash e)), y)$ by Proposition 21. Then $K(K(y\backslash(y/(x\backslash e)), y), y\backslash T(y, R(T(z, y), T(z, y)\backslash e, y))) = e$ by Proposition 22. Then $K(K(y\backslash(y/(x\backslash e)), y), y\backslash((T(z, y)\backslash z) \cdot y)) = e$ by Theorem 591. Then $K(K(y\backslash(y/(x\backslash e)), y), T(T(z, y)\backslash z, y)) = e$ by Definition 3. Then $K(K(y\backslash(y/(x\backslash e)), y), T(z, y)\backslash z) = e$ by Theorem 738. Hence we are done by Theorem 198. □

Theorem 742. $T(x, R(T(y, x), T(y, x)\backslash e, x)) = x \cdot (T(y, x)\backslash y)$.

Proof. We have $T(x, R(T(y, x), T(y, x)\backslash e, x)) = (T(y, x)\backslash y) \cdot x$ by Theorem 591. Hence we are done by Theorem 740. □

Theorem 743. $K(T(y, x), (y \cdot x)\backslash x) = K(y, y\backslash e)$.

Proof. We have

$$K(x\backslash(y \cdot x), (y \cdot x)\backslash x) \cdot x = (y \cdot x) \cdot (e/(e/((y \cdot x)\backslash x))) \tag{347}$$

by Theorem 731. We have $K(y, y\backslash e) \cdot x = (y \cdot x) \cdot (e/(e/((y \cdot x)\backslash x)))$ by Theorem 714. Then $K(x\backslash(y \cdot x), (y \cdot x)\backslash x) = K(y, y\backslash e)$ by (347) and Proposition 8. Hence we are done by Definition 3. □

Theorem 744. $K(T(y, x), T(y, x)\backslash e) = K(y, y\backslash e)$.

Proof. We have $K(T(y, x), (x \cdot T(y, x))\backslash x) = K(T(y, x), T(y, x)\backslash e)$ by Theorem 392. Then $K(T(y, x), (y \cdot x)\backslash x) = K(T(y, x), T(y, x)\backslash e)$ by Proposition 46. Hence we are done by Theorem 743. □

Theorem 745. $T(x, y)\backslash K(x, x\backslash e) = e/T(x, y)$.

Proof. We have $T(x, y)\backslash K(T(x, y), T(x, y)\backslash e) = e/T(x, y)$ by Theorem 251. Hence we are done by Theorem 744. □

Theorem 746. $(T(x, y)\backslash e) \cdot K(x, x\backslash e) = e/T(x, y)$.

Proof. We have $(T(x, y)\backslash e) \cdot K(T(x, y), T(x, y)\backslash e) = e/T(x, y)$ by Theorem 553. Hence we are done by Theorem 744. □

Theorem 747. $e/T(y, x) = (e/y) \cdot (T(y, x)\backslash y)$.

Proof. We have $(e/y) \cdot (y \cdot (T(y, x)\backslash e)) = L(T(y, x)\backslash e, y, e/y)$ by Proposition 67. Then $(e/y) \cdot (T(y, x)\backslash y) = L(T(y, x)\backslash e, y, e/y)$ by Theorem 175. Then

$$L(T(y, x)\backslash e, y, y\backslash e) = (e/y) \cdot (T(y, x)\backslash y) \tag{348}$$

by Theorem 683. We have $T(y, x) \cdot L(T(y, x)\backslash e, y, y\backslash e) = T(y \cdot L(y\backslash e, y, y\backslash e), x)$ by Theorem 351. Then $T(y, x) \cdot L(T(y, x)\backslash e, y, y\backslash e) = T(y \cdot ((e/y) \cdot e), x)$ by Theorem 434. Then $T(y, x) \cdot ((e/y) \cdot (T(y, x)\backslash y)) = T(y \cdot ((e/y) \cdot e), x)$ by (348). Then $T(y, x) \cdot ((e/y) \cdot (T(y, x)\backslash y)) = T(y \cdot (e/y), x)$ by Axiom 2. Then $T(y, x) \cdot ((e/y) \cdot (T(y, x)\backslash y)) = T(K(y, y\backslash e), x)$ by Theorem 250. Then $T(y, x)\backslash T(K(y, y\backslash e), x) = (e/y) \cdot (T(y, x)\backslash y)$ by Proposition 2. Then $T(y, x)\backslash K(y, y\backslash e) = (e/y) \cdot (T(y, x)\backslash y)$ by Theorem 723. Hence we are done by Theorem 745. □

Theorem 748. $K(x\backslash e, x) \cdot (e/T(x, y)) = T(x, y)\backslash e$.

Proof.

$$\begin{array}{lcl} & (K(x\backslash e, x) \cdot (e/T(x, y))) \cdot K(x, x\backslash e) & \\ = & e/T(x, y) & \text{by Theorem 720} \\ = & (T(x, y)\backslash e) \cdot K(x, x\backslash e) & \text{by Theorem 746.} \end{array}$$

Then $(K(x\backslash e, x) \cdot (e/T(x, y))) \cdot K(x, x\backslash e) = (T(x, y)\backslash e) \cdot K(x, x\backslash e)$. Hence we are done by Proposition 10. □

Theorem 749. $(y\backslash T(y, x))\backslash e = T(y, x)\backslash y$.

Proof. We have $T(y, x)\backslash K(y, e/y) = y \cdot (T(y, x)\backslash(e/y))$ by Theorem 523. Then $T(y, x)\backslash K(y, y\backslash e) = y \cdot (T(y, x)\backslash(e/y))$ by Theorem 248. Then

$$e/T(y, x) = y \cdot (T(y, x)\backslash(e/y)) \tag{349}$$

by Theorem 745. We have $(y\backslash e)\backslash(T(y, x)\backslash e) = (y\backslash T(y, x))\backslash e$ by Theorem 638. Then $(e/y)\backslash(K(y, y\backslash e) \cdot (T(y, x)\backslash e)) = (y\backslash T(y, x))\backslash e$ by Theorem 692. Then $(e/y)\backslash(y \cdot (T(y, x)\backslash(e/y))) = (y\backslash T(y, x))\backslash e$ by Theorem 688. Then

$$(e/y)\backslash(e/T(y, x)) = (y\backslash T(y, x))\backslash e \tag{350}$$

by (349). We have $(e/y) \cdot (T(y, x)\backslash y) = e/T(y, x)$ by Theorem 747. Then $(e/y)\backslash(e/T(y, x)) = T(y, x)\backslash y$ by Proposition 2. Hence we are done by (350). □

Theorem 750. $(y \cdot T(y\backslash e, x))\backslash e = T(y\backslash e, x)\backslash(y\backslash e)$.

Proof. We have $((y\backslash e)\backslash T(y\backslash e, x))\backslash e = T(y\backslash e, x)\backslash(y\backslash e)$ by Theorem 749. Hence we are done by Theorem 729. □

Theorem 751. $e/(y\backslash T(y, x)) = T(y, x)\backslash y$.

Proof. We have $(y\backslash T(y, x))\backslash K(y\backslash T(y, x), T(y, x)\backslash y) = e/(y\backslash T(y, x))$ by Theorem 613. Then $(y\backslash T(y, x))\backslash e = e/(y\backslash T(y, x))$ by Theorem 741. Hence we are done by Theorem 749. □

Theorem 752. $T(x, y)\backslash e = x\backslash(x/T(x, y))$.

Proof. We have $K(x\backslash e, x) \cdot (e/T(x, y)) = x\backslash(x/T(x, y))$ by Theorem 715. Hence we are done by Theorem 748. □

Theorem 753. $y/T(y, x) = T(y, x)\backslash y$.

Proof. We have $y \cdot (y\backslash(y/T(y, x))) = y/T(y, x)$ by Axiom 4. Then

$$y \cdot (T(y, x)\backslash e) = y/T(y, x) \tag{351}$$

by Theorem 752. We have $y \cdot (T(y, x)\backslash e) = T(y, x)\backslash y$ by Theorem 175. Hence we are done by (351). □

Theorem 754. $K(x, y) = T(K(x, y), y)$.

Proof. We have $y \cdot (T(y, x)\backslash y) = (y/T(y, x)) \cdot y$ by Theorem 572. Then

$$T(y/T(y, x), y) = T(y, x)\backslash y \tag{352}$$

by Theorem 11. We have $T(y/T(y, x), y) \cdot T(y, x) = T(y, x) \cdot T(T(y, x)\backslash y, y)$ by Proposition 61. Then

$$(T(y, x)\backslash y) \cdot T(y, x) = T(y, x) \cdot T(T(y, x)\backslash y, y) \tag{353}$$

by (352). We have $T(y,x) \cdot T(T(y,x)\backslash y, y) = L(y, T(y,x)\backslash y, T(y,x))$ by Theorem 527. Then $T(y,x) \cdot T(T(y,x)\backslash y, y) = L(y, T(y,x)\backslash y, y)$ by Theorem 609. Then

$$L(y, T(y,x)\backslash y, y) = (T(y,x)\backslash y) \cdot T(y,x) \tag{354}$$

by (353). We have $(y/T(y,x)) \cdot T(y,x) = y$ by Axiom 6. Then $(T(y,x)\backslash y) \cdot T(y,x) = y$ by Theorem 753. Then

$$L(y, T(y,x)\backslash y, y) = y \tag{355}$$

by (354). We have $(y \cdot x) \cdot L(T(T(y,x)\backslash y, y), T(y,x), x) = x \cdot (T(y,x) \cdot T(T(y,x)\backslash y, y))$ by Theorem 58. Then $(y \cdot x) \cdot L(T(T(y,x)\backslash y, y), T(y,x), x) = x \cdot L(y, T(y,x)\backslash y, T(y,x))$ by Theorem 527. Then $(y \cdot x) \cdot T((y \cdot x)\backslash(x \cdot y), y) = x \cdot L(y, T(y,x)\backslash y, T(y,x))$ by Theorem 487. Then $(y \cdot x) \cdot T(K(x,y), y) = x \cdot L(y, T(y,x)\backslash y, T(y,x))$ by Definition 2. Then $x \cdot L(y, T(y,x)\backslash y, y) = (y \cdot x) \cdot T(K(x,y), y)$ by Theorem 609. Then $x \cdot y = (y \cdot x) \cdot T(K(x,y), y)$ by (355). Hence we are done by Theorem 460. □

Theorem 755. $T(y, K(x,y)) = y$.

Proof. We have $T(K(x,y), y) = K(x,y)$ by Theorem 754. Hence we are done by Proposition 21. □

Theorem 756. $T(x,y) \cdot K(x\backslash e, x) = T(e/(e/x), y)$.

Proof. We have $T(e/(e/x), y)\backslash T(x,y) = K(T(x,y), T(x,y)\backslash e)$ by Theorem 712. Then

$$T(e/(e/x), y)\backslash T(x,y) = K(x, x\backslash e) \tag{356}$$

by Theorem 744. We have $T(e/(e/x), y) \cdot (T(e/(e/x), y)\backslash T(x,y)) = T(x,y)$ by Axiom 4. Then

$$T(e/(e/x), y) \cdot K(x, x\backslash e) = T(x,y) \tag{357}$$

by (356). We have $(T(x,y) \cdot K(x\backslash e, x)) \cdot K(x, x\backslash e) = T(x,y)$ by Theorem 426. Then $(T(x,y) \cdot K(x\backslash e, x)) \cdot K(x, x\backslash e) = T(e/(e/x), y) \cdot K(x, x\backslash e)$ by (357). Hence we are done by Proposition 10. □

Theorem 757. $T(y,x) = y \cdot (T(y,x)/y)$.

Proof. We have $L(T(y,x), e/y, y) = (y \cdot (e/y))\backslash(y \cdot ((e/y) \cdot T(y,x)))$ by Definition 4. Then $L(T(y,x), e/y, y) = K(y, e/y)\backslash(y \cdot ((e/y) \cdot T(y,x)))$ by Proposition 77. Then

$$K(y, e/y)\backslash(y \cdot (T(y,x)/y)) = L(T(y,x), e/y, y) \tag{358}$$

by Theorem 150. We have $L(T(y,x), e/y, y) = T(L(y, e/y, y), x)$ by Axiom 8. Then $L(T(y,x), e/y, y) = T(e/(e/y), x)$ by Theorem 266. Then

$$K(y, e/y)\backslash(y \cdot (T(y,x)/y)) = T(e/(e/y), x) \tag{359}$$

by (358). We have $K(y, e/y) \cdot (K(y, e/y)\backslash(y \cdot (T(y,x)/y))) = y \cdot (T(y,x)/y)$ by Axiom 4. Then $K(y, e/y) \cdot T(e/(e/y), x) = y \cdot (T(y,x)/y)$ by (359). Then $K(y, y\backslash e) \cdot T(e/(e/y), x) = y \cdot (T(y,x)/y)$ by Theorem 248. Then

$$K(y, y\backslash e) \cdot T(K(y, y\backslash e)\backslash y, x) = y \cdot (T(y,x)/y) \tag{360}$$

by Theorem 558. We have $(K(y, y\backslash e) \cdot T(K(y, y\backslash e)\backslash y, x)) \cdot (K(y, y\backslash e)\backslash e) = T(y \cdot (K(y, y\backslash e)\backslash e), x)$ by Theorem 506. Then $(y \cdot (T(y,x)/y)) \cdot (K(y, y\backslash e)\backslash e) = T(y \cdot (K(y, y\backslash e)\backslash e), x)$ by (360). Then $(y \cdot (T(y,x)/y)) \cdot K(y\backslash e, y) = T(y \cdot (K(y, y\backslash e)\backslash e), x)$ by Theorem 565. Then $(y \cdot (T(y,x)/y)) \cdot K(y\backslash e, y) = T(y \cdot K(y\backslash e, y), x)$ by Theorem 565. Then

$$(y \cdot (T(y,x)/y)) \cdot K(y\backslash e, y) = T(e/(e/y), x) \tag{361}$$

by Theorem 262. We have $T(y,x) \cdot K(y\backslash e, y) = T(e/(e/y), x)$ by Theorem 756. Hence we are done by (361) and Proposition 8. □

Theorem 758. $y\backslash T(y,x) = T(y,x)/y$.

Proof. We have $y \cdot (T(y,x)/y) = T(y,x)$ by Theorem 757. Hence we are done by Proposition 2. □

Theorem 759. $(x\backslash T(x,y)) \cdot x = T(x,y)$.

Proof. We have $(T(x,y)/x) \cdot x = T(x,y)$ by Axiom 6. Then $(x\backslash(x \cdot (T(x,y)/x))) \cdot x = T(x,y)$ by Axiom 3. Hence we are done by Theorem 757. □

Theorem 760. $y\backslash T(y, R(x, x\backslash e, y)) = x\backslash((y \cdot x)/y)$.

Proof. We have $T(y, R(x, x\backslash e, y))/y = x\backslash((y \cdot x)/y)$ by Theorem 590. Hence we are done by Theorem 758. □

Theorem 761. $T(y, y\backslash T(y,x)) = y$.

Proof. We have $(y\backslash T(y,x)) \cdot y = T(y,x)$ by Theorem 759. Hence we are done by Theorem 20. □

Theorem 762. $x\backslash e = T(e/x, T(x,y))$.

Proof. We have $T(x, x\backslash T(x,y)) = x$ by Theorem 761. Then $K(x, x\backslash T(x,y)) = e$ by Proposition 22. Then

$$K(x, T(x,y)/x) = e \tag{362}$$

by Theorem 758. We have $x \cdot T(e/x, T(x,y)) = K(x, T(x,y)/x)$ by Theorem 529. Then $x\backslash K(x, T(x,y)/x) = T(e/x, T(x,y))$ by Proposition 2. Hence we are done by (362). □

Theorem 763. $K(x,y)\backslash e = T(L(x\backslash e, x, y)\backslash e, y)\backslash(L(x\backslash e, x, y)\backslash e)$.

Proof. We have $(L(x\backslash e, x, y) \cdot T(L(x\backslash e, x, y)\backslash e, y))\backslash e = T(L(x\backslash e, x, y)\backslash e, y)\backslash(L(x\backslash e, x, y)\backslash e)$ by Theorem 750. Hence we are done by Theorem 542. □

Theorem 764. $(y\backslash(z \cdot T(z\backslash y, x)))\backslash e = T((y\backslash z)\backslash e, x)\backslash((y\backslash z)\backslash e)$.

Proof. We have $((y\backslash z) \cdot T((y\backslash z)\backslash e, x))\backslash e = T((y\backslash z)\backslash e, x)\backslash((y\backslash z)\backslash e)$ by Theorem 750. Hence we are done by Theorem 568. □

Theorem 765. $(K(x,y)\backslash y) \cdot K(x,y) = y$.

Proof. We have $x \cdot (K(x,y) \cdot T(K(x,y)\backslash y, K(x,y))) = (T(y, K(x,y)) \cdot x) \cdot K(x,y)$ by Theorem 634. Then $x \cdot ((K(x,y)\backslash y) \cdot K(x,y)) = (T(y, K(x,y)) \cdot x) \cdot K(x,y)$ by Proposition 46. Then

$$(y \cdot x) \cdot K(x,y) = x \cdot ((K(x,y)\backslash y) \cdot K(x,y)) \tag{363}$$

by Theorem 755. We have $x \cdot y = (y \cdot x) \cdot K(x,y)$ by Proposition 82. Hence we are done by (363) and Proposition 7. □

Theorem 766. $y/K(x,y) = K(x,y)\backslash y$.

Proof. We have $(K(x,y)\backslash y) \cdot K(x,y) = y$ by Theorem 765. Hence we are done by Proposition 1. □

Theorem 767. $y\backslash((x \cdot y)/x) = (((x \cdot y)/x)\backslash y)\backslash e$.

Proof. We have $(((x \cdot y)/x)\backslash T((x \cdot y)/x, x))\backslash e = T((x \cdot y)/x, x)\backslash((x \cdot y)/x)$ by Theorem 749. Then $(((x \cdot y)/x)\backslash(x\backslash(x \cdot y)))\backslash e = T((x \cdot y)/x, x)\backslash((x \cdot y)/x)$ by Proposition 47. Then $(x\backslash(x \cdot y))\backslash((x \cdot y)/x) = (((x \cdot y)/x)\backslash(x\backslash(x \cdot y)))\backslash e$ by Proposition 47. Then $(((x \cdot y)/x)\backslash y)\backslash e = (x\backslash(x \cdot y))\backslash((x \cdot y)/x)$ by Axiom 3. Hence we are done by Axiom 3. □

Theorem 768. $e/(y\backslash((x \cdot y)/x)) = ((x \cdot y)/x)\backslash y$.

Proof. We have $e/((((x \cdot y)/x)\backslash y)\backslash e) = ((x \cdot y)/x)\backslash y$ by Proposition 24. Hence we are done by Theorem 767. □

Theorem 769. $T(x, R(y, y\backslash e, x))\backslash x = ((x \cdot y)/x)\backslash y$.

Proof. We have $e/(x\backslash T(x, R(y, y\backslash e, x))) = T(x, R(y, y\backslash e, x))\backslash x$ by Theorem 751. Then $e/(y\backslash((x \cdot y)/x)) = T(x, R(y, y\backslash e, x))\backslash x$ by Theorem 760. Hence we are done by Theorem 768. ☐

Theorem 770. *$T(y, R(T(x, y), T(x, y)\backslash e, y))\backslash y = x\backslash T(x, y)$.*

Proof. We have $T(y, R(T(x, y), T(x, y)\backslash e, y))\backslash y = ((y \cdot T(x, y))/y)\backslash T(x, y)$ by Theorem 769. Hence we are done by Proposition 48. ☐

Theorem 771. *$(y \cdot (T(x, y)\backslash x))\backslash y = x\backslash T(x, y)$.*

Proof. We have $T(y, R(T(x, y), T(x, y)\backslash e, y))\backslash y = x\backslash T(x, y)$ by Theorem 770. Hence we are done by Theorem 742. ☐

Theorem 772. *$T(y, K(x, y)\backslash e) = y$.*

Proof. We have $T(y, T(L(x\backslash e, x, y)\backslash e, y)\backslash(L(x\backslash e, x, y)\backslash e)) = y$ by Theorem 739. Hence we are done by Theorem 763. ☐

Theorem 773. *$(x\backslash R(x, y, z))\backslash(x\backslash e) = R(x, y, z)\backslash e$.*

Proof. We have $R(T(x, x\backslash e)\cdot T(T(x, x\backslash e)\backslash e, x\backslash R(x, y, z)), y, z) = K((x\backslash R(x, y, z))\backslash((x\backslash R(x, y, z))/R(T(x, x\backslash e), y, z)), x\backslash R(x, y, z$ by Theorem 636. Then $R(T(x, x\backslash e)\cdot T(T(x, x\backslash e)\backslash e, x\backslash R(x, y, z)), y, z) = K((x\backslash R(x, y, z))\backslash(x\backslash e), x\backslash R(x, y, z))$ by Theorem 532. Then

$$R(T(x, x\backslash e) \cdot T(T(x\backslash e, x), x\backslash R(x, y, z)), y, z) = K((x\backslash R(x, y, z))\backslash(x\backslash e), x\backslash R(x, y, z)) \tag{364}$$

by Theorem 208. We have $T(T(x\backslash e, x\backslash R(x, y, z)), x) = T(T(x\backslash e, x), x\backslash R(x, y, z))$ by Axiom 7. Then $T(x\backslash e, x) = T(T(x\backslash e, x), x\backslash R(x, y, z))$ by Theorem 578. Then $R(T(x, x\backslash e)\cdot T(x\backslash e, x), y, z) = K((x\backslash R(x, y, z))\backslash(x\backslash e), x\backslash R(x, y, z))$ by (364). Then $K((x\backslash R(x, y, z))\backslash(x\backslash e), x\backslash R(x, y, z)) = R((x\backslash e)\cdot T(x, x\backslash e), y, z)$ by Theorem 726. Then

$$K((x\backslash R(x, y, z))\backslash(x\backslash e), x\backslash R(x, y, z)) = R(e, y, z) \tag{365}$$

by Theorem 8. We have $R(e, y, z) = ((e \cdot y) \cdot z)/(y \cdot z)$ by Definition 5. Then

$$R(e, y, z) = (y \cdot z)/(y \cdot z) \tag{366}$$

by Axiom 1. We have $(y \cdot z)/(y \cdot z) = e$ by Proposition 29. Then $R(e, y, z) = e$ by (366). Then

$$K((x\backslash R(x, y, z))\backslash(x\backslash e), x\backslash R(x, y, z)) = e \tag{367}$$

by (365). We have $(x\backslash e) \cdot K((x\backslash R(x, y, z))\backslash(x\backslash e), x\backslash R(x, y, z)) = ((x\backslash R(x, y, z))\backslash(x\backslash e)) \cdot (x\backslash R(x, y, z))$ by Theorem 17. Then

$$(x\backslash e) \cdot e = ((x\backslash R(x, y, z))\backslash(x\backslash e)) \cdot (x\backslash R(x, y, z)) \tag{368}$$

by (367). We have $(((x\backslash R(x, y, z))\backslash(x\backslash e)) \cdot (x\backslash R(x, y, z))) \cdot e = ((x\backslash R(x, y, z))\backslash(x\backslash e)) \cdot (x\backslash R(x, y, z))$ by Axiom 2. Then $(x\backslash e) \cdot e = (((x\backslash R(x, y, z))\backslash(x\backslash e)) \cdot (x\backslash R(x, y, z))) \cdot e$ by (368). Then $x\backslash e = ((x\backslash R(x, y, z))\backslash(x\backslash e)) \cdot (x\backslash R(x, y, z))$ by Proposition 10. Then

$$(x\backslash e)/(x\backslash R(x, y, z)) = (x\backslash R(x, y, z))\backslash(x\backslash e) \tag{369}$$

by Proposition 1. We have $((x\backslash e)/(R(x, y, z)\cdot(x\backslash e)))\cdot R(x, y, z) = R(x, y, z)\cdot R(R(x, y, z)\backslash e, R(x, y, z), x\backslash e)$ by Theorem 90. Then $((x\backslash e)/(x\backslash R(x, y, z))) \cdot R(x, y, z) = R(x, y, z) \cdot R(R(x, y, z)\backslash e, R(x, y, z), x\backslash e)$ by Proposition 89. Then $R(x \cdot R(x\backslash e, R(x, y, z), x\backslash e), y, z) = ((x\backslash e)/(x\backslash R(x, y, z))) \cdot R(x, y, z)$ by Theorem 641. Then

$$R(x \cdot T(x\backslash e, R(x, y, z)), y, z) = ((x\backslash e)/(x\backslash R(x, y, z))) \cdot R(x, y, z) \tag{370}$$

by Theorem 580. We have $R(x \cdot T(x\backslash e, R(x, y, z)), y, z) = K(R(x, y, z)\backslash e, R(x, y, z))$ by Theorem 637. Then

$$((x\backslash e)/(x\backslash R(x, y, z))) \cdot R(x, y, z) = K(R(x, y, z)\backslash e, R(x, y, z)) \tag{371}$$

by (370). We have $(R(x, y, z)\backslash e) \cdot R(x, y, z) = K(R(x, y, z)\backslash e, R(x, y, z))$ by Proposition 76. Then $(x\backslash e)/(x\backslash R(x, y, z)) = R(x, y, z)\backslash e$ by (371) and Proposition 8. Hence we are done by (369). ☐

Theorem 774. $a(R(x,y,z),e/x,x) = x\cdot((x\backslash R(x,y,z))/R(x,y,z))$.

Proof. We have $R(x,y,z)\backslash T(R(x,y,z),R(x\backslash e,(x\backslash e)\backslash e,R(x,y,z))) = (x\backslash e)\backslash((R(x,y,z)\cdot(x\backslash e))/R(x,y,z))$ by Theorem 760. Then $R(x,y,z)\backslash T(R(x,y,z),R(x\backslash e,(x\backslash e)\backslash e,R(x,y,z))) = (x\backslash e)\backslash((x\backslash R(x,y,z))/R(x,y,z))$ by Proposition 89. Then $R(x,y,z)\backslash T(R(x,y,z),R(e/x,x,R(x,y,z))) = (x\backslash e)\backslash((x\backslash R(x,y,z))/R(x,y,z))$ by Theorem 727. Then $R(x,y,z)\backslash R(T(x,R(e/x,x,R(x,y,z))),y,z) = (x\backslash e)\backslash((x\backslash R(x,y,z))/R(x,y,z))$ by Axiom 9. Then

$$R(x,y,z)\backslash R(T(x,x\backslash e),y,z) = (x\backslash e)\backslash((x\backslash R(x,y,z))/R(x,y,z)) \tag{372}$$

by Theorem 361. We have $R(x,y,z)\backslash R(T(x,x\backslash e),y,z) = a(R(x,y,z),e/x,x)$ by Theorem 545. Then

$$(x\backslash e)\backslash((x\backslash R(x,y,z))/R(x,y,z)) = a(R(x,y,z),e/x,x) \tag{373}$$

by (372). We have $(x\backslash e)\backslash((x\backslash R(x,y,z))/R(x,y,z)) = x\cdot((x\backslash R(x,y,z))/R(x,y,z))$ by Theorem 709. Hence we are done by (373). □

Theorem 775. $z\cdot L(z\backslash e,R(z,x,y),e/z) = a(R(z,x,y),e/z,z)$.

Proof. We have $T((R(z,x,y)\cdot(e/z))/R(z,x,y),z) = (R(z,x,y)\cdot(z\backslash e))/R(z,x,y)$ by Theorem 94. Then $T((R(z,x,y)\cdot(e/z))/R(z,x,y),z) = (z\backslash R(z,x,y))/R(z,x,y)$ by Proposition 89. Then

$$T(L(e/z,R(z,x,y),e/z),z) = (z\backslash R(z,x,y))/R(z,x,y) \tag{374}$$

by Theorem 579. We have $T(L(e/z,R(z,x,y),e/z),z) = L(z\backslash e,R(z,x,y),e/z)$ by Proposition 72. Then

$$(z\backslash R(z,x,y))/R(z,x,y) = L(z\backslash e,R(z,x,y),e/z) \tag{375}$$

by (374). We have $z\cdot((z\backslash R(z,x,y))/R(z,x,y)) = a(R(z,x,y),e/z,z)$ by Theorem 774. Hence we are done by (375). □

Theorem 776. $y/K(x,y) = y\cdot(T(((y\cdot x)\backslash y)\backslash e,y)\backslash(((y\cdot x)\backslash y)\backslash e))$.

Proof. We have $y/((y\cdot(T(((y\cdot x)\backslash y)\backslash e,y)\backslash(((y\cdot x)\backslash y)\backslash e)))\backslash y) = y\cdot(T(((y\cdot x)\backslash y)\backslash e,y)\backslash(((y\cdot x)\backslash y)\backslash e))$ by Proposition 24. Then $y/((((y\cdot x)\backslash y)\backslash e)\backslash T(((y\cdot x)\backslash y)\backslash e,y)) = y\cdot(T(((y\cdot x)\backslash y)\backslash e,y)\backslash(((y\cdot x)\backslash y)\backslash e))$ by Theorem 771. Then $y/((((y\cdot x)\backslash y)\backslash e)\backslash(((y\cdot x)\backslash y)\backslash K(x,y))) = y\cdot(T(((y\cdot x)\backslash y)\backslash e,y)\backslash(((y\cdot x)\backslash y)\backslash e))$ by Theorem 538. Hence we are done by Theorem 735. □

Theorem 777. $y\cdot(K(x,y)\backslash e) = K(x,y)\backslash y$.

Proof. We have $y\cdot(T(((y\cdot x)\backslash y)\backslash e,y)\backslash(((y\cdot x)\backslash y)\backslash e)) = y/K(x,y)$ by Theorem 776. Then $y\cdot(((y\cdot x)\backslash(y\cdot T(y\backslash(y\cdot x),y)))\backslash e) = y/K(x,y)$ by Theorem 764. Then $y\cdot(((y\cdot x)\backslash(y\cdot T(x,y)))\backslash e) = y/K(x,y)$ by Axiom 3. Then $y\cdot(((y\cdot x)\backslash(x\cdot y))\backslash e) = y/K(x,y)$ by Proposition 46. Then

$$y\cdot(K(x,y)\backslash e) = y/K(x,y) \tag{376}$$

by Definition 2. We have $y/K(x,y) = K(x,y)\backslash y$ by Theorem 766. Hence we are done by (376). □

Theorem 778. $(K(x,y)\backslash e)\cdot T(y,z) = K(x,y)\backslash T(y,z)$.

Proof. We have $((K(x,y)\backslash e)\cdot T((K(x,y)\backslash e)\backslash(K(x,y)\backslash y),K(x,y)\backslash e))/(K(x,y)\backslash e) = (K(x,y)\backslash e)\backslash(K(x,y)\backslash y)$ by Proposition 48. Then $(K(x,y)\backslash T(y,K(x,y)\backslash e))/(K(x,y)\backslash e) = (K(x,y)\backslash e)\backslash(K(x,y)\backslash y)$ by Theorem 496. Then

$$(K(x,y)\backslash y)/(K(x,y)\backslash e) = (K(x,y)\backslash e)\backslash(K(x,y)\backslash y) \tag{377}$$

by Theorem 772. We have $y\cdot(K(x,y)\backslash e) = K(x,y)\backslash y$ by Theorem 777. Then $(K(x,y)\backslash y)/(K(x,y)\backslash e) = y$ by Proposition 1. Then

$$y = (K(x,y)\backslash e)\backslash(K(x,y)\backslash y) \tag{378}$$

by (377). We have $(K(x,y)\backslash e)\cdot T((K(x,y)\backslash e)\backslash(K(x,y)\backslash y),z) = K(x,y)\backslash T(y,z)$ by Theorem 496. Hence we are done by (378). □

Theorem 779. $K(x,y) = K(x,(y\backslash e)\backslash e)$.

Proof. We have $L(y\backslash e, y, T(y, R(T(L(x\backslash e,x,y)\backslash e,y), T(L(x\backslash e,x,y)\backslash e,y)\backslash e,y))/y) = T(e/y, T(y, R(T(L(x\backslash e,x,y)\backslash e,y), T(L(x\backslash e$ by Theorem 503. Then $L(y\backslash e, y, T(L(x\backslash e,x,y)\backslash e,y)\backslash((y\cdot T(L(x\backslash e,x,y)\backslash e,y))/y)) = T(e/y, T(y, R(T(L(x\backslash e,x,y)\backslash e,y), T(L(x\backslash e,$ by Theorem 590. Then $y\backslash e = L(y\backslash e, y, T(L(x\backslash e,x,y)\backslash e,y)\backslash((y\cdot T(L(x\backslash e,x,y)\backslash e,y))/y))$ by Theorem 762. Then $L(y\backslash e, y, T(L(x\backslash e,x,y)\backslash e,y)\backslash(L(x\backslash e,x,y)\backslash e)) = y\backslash e$ by Proposition 48. Then

$$L(y\backslash e, y, K(x,y)\backslash e) = y\backslash e \tag{379}$$

by Theorem 763. We have $(K(x,y)\backslash e)\cdot T(y, K(x,y)\backslash e) = y\cdot(K(x,y)\backslash e)$ by Proposition 46. Then

$$(K(x,y)\backslash e)\cdot y = y\cdot(K(x,y)\backslash e) \tag{380}$$

by Theorem 772. We have $L(y\backslash e, y, K(x,y)\backslash e) = ((K(x,y)\backslash e)\cdot y)\backslash(K(x,y)\backslash e)$ by Proposition 78. Then $L(y\backslash e, y, K(x,y)\backslash e) = (y\cdot(K(x,y)\backslash e))\backslash(K(x,y)\backslash e)$ by (380). Then $(y\cdot(K(x,y)\backslash e))\backslash(K(x,y)\backslash e) = y\backslash e$ by (379). Then

$$(K(x,y)\backslash y)\backslash(K(x,y)\backslash e) = y\backslash e \tag{381}$$

by Theorem 777. We have $(K(x,y)\backslash y)\cdot((K(x,y)\backslash y)\backslash(K(x,y)\backslash e)) = K(x,y)\backslash e$ by Axiom 4. Then

$$(K(x,y)\backslash y)\cdot(y\backslash e) = K(x,y)\backslash e \tag{382}$$

by (381). We have $((K(x,y)\backslash y)\cdot(y\backslash e))\cdot((y\backslash e)\backslash e) = R(K(x,y)\backslash y, y\backslash e, y)$ by Theorem 662. Then

$$(K(x,y)\backslash e)\cdot((y\backslash e)\backslash e) = R(K(x,y)\backslash y, y\backslash e, y) \tag{383}$$

by (382). We have $(K(x,y)\backslash e)\cdot T(y, y\backslash e) = K(x,y)\backslash T(y, y\backslash e)$ by Theorem 778. Then $(K(x,y)\backslash e)\cdot((y\backslash e)\backslash e) = K(x,y)\backslash T(y, y\backslash e)$ by Proposition 49. Then $K(x,y)\backslash((y\backslash e)\backslash e) = (K(x,y)\backslash e)\cdot((y\backslash e)\backslash e)$ by Proposition 49. Then

$$R(K(x,y)\backslash y, y\backslash e, y) = K(x,y)\backslash((y\backslash e)\backslash e) \tag{384}$$

by (383). We have $R((x\cdot y)/(x\cdot K(x,y)), y\backslash e, y)\cdot(x\cdot K(x,y)) = (x\cdot K(x,y))\cdot R((x\cdot K(x,y))\backslash(x\cdot y), y\backslash e, y)$ by Proposition 83. Then

$$R(R(y,x,K(x,y)), y\backslash e, y)\cdot(x\cdot K(x,y)) = (x\cdot K(x,y))\cdot R((x\cdot K(x,y))\backslash(x\cdot y), y\backslash e, y) \tag{385}$$

by Theorem 461. We have $R(R(y,x,K(x,y)), y\backslash e, y)\cdot(x\cdot K(x,y)) = (R(y, y\backslash e, y)\cdot x)\cdot K(x,y)$ by Theorem 468. Then

$$(x\cdot K(x,y))\cdot R((x\cdot K(x,y))\backslash(x\cdot y), y\backslash e, y) = (R(y, y\backslash e, y)\cdot x)\cdot K(x,y) \tag{386}$$

by (385). We have $(x\cdot K(x,y))\cdot R((x\cdot K(x,y))\backslash(x\cdot y), y\backslash e, y) = x\cdot(K(x,y)\cdot R(K(x,y)\backslash y, y\backslash e, y))$ by Theorem 516. Then $(R(y, y\backslash e, y)\cdot x)\cdot K(x,y) = x\cdot(K(x,y)\cdot R(K(x,y)\backslash y, y\backslash e, y))$ by (386). Then $x\cdot(K(x,y)\cdot R(K(x,y)\backslash y, y\backslash e, y)) = (T(y, y\backslash e)\cdot x)\cdot K(x,y)$ by Theorem 207. Then

$$x\cdot(K(x,y)\cdot(K(x,y)\backslash((y\backslash e)\backslash e))) = (T(y, y\backslash e)\cdot x)\cdot K(x,y) \tag{387}$$

by (384). We have $x\backslash(x\cdot((y\backslash e)\backslash e)) = (y\backslash e)\backslash e$ by Axiom 3. Then

$$x\backslash((T(y, y\backslash e)\cdot x)\cdot((T(y, y\backslash e)\cdot x)\backslash(x\cdot((y\backslash e)\backslash e)))) = (y\backslash e)\backslash e \tag{388}$$

by Axiom 4. We have $x\cdot(x\backslash((T(y, y\backslash e)\cdot x)\cdot((T(y, y\backslash e)\cdot x)\backslash(x\cdot((y\backslash e)\backslash e))))) = (T(y, y\backslash e)\cdot x)\cdot((T(y, y\backslash e)\cdot x)\backslash(x\cdot((y\backslash e)\backslash e)))$ by Axiom 4. Then $x\cdot(K(x,y)\cdot(K(x,y)\backslash(x\backslash((T(y, y\backslash e)\cdot x)\cdot((T(y, y\backslash e)\cdot x)\backslash(x\cdot((y\backslash e)\backslash e))))))) = (T(y, y\backslash e)\cdot x)\cdot((T(y, y\backslash e)\cdot x)\backslash(x\cdot((y\backslash e)\backslash e)))$ by Axiom 4. Then $x\cdot(K(x,y)\cdot(K(x,y)\backslash((y\backslash e)\backslash e))) = (T(y, y\backslash e)\cdot x)\cdot((T(y, y\backslash e)\cdot x)\backslash(x\cdot((y\backslash e)\backslash e)))$ by (388). Then $(T(y, y\backslash e)\cdot x)\cdot((T(y, y\backslash e)\cdot x)\backslash(x\cdot((y\backslash e)\backslash e))) = (T(y, y\backslash e)\cdot x)\cdot K(x,y)$ by (387). Then

$$(T(y, y\backslash e)\cdot x)\backslash(x\cdot((y\backslash e)\backslash e)) = K(x,y) \tag{389}$$

by Proposition 9. We have $(T(y, y\backslash e)\cdot x)\backslash(x\cdot((y\backslash e)\backslash e)) = K(x,(y\backslash e)\backslash e)$ by Theorem 478. Hence we are done by (389). ∎

Theorem 780. $K(x, y\backslash e) = K(x, e/y)$.

Proof. We have $K(x, ((e/y)\backslash e)\backslash e) = K(x, e/y)$ by Theorem 779. Hence we are done by Proposition 25. $\square$

Theorem 781. $z \cdot ((z\backslash R(z, x, y))/R(z, x, y)) = K(R(z, x, y), z\backslash e)$.

Proof. We have $K(R(z, x, y), (R(z, x, y)/z)/R(z, x, y)) = z\cdot((R(z, x, y)\cdot(z\backslash e))/R(z, x, y))$ by Theorem 333. Then $K(R(z, x, y), e/z) = z\cdot((R(z, x, y)\cdot(z\backslash e))/R(z, x, y))$ by Theorem 533. Then $z\cdot((z\backslash R(z, x, y))/R(z, x, y)) = K(R(z, x, y), e/z)$ by Proposition 89. Hence we are done by Theorem 780. $\square$

Theorem 782. $K(R(z, x, y), z\backslash e) = a(R(z, x, y), e/z, z)$.

Proof. We have $z \cdot ((z\backslash R(z, x, y))/R(z, x, y)) = a(R(z, x, y), e/z, z)$ by Theorem 774. Hence we are done by Theorem 781. $\square$

Theorem 783. $R(e/(e/x), y, z)\backslash e = e/R(x, y, z)$.

Proof. We have $(((x\cdot y)\cdot z)/(y\cdot z))\cdot L((((x\cdot y)\cdot z)/(y\cdot z))\backslash e, R(x, y, z), e/x) = (((x\cdot y)\cdot z)\cdot L(((x\cdot y)\cdot z)\backslash(y\cdot z), R(x, y, z), e/x))/(y \cdot z)$ by Theorem 598. Then $R(x, y, z) \cdot L((((x \cdot y) \cdot z)/(y \cdot z))\backslash e, R(x, y, z), e/x) = (((x \cdot y) \cdot z) \cdot L(((x \cdot y) \cdot z)\backslash(y \cdot z), R(x, y, z), e/x))/(y \cdot z)$ by Definition 5. Then $(((x \cdot y) \cdot z) \cdot L(((x \cdot y) \cdot z)\backslash(y\cdot z), R(x, y, z), e/x))/(y\cdot z) = R(x, y, z)\cdot L(R(x, y, z)\backslash e, R(x, y, z), e/x)$ by Definition 5. Then $(((x\cdot y) \cdot L((x \cdot y)\backslash y, R(x, y, z), e/x)) \cdot z)/(y \cdot z) = R(x, y, z) \cdot L(R(x, y, z)\backslash e, R(x, y, z), e/x)$ by Theorem 332. Then

$$(((x \cdot L(x\backslash e, R(x, y, z), e/x)) \cdot y) \cdot z)/(y \cdot z) = R(x, y, z) \cdot L(R(x, y, z)\backslash e, R(x, y, z), e/x) \quad (390)$$

by Theorem 294. We have $R(x\cdot L(x\backslash e, R(x, y, z), e/x), y, z) = (((x\cdot L(x\backslash e, R(x, y, z), e/x))\cdot y)\cdot z)/(y\cdot z)$ by Definition 5. Then $R(x \cdot L(x\backslash e, R(x, y, z), e/x), y, z) = R(x, y, z) \cdot L(R(x, y, z)\backslash e, R(x, y, z), e/x)$ by (390). Then $R(x, y, z)\cdot((R(x, y, z)/x)\backslash((e/x)\cdot e)) = R(x\cdot L(x\backslash e, R(x, y, z), e/x), y, z)$ by Theorem 534. Then $R(x, y, z)\cdot((R(x, y, z)/x)\backslash(e/x)) = R(x\cdot L(x\backslash e, R(x, y, z), e/x), y, z)$ by Axiom 2. Then $R(a(R(x, y, z), e/x, x), y, z) = R(x, y, z) \cdot ((R(x, y, z)/x)\backslash(e/x))$ by Theorem 775. Then

$$R(x, y, z) \cdot ((R(x, y, z)/x)\backslash(e/x)) = R(K(R(x, y, z), x\backslash e), y, z) \quad (391)$$

by Theorem 782. We have $R(x, y, z) \cdot ((R(x, y, z) \cdot (R(x, y, z)\backslash e))/R(x, y, z)) = R(x \cdot ((R(x, y, z) \cdot (x\backslash e))/R(x, y, z)), y, z)$ by Theorem 642. Then

$$R(x, y, z) \cdot (e/R(x, y, z)) = R(x \cdot ((R(x, y, z) \cdot (x\backslash e))/R(x, y, z)), y, z) \quad (392)$$

by Axiom 4. We have

$$K(R(x, y, z), R(x, y, z)\backslash e) = R(x, y, z) \cdot (e/R(x, y, z)) \quad (393)$$

by Theorem 250. Then $K(R(x, y, z), R(x, y, z)\backslash e) = R(x \cdot ((R(x, y, z) \cdot (x\backslash e))/R(x, y, z)), y, z)$ by (392). Then $R(x\cdot((x\backslash R(x, y, z))/R(x, y, z)), y, z) = K(R(x, y, z), R(x, y, z)\backslash e)$ by Proposition 89. Then $R(K(R(x, y, z), x\backslash e), y, z) = K(R(x, y, z), R(x, y, z)\backslash e)$ by Theorem 781. Then $R(x, y, z)\cdot((R(x, y, z)/x)\backslash(e/x)) = K(R(x, y, z), R(x, y, z)\backslash e)$ by (391). Then

$$(R(x, y, z)/x)\backslash(e/x) = e/R(x, y, z) \quad (394)$$

by (393) and Proposition 7. We have $((e/(e/x))\backslash R(e/(e/x), y, z))\backslash((e/(e/x))\backslash e) = R(e/(e/x), y, z)\backslash e$ by Theorem 773. Then $((e/(e/x))\backslash R(e/(e/x), y, z))\backslash(e/x) = R(e/(e/x), y, z)\backslash e$ by Proposition 25. Then

$$((e/x) \cdot R((e/x)\backslash e, y, z))\backslash(e/x) = R(e/(e/x), y, z)\backslash e \quad (395)$$

by Theorem 187. We have $R(e/(e/x), y, z) \cdot (e/x) = (e/x) \cdot R((e/x)\backslash e, y, z)$ by Proposition 83. Then $R(x, y, z)/x = (e/x)\cdot R((e/x)\backslash e, y, z)$ by Theorem 191. Then $(R(x, y, z)/x)\backslash(e/x) = R(e/(e/x), y, z)\backslash e$ by (395). Hence we are done by (394). $\square$

The following was first conjectured in [9].

Theorem 784. $e/(e/R(x,y,z)) = R(e/(e/x),y,z)$.

Proof. We have $e/(R(e/(e/x),y,z)\backslash e) = R(e/(e/x),y,z)$ by Proposition 24. Hence we are done by Theorem 783. ☐

Theorem 785. $R(x,x\backslash y,z) = (y\cdot z)/((x\backslash y)\cdot z)$.

Proof. We have $R(x,x\backslash y,z) = ((x\cdot(x\backslash y))\cdot z)/((x\backslash y)\cdot z)$ by Definition 5. Hence we are done by Axiom 4. ☐

Theorem 786. $R(x,y,(x\cdot y)\backslash z) = z/(y\cdot((x\cdot y)\backslash z))$.

Proof. We have $R(x,y,(x\cdot y)\backslash z) = ((x\cdot y)\cdot((x\cdot y)\backslash z))/(y\cdot((x\cdot y)\backslash z))$ by Definition 5. Hence we are done by Axiom 4. ☐

Theorem 787. $R(x,z,T(y,z)) = ((x\cdot z)\cdot T(y,z))/(y\cdot z)$.

Proof. We have $R(x,z,T(y,z)) = ((x\cdot z)\cdot T(y,z))/(z\cdot T(y,z))$ by Definition 5. Hence we are done by Proposition 46. ☐

Theorem 788. $T(y,T(y,x)) = y$.

Proof. We have $T(T(y,x),y) = T(y,x)$ by Theorem 451. Hence we are done by Proposition 21. ☐

Theorem 789. $T(y,x) = T(T(y,x),T(y,z))$.

Proof. We have $T(T(y,T(y,z)),x) = T(T(y,x),T(y,z))$ by Axiom 7. Hence we are done by Theorem 788. ☐

Theorem 790. $x\backslash((x\cdot y)\cdot L(z,y,x)) = y\cdot z$.

Proof. We have $x\backslash(x\cdot(y\cdot z)) = y\cdot z$ by Axiom 3. Then $x\backslash((x\cdot y)\cdot((x\cdot y)\backslash(x\cdot(y\cdot z)))) = y\cdot z$ by Axiom 4. Hence we are done by Definition 4. ☐

Theorem 791. $R(x\backslash e,x,y) = (K(x\backslash e,x)\cdot y)/(x\cdot y)$.

Proof. We have $R(x\backslash e,x,y) = (((x\backslash e)\cdot x)\cdot y)/(x\cdot y)$ by Definition 5. Hence we are done by Proposition 76. ☐

Theorem 792. $T(x\backslash e,x) = x\backslash K(x\backslash e,x)$.

Proof. We have $T(x\backslash e,x) = x\backslash((x\backslash e)\cdot x)$ by Definition 3. Hence we are done by Proposition 76. ☐

Theorem 793. $T(y,z)\cdot T(y,x) = T(y,x)\cdot T(y,z)$.

Proof. We have $T(y,z)\cdot T(T(y,x),T(y,z)) = T(y,x)\cdot T(y,z)$ by Proposition 46. Hence we are done by Theorem 789. ☐

Theorem 794. $R(y/x,x,y\backslash z) = z/(x\cdot(y\backslash z))$.

Proof. We have $(z/(y\backslash z))\cdot(y\backslash z) = z$ by Axiom 6. Then $(((z/(y\backslash z))/x)\cdot x)\cdot(y\backslash z) = z$ by Axiom 6. Then

$$((y/x)\cdot x)\cdot(y\backslash z) = z \tag{396}$$

by Proposition 24. We have $R(y/x,x,y\backslash z) = (((y/x)\cdot x)\cdot(y\backslash z))/(x\cdot(y\backslash z))$ by Definition 5. Hence we are done by (396). ☐

Theorem 795. $(y\backslash z)\backslash z = x\cdot(x\backslash T(y,y\backslash z))$.

Proof. We have $(z/T(y,y\backslash z))\backslash z = x\cdot(x\backslash T(y,y\backslash z))$ by Theorem 19. Hence we are done by Theorem 15. ☐

Theorem 796. $x/L(y, e/y, x) = x \cdot (e/y)$.

Proof. We have $x/L((e/y)\backslash e, e/y, x) = x \cdot (e/y)$ by Theorem 33. Hence we are done by Proposition 25. □

Theorem 797. $R(z \cdot y, L(x, y, z), w) = ((z \cdot (y \cdot x)) \cdot w)/(L(x, y, z) \cdot w)$.

Proof. We have $R(z \cdot y, L(x, y, z), w) = (((z \cdot y) \cdot L(x, y, z)) \cdot w)/(L(x, y, z) \cdot w)$ by Definition 5. Hence we are done by Proposition 52. □

Theorem 798. $L(T(x, y)\backslash z, T(x, y), y) = (x \cdot y)\backslash(y \cdot z)$.

Proof. We have $L(T(x, y)\backslash z, T(x, y), y) = (y \cdot T(x, y))\backslash(y \cdot z)$ by Proposition 53. Hence we are done by Proposition 46. □

Theorem 799. $L(x\backslash((y \cdot x)/y), x, y) = K(y, (y \cdot x)/y)$.

Proof. We have $(y \cdot x)\backslash(y \cdot ((y \cdot x)/y)) = K(y, (y \cdot x)/y)$ by Theorem 3. Hence we are done by Proposition 53. □

Theorem 800. $R(x/y, y, z)\backslash(x \cdot z) = y \cdot z$.

Proof. We have $((x \cdot z)/(y \cdot z))\backslash(x \cdot z) = y \cdot z$ by Proposition 25. Hence we are done by Proposition 55. □

Theorem 801. $R(z/x, x, T(y, z))\backslash(y \cdot z) = x \cdot T(y, z)$.

Proof. We have $R(z/x, x, T(y, z))\backslash(z \cdot T(y, z)) = x \cdot T(y, z)$ by Theorem 800. Hence we are done by Proposition 46. □

Theorem 802. $R(x \cdot (w\backslash e), y, z) = R(R(x/w, y, z), w, w\backslash e)$.

Proof. We have $R(R(x/w, w, w\backslash e), y, z) = R(R(x/w, y, z), w, w\backslash e)$ by Axiom 12. Hence we are done by Proposition 71. □

Theorem 803. $x\backslash L(L(y, x\backslash e, x), z, w) = (x\backslash e) \cdot L(y, z, w)$.

Proof. We have $x\backslash L(L(y, z, w), x\backslash e, x) = (x\backslash e) \cdot L(y, z, w)$ by Proposition 57. Hence we are done by Axiom 11. □

Theorem 804. $(z\backslash w) \cdot L((z\backslash w)\backslash w, x, y) = L(z, x, y) \cdot (z\backslash w)$.

Proof. We have $(z\backslash w) \cdot L(T(z, z\backslash w), x, y) = L(z, x, y) \cdot (z\backslash w)$ by Proposition 58. Hence we are done by Proposition 49. □

Theorem 805. $y\backslash(L(x, z, y) \cdot (y \cdot z)) = z \cdot T(x, y \cdot z)$.

Proof. We have $y\backslash((y \cdot z) \cdot L(T(x, y \cdot z), z, y)) = z \cdot T(x, y \cdot z)$ by Theorem 790. Hence we are done by Proposition 58. □

Theorem 806. $x \cdot ((y\backslash z)\backslash z) = x \cdot T(y, y\backslash z)$.

Proof. We have $x \cdot (x\backslash(x \cdot T(y, y\backslash z))) = x \cdot T(y, y\backslash z)$ by Axiom 4. Then $x \cdot (e \cdot (e\backslash(x\backslash(x \cdot T(y, y\backslash z))))) = x \cdot T(y, y\backslash z)$ by Axiom 4. Then $x \cdot (e \cdot (e\backslash T(y, y\backslash z))) = x \cdot T(y, y\backslash z)$ by Axiom 3. Hence we are done by Theorem 795. □

Theorem 807. $(z \cdot T(x, x\backslash y))/((x\backslash y)\backslash y) = z$.

Proof. We have

$$((z \cdot T(x, x\backslash y))/((x\backslash y)\backslash y)) \cdot T(x, x\backslash y) = ((z \cdot T(x, x\backslash y))/((x\backslash y)\backslash y)) \cdot ((x\backslash y)\backslash y) \quad (397)$$

by Theorem 806. We have $z \cdot T(x, x\backslash y) = ((z \cdot T(x, x\backslash y))/((x\backslash y)\backslash y)) \cdot ((x\backslash y)\backslash y)$ by Axiom 6. Hence we are done by (397) and Proposition 8. □

Theorem 808. $R(x, x\backslash(y/z), z)\backslash y = (x\backslash(y/z)) \cdot z$.

Proof. We have $e \cdot y = y$ by Axiom 1. Then

$$y \cdot (y\backslash(e \cdot y)) = y \tag{398}$$

by Axiom 4. We have $y \cdot (y\backslash(e \cdot y)) = e \cdot y$ by Axiom 4. Then $((y \cdot (y\backslash(e \cdot y)))/z) \cdot z = e \cdot y$ by Axiom 6. Then

$$(y/z) \cdot z = e \cdot y \tag{399}$$

by (398). We have $e\backslash(e \cdot y) = y$ by Axiom 3. Then

$$e\backslash(((e \cdot y)/((x\backslash(y/z)) \cdot z)) \cdot ((x\backslash(y/z)) \cdot z)) = y \tag{400}$$

by Axiom 6. We have $((e \cdot y)/((x\backslash(y/z)) \cdot z)) \cdot ((x\backslash(y/z)) \cdot z) = e\backslash(((e \cdot y)/((x\backslash(y/z)) \cdot z)) \cdot ((x\backslash(y/z)) \cdot z))$ by Proposition 26. Then $(x\backslash(y/z)) \cdot z = ((e \cdot y)/((x\backslash(y/z)) \cdot z))\backslash(e\backslash(((e \cdot y)/((x\backslash(y/z)) \cdot z)) \cdot ((x\backslash(y/z)) \cdot z)))$ by Proposition 2. Then $((e \cdot y)/((x\backslash(y/z)) \cdot z))\backslash y = (x\backslash(y/z)) \cdot z$ by (400). Then $(((y/z) \cdot z)/((x\backslash(y/z)) \cdot z))\backslash y = (x\backslash(y/z)) \cdot z$ by (399). Hence we are done by Theorem 785. ☐

Theorem 809. $R(T(y, x), x/y, y) = T(y, x/y)$.

Proof. We have $R(T(y, (x/y) \cdot y), x/y, y) = T(y, x/y)$ by Theorem 113. Hence we are done by Axiom 6. ☐

Theorem 810. $K(x\backslash e, x)\backslash x = T(x, x\backslash e)$.

Proof. We have $e \cdot x = x$ by Axiom 1. Then

$$(x \cdot (x\backslash e)) \cdot x = x \tag{401}$$

by Axiom 4. We have $((x\backslash e) \cdot x) \cdot T(x, x\backslash e) = (x \cdot (x\backslash e)) \cdot x$ by Theorem 114. Then $((x\backslash e) \cdot x)\backslash((x \cdot (x\backslash e)) \cdot x) = T(x, x\backslash e)$ by Proposition 2. Then $T(x, x\backslash e) = ((x\backslash e) \cdot x)\backslash x$ by (401). Hence we are done by Proposition 76. ☐

Theorem 811. $R(R(y/x, z, w), x, x\backslash e)/(x\backslash e) = x \cdot R(x\backslash y, z, w)$.

Proof. We have $R(R(y/x, z, w), x, x\backslash e)/(x\backslash e) = R(y/x, z, w) \cdot x$ by Theorem 73. Hence we are done by Proposition 83. ☐

Theorem 812. $z \cdot R(x, y, y\backslash z) = (((z \cdot x)/z) \cdot y) \cdot (y\backslash z)$.

Proof. We have $R((z \cdot x)/z, y, y\backslash z) \cdot z = (((z \cdot x)/z) \cdot y) \cdot (y\backslash z)$ by Theorem 25. Hence we are done by Theorem 87. ☐

Theorem 813. $((x/y) \cdot R(y, z, w))/(x/y) = R(x/(x/y), z, w)$.

Proof. We have $((x/y) \cdot R((x/y)\backslash x, z, w))/(x/y) = R(x/(x/y), z, w)$ by Theorem 88. Hence we are done by Proposition 25. ☐

Theorem 814. $K(x, e/x)\backslash(e/x) = x\backslash e$.

Proof. We have $T(e/x, (e/x)\backslash e) = ((e/x)\backslash e)\backslash e$ by Proposition 49. Then $K((e/x)\backslash e, e/x)\backslash(e/x) = ((e/x)\backslash e)\backslash e$ by Theorem 810. Then $K(x, e/x)\backslash(e/x) = ((e/x)\backslash e)\backslash e$ by Proposition 25. Hence we are done by Proposition 25. ☐

Theorem 815. $(x \cdot y) \cdot ((w \cdot L(z, y, x))/w) = x \cdot (y \cdot ((w \cdot z)/w))$.

Proof. We have $(x \cdot y) \cdot L((w \cdot z)/w, y, x) = x \cdot (y \cdot ((w \cdot z)/w))$ by Proposition 52. Hence we are done by Theorem 122. ☐

Theorem 816. $(x \cdot y) \cdot L(L(z, T(x, y), y), w, u) = y \cdot (T(x, y) \cdot L(z, w, u))$.

Proof. We have $(x \cdot y) \cdot L(L(z,w,u),T(x,y),y) = y \cdot (T(x,y) \cdot L(z,w,u))$ by Theorem 58. Hence we are done by Axiom 11. ☐

Theorem 817. $R((w \cdot u)\backslash(u \cdot x),y,z) = L(R(T(w,u)\backslash x,y,z),T(w,u),u)$.

Proof. We have $R(L(T(w,u)\backslash x,T(w,u),u),y,z) = L(R(T(w,u)\backslash x,y,z),T(w,u),u)$ by Axiom 10. Hence we are done by Theorem 798. ☐

Theorem 818. $(x \cdot y)/x = (y/x) \cdot T(x,y)$.

Proof. We have $(x \cdot T((y/x) \cdot T(x,y),x))/x = (y/x) \cdot T(x,y)$ by Proposition 48. Hence we are done by Theorem 493. ☐

Theorem 819. $T(x,y)/(z\backslash e) = T((x/(z\backslash e))/z,y) \cdot z$.

Proof. We have $T((x/(z\backslash e)) \cdot (z\backslash e),y)/(z\backslash e) = T((x/(z\backslash e))/z,y) \cdot z$ by Theorem 490. Hence we are done by Axiom 6. ☐

Theorem 820. $y\backslash T(x,y\backslash e) = ((y\backslash e)\backslash(y\backslash x)) \cdot (y\backslash e)$.

Proof. We have $(y\backslash e) \cdot T((y\backslash e)\backslash(y\backslash x),y\backslash e) = ((y\backslash e)\backslash(y\backslash x)) \cdot (y\backslash e)$ by Proposition 46. Hence we are done by Theorem 496. ☐

Theorem 821. $(y\backslash(x/(y\backslash e))) \cdot y = T(x,y)/(y\backslash e)$.

Proof. We have $T((x/(y\backslash e))/y,y) \cdot y = T(x,y)/(y\backslash e)$ by Theorem 819. Hence we are done by Proposition 47. ☐

Theorem 822. $R(y,x,T(y,x)) = x\backslash T(y \cdot x,y)$.

Proof. We have $((y \cdot x) \cdot T(y,x))/(y \cdot x) = T(((y \cdot x) \cdot y)/(y \cdot x),x)$ by Theorem 50. Then $R(y,x,T(y,x)) = T(((y \cdot x) \cdot y)/(y \cdot x),x)$ by Theorem 787. Then

$$T(T(y \cdot x,y)/x,x) = R(y,x,T(y,x)) \tag{402}$$

by Theorem 508. We have $T(T(y \cdot x,y)/x,x) = x\backslash T(y \cdot x,y)$ by Proposition 47. Hence we are done by (402). ☐

Theorem 823. $L(R(y,x,T(y,x)),x,y) = y$.

Proof. We have

$$(y \cdot x) \cdot L(x\backslash T(y \cdot x,y),x,y) = y \cdot (x \cdot (x\backslash T(y \cdot x,y))) \tag{403}$$

by Proposition 52. We have $(y \cdot x) \cdot y = y \cdot (x \cdot (x\backslash T(y \cdot x,y)))$ by Theorem 9. Then $L(x\backslash T(y \cdot x,y),x,y) = y$ by (403) and Proposition 7. Hence we are done by Theorem 822. ☐

Theorem 824. $(x/y) \cdot ((x \cdot y)/x) = T(x,x/(x/y))$.

Proof. We have $(x/y) \cdot ((x \cdot ((x/y)\backslash x))/x) = T(x,x/(x/y))$ by Theorem 515. Hence we are done by Proposition 25. ☐

Theorem 825. $(y/T(x,y)) \cdot x = T(y,y/(y/T(x,y)))$.

Proof. We have $(y/T(x,y)) \cdot ((y \cdot T(x,y))/y) = T(y,y/(y/T(x,y)))$ by Theorem 824. Hence we are done by Proposition 48. ☐

Theorem 826. $x\backslash R(x \cdot y,z,w) = (x\backslash e) \cdot R((x\backslash e)\backslash y,z,w)$.

Proof. We have $x\backslash R(L((x\backslash e)\backslash y,x\backslash e,x),z,w) = (x\backslash e) \cdot R((x\backslash e)\backslash y,z,w)$ by Theorem 474. Hence we are done by Theorem 62. ☐

Theorem 827. $x\backslash L(x \cdot y,z,w) = (x\backslash e) \cdot L((x\backslash e)\backslash y,z,w)$.

Proof. We have $x\backslash L(L((x\backslash e)\backslash y, x\backslash e, x), z, w) = (x\backslash e) \cdot L((x\backslash e)\backslash y, z, w)$ by Theorem 803. Hence we are done by Theorem 62. □

Theorem 828. $(y \cdot R(x, z, z\backslash y))/(z\backslash y) = ((y \cdot x)/y) \cdot z$.

Proof. We have $(((y \cdot x)/y) \cdot z) \cdot (z\backslash y) = y \cdot R(x, z, z\backslash y)$ by Theorem 812. Hence we are done by Proposition 1. □

Theorem 829. $(x \cdot y) \cdot L((x \cdot (y\backslash e))/x, y, x) = T(x, x/(x \cdot y))$.

Proof. We have $(x \cdot y) \cdot ((x \cdot ((x \cdot y)\backslash x))/x) = T(x, x/(x \cdot y))$ by Theorem 515. Hence we are done by Theorem 127. □

Theorem 830. $T(y, x) \cdot z = y \cdot (T(y, x) \cdot (y\backslash z))$.

Proof. We have $T(y, x) \cdot (y \cdot (y\backslash z)) = y \cdot (T(y, x) \cdot (y\backslash z))$ by Theorem 172. Hence we are done by Axiom 4. □

Theorem 831. $(T(z, x) \cdot y)/(T(z, x) \cdot (z\backslash y)) = z$.

Proof. We have $(T(z, x) \cdot y)/(z\backslash (T(z, x) \cdot y)) = z$ by Proposition 24. Hence we are done by Theorem 173. □

Theorem 832. $z \cdot (T(z, y)\backslash ((z\backslash T(z, y)) \cdot x)) = L(x, z\backslash T(z, y), z)$.

Proof. We have $T(z, y)\backslash (z \cdot ((z\backslash T(z, y)) \cdot x)) = L(x, z\backslash T(z, y), z)$ by Theorem 449. Hence we are done by Theorem 174. □

Theorem 833. $T(y, z) \cdot (y\backslash T(y, x)) = T(y, x) \cdot (y\backslash T(y, z))$.

Proof. We have $y\backslash (T(y, z) \cdot T(y, x)) = T(y, z) \cdot (y\backslash T(y, x))$ by Theorem 173. Then

$$y\backslash (T(y, x) \cdot T(y, z)) = T(y, z) \cdot (y\backslash T(y, x)) \tag{404}$$

by Theorem 793. We have $y\backslash (T(y, x) \cdot T(y, z)) = T(y, x) \cdot (y\backslash T(y, z))$ by Theorem 173. Hence we are done by (404). □

Theorem 834. $(x\backslash R(x, y, z))/(x\backslash e) = R(x, y, z)$.

Proof. We have $R(x, y, z) \cdot (x\backslash e) = x\backslash R(x, y, z)$ by Proposition 89. Hence we are done by Proposition 1. □

Theorem 835. $R(z, x, y)\backslash (z\backslash R(z, x, y)) = z\backslash e$.

Proof. We have $R(z, x, y) \cdot (z\backslash e) = z\backslash R(z, x, y)$ by Proposition 89. Hence we are done by Proposition 2. □

Theorem 836. $(x\backslash e) \cdot L((x\backslash e)\backslash e, y, z) = x\backslash L(x, y, z)$.

Proof. We have $(x\backslash e) \cdot L((x\backslash e)\backslash e, y, z) = x\backslash L(x \cdot e, y, z)$ by Theorem 827. Hence we are done by Axiom 2. □

Theorem 837. $z\backslash L(z, x, y) = L(z, x, y) \cdot (z\backslash e)$.

Proof. We have $(z\backslash e) \cdot L((z\backslash e)\backslash e, x, y) = L(z, x, y) \cdot (z\backslash e)$ by Theorem 804. Hence we are done by Theorem 836. □

Theorem 838. $((z\backslash e)/(z\backslash L(z, x, y))) \cdot L(z, x, y) = L(z, x, y) \cdot R(L(z, x, y)\backslash e, L(z, x, y), z\backslash e)$.

Proof. We have $((z\backslash e)/(L(z, x, y) \cdot (z\backslash e))) \cdot L(z, x, y) = L(z, x, y) \cdot R(L(z, x, y)\backslash e, L(z, x, y), z\backslash e)$ by Theorem 90. Hence we are done by Theorem 837. □

Theorem 839. $T(e/z, R(z,x,y)) = R(z,x,y)\backslash(R(z,x,y)/z)$.

Proof. We have $T(e/z, R(z,x,y)) = R(z,x,y)\backslash((e/z) \cdot R(z,x,y))$ by Definition 3. Hence we are done by Theorem 192. ☐

Theorem 840. $((y\backslash(y/x)) \cdot y) \cdot x = y \cdot (x \cdot T(x\backslash e, y))$.

Proof. We have $(y \cdot x) \cdot T((y \cdot x)\backslash y, y) = y \cdot (x \cdot T(x\backslash e, y))$ by Theorem 98. Hence we are done by Theorem 530. ☐

Theorem 841. $(x \cdot T(x\backslash e, z)) \cdot (x\backslash y) = y \cdot T(y\backslash(x\backslash y), z)$.

Proof. We have $(T(e/x, z) \cdot x) \cdot (x\backslash y) = y \cdot T(y\backslash(x\backslash y), z)$ by Theorem 183. Hence we are done by Proposition 61. ☐

Theorem 842. $K(x\backslash e, x) \cdot x = x \cdot K(x\backslash e, x)$.

Proof. We have $K(x\backslash e, x) \cdot T(x, K(x\backslash e, x)) = x \cdot K(x\backslash e, x)$ by Proposition 46. Hence we are done by Theorem 755. ☐

Theorem 843. $(x \cdot T(x\backslash(y\backslash x), z))/(y\backslash x) = y \cdot T(y\backslash e, z)$.

Proof. We have $(y \cdot T(y\backslash e, z)) \cdot (y\backslash x) = x \cdot T(x\backslash(y\backslash x), z)$ by Theorem 841. Hence we are done by Proposition 1. ☐

Theorem 844. $(y \cdot x) \cdot ((w \cdot ((y \cdot x)\backslash z))/w) = y \cdot (x \cdot ((w \cdot (x\backslash(y\backslash z)))/w))$.

Proof. We have $(y \cdot x) \cdot ((w \cdot L(x\backslash(y\backslash z), x, y))/w) = y \cdot (x \cdot ((w \cdot (x\backslash(y\backslash z)))/w))$ by Theorem 815. Hence we are done by Proposition 70. ☐

Theorem 845. $T(y, y/(y \cdot x)) = y \cdot (x \cdot ((y \cdot (x\backslash e))/y))$.

Proof. We have $(y \cdot x) \cdot L((y \cdot (x\backslash e))/y, x, y) = y \cdot (x \cdot ((y \cdot (x\backslash e))/y))$ by Proposition 52. Hence we are done by Theorem 829. ☐

Theorem 846. $y\backslash T(y, y/(y \cdot x)) = x \cdot ((y \cdot (x\backslash e))/y)$.

Proof. We have $y\backslash((y \cdot x) \cdot L((y \cdot (x\backslash e))/y, x, y)) = x \cdot ((y \cdot (x\backslash e))/y)$ by Theorem 790. Hence we are done by Theorem 829. ☐

Theorem 847. $x \cdot y = ((x/T(y,z)) \cdot y) \cdot T(y,z)$.

Proof. We have $((x/T(y,z)) \cdot T(y,z)) \cdot y = ((x/T(y,z)) \cdot y) \cdot T(y,z)$ by Theorem 238. Hence we are done by Axiom 6. ☐

Theorem 848. $y \cdot (y \cdot x) = T(y, y/(y/T(x,y))) \cdot (x \cdot y)$.

Proof. We have $((y/T(x,y)) \cdot x) \cdot T(x,y) = y \cdot x$ by Theorem 847. Then

$$T(y, y/(y/T(x,y))) \cdot T(x,y) = y \cdot x \tag{405}$$

by Theorem 825. We have $y \cdot (T(y, y/(y/T(x,y))) \cdot T(x,y)) = T(y, y/(y/T(x,y))) \cdot (x \cdot y)$ by Theorem 520. Hence we are done by (405). ☐

Theorem 849. $K(x, x\backslash e) \cdot (x\backslash e) = e/x$.

Proof. We have $K(x, e/x) \cdot (K(x, e/x)\backslash(e/x)) = e/x$ by Axiom 4. Then $K(x, e/x) \cdot (x\backslash e) = e/x$ by Theorem 814. Hence we are done by Theorem 780. ☐

Theorem 850. $(e/(e/x))/(x \cdot x) = R(x\backslash e, x, x)$.

Proof. We have $(K(x\backslash e, x) \cdot x)/(x \cdot x) = R(x\backslash e, x, x)$ by Theorem 791. Then $(x \cdot K(x\backslash e, x))/(x \cdot x) = R(x\backslash e, x, x)$ by Theorem 842. Hence we are done by Theorem 262. □

Theorem 851. $R(e/(e/z), x, y) = L(R(z, x, y), z\backslash e, z)$.

Proof. We have $R(L(z, z\backslash e, z), x, y) = L(R(z, x, y), z\backslash e, z)$ by Axiom 10. Hence we are done by Theorem 263. □

Theorem 852. $K(z\backslash(z/(x\backslash(x/y))), z) = x\backslash((x/y) \cdot T(y, z))$.

Proof. We have $K(z\backslash(z/(x\backslash(x/y))), z) = x\backslash((x/y) \cdot T((x/y)\backslash x, z))$ by Theorem 285. Hence we are done by Proposition 25. □

Theorem 853. $K(x\backslash(x/(x\backslash(x/y))), x) = x\backslash((y \cdot x)/y)$.

Proof. We have $K(x\backslash(x/(x\backslash(x/y))), x) = x\backslash((x/y) \cdot T(y, x))$ by Theorem 852. Hence we are done by Theorem 818. □

Theorem 854. $y \cdot (((x \cdot y)/x) \cdot z) = ((x \cdot y)/x) \cdot (y \cdot z)$.

Proof. We have $((x \cdot y)/x) \cdot (T((x \cdot y)/x, x) \cdot z) = (x\backslash(x \cdot y)) \cdot (((x \cdot y)/x) \cdot z)$ by Theorem 521. Then $((x \cdot y)/x) \cdot (y \cdot z) = (x\backslash(x \cdot y)) \cdot (((x \cdot y)/x) \cdot z)$ by Theorem 7. Hence we are done by Axiom 3. □

Theorem 855. $y \cdot (((x \cdot y)/x)\backslash z) = ((x \cdot y)/x)\backslash(y \cdot z)$.

Proof. We have $y\backslash(y \cdot z) = z$ by Axiom 3. Then

$$y\backslash(((x \cdot y)/x) \cdot (((x \cdot y)/x)\backslash(y \cdot z))) = z \tag{406}$$

by Axiom 4. We have $y \cdot (y\backslash(((x \cdot y)/x) \cdot (((x \cdot y)/x)\backslash(y \cdot z)))) = ((x \cdot y)/x) \cdot (((x \cdot y)/x)\backslash(y \cdot z))$ by Axiom 4. Then $y \cdot (((x \cdot y)/x) \cdot (((x \cdot y)/x)\backslash(y\backslash(((x \cdot y)/x) \cdot (((x \cdot y)/x)\backslash(y \cdot z)))))) = ((x \cdot y)/x) \cdot (((x \cdot y)/x)\backslash(y \cdot z))$ by Axiom 4. Then

$$y \cdot (((x \cdot y)/x) \cdot (((x \cdot y)/x)\backslash z)) = ((x \cdot y)/x) \cdot (((x \cdot y)/x)\backslash(y \cdot z)) \tag{407}$$

by (406). We have $((x \cdot y)/x) \cdot (y \cdot (((x \cdot y)/x)\backslash z)) = y \cdot (((x \cdot y)/x) \cdot (((x \cdot y)/x)\backslash z))$ by Theorem 854. Hence we are done by (407) and Proposition 7. □

Theorem 856. $y \cdot ((x/(x/y)) \cdot z) = (x/(x/y)) \cdot (y \cdot z)$.

Proof. We have $(x/(x/y)) \cdot (T(x/(x/y), x/y) \cdot z) = ((x/y)\backslash x) \cdot ((x/(x/y)) \cdot z)$ by Theorem 521. Then $(x/(x/y)) \cdot (y \cdot z) = ((x/y)\backslash x) \cdot ((x/(x/y)) \cdot z)$ by Theorem 14. Hence we are done by Proposition 25. □

Theorem 857. $y \cdot ((x/(x/y))\backslash z) = (x/(x/y))\backslash(y \cdot z)$.

Proof. We have $y\backslash(y \cdot z) = z$ by Axiom 3. Then

$$y\backslash((x/(x/y)) \cdot ((x/(x/y))\backslash(y \cdot z))) = z \tag{408}$$

by Axiom 4. We have $y \cdot (y\backslash((x/(x/y)) \cdot ((x/(x/y))\backslash(y \cdot z)))) = (x/(x/y)) \cdot ((x/(x/y))\backslash(y \cdot z))$ by Axiom 4. Then $y \cdot ((x/(x/y)) \cdot ((x/(x/y))\backslash(y\backslash((x/(x/y)) \cdot ((x/(x/y))\backslash(y \cdot z)))))) = (x/(x/y)) \cdot ((x/(x/y))\backslash(y \cdot z))$ by Axiom 4. Then

$$y \cdot ((x/(x/y)) \cdot ((x/(x/y))\backslash z)) = (x/(x/y)) \cdot ((x/(x/y))\backslash(y \cdot z)) \tag{409}$$

by (408). We have $(x/(x/y)) \cdot (y \cdot ((x/(x/y))\backslash z)) = y \cdot ((x/(x/y)) \cdot ((x/(x/y))\backslash z))$ by Theorem 856. Hence we are done by (409) and Proposition 7. □

Theorem 858. $(e/y) \cdot ((x \cdot y)/x) = ((x \cdot y)/x)/y$.

Proof. We have $(e/y) \cdot ((x \cdot ((e/y)\backslash e))/x) = ((x \cdot y)/x)/y$ by Theorem 290. Hence we are done by Proposition 25. ☐

Theorem 859. $y \cdot ((x \cdot (y\backslash e))/x) = ((x \cdot (y\backslash e))/x)/(y\backslash e)$.

Proof. We have $(e/(y\backslash e)) \cdot ((x \cdot (y\backslash e))/x) = ((x \cdot (y\backslash e))/x)/(y\backslash e)$ by Theorem 858. Hence we are done by Proposition 24. ☐

Theorem 860. $(x \cdot z) \cdot ((y \cdot z)/y) = (x \cdot ((y \cdot z)/y)) \cdot z$.

Proof. We have $(x \cdot ((y \cdot z)/y)) \cdot T((y \cdot z)/y, y) = (x \cdot (y\backslash(y \cdot z))) \cdot ((y \cdot z)/y)$ by Theorem 240. Then $(x \cdot ((y \cdot z)/y)) \cdot z = (x \cdot (y\backslash(y \cdot z))) \cdot ((y \cdot z)/y)$ by Theorem 7. Hence we are done by Axiom 3. ☐

Theorem 861. $T(y, y/(y/x)) \cdot (y\backslash x) = (y \cdot x)/y$.

Proof. We have $((y/x) \cdot ((y \cdot x)/y)) \cdot x = ((y/x) \cdot x) \cdot ((y \cdot x)/y)$ by Theorem 860. Then

$$T(y, y/(y/x)) \cdot x = ((y/x) \cdot x) \cdot ((y \cdot x)/y) \tag{410}$$

by Theorem 824.

$$\begin{array}{lll} & y \cdot (T(y, y/(y/x)) \cdot (y\backslash x)) & \\ = & T(y, y/(y/x)) \cdot x & \text{by Theorem 830} \\ = & y \cdot ((y \cdot x)/y) & \text{by (410), Axiom 6.} \end{array}$$

Then $y \cdot (T(y, y/(y/x)) \cdot (y\backslash x)) = y \cdot ((y \cdot x)/y)$. Hence we are done by Proposition 9. ☐

Theorem 862. $R(x\backslash e, L(x, y, z), x\backslash e) = T(x\backslash e, L(x, y, z))$.

Proof. We have $T(x\backslash e, (x\backslash e) \cdot L((x\backslash e)\backslash e, y, z)) = x\backslash e$ by Theorem 304. Then

$$T(x\backslash e, L(x, y, z) \cdot (x\backslash e)) = x\backslash e \tag{411}$$

by Theorem 804. We have $R(T(x\backslash e, L(x, y, z) \cdot (x\backslash e)), L(x, y, z), x\backslash e) = T(x\backslash e, L(x, y, z))$ by Theorem 113. Hence we are done by (411). ☐

Theorem 863. $R(y \cdot (x\backslash e), z, w)/(x\backslash e) = x \cdot R(x\backslash y, z, w)$.

Proof. We have $R(R(y/x, z, w), x, x\backslash e)/(x\backslash e) = x \cdot R(x\backslash y, z, w)$ by Theorem 811. Hence we are done by Theorem 802. ☐

Theorem 864. $z \cdot R(e/z, x, y) = R(e/z, x, y)/(z\backslash e)$.

Proof. We have $R(K(z, z\backslash e) \cdot (z\backslash e), x, y)/(z\backslash e) = z \cdot R(z\backslash K(z, z\backslash e), x, y)$ by Theorem 863. Then $R(e/z, x, y)/(z\backslash e) = z \cdot R(z\backslash K(z, z\backslash e), x, y)$ by Theorem 849. Hence we are done by Theorem 251. ☐

Theorem 865. $T(y, T(e/y, x)) = R(y, T(e/y, x), y)$.

Proof. We have $R(T(y, y \cdot T(y\backslash e, x)), (y \cdot T(y\backslash e, x))/y, y) = T(y, (y \cdot T(y\backslash e, x))/y)$ by Theorem 809. Then $R(y, (y \cdot T(y\backslash e, x))/y, y) = T(y, (y \cdot T(y\backslash e, x))/y)$ by Theorem 203. Then $R(y, T(e/y, x), y) = T(y, (y \cdot T(y\backslash e, x))/y)$ by Theorem 49. Hence we are done by Theorem 49. ☐

Theorem 866. $(x/y) \cdot (y/x) = (y/x)/((x/y)\backslash e)$.

Proof. We have $R(e/(x/y), x/y, y)/((x/y)\backslash e) = (x/y) \cdot R(e/(x/y), x/y, y)$ by Theorem 864. Then $(y/x)/((x/y)\backslash e) = (x/y) \cdot R(e/(x/y), x/y, y)$ by Theorem 38. Hence we are done by Theorem 38. ☐

Theorem 867. $((x\backslash y)/y) \cdot x = x/(((x\backslash y)/y)\backslash e)$.

Proof. We have $(y/(x\backslash y))/(((x\backslash y)/y)\backslash e) = ((x\backslash y)/y)\cdot(y/(x\backslash y))$ by Theorem 866. Then $x/(((x\backslash y)/y)\backslash e) = ((x\backslash y)/y) \cdot (y/(x\backslash y))$ by Proposition 24. Hence we are done by Proposition 24. □

Theorem 868. $(x \cdot L(y, z, w)) \cdot y = (x \cdot y) \cdot L(y, z, w)$.

Proof. We have $L(((x \cdot y) \cdot y)/(x \cdot y), z, w) \cdot (x \cdot y) = (x \cdot y) \cdot L(y, z, w)$ by Theorem 86. Then $(L((x \cdot y)/x, z, w) \cdot x) \cdot y = (x \cdot y) \cdot L(y, z, w)$ by Theorem 157. Hence we are done by Theorem 86. □

Theorem 869. $z/L(z, x, y) = (e/L(z, x, y)) \cdot z$.

Proof. We have $e \cdot z = z$ by Axiom 1. Then

$$z \cdot (z\backslash(e \cdot z)) = z \tag{412}$$

by Axiom 4. We have $z\cdot(z\backslash(e\cdot z)) = e\cdot z$ by Axiom 4. Then $((z\cdot(z\backslash(e\cdot z)))/L(z, x, y))\cdot L(z, x, y) = e\cdot z$ by Axiom 6. Then $(z/L(z, x, y)) \cdot L(z, x, y) = e \cdot z$ by (412). Then

$$(z/L(z, x, y)) \cdot L(z, x, y) = ((e/L(z, x, y)) \cdot L(z, x, y)) \cdot z \tag{413}$$

by Axiom 6. We have $((e/L(z, x, y))\cdot L(z, x, y))\cdot z = ((e/L(z, x, y))\cdot z)\cdot L(z, x, y)$ by Theorem 868. Then $(z/L(z, x, y))\cdot L(z, x, y) = ((e/L(z, x, y))\cdot z)\cdot L(z, x, y)$ by (413). Hence we are done by Proposition 10. □

Theorem 870. $x \cdot K(x,(x/y)/x) = T(x, x/(x \cdot y))$.

Proof. We have $x \cdot (y \cdot ((x \cdot (y\backslash e))/x)) = T(x, x/(x \cdot y))$ by Theorem 845. Hence we are done by Theorem 333. □

Theorem 871. $T(x, R(y, x,(y \cdot x)\backslash x)) = x \cdot K(x, y)$.

Proof. We have $x\cdot K(x,(x/((y\cdot x)\backslash x))/x) = T(x, x/(x\cdot((y\cdot x)\backslash x)))$ by Theorem 870. Then $x\cdot K(x,(y\cdot x)/x) = T(x, x/(x \cdot ((y \cdot x)\backslash x)))$ by Proposition 24. Then $T(x, R(y, x,(y \cdot x)\backslash x)) = x \cdot K(x,(y \cdot x)/x)$ by Theorem 786. Hence we are done by Axiom 5. □

Theorem 872. $(y \cdot L(y\backslash(z/x), w, u)) \cdot x = y \cdot (x \cdot L(x\backslash(y\backslash z), w, u))$.

Proof. We have $L(R((z/x)/y, y, x), w, u) \cdot (y \cdot x) = (L((z/x)/y, w, u) \cdot y) \cdot x$ by Theorem 65. Then

$$L(z/(y \cdot x), w, u) \cdot (y \cdot x) = (L((z/x)/y, w, u) \cdot y) \cdot x \tag{414}$$

by Proposition 74. We have $L(z/(y\cdot x), w, u)\cdot(y\cdot x) = (y\cdot x)\cdot L((y\cdot x)\backslash z, w, u)$ by Theorem 85. Then $(L((z/x)/y, w, u) \cdot y) \cdot x = (y \cdot x) \cdot L((y \cdot x)\backslash z, w, u)$ by (414). Then

$$(y \cdot L(y\backslash(z/x), w, u)) \cdot x = (y \cdot x) \cdot L((y \cdot x)\backslash z, w, u) \tag{415}$$

by Theorem 85. We have $(y \cdot x) \cdot L((y \cdot x)\backslash z, w, u) = y \cdot (x \cdot L(x\backslash(y\backslash z), w, u))$ by Theorem 155. Hence we are done by (415). □

Theorem 873. $(x \cdot R(x\backslash y, w, u)) \cdot z = x \cdot (z \cdot R(z\backslash(x\backslash(y \cdot z)), w, u))$.

Proof. We have $(x \cdot z) \cdot R((x \cdot z)\backslash(y \cdot z), w, u) = x \cdot (z \cdot R(z\backslash(x\backslash(y \cdot z)), w, u))$ by Proposition 87. Hence we are done by Theorem 594. □

Theorem 874. $T(y \cdot L(y\backslash(z/x), w, u), x) = x\backslash(y \cdot (x \cdot L(x\backslash(y\backslash z), w, u)))$.

Proof. We have $T(y \cdot L(y\backslash(z/x), w, u), x) = x\backslash((y \cdot L(y\backslash(z/x), w, u)) \cdot x)$ by Definition 3. Hence we are done by Theorem 872. □

Theorem 875. $x\backslash(y \cdot ((w \cdot (y\backslash(x \cdot z)))/w)) = (x\backslash y) \cdot ((w \cdot ((x\backslash y)\backslash z))/w)$.

Proof. We have $x\cdot((x\backslash y)\cdot((w\cdot((x\backslash y)\backslash z))/w)) = y\cdot((w\cdot(y\backslash(x\cdot z)))/w)$ by Theorem 318. Hence we are done by Proposition 2. □

Theorem 876. $x\backslash(y\cdot((z\cdot(y\backslash x))/z)) = (x\backslash y)\cdot((z\cdot((x\backslash y)\backslash e))/z)$.

Proof. We have $x\backslash(y\cdot((z\cdot(y\backslash(x\cdot e)))/z)) = (x\backslash y)\cdot((z\cdot((x\backslash y)\backslash e))/z)$ by Theorem 875. Hence we are done by Axiom 2. □

Theorem 877. $(L(x\backslash e,x,y)\cdot y)/L(x\backslash e,x,y) = y\cdot(x\cdot T(x\backslash e,y))$.

Proof. We have $(y/L(x\backslash e,x,y))\cdot T(L(x\backslash e,x,y),y) = (L(x\backslash e,x,y)\cdot y)/L(x\backslash e,x,y)$ by Theorem 818. Then

$$(y\cdot x)\cdot T(L(x\backslash e,x,y),y) = (L(x\backslash e,x,y)\cdot y)/L(x\backslash e,x,y) \tag{416}$$

by Theorem 33. We have $(y\cdot x)\cdot T(L(x\backslash e,x,y),y) = y\cdot(x\cdot T(x\backslash e,y))$ by Theorem 55. Hence we are done by (416). □

Theorem 878. $y\cdot(T(x,y)\cdot L(T(x,y)\backslash z,w,u)) = x\cdot(y\cdot L(y\backslash(x\backslash(y\cdot z)),w,u))$.

Proof. We have $(x\cdot y)\cdot L(L(T(x,y)\backslash z,T(x,y),y),w,u) = y\cdot(T(x,y)\cdot L(T(x,y)\backslash z,w,u))$ by Theorem 816. Then

$$(x\cdot y)\cdot L((x\cdot y)\backslash(y\cdot z),w,u) = y\cdot(T(x,y)\cdot L(T(x,y)\backslash z,w,u)) \tag{417}$$

by Theorem 798. We have $(x\cdot y)\cdot L((x\cdot y)\backslash(y\cdot z),w,u) = x\cdot(y\cdot L(y\backslash(x\backslash(y\cdot z)),w,u))$ by Theorem 155. Hence we are done by (417). □

Theorem 879. $T(x,u)\cdot L(T(x,u)\backslash y,z,w) = T(x\cdot L(x\backslash((u\cdot y)/u),z,w),u)$.

Proof. We have $u\cdot(T(x,u)\cdot L(T(x,u)\backslash y,z,w)) = x\cdot(u\cdot L(u\backslash(x\backslash(u\cdot y)),z,w))$ by Theorem 878. Then

$$u\backslash(x\cdot(u\cdot L(u\backslash(x\backslash(u\cdot y)),z,w))) = T(x,u)\cdot L(T(x,u)\backslash y,z,w) \tag{418}$$

by Proposition 2. We have $u\backslash(x\cdot(u\cdot L(u\backslash(x\backslash(u\cdot y)),z,w))) = T(x\cdot L(x\backslash((u\cdot y)/u),z,w),u)$ by Theorem 874. Hence we are done by (418). □

Theorem 880. $(z\cdot((y/x)/z))\cdot x = (z\cdot x)\cdot((z\cdot((z\cdot x)\backslash y))/z)$.

Proof. We have $(z\cdot x)\cdot((z\cdot((z\cdot x)\backslash((y/x)\cdot x)))/z) = (((z\cdot((y/x)/z))/z)\cdot z)\cdot x$ by Theorem 608. Then $(z\cdot x)\cdot((z\cdot((z\cdot x)\backslash y))/z) = (((z\cdot((y/x)/z))/z)\cdot z)\cdot x$ by Axiom 6. Hence we are done by Axiom 6. □

Theorem 881. $(x\cdot((x/y)/x))\cdot y = T(x,x/(x\cdot y))$.

Proof. We have $(x\cdot y)\cdot((x\cdot((x\cdot y)\backslash x))/x) = T(x,x/(x\cdot y))$ by Theorem 515. Hence we are done by Theorem 880. □

Theorem 882. $(z\cdot((y/x)/z))\cdot x = z\cdot(x\cdot((z\cdot(x\backslash(z\backslash y)))/z))$.

Proof. We have $(z\cdot x)\cdot((z\cdot((z\cdot x)\backslash y))/z) = z\cdot(x\cdot((z\cdot(x\backslash(z\backslash y)))/z))$ by Theorem 844. Hence we are done by Theorem 880. □

Theorem 883. $(L(y,e/y,x)\cdot x)/L(y,e/y,x) = x\cdot(T(y,x)/y)$.

Proof. We have $(x/L(y,e/y,x))\cdot T(L(y,e/y,x),x) = (L(y,e/y,x)\cdot x)/L(y,e/y,x)$ by Theorem 818. Then

$$(x\cdot(e/y))\cdot T(L(y,e/y,x),x) = (L(y,e/y,x)\cdot x)/L(y,e/y,x) \tag{419}$$

by Theorem 796. We have $(x\cdot(e/y))\cdot T(L(y,e/y,x),x) = x\cdot((e/y)\cdot T(y,x))$ by Theorem 55. Then $(L(y,e/y,x)\cdot x)/L(y,e/y,x) = x\cdot((e/y)\cdot T(y,x))$ by (419). Hence we are done by Theorem 150. □

Theorem 884. $(z\cdot(y\cdot T(y\backslash e,w)))\cdot x = (z\cdot y)\cdot(x\cdot T(x\backslash L(y\backslash x,y,z),w))$.

Proof. We have $(z \cdot y) \cdot (x \cdot T(x\backslash((z \cdot y)\backslash(z \cdot x)), w)) = ((z \cdot y) \cdot T((z \cdot y)\backslash z, w)) \cdot x$ by Theorem 301. Then $(z \cdot y) \cdot (x \cdot T(x\backslash L(y\backslash x, y, z), w)) = ((z \cdot y) \cdot T((z \cdot y)\backslash z, w)) \cdot x$ by Proposition 53. Hence we are done by Theorem 98. ☐

Theorem 885. $(z\backslash e) \cdot R(z, x, y) = T((z\backslash e) \cdot R(z, x, y), z\backslash e)$.

Proof. We have $T(z\backslash e, z\backslash R(z, x, y)) = z\backslash e$ by Theorem 578. Then

$$T(z\backslash R(z, x, y), z\backslash e) = z\backslash R(z, x, y) \tag{420}$$

by Proposition 21. We have $(z\backslash e) \cdot (T(R(z, x, y) \cdot (z\backslash e), z\backslash e)/(z\backslash e)) = T((z\backslash e) \cdot R(z, x, y), z\backslash e)$ by Theorem 584. Then $(z\backslash e) \cdot (T(z\backslash R(z, x, y), z\backslash e)/(z\backslash e)) = T((z\backslash e) \cdot R(z, x, y), z\backslash e)$ by Proposition 89. Then $(z\backslash e) \cdot ((z\backslash R(z, x, y))/(z\backslash e)) = T((z\backslash e) \cdot R(z, x, y), z\backslash e)$ by (420). Hence we are done by Theorem 834. ☐

Theorem 886. $R(T(x, T(e/x, R(x, y, z))), y, z) = R(T(x, x\backslash e), y, z)$.

Proof. We have $L((e/(e/(R(x,y,z)\backslash(R(x,y,z)/x))))/((R(x,y,z)\backslash(R(x,y,z)/x))\cdot(R(x,y,z)\backslash(R(x,y,z)/x))), R(x,y,z)\backslash(R(x,y,z$
$((R(x,y,z)\backslash(R(x,y,z)/x))\cdot(R(x,y,z)\backslash(R(x,y,z)/x))) = ((R(x,y,z)\backslash(R(x,y,z)/x))\cdot(R(x,y,z)\backslash(R(x,y,z)/x)))\cdot$
$L(((R(x,y,z)\backslash(R(x,y,z)/x))\cdot(R(x,y,z)\backslash(R(x,y,z)/x)))\backslash(e/(e/(R(x,y,z)\backslash(R(x,y,z)/x)))), R(x,y,z)\backslash(R(x,y,z)/x), R(x,y,z))$
by Theorem 85. Then

$$L(R((R(x,y,z)\backslash(R(x,y,z)/x))\backslash e, R(x,y,z)\backslash(R(x,y,z)/x), R(x,y,z)\backslash(R(x,y,z)/x)), R(x,y,z)\backslash(R(x,y,z)/x), R(x,y,z))\cdot((R(x,y \tag{421}$$

by Theorem 850. We have $L(R((R(x,y,z)\backslash(R(x,y,z)/x))\backslash e, R(x,y,z)\backslash(R(x,y,z)/x), R(x,y,z)\backslash(R(x,y,z)/x)), R(x,y,z)\backslash(R(x,y$
$((R(x,y,z)\backslash(R(x,y,z)/x))\cdot(R(x,y,z)\backslash(R(x,y,z)/x))) = (L((R(x,y,z)\backslash(R(x,y,z)/x))\backslash e, R(x,y,z)\backslash(R(x,y,z)/x), R(x,y,z))\cdot$
$(R(x,y,z)\backslash(R(x,y,z)/x))) \cdot (R(x,y,z)\backslash(R(x,y,z)/x))$ by Theorem 65. Then

$$((R(x,y,z)\backslash(R(x,y,z)/x))\cdot(R(x,y,z)\backslash(R(x,y,z)/x)))\cdot L(((R(x,y,z)\backslash(R(x,y,z)/x))\cdot(R(x,y,z)\backslash(R(x,y,z)/x)))\backslash(e/(e/(R(x,y,z) \tag{422}$$

by (421). We have $((R(x,y,z)\backslash(R(x,y,z)/x))\cdot(R(x,y,z)\backslash(R(x,y,z)/x)))\cdot L(((R(x,y,z)\backslash(R(x,y,z)/x))\cdot$
$(R(x,y,z)\backslash(R(x,y,z)/x)))\backslash(e/(e/(R(x,y,z)\backslash(R(x,y,z)/x)))), R(x,y,z)\backslash(R(x,y,z)/x), R(x,y,z)) = (R(x,y,z)\backslash(R(x,y,z)/x))\cdot$
$((R(x,y,z)\backslash(R(x,y,z)/x))\cdot L((R(x,y,z)\backslash(R(x,y,z)/x))\backslash((R(x,y,z)\backslash(R(x,y,z)/x))\backslash(e/(e/(R(x,y,z)\backslash(R(x,y,z)/x))))), R(x,y,z$
by Theorem 155. Then $(L((R(x,y,z)\backslash(R(x,y,z)/x))\backslash e, R(x,y,z)\backslash(R(x,y,z)/x), R(x,y,z))\cdot(R(x,y,z)\backslash(R(x,y,z)/x)))\cdot$
$(R(x,y,z)\backslash(R(x,y,z)/x)) = (R(x,y,z)\backslash(R(x,y,z)/x))\cdot((R(x,y,z)\backslash(R(x,y,z)/x))\cdot L((R(x,y,z)\backslash(R(x,y,z)/x))\backslash((R(x,y,z)\backslash(R($
by (422). Then $(R(x,y,z)\backslash(R(x,y,z)/x))\cdot((R(x,y,z)\backslash(R(x,y,z)/x))\cdot L((R(x,y,z)\backslash(R(x,y,z)/x))\backslash K((R(x,y,z)\backslash(R(x,y,z)/x))\backslash$
$(L((R(x,y,z)\backslash(R(x,y,z)/x))\backslash e, R(x,y,z)\backslash(R(x,y,z)/x), R(x,y,z))\cdot(R(x,y,z)\backslash(R(x,y,z)/x)))\cdot(R(x,y,z)\backslash(R(x,y,z)/x))$
by Theorem 257. Then $(R(x,y,z)\backslash(R(x,y,z)/x))\cdot((R(x,y,z)\backslash(R(x,y,z)/x))\cdot L(T((R(x,y,z)\backslash(R(x,y,z)/x))\backslash e, R(x,y,z)\backslash(R(x,y$
$(L((R(x,y,z)\backslash(R(x,y,z)/x))\backslash e, R(x,y,z)\backslash(R(x,y,z)/x), R(x,y,z))\cdot(R(x,y,z)\backslash(R(x,y,z)/x)))\cdot(R(x,y,z)\backslash(R(x,y,z)/x))$
by Theorem 792. Then $(R(x,y,z)\backslash(R(x,y,z)/x))\cdot(L((R(x,y,z)\backslash(R(x,y,z)/x))\backslash e, R(x,y,z)\backslash(R(x,y,z)/x), R(x,y,z))\cdot$
$(R(x,y,z)\backslash(R(x,y,z)/x))) = (L((R(x,y,z)\backslash(R(x,y,z)/x))\backslash e, R(x,y,z)\backslash(R(x,y,z)/x), R(x,y,z))\cdot(R(x,y,z)\backslash(R(x,y,z)/x)))\cdot$
$(R(x,y,z)\backslash(R(x,y,z)/x))$ by Proposition 58. Then $T(R(x,y,z)\backslash(R(x,y,z)/x), L((R(x,y,z)\backslash(R(x,y,z)/x))\backslash e, R(x,y,z)\backslash(R(x,y,$
$(R(x,y,z)\backslash(R(x,y,z)/x))) = R(x,y,z)\backslash(R(x,y,z)/x)$ by Theorem 11. Then $T(R(x,y,z)\backslash(R(x,y,z)/x), ((R(x,y,z)/x)\backslash R(x,y,z$
$(R(x,y,z)\backslash(R(x,y,z)/x))) = R(x,y,z)\backslash(R(x,y,z)/x)$ by Theorem 28. Then

$$T(R(x,y,z)\backslash(R(x,y,z)/x), x\cdot(R(x,y,z)\backslash(R(x,y,z)/x))) = R(x,y,z)\backslash(R(x,y,z)/x) \tag{423}$$

by Proposition 25. We have $R(T(R(x,y,z)\backslash(R(x,y,z)/x), x\cdot(R(x,y,z)\backslash(R(x,y,z)/x))), x, R(x,y,z)\backslash(R(x,y,z)/x)) = T(R(x,y,z)\backslash(R(x,y,z)/x), x)$ by Theorem 113. Then

$$R(R(x,y,z)\backslash(R(x,y,z)/x), x, R(x,y,z)\backslash(R(x,y,z)/x)) = T(R(x,y,z)\backslash(R(x,y,z)/x), x) \tag{424}$$

by (423). We have $R(R(x,y,z)\backslash(R(x,y,z)/x), x, R(x,y,z)\backslash(R(x,y,z)/x))\backslash T((R(x,y,z)\backslash(R(x,y,z)/x))\cdot x, R(x,y,z)\backslash(R(x,y,z)/x)) = T(x, R(x,y,z)\backslash(R(x,y,z)/x))$ by Theorem 231. Then

$$T(R(x,y,z)\backslash(R(x,y,z)/x), x)\backslash T((R(x,y,z)\backslash(R(x,y,z)/x))\cdot x, R(x,y,z)\backslash(R(x,y,z)/x)) = T(x, R(x,y,z)\backslash(R(x,y,z)/x)) \tag{425}$$

by (424). We have $T(R(x,y,z)\backslash(R(x,y,z)/x),(R(x,y,z)\backslash(R(x,y,z)/x))\cdot((R(x,y,z)/x)\backslash R(x,y,z))) = R(x,y,z)\backslash(R(x,y,z)/x)$ by Theorem 306. Then $T(R(x,y,z)\backslash(R(x,y,z)/x),(R(x,y,z)\backslash(R(x,y,z)/x))\cdot x) = R(x,y,z)\backslash(R(x,y,z)/x)$ by Proposition 25. Then $T((R(x,y,z)\backslash(R(x,y,z)/x))\cdot x, R(x,y,z)\backslash(R(x,y,z)/x)) = (R(x,y,z)\backslash(R(x,y,z)/x))\cdot x$ by Proposition 21. Then

$$T(R(x,y,z)\backslash(R(x,y,z)/x),x)\backslash((R(x,y,z)\backslash(R(x,y,z)/x))\cdot x) = T(x,R(x,y,z)\backslash(R(x,y,z)/x)) \quad (426)$$

by (425). We have $T(R(x,y,z)\backslash(R(x,y,z)/x),x)\backslash((R(x,y,z)\backslash(R(x,y,z)/x))\cdot x) = T(x,T(R(x,y,z)\backslash(R(x,y,z)/x),x))$ by Theorem 13. Then $T(x,R(x,y,z)\backslash(R(x,y,z)/x)) = T(x,T(R(x,y,z)\backslash(R(x,y,z)/x),x))$ by (426). Then $T(x,T(T(e/x,R(x,y,z)),x)) = T(x,R(x,y,z)\backslash(R(x,y,z)/x))$ by Theorem 839. Then $T(x,R(x,y,z)\backslash(R(x,y,z)/x)) = T(x,T(x\backslash e,R(x,y,z)))$ by Proposition 51. Then

$$T(x,T(e/x,R(x,y,z))) = T(x,T(x\backslash e,R(x,y,z))) \quad (427)$$

by Theorem 839. We have $R(x\backslash e,R(x,y,z),x\backslash e)\backslash T((x\backslash e)\cdot R(x,y,z),x\backslash e) = T(R(x,y,z),x\backslash e)$ by Theorem 231. Then $T(x\backslash e,R(x,y,z))\backslash T((x\backslash e)\cdot R(x,y,z),x\backslash e) = T(R(x,y,z),x\backslash e)$ by Theorem 580. Then

$$T(x\backslash e,R(x,y,z))\backslash((x\backslash e)\cdot R(x,y,z)) = T(R(x,y,z),x\backslash e) \quad (428)$$

by Theorem 885. We have $T(x\backslash e,R(x,y,z))\backslash((x\backslash e)\cdot R(x,y,z)) = T(R(x,y,z),T(x\backslash e,R(x,y,z)))$ by Theorem 13. Then

$$T(R(x,y,z),x\backslash e) = T(R(x,y,z),T(x\backslash e,R(x,y,z))) \quad (429)$$

by (428). We have $R(T(x,T(x\backslash e,R(x,y,z))),y,z) = T(R(x,y,z),T(x\backslash e,R(x,y,z)))$ by Axiom 9. Then

$$R(T(x,T(x\backslash e,R(x,y,z))),y,z) = T(R(x,y,z),x\backslash e) \quad (430)$$

by (429). We have $R(T(x,x\backslash e),y,z) = T(R(x,y,z),x\backslash e)$ by Axiom 9. Then $R(T(x,x\backslash e),y,z) = R(T(x,T(x\backslash e,R(x,y,z))),y,z)$ by (430). Hence we are done by (427). □

Theorem 887. $T(x\cdot y,y\backslash e) = (((y\backslash e)\backslash x)\cdot(y\backslash e))\cdot y$.

Proof. We have $(((y\backslash e)\backslash x)\cdot(y\backslash e))\cdot(e/(y\backslash e)) = T(x\cdot(e/(y\backslash e)),y\backslash e)$ by Theorem 610. Then $(((y\backslash e)\backslash x)\cdot(y\backslash e))\cdot y = T(x\cdot(e/(y\backslash e)),y\backslash e)$ by Proposition 24. Hence we are done by Proposition 24. □

Theorem 888. $(y\backslash T(x,y\backslash e))\cdot y = T((y\backslash x)\cdot y,y\backslash e)$.

Proof. We have $(((y\backslash e)\backslash(y\backslash x))\cdot(y\backslash e))\cdot y = T((y\backslash x)\cdot y,y\backslash e)$ by Theorem 887. Hence we are done by Theorem 820. □

Theorem 889. $(y\backslash((y\backslash e)\backslash x))\cdot y = (y\backslash e)\backslash T(x,y)$.

Proof. We have $T((y\backslash(x/(y\backslash e)))\cdot y,y\backslash e) = (y\backslash T(x/(y\backslash e),y\backslash e))\cdot y$ by Theorem 888. Then $T(T(x,y)/(y\backslash e),y\backslash e) = (y\backslash T(x/(y\backslash e),y\backslash e))\cdot y$ by Theorem 821. Then

$$(y\backslash((y\backslash e)\backslash x))\cdot y = T(T(x,y)/(y\backslash e),y\backslash e) \quad (431)$$

by Proposition 47. We have $T(T(x,y)/(y\backslash e),y\backslash e) = (y\backslash e)\backslash T(x,y)$ by Proposition 47. Hence we are done by (431). □

Theorem 890. $(y\backslash x)\cdot y = (y\backslash e)\backslash T((y\backslash e)\cdot x,y)$.

Proof. We have $(y\backslash((y\backslash e)\backslash((y\backslash e)\cdot x)))\cdot y = (y\backslash e)\backslash T((y\backslash e)\cdot x,y)$ by Theorem 889. Hence we are done by Axiom 3. □

Theorem 891. $(y\backslash e)\cdot((y\backslash x)\cdot y) = T((y\backslash e)\cdot x,y)$.

Proof. We have $(y\backslash e)\cdot((y\backslash e)\backslash T((y\backslash e)\cdot x,y)) = T((y\backslash e)\cdot x,y)$ by Axiom 4. Hence we are done by Theorem 890. □

Theorem 892. $(y\backslash e)\cdot(x\cdot y) = T((y\backslash e)\cdot(y\cdot x), y)$.

Proof. We have $(y\backslash e)\cdot((y\backslash(y\cdot x))\cdot y) = T((y\backslash e)\cdot(y\cdot x), y)$ by Theorem 891. Hence we are done by Axiom 3. □

Theorem 893. $((y\backslash e)\cdot(y\cdot x))\cdot y = L(x\cdot y, y\backslash e, y)$.

Proof. We have $y\cdot T((y\backslash e)\cdot(y\cdot x), y) = ((y\backslash e)\cdot(y\cdot x))\cdot y$ by Proposition 46. Then

$$y\cdot((y\backslash e)\cdot(x\cdot y)) = ((y\backslash e)\cdot(y\cdot x))\cdot y \tag{432}$$

by Theorem 892. We have $y\cdot((y\backslash e)\cdot(x\cdot y)) = L(x\cdot y, y\backslash e, y)$ by Proposition 56. Hence we are done by (432). □

Theorem 894. $L(y\cdot x, x\backslash e, x)/x = (x\backslash e)\cdot(x\cdot y)$.

Proof. We have $((x\backslash e)\cdot(x\cdot y))\cdot x = L(y\cdot x, x\backslash e, x)$ by Theorem 893. Hence we are done by Proposition 1. □

Theorem 895. $K(y, x\backslash(x/y)) = K(y, y\backslash e)$.

Proof. We have $K(y, ((x/y)\cdot y)\backslash(x/y)) = K(y, y\backslash e)$ by Theorem 392. Hence we are done by Axiom 6. □

Theorem 896. $(y\cdot K(x,y))\backslash K(x,y) = y\backslash e$.

Proof. We have $((L(L(x\backslash e,x,y)\backslash e, L(x\backslash e,x,y),y)\cdot y)/L(L(x\backslash e,x,y)\backslash e, L(x\backslash e,x,y),y))\cdot(y\backslash e) = y\backslash((L(L(x\backslash e,x,y)\backslash e, L(x\backslash e,x,y),y)\cdot y)/L(L(x\backslash e,x,y)\backslash e, L(x\backslash e,x,y),y))$ by Theorem 195. Then $(y\cdot(L(x\backslash e,x,y)\cdot T(L(x\backslash e,x,y)\backslash e,y)))\cdot(y\backslash e) = y\backslash((L(L(x\backslash e,x,y)\backslash e, L(x\backslash e,x,y),y)\cdot y)/L(L(x\backslash e,x,y)\backslash e, L(x\backslash e,x,y),y))$ by Theorem 877. Then

$$y\backslash(y\cdot(L(x\backslash e,x,y)\cdot T(L(x\backslash e,x,y)\backslash e,y))) = (y\cdot(L(x\backslash e,x,y)\cdot T(L(x\backslash e,x,y)\backslash e,y)))\cdot(y\backslash e) \tag{433}$$

by Theorem 877. We have $y\backslash(y\cdot(L(x\backslash e,x,y)\cdot T(L(x\backslash e,x,y)\backslash e,y))) = L(x\backslash e,x,y)\cdot T(L(x\backslash e,x,y)\backslash e,y)$ by Axiom 3. Then $(y\cdot(L(x\backslash e,x,y)\cdot T(L(x\backslash e,x,y)\backslash e,y)))\cdot(y\backslash e) = L(x\backslash e,x,y)\cdot T(L(x\backslash e,x,y)\backslash e,y)$ by (433). Then $(y\cdot(L(x\backslash e,x,y)\cdot T(L(x\backslash e,x,y)\backslash e,y)))\backslash(L(x\backslash e,x,y)\cdot T(L(x\backslash e,x,y)\backslash e,y)) = y\backslash e$ by Proposition 2. Then $(y\cdot K(x,y))\backslash(L(x\backslash e,x,y)\cdot T(L(x\backslash e,x,y)\backslash e,y)) = y\backslash e$ by Theorem 542. Hence we are done by Theorem 542. □

Theorem 897. $(y\cdot K(x,y))\cdot(y\backslash e) = K(x,y)$.

Proof. We have $(y\cdot K(x,y))\cdot((y\cdot K(x,y))\backslash K(x,y)) = K(x,y)$ by Axiom 4. Hence we are done by Theorem 896. □

Theorem 898. $K(x,y)/(y\backslash e) = y\cdot K(x,y)$.

Proof. We have $K(x,y)/((y\cdot K(x,y))\backslash K(x,y)) = y\cdot K(x,y)$ by Proposition 24. Hence we are done by Theorem 896. □

Theorem 899. $T(K(y,z), K(x,y)) = K(y,z)$.

Proof. We have $L(T(y\backslash T(y,z), K(x,y)), y, z) = T((z\cdot y)\backslash(y\cdot z), K(x,y))$ by Theorem 103. Then

$$L(y\backslash T(y,z), y, z) = T((z\cdot y)\backslash(y\cdot z), K(x,y)) \tag{434}$$

by Theorem 619. We have $L(y\backslash T(y,z), y, z) = (z\cdot y)\backslash(y\cdot z)$ by Proposition 73. Then

$$T((z\cdot y)\backslash(y\cdot z), K(x,y)) = (z\cdot y)\backslash(y\cdot z) \tag{435}$$

by (434). We have $K(y,z) = (z\cdot y)\backslash(y\cdot z)$ by Definition 2. Then $K(y,z) = T((z\cdot y)\backslash(y\cdot z), K(x,y))$ by (435). Hence we are done by Definition 2. □

Theorem 900. $K(x,y) = (K(x,y)\backslash e)\backslash e$.

Proof. We have $T(K(x,y),(x\cdot y)\backslash((x\cdot y)/K(x,y))) = T(K(x,y),K(x,y)\backslash e)$ by Theorem 383. Then $T(K(x,y),(x\cdot y)\backslash(y\cdot x)) = T(K(x,y),K(x,y)\backslash e)$ by Theorem 452. Then

$$T(K(x,y),K(y,x)) = T(K(x,y),K(x,y)\backslash e) \tag{436}$$

by Definition 2. We have $T(K(x,y),K(x,y)\backslash e) = (K(x,y)\backslash e)\backslash e$ by Proposition 49. Then $T(K(x,y),K(y,x)) = (K(x,y)\backslash e)\backslash e$ by (436). Hence we are done by Theorem 899. ☐

Theorem 901. $e/K(x,y) = K(x,y)\backslash e$.

Proof. We have $e/((K(x,y)\backslash e)\backslash e) = K(x,y)\backslash e$ by Proposition 24. Hence we are done by Theorem 900. ☐

Theorem 902. $L(K(y\backslash e,y),x,K(y\backslash e,y)) = K(y\backslash e,y)$.

Proof. We have $L(y\cdot T(y\backslash e,y),x,K(y\backslash e,y)) = K(y\backslash(y/L(y,x,K(y\backslash e,y))),y)$ by Theorem 635. Then $L((y\backslash e)\cdot y,x,K(y\backslash e,y)) = K(y\backslash(y/L(y,x,K(y\backslash e,y))),y)$ by Proposition 46. Then

$$K(y\backslash(y/L(y,x,K(y\backslash e,y))),y) = L(K(y\backslash e,y),x,K(y\backslash e,y)) \tag{437}$$

by Proposition 76. We have $T(e/L(y,x,K(y\backslash e,y)),y) = y\backslash((e/L(y,x,K(y\backslash e,y)))\cdot y)$ by Definition 3. Then $T(e/L(y,x,K(y\backslash e,y)),y) = y\backslash(y/L(y,x,K(y\backslash e,y)))$ by Theorem 869. Then $K(T(e/L(y,x,K(y\backslash e,y)),y),y) = L(K(y\backslash e,y),x,K(y\backslash e,y))$ by (437). Then $K(T(e/y,y),y) = L(K(y\backslash e,y),x,K(y\backslash e,y))$ by Theorem 629. Hence we are done by Proposition 47. ☐

Theorem 903. $T(K(y\backslash e,y)\cdot x,K(y\backslash e,y)) = x\cdot K(y\backslash e,y)$.

Proof. We have $(K(y\backslash e,y)\cdot x)\cdot L(K(y\backslash e,y),x,K(y\backslash e,y)) = K(y\backslash e,y)\cdot(x\cdot K(y\backslash e,y))$ by Proposition 52. Then $(K(y\backslash e,y)\cdot x)\cdot K(y\backslash e,y) = K(y\backslash e,y)\cdot(x\cdot K(y\backslash e,y))$ by Theorem 902. Hence we are done by Theorem 11. ☐

Theorem 904. $(x\cdot y)\cdot(L(z,y,x)\cdot R(L(z,y,x)\backslash e,w,u)) = x\cdot(y\cdot(z\cdot R(z\backslash e,w,u)))$.

Proof. We have $((x\cdot y)\cdot L(z,y,x))\cdot R(((x\cdot y)\cdot L(z,y,x))\backslash(x\cdot y),w,u) = (x\cdot y)\cdot(L(z,y,x)\cdot R(L(z,y,x)\backslash e,w,u))$ by Theorem 133. Then $(x\cdot(y\cdot z))\cdot R(((x\cdot y)\cdot L(z,y,x))\backslash(x\cdot y),w,u) = (x\cdot y)\cdot(L(z,y,x)\cdot R(L(z,y,x)\backslash e,w,u))$ by Proposition 52. Then

$$(x\cdot(y\cdot z))\cdot R((x\cdot(y\cdot z))\backslash(x\cdot y),w,u) = (x\cdot y)\cdot(L(z,y,x)\cdot R(L(z,y,x)\backslash e,w,u)) \tag{438}$$

by Proposition 52. We have $(x\cdot(y\cdot z))\cdot R((x\cdot(y\cdot z))\backslash(x\cdot y),w,u) = x\cdot((y\cdot z)\cdot R((y\cdot z)\backslash y,w,u))$ by Theorem 516. Then $(x\cdot y)\cdot(L(z,y,x)\cdot R(L(z,y,x)\backslash e,w,u)) = x\cdot((y\cdot z)\cdot R((y\cdot z)\backslash y,w,u))$ by (438). Hence we are done by Theorem 133. ☐

Theorem 905. $L(x\cdot R(x\backslash e,w,u),y,z) = L(x,y,z)\cdot R(L(x,y,z)\backslash e,w,u)$.

Proof. We have $(z\cdot y)\cdot(L(x,y,z)\cdot R(L(x,y,z)\backslash e,w,u)) = z\cdot(y\cdot(x\cdot R(x\backslash e,w,u)))$ by Theorem 904. Hence we are done by Theorem 54. ☐

Theorem 906. $(x\backslash e)/(x\backslash L(x,y,z)) = L(x,y,z)\backslash e$.

Proof. We have $L(x,y,z)\cdot R(L(x,y,z)\backslash e,L(x,y,z),x\backslash e) = ((x\backslash e)/(x\backslash L(x,y,z)))\cdot L(x,y,z)$ by Theorem 838. Then $L(x\cdot R(x\backslash e,L(x,y,z),x\backslash e),y,z) = ((x\backslash e)/(x\backslash L(x,y,z)))\cdot L(x,y,z)$ by Theorem 905. Then

$$L(x\cdot T(x\backslash e,L(x,y,z)),y,z) = ((x\backslash e)/(x\backslash L(x,y,z)))\cdot L(x,y,z) \tag{439}$$

by Theorem 862. We have $L(x\cdot T(x\backslash e,L(x,y,z)),y,z) = K(L(x,y,z)\backslash(L(x,y,z)/L(x,y,z)),L(x,y,z))$ by Theorem 635. Then $L(x\cdot T(x\backslash e,L(x,y,z)),y,z) = K(L(x,y,z)\backslash e,L(x,y,z))$ by Proposition 29. Then

$$((x\backslash e)/(x\backslash L(x,y,z)))\cdot L(x,y,z) = K(L(x,y,z)\backslash e,L(x,y,z)) \tag{440}$$

by (439). We have $(L(x,y,z)\backslash e)\cdot L(x,y,z) = K(L(x,y,z)\backslash e,L(x,y,z))$ by Proposition 76. Hence we are done by (440) and Proposition 8. ☐

Theorem 907. $x \cdot K(y\backslash e, y) = (x \cdot y)/((y\backslash e)\backslash e)$.

Proof. We have $((x \cdot y)/y) \cdot K(y\backslash e, y) = (x \cdot y)/((y\backslash e)\backslash e)$ by Theorem 423. Hence we are done by Axiom 5. $\square$

Theorem 908. $(x \cdot K(y\backslash e, y)) \cdot ((y\backslash e)\backslash e) = x \cdot y$.

Proof. We have $((x{\cdot}y)/((y\backslash e)\backslash e)){\cdot}((y\backslash e)\backslash e) = x{\cdot}y$ by Axiom 6. Hence we are done by Theorem 907. $\square$

Theorem 909. $x \cdot y = (x \cdot T(y, y\backslash e)) \cdot K(y\backslash e, y)$.

Proof. We have $((x \cdot T(y, y\backslash e))/((y\backslash e)\backslash e)) \cdot y = (x \cdot T(y, y\backslash e)) \cdot K(y\backslash e, y)$ by Theorem 424. Hence we are done by Theorem 807. $\square$

Theorem 910. $y \cdot K(x\backslash y, y\backslash x) = x \cdot (((x\backslash y)\backslash e)\backslash e)$.

Proof. We have $x{\cdot}T(x\backslash y, y\backslash x) = y{\cdot}K((y\backslash x)\backslash((y\backslash x)/(y\backslash x)), y\backslash x)$ by Theorem 284. Then $x{\cdot}T(x\backslash y, y\backslash x) = y \cdot K((y\backslash x)\backslash e, y\backslash x)$ by Proposition 29. Then $x \cdot (((x\backslash y)\backslash e)\backslash e) = y \cdot K((y\backslash x)\backslash e, y\backslash x)$ by Theorem 387. Hence we are done by Theorem 666. $\square$

Theorem 911. $(x \cdot (e/y)) \cdot y = R(x, y\backslash e, y)$.

Proof. We have $((x \cdot (e/y)) \cdot K(y\backslash e, y)) \cdot ((y\backslash e)\backslash e) = (x \cdot (e/y)) \cdot y$ by Theorem 908. Then

$$(x \cdot (y\backslash e)) \cdot ((y\backslash e)\backslash e) = (x \cdot (e/y)) \cdot y \tag{441}$$

by Theorem 412. We have $(x \cdot (y\backslash e)) \cdot ((y\backslash e)\backslash e) = R(x, y\backslash e, y)$ by Theorem 662. Hence we are done by (441). $\square$

Theorem 912. $x \cdot y = R(x/(e/y), y\backslash e, y)$.

Proof. We have $((x/(e/y)) \cdot (e/y)) \cdot y = R(x/(e/y), y\backslash e, y)$ by Theorem 911. Hence we are done by Axiom 6. $\square$

Theorem 913. $(e/x)\backslash L(y, x, x\backslash e) = x \cdot y$.

Proof. We have $(e/x)\backslash L(y, x, e/x) = x \cdot y$ by Theorem 71. Hence we are done by Theorem 683. $\square$

Theorem 914. $K(x\backslash e, x) \cdot (x \cdot y) = (e/(e/x)) \cdot y$.

Proof. We have

$$K(x, x\backslash e) \cdot (K(x\backslash e, x) \cdot (x \cdot y)) = x \cdot y \tag{442}$$

by Theorem 442. We have $K(x, x\backslash e) \cdot ((e/(e/x)) \cdot y) = x \cdot y$ by Theorem 685. Hence we are done by (442) and Proposition 7. $\square$

Theorem 915. $K(x\backslash e, x) \cdot (x\backslash y) = T(x, x\backslash e)\backslash y$.

Proof. We have $T(x, x\backslash e) \cdot (x\backslash(T(x, x\backslash e)\backslash y)) = x\backslash y$ by Theorem 522. Then

$$K(x, x\backslash e) \cdot (T(x, x\backslash e)\backslash y) = x\backslash y \tag{443}$$

by Theorem 431. We have $K(x, x\backslash e) \cdot (K(x\backslash e, x) \cdot (x\backslash y)) = x\backslash y$ by Theorem 442. Hence we are done by (443) and Proposition 7. $\square$

Theorem 916. $T((e/(e/y)) \cdot x, K(y\backslash e, y)) = (y \cdot x) \cdot K(y\backslash e, y)$.

Proof. We have $T(K(y\backslash e, y) \cdot (y \cdot x), K(y\backslash e, y)) = (y \cdot x) \cdot K(y\backslash e, y)$ by Theorem 903. Hence we are done by Theorem 914. $\square$

Theorem 917. $T(e/x, y)\backslash e = e/T(x\backslash e, y)$.

Proof. We have $T(e/(e/(x\backslash e)), y)\backslash e = e/T(x\backslash e, y)$ by Theorem 446. Hence we are done by Proposition 24. ☐

Theorem 918. $e/T((x\backslash e)\backslash e, y) = T(x, y)\backslash e$.

Proof. We have $e/T((x\backslash e)\backslash e, y) = T(e/(x\backslash e), y)\backslash e$ by Theorem 917. Hence we are done by Proposition 24. ☐

Theorem 919. $(T(x, y)\backslash e)\backslash e = T((x\backslash e)\backslash e, y)$.

Proof. We have $(e/T((x\backslash e)\backslash e, y))\backslash e = T((x\backslash e)\backslash e, y)$ by Proposition 25. Hence we are done by Theorem 918. ☐

Theorem 920. $((y\backslash x)\backslash e)\backslash e = T(((x/y)\backslash e)\backslash e, y)$.

Proof. We have $(T(x/y, y)\backslash e)\backslash e = T(((x/y)\backslash e)\backslash e, y)$ by Theorem 919. Hence we are done by Proposition 47. ☐

Theorem 921. $y \cdot (((y\backslash x)\backslash e)\backslash e) = (((x/y)\backslash e)\backslash e) \cdot y$.

Proof. We have $y \cdot T(((x/y)\backslash e)\backslash e, y) = (((x/y)\backslash e)\backslash e) \cdot y$ by Proposition 46. Hence we are done by Theorem 920. ☐

Theorem 922. $(((x/y)\backslash e)\backslash e)\backslash x = x \cdot (((x\backslash y)\backslash e)\backslash e)$.

Proof. We have $R((y/x)\backslash e, y/x, x)\backslash x = (((y/x)\backslash e)\backslash e) \cdot x$ by Theorem 702. Then $R((y/x)\backslash e, y/x, x)\backslash x = T(y/x, (y/x)\backslash e) \cdot x$ by Proposition 49. Then $(((x/((y/x) \cdot x))\backslash e)\backslash e)\backslash x = T(y/x, (y/x)\backslash e) \cdot x$ by Theorem 670. Then $(((x/y)\backslash e)\backslash e)\backslash x = T(y/x, (y/x)\backslash e) \cdot x$ by Axiom 6. Then $(((y/x)\backslash e)\backslash e) \cdot x = (((x/y)\backslash e)\backslash e)\backslash x$ by Proposition 49. Hence we are done by Theorem 921. ☐

Theorem 923. $y \cdot K(x\backslash e, x) = K(x\backslash e, x) \cdot y$.

Proof. We have $y \cdot T(K(x\backslash e, x), y) = K(x\backslash e, x) \cdot y$ by Proposition 46. Hence we are done by Theorem 716. ☐

Theorem 924. $(K(x, x\backslash e) \cdot y) \cdot K(x\backslash e, x) = y$.

Proof. We have $(K(x\backslash e, x)\backslash y) \cdot T(K(x\backslash e, x), K(x\backslash e, x)\backslash y) = y$ by Theorem 8. Then $(K(x\backslash e, x)\backslash y) \cdot T(K(x\backslash e, x), y) = y$ by Theorem 631. Then $(K(x, x\backslash e) \cdot y) \cdot T(K(x\backslash e, x), y) = y$ by Theorem 696. Hence we are done by Theorem 716. ☐

Theorem 925. $x \cdot T(y, y\backslash e) = K(y, y\backslash e) \cdot (x \cdot y)$.

Proof.

$$\begin{aligned} & (x \cdot T(y, y\backslash e)) \cdot K(y\backslash e, y) & \\ = {} & x \cdot y & \text{by Theorem 909} \\ = {} & (K(y, y\backslash e) \cdot (x \cdot y)) \cdot K(y\backslash e, y) & \text{by Theorem 924.} \end{aligned}$$

Then $(x \cdot T(y, y\backslash e)) \cdot K(y\backslash e, y) = (K(y, y\backslash e) \cdot (x \cdot y)) \cdot K(y\backslash e, y)$. Hence we are done by Proposition 10. ☐

Theorem 926. $(x \cdot ((y \cdot (x\backslash e))/y)) \cdot y = T(y, R(x\backslash e, x, y))$.

Proof. We have $(x \cdot ((y \cdot (x\backslash e))/y)) \cdot y = T(y, R(e/x, x, y))$ by Theorem 289. Hence we are done by Theorem 725. ☐

Theorem 927. $K(w\backslash e, w) \cdot R(x \cdot T(w, w\backslash e), y, z) = (w\backslash R(w \cdot x, y, z)) \cdot w$.

Proof. We have $K(w\backslash e, w)\cdot L(R(w\backslash((w\backslash e)\backslash(x\cdot w)), y, z), w, w\backslash e) = (w\backslash e)\cdot(w\cdot R(w\backslash((w\backslash e)\backslash(x\cdot w)), y, z))$ by Theorem 59. Then $K(w\backslash e, w) \cdot R(((w\backslash e) \cdot w)\backslash(x \cdot w), y, z) = (w\backslash e) \cdot (w \cdot R(w\backslash((w\backslash e)\backslash(x \cdot w)), y, z))$ by Proposition 85. Then

$$K(w\backslash e, w) \cdot R(K(w\backslash e, w)\backslash(x \cdot w), y, z) = (w\backslash e) \cdot (w \cdot R(w\backslash((w\backslash e)\backslash(x \cdot w)), y, z)) \tag{444}$$

by Proposition 76. We have $(w\backslash e) \cdot (w \cdot R(w\backslash((w\backslash e)\backslash(x \cdot w)), y, z)) = ((w\backslash e) \cdot R((w\backslash e)\backslash x, y, z)) \cdot w$ by Theorem 873. Then $K(w\backslash e, w) \cdot R(K(w\backslash e, w)\backslash(x \cdot w), y, z) = ((w\backslash e) \cdot R((w\backslash e)\backslash x, y, z)) \cdot w$ by (444). Then $K(w\backslash e, w) \cdot R(K(w\backslash e, w)\backslash(x \cdot w), y, z) = (w\backslash R(w \cdot x, y, z)) \cdot w$ by Theorem 826. Then $K(w\backslash e, w) \cdot R(K(w, w\backslash e) \cdot (x \cdot w), y, z) = (w\backslash R(w \cdot x, y, z)) \cdot w$ by Theorem 696. Hence we are done by Theorem 925. □

Theorem 928. $y \cdot K(y, x) = K(y, x) \cdot y$.

Proof. We have $y \cdot T(K(y, x), y) = K(y, x) \cdot y$ by Proposition 46. Then $y \cdot K((y\backslash e)\backslash e, x) = K(y, x) \cdot y$ by Theorem 674. Hence we are done by Theorem 733. □

Theorem 929. $K(T(y, x)\backslash e, T(y, x)) = K(y\backslash e, y)$.

Proof.

$$\begin{array}{lcl} & K(T(y,x)\backslash e, T(y,x)) \cdot (e/T(y,x)) & \\ = & T(y,x)\backslash e & \text{by Theorem 255} \\ = & K(y\backslash e, y) \cdot (e/T(y,x)) & \text{by Theorem 748.} \end{array}$$

Then $K(T(y, x)\backslash e, T(y, x))\cdot(e/T(y, x)) = K(y\backslash e, y)\cdot(e/T(y, x))$. Hence we are done by Proposition 10. □

Theorem 930. $(T(x, y)\backslash x)\backslash x = T(x, y)$.

Proof. We have $(x/T(x, y))\backslash x = T(x, y)$ by Proposition 25. Hence we are done by Theorem 753. □

Theorem 931. $(T(y, x)\backslash y) \cdot y = y \cdot (T(y, x)\backslash y)$.

Proof. We have $(y/T(y, x)) \cdot y = y \cdot (T(y, x)\backslash y)$ by Theorem 572. Hence we are done by Theorem 753. □

Theorem 932. $K(y\backslash(y/((x\backslash y)/T(y, x))), y) = T(y, x)\backslash y$.

Proof. We have $K(y\backslash(y/((x\backslash y)/T(y, x))), y) = ((x\backslash y) \cdot T((x\backslash y)\backslash T(y, x), y))/T(y, x)$ by Theorem 287. Then $K(y\backslash(y/((x\backslash y)/T(y, x))), y) = ((x\backslash y)\cdot((x\backslash y)\backslash y))/T(y, x)$ by Theorem 514. Then $K(y\backslash(y/((x\backslash y)/T(y, x))), y) = y/T(y, x)$ by Axiom 4. Hence we are done by Theorem 753. □

Theorem 933. $y \cdot K(x, y) = K(x, y) \cdot y$.

Proof. We have $y\cdot T(K(x, y), y) = K(x, y)\cdot y$ by Proposition 46. Hence we are done by Theorem 754. □

Theorem 934. $(y \cdot K(x, y))/y = K(x, y)$.

Proof. We have $(y\cdot T(K(x, y), y))/y = K(x, y)$ by Proposition 48. Hence we are done by Theorem 754. □

Theorem 935. $T(z, x) = T(T(z, x), K(y, z))$.

Proof. We have $T(T(z, K(y, z)), x) = T(T(z, x), K(y, z))$ by Axiom 7. Hence we are done by Theorem 755. □

Theorem 936. $T(y, x\backslash T(x, y)) = y$.

Proof. We have $T(y, K(y\backslash(y/(x\backslash e)), y)) = y$ by Theorem 755. Hence we are done by Theorem 198. □

Theorem 937. $(x\backslash T(x,y)) \cdot y = y \cdot (x\backslash T(x,y))$.

Proof. We have $(x\backslash T(x,y)) \cdot T(y, x\backslash T(x,y)) = y \cdot (x\backslash T(x,y))$ by Proposition 46. Hence we are done by Theorem 936. ☐

Theorem 938. $R(x, K(y,z), z) \cdot (z \cdot K(y,z)) = (x \cdot K(y,z)) \cdot z$.

Proof. We have $R(x, K(y,z), z) \cdot (K(y,z) \cdot z) = (x \cdot K(y,z)) \cdot z$ by Proposition 54. Hence we are done by Theorem 933. ☐

Theorem 939. $T(K(y,z), T(z,x)) = K(y,z)$.

Proof. We have $T(T(z,x), K(y,z)) = T(z,x)$ by Theorem 935. Hence we are done by Proposition 21. ☐

Theorem 940. $((y \cdot x)/y)/x = x\backslash((y \cdot x)/y)$.

Proof. We have $T(x, K(x\backslash(x/(x\backslash(x/y))), x)) = x$ by Theorem 755. Then

$$T(x, x\backslash((y \cdot x)/y)) = x \tag{445}$$

by Theorem 853. We have $((y \cdot x)/y)/T(x, x\backslash((y \cdot x)/y)) = x\backslash((y \cdot x)/y)$ by Theorem 15. Hence we are done by (445). ☐

Theorem 941. $K(x\backslash e, x) = K((x \cdot y)\backslash y, T(x,y))$.

Proof. We have $K((y \cdot T(x,y))\backslash y, T(x,y)) = K(T(x,y)\backslash e, T(x,y))$ by Theorem 665. Then $K((x \cdot y)\backslash y, T(x,y)) = K(T(x,y)\backslash e, T(x,y))$ by Proposition 46. Hence we are done by Theorem 929. ☐

Theorem 942. $x \cdot (((y \cdot x)/y)/x) = (y \cdot x)/y$.

Proof. We have $x \cdot (x\backslash((y \cdot x)/y)) = (y \cdot x)/y$ by Axiom 4. Hence we are done by Theorem 940. ☐

Theorem 943. $R(y\backslash T(y,x), y, z) = (T(y,x) \cdot z)/(y \cdot z)$.

Proof. We have $R(T(y,x)/y, y, z) = (T(y,x) \cdot z)/(y \cdot z)$ by Proposition 55. Hence we are done by Theorem 758. ☐

Theorem 944. $T(x, R(e/y, y, x)) = T(x, x/(x \cdot y))$.

Proof. We have $(y \cdot ((x \cdot (y\backslash e))/x)) \cdot x = T(x, R(e/y, y, x))$ by Theorem 289. Then $(x\backslash T(x, x/(x \cdot y))) \cdot x = T(x, R(e/y, y, x))$ by Theorem 846. Hence we are done by Theorem 759. ☐

Theorem 945. $T(z/(y \cdot z), T(y,x)) = R(y\backslash e, y, z)$.

Proof. We have $T(z/(y \cdot z), T(y,x)) = R(T(e/y, T(y,x)), y, z)$ by Theorem 39. Hence we are done by Theorem 762. ☐

Theorem 946. $(x \cdot y)/x = (((x \cdot y)/x)\backslash y)\backslash y$.

Proof. We have $T((x \cdot y)/x, ((x \cdot y)/x)\backslash T((x \cdot y)/x, x)) = (x \cdot y)/x$ by Theorem 761. Then

$$T((x \cdot y)/x, K(x\backslash(x/(((x \cdot y)/x)\backslash e)), x)) = (x \cdot y)/x \tag{446}$$

by Theorem 198. We have $T((x \cdot y)/x, ((x \cdot y)/x)\backslash y) = (((x \cdot y)/x)\backslash y)\backslash y$ by Proposition 49. Then $T((x \cdot y)/x, K(x\backslash(x/(((x \cdot y)/x)\backslash e)), x)) = (((x \cdot y)/x)\backslash y)\backslash y$ by Theorem 200. Hence we are done by (446). ☐

Theorem 947. $K(x\backslash e, x) \cdot y = (x \cdot y) \cdot ((((x \cdot y)\backslash y)\backslash e)\backslash e)$.

Proof. We have $K((x \cdot y)\backslash y, T(x, y)) = K(x\backslash e, x)$ by Theorem 941. Then

$$K((x \cdot y)\backslash y, y\backslash(x \cdot y)) = K(x\backslash e, x) \tag{447}$$

by Definition 3. We have $y \cdot K((x \cdot y)\backslash y, y\backslash(x \cdot y)) = (x \cdot y) \cdot ((((x \cdot y)\backslash y)\backslash e)\backslash e)$ by Theorem 910. Then $K((x \cdot y)\backslash y, y\backslash(x \cdot y)) \cdot y = (x \cdot y) \cdot ((((x \cdot y)\backslash y)\backslash e)\backslash e)$ by Theorem 724. Hence we are done by (447). □

Theorem 948. $(x \cdot ((y \cdot (x\backslash e))/y)) \cdot y = T(y, R(x\backslash e, (x\backslash e)\backslash e, y))$.

Proof. We have $(x \cdot ((y \cdot (x\backslash e))/y)) \cdot y = T(y, R(x\backslash e, x, y))$ by Theorem 926. Hence we are done by Theorem 704. □

Theorem 949. $T(x, R(y, x, (y \cdot x)\backslash x))\backslash x = K(x, y)\backslash e$.

Proof. We have $(x\backslash T(x, R(y, x, (y \cdot x)\backslash x)))\backslash e = T(x, R(y, x, (y \cdot x)\backslash x))\backslash x$ by Theorem 749. Then $(x\backslash(x \cdot K(x, y)))\backslash e = T(x, R(y, x, (y \cdot x)\backslash x))\backslash x$ by Theorem 871. Hence we are done by Axiom 3. □

Theorem 950. $z \cdot L(x, z, z\backslash T(z, y)) = L(z \cdot x, z\backslash T(z, y), z)$.

Proof. We have $L(x, z, z\backslash T(z, y)) = ((z\backslash T(z, y)) \cdot z)\backslash((z\backslash T(z, y)) \cdot (z \cdot x))$ by Definition 4. Then

$$L(x, z, z\backslash T(z, y)) = T(z, y)\backslash((z\backslash T(z, y)) \cdot (z \cdot x)) \tag{448}$$

by Theorem 759. We have $z \cdot (T(z, y)\backslash((z\backslash T(z, y)) \cdot (z \cdot x))) = L(z \cdot x, z\backslash T(z, y), z)$ by Theorem 832. Hence we are done by (448). □

Theorem 951. $L(x, y\backslash T(y, z), y) = y \cdot L(y\backslash x, y, y\backslash T(y, z))$.

Proof. We have $L(y \cdot (y\backslash x), y\backslash T(y, z), y) = y \cdot L(y\backslash x, y, y\backslash T(y, z))$ by Theorem 950. Hence we are done by Axiom 4. □

Theorem 952. $T(x, y)\backslash x = (y \cdot (x\backslash T(x, y)))\backslash y$.

Proof. We have $(y \cdot (T(y, R(T(x, y), T(x, y)\backslash e, y))\backslash y))\backslash(y \cdot y) = T(y, R(T(x, y), T(x, y)\backslash e, y))$ by Theorem 570. Then

$$(y \cdot (((T(x, y)\backslash x) \cdot y)\backslash y))\backslash(y \cdot y) = T(y, R(T(x, y), T(x, y)\backslash e, y)) \tag{449}$$

by Theorem 591. We have $T(y, R(T(x, y), T(x, y)\backslash e, y)) = (T(x, y)\backslash x) \cdot y$ by Theorem 591. Then $(y \cdot (((T(x, y)\backslash x) \cdot y)\backslash y))\backslash(y \cdot y) = (T(x, y)\backslash x) \cdot y$ by (449). Then

$$(y \cdot ((y \cdot (T(x, y)\backslash x))\backslash y))\backslash(y \cdot y) = (T(x, y)\backslash x) \cdot y \tag{450}$$

by Theorem 740. We have $(T(x, y)\backslash x) \cdot y = y \cdot (T(x, y)\backslash x)$ by Theorem 740. Then $(y \cdot ((y \cdot (T(x, y)\backslash x))\backslash y))\backslash(y \cdot y) = y \cdot (T(x, y)\backslash x)$ by (450). Then

$$(y \cdot (x\backslash T(x, y)))\backslash(y \cdot y) = y \cdot (T(x, y)\backslash x) \tag{451}$$

by Theorem 771. We have $((L(x, e/x, y) \cdot y)/L(x, e/x, y))\backslash(y \cdot y) = y \cdot (((L(x, e/x, y) \cdot y)/L(x, e/x, y))\backslash y)$ by Theorem 855. Then $(y \cdot (T(x, y)/x))\backslash(y \cdot y) = y \cdot (((L(x, e/x, y) \cdot y)/L(x, e/x, y))\backslash y)$ by Theorem 883. Then $y \cdot ((y \cdot (T(x, y)/x))\backslash y) = (y \cdot (T(x, y)/x))\backslash(y \cdot y)$ by Theorem 883. Then $(y \cdot (x\backslash T(x, y)))\backslash(y \cdot y) = y \cdot ((y \cdot (T(x, y)/x))\backslash y)$ by Theorem 758. Then $(y \cdot (x\backslash T(x, y)))\backslash(y \cdot y) = y \cdot ((y \cdot (x\backslash T(x, y)))\backslash y)$ by Theorem 758. Then $y \cdot (T(x, y)\backslash x) = y \cdot ((y \cdot (x\backslash T(x, y)))\backslash y)$ by (451). Hence we are done by Proposition 9. □

Theorem 953. $T(z, R((y\backslash x)\backslash e, ((y\backslash x)\backslash e)\backslash e, z)) = (y\backslash(x \cdot ((z \cdot (x\backslash y))/z))) \cdot z$.

Proof. We have $T(z, R((y\backslash x)\backslash e, ((y\backslash x)\backslash e)\backslash e, z)) = ((y\backslash x) \cdot ((z \cdot ((y\backslash x)\backslash e))/z)) \cdot z$ by Theorem 948. Hence we are done by Theorem 876. □

Theorem 954. $T(y, R((x\backslash y)\backslash e, ((x\backslash y)\backslash e)\backslash e, y)) = y \cdot K(y, x/y)$.

Proof. We have $T(y, R((x\backslash y)\backslash e, ((x\backslash y)\backslash e)\backslash e, y)) = (x\backslash(y \cdot ((y \cdot (y\backslash x))/y))) \cdot y$ by Theorem 953. Then $T(y, R((x\backslash y)\backslash e, ((x\backslash y)\backslash e)\backslash e, y)) = (x\backslash(y \cdot (x/y))) \cdot y$ by Axiom 4. Then

$$T(y, R((x\backslash y)\backslash e, ((x\backslash y)\backslash e)\backslash e, y)) = K(y, x/y) \cdot y \tag{452}$$

by Theorem 3. We have $K(y, x/y) \cdot y = y \cdot K(y, x/y)$ by Theorem 928. Hence we are done by (452). □

Theorem 955. $L(z, x\backslash T(x, y), x) = (x\backslash T(x, y)) \cdot (x \cdot (T(x, y)\backslash z))$.

Proof. We have $((x\backslash T(x, y)) \cdot x) \cdot L(T(x, y)\backslash z, x, x\backslash T(x, y)) = (x\backslash T(x, y)) \cdot (x \cdot (T(x, y)\backslash z))$ by Proposition 52. Then

$$T(x, y) \cdot L(T(x, y)\backslash z, x, x\backslash T(x, y)) = (x\backslash T(x, y)) \cdot (x \cdot (T(x, y)\backslash z)) \tag{453}$$

by Theorem 759. We have $T(x \cdot L(x\backslash((y \cdot z)/y), x, x\backslash T(x, y)), y) = T(x, y) \cdot L(T(x, y)\backslash z, x, x\backslash T(x, y))$ by Theorem 879. Then $T(x \cdot L(x\backslash((y \cdot z)/y), x, x\backslash T(x, y)), y) = (x\backslash T(x, y)) \cdot (x \cdot (T(x, y)\backslash z))$ by (453). Then

$$T(L((y \cdot z)/y, x\backslash T(x, y), x), y) = (x\backslash T(x, y)) \cdot (x \cdot (T(x, y)\backslash z)) \tag{454}$$

by Theorem 951. We have $T(L((y \cdot z)/y, x\backslash T(x, y), x), y) = L(y\backslash(y \cdot z), x\backslash T(x, y), x)$ by Proposition 72. Then $(x\backslash T(x, y)) \cdot (x \cdot (T(x, y)\backslash z)) = L(y\backslash(y \cdot z), x\backslash T(x, y), x)$ by (454). Hence we are done by Axiom 3. □

Theorem 956. $(y\backslash T(y, x))\backslash y = y \cdot (T(y, x)\backslash y)$.

Proof. We have $(y\backslash T(y, x)) \cdot T(T(y, y\backslash T(y, x)), y\backslash T(y, x)) = T(y, y\backslash T(y, x)) \cdot (y\backslash T(y, x))$ by Proposition 46. Then

$$(y\backslash T(y, x)) \cdot T(y, y\backslash T(y, x)) = T(y, y\backslash T(y, x)) \cdot (y\backslash T(y, x)) \tag{455}$$

by Theorem 761. We have $T(y, y\backslash T(y, x)) \cdot (y\backslash T(y, x)) = T(y, x) \cdot (y\backslash T(y, y\backslash T(y, x)))$ by Theorem 833. Then

$$(y\backslash T(y, x)) \cdot T(y, y\backslash T(y, x)) = T(y, x) \cdot (y\backslash T(y, y\backslash T(y, x))) \tag{456}$$

by (455). We have $(T(y, x) \cdot T(y, y\backslash T(y, x)))/(T(y, x) \cdot (y\backslash T(y, y\backslash T(y, x)))) = y$ by Theorem 831. Then

$$(T(y, x) \cdot T(y, y\backslash T(y, x)))/((y\backslash T(y, x)) \cdot T(y, y\backslash T(y, x))) = y \tag{457}$$

by (456). We have $R(y, y\backslash T(y, x), T(y, y\backslash T(y, x))) = (T(y, x) \cdot T(y, y\backslash T(y, x)))/((y\backslash T(y, x)) \cdot T(y, y\backslash T(y, x)))$ by Theorem 785. Then

$$R(y, y\backslash T(y, x), T(y, y\backslash T(y, x))) = y \tag{458}$$

by (457). We have $L(R(y, y\backslash T(y, x), T(y, y\backslash T(y, x))), y\backslash T(y, x), y) = y$ by Theorem 823. Then $L(y, y\backslash T(y, x), y) = y$ by (458). Then $(y\backslash T(y, x)) \cdot (y \cdot (T(y, x)\backslash y)) = y$ by Theorem 955. Hence we are done by Proposition 2. □

Theorem 957. $x\backslash(K(x, y)\backslash x) = K(x, y)\backslash e$.

Proof. We have $(x\backslash T(x, R(y, x, (y \cdot x)\backslash x)))\backslash x = x \cdot (T(x, R(y, x, (y \cdot x)\backslash x))\backslash x)$ by Theorem 956. Then $(x\backslash(x \cdot K(x, y)))\backslash x = x \cdot (T(x, R(y, x, (y \cdot x)\backslash x))\backslash x)$ by Theorem 871. Then $x \cdot (T(x, R(y, x, (y \cdot x)\backslash x))\backslash x) = K(x, y)\backslash x$ by Axiom 3. Then $x \cdot (K(x, y)\backslash e) = K(x, y)\backslash x$ by Theorem 949. Hence we are done by Proposition 2. □

Theorem 958. $y\backslash(K(x, y)\backslash y) = K(x, y)\backslash e$.

Proof. We have $y \cdot (K(x, y)\backslash e) = K(x, y)\backslash y$ by Theorem 777. Hence we are done by Proposition 2. □

Theorem 959. $y \cdot (T(L(x\backslash e, x, y)\backslash e, y)\backslash(L(x\backslash e, x, y)\backslash e)) = K(x, y)\backslash y$.

Proof. We have $y \cdot (K(x, y)\backslash e) = K(x, y)\backslash y$ by Theorem 777. Hence we are done by Theorem 763. □

Theorem 960. $R(T(x, x\backslash e), y, z)\backslash R(x, y, z) = K(x\backslash e, R(x, y, z))$.

Proof. We have $R((x/(((x\backslash R(x,y,z))/R(x,y,z))\backslash e))\cdot(((x\backslash R(x,y,z))/R(x,y,z))\backslash e),y,z)/(((x\backslash R(x,y,z))/R(x,y,z))\backslash e) = ((x\backslash R(x,y,z))/R(x,y,z))\cdot R(((x\backslash R(x,y,z))/R(x,y,z))\backslash(x/(((x\backslash R(x,y,z))/R(x,y,z))\backslash e)),y,z)$ by Theorem 863. Then $R(x,y,z)/(((x\backslash R(x,y,z))/R(x,y,z))\backslash e) = ((x\backslash R(x,y,z))/R(x,y,z))\cdot R(((x\backslash R(x,y,z))/R(x,y,z))\backslash(x/(((x\backslash R(x,y,z))/R(x$ by Axiom 6. Then $((x\backslash R(x,y,z))/R(x,y,z))\cdot R(((x\backslash R(x,y,z))/R(x,y,z))\backslash(((x\backslash R(x,y,z))/R(x,y,z))\cdot x),y,z) = R(x,y,z)/(((x\backslash R(x,y,z))/R(x,y,z))\backslash e)$ by Theorem 867. Then

$$((x\backslash R(x,y,z))/R(x,y,z))\cdot R(x,y,z) = R(x,y,z)/(((x\backslash R(x,y,z))/R(x,y,z))\backslash e) \tag{459}$$

by Axiom 3. We have $K(R(x,y,z)\backslash(R(x,y,z)/(((x\backslash R(x,y,z))/R(x,y,z))\backslash e)),R(x,y,z)) = ((x\backslash R(x,y,z))/R(x,y,z))\backslash T((x\backslash R(x$ by Theorem 198. Then $K(R(x,y,z)\backslash(R(x,y,z)/(((x\backslash R(x,y,z))/R(x,y,z))\backslash e)),R(x,y,z)) = ((x\backslash R(x,y,z))/R(x,y,z))\backslash(R(x,y,$ by Proposition 47. Then $K(R(x,y,z)\backslash(R(x,y,z)/(((x\backslash R(x,y,z))/R(x,y,z))\backslash e)),R(x,y,z)) = ((x\backslash R(x,y,z))/R(x,y,z))\backslash(x\backslash e)$ by Theorem 835. Then $K(R(x,y,z)\backslash(((x\backslash R(x,y,z))/R(x,y,z))\cdot R(x,y,z)),R(x,y,z)) = ((x\backslash R(x,y,z))/R(x,y,z))\backslash(x\backslash e)$ by (459). Then $K(R(x,y,z)\backslash(x\backslash R(x,y,z)),R(x,y,z)) = ((x\backslash R(x,y,z))/R(x,y,z))\backslash(x\backslash e)$ by Axiom 6. Then

$$K(x\backslash e,R(x,y,z)) = ((x\backslash R(x,y,z))/R(x,y,z))\backslash(x\backslash e) \tag{460}$$

by Theorem 835. We have $(((R(x,y,z)\cdot(x\backslash e))/R(x,y,z))\backslash(x\backslash e))\backslash(x\backslash e) = (R(x,y,z)\cdot(x\backslash e))/R(x,y,z)$ by Theorem 946. Then $(((x\backslash R(x,y,z))/R(x,y,z))\backslash(x\backslash e))\backslash(x\backslash e) = (R(x,y,z)\cdot(x\backslash e))/R(x,y,z)$ by Proposition 89. Then $(((x\backslash R(x,y,z))/R(x,y,z))\backslash(x\backslash e))\backslash(x\backslash e) = (x\backslash R(x,y,z))/R(x,y,z)$ by Proposition 89. Then

$$K(x\backslash e,R(x,y,z))\backslash(x\backslash e) = (x\backslash R(x,y,z))/R(x,y,z) \tag{461}$$

by (460). We have $(x\backslash e)\backslash(K(x\backslash e,R(x,y,z))\backslash(x\backslash e)) = K(x\backslash e,R(x,y,z))\backslash e$ by Theorem 957. Then

$$(x\backslash e)\backslash((x\backslash R(x,y,z))/R(x,y,z)) = K(x\backslash e,R(x,y,z))\backslash e \tag{462}$$

by (461). We have $(x\backslash e)\backslash((x\backslash R(x,y,z))/R(x,y,z)) = x\cdot((x\backslash R(x,y,z))/R(x,y,z))$ by Theorem 709. Then $K(x\backslash e,R(x,y,z))\backslash e = x\cdot((x\backslash R(x,y,z))/R(x,y,z))$ by (462). Then

$$K(R(x,y,z),x\backslash e) = K(x\backslash e,R(x,y,z))\backslash e \tag{463}$$

by Theorem 781. We have $(K(x\backslash e,R(x,y,z))\backslash e)\backslash e = K(x\backslash e,R(x,y,z))$ by Theorem 900. Then $K(R(x,y,z),x\backslash e)\backslash e = K(x\backslash e,R(x,y,z))$ by (463). Then

$$a(R(x,y,z),e/x,x)\backslash e = K(x\backslash e,R(x,y,z)) \tag{464}$$

by Theorem 782. We have $(R(x,y,z)\backslash T(R(x,y,z),x\backslash e))\backslash e = T(R(x,y,z),x\backslash e)\backslash R(x,y,z)$ by Theorem 749. Then $(R(x,y,z)\backslash R(T(x,x\backslash e),y,z))\backslash e = T(R(x,y,z),x\backslash e)\backslash R(x,y,z)$ by Axiom 9. Then $(R(x,y,z)\backslash R(T(x,x\backslash e),y,z))\backslash e = R(T(x,x\backslash e),y,z)\backslash R(x,y,z)$ by Axiom 9. Then $a(R(x,y,z),e/x,x)\backslash e = R(T(x,x\backslash e),y,z)\backslash R(x,y,z)$ by Theorem 545. Hence we are done by (464). □

Theorem 961. $K(x\backslash e,R(x,y,z)) = x\cdot T(x\backslash e,R(x,y,z))$.

Proof. We have $R(R(x,T(e/x,R(x,y,z)),x),y,z) = R(R(x,y,z),T(e/x,R(x,y,z)),x)$ by Axiom 12. Then

$$R(T(x,T(e/x,R(x,y,z))),y,z) = R(R(x,y,z),T(e/x,R(x,y,z)),x) \tag{465}$$

by Theorem 865. We have $R(R(x,y,z),R(x,y,z)\backslash(R(x,y,z)/x),x)\backslash R(x,y,z) = (R(x,y,z)\backslash(R(x,y,z)/x))\cdot x$ by Theorem 808. Then $R(R(x,y,z),T(e/x,R(x,y,z)),x)\backslash R(x,y,z) = (R(x,y,z)\backslash(R(x,y,z)/x))\cdot x$ by Theorem 839. Then $R(T(x,T(e/x,R(x,y,z))),y,z)\backslash R(x,y,z) = (R(x,y,z)\backslash(R(x,y,z)/x))\cdot x$ by (465). Then $R(T(x,x\backslash e),y,z)\backslash R(x,y,z) = (R(x,y,z)\backslash(R(x,y,z)/x))\cdot x$ by Theorem 886. Then

$$R(T(x,x\backslash e),y,z)\backslash R(x,y,z) = T(e/x,R(x,y,z))\cdot x \tag{466}$$

by Theorem 839. We have $T(e/x,R(x,y,z))\cdot x = x\cdot T(x\backslash e,R(x,y,z))$ by Proposition 61. Then $R(T(x,x\backslash e),y,z)\backslash R(x,y,z) = x\cdot T(x\backslash e,R(x,y,z))$ by (466). Hence we are done by Theorem 960. □

Theorem 962. $K(R(x,y,z)\backslash e,R(x,y,z))\cdot(y\cdot z) = (K(x\backslash e,R(x,y,z))\cdot y)\cdot z$.

Proof. We have $R(x \cdot T(x\backslash e, R(x,y,z)), y, z) \cdot (y \cdot z) = ((x \cdot T(x\backslash e, R(x,y,z))) \cdot y) \cdot z$ by Proposition 54. Then $K(R(x,y,z)\backslash e, R(x,y,z)) \cdot (y \cdot z) = ((x \cdot T(x\backslash e, R(x,y,z))) \cdot y) \cdot z$ by Theorem 637. Hence we are done by Theorem 961. □

Theorem 963. $R(e/x, y, z)\backslash e = e/R(x\backslash e, y, z)$.

Proof. We have $R(e/(e/(x\backslash e)), y, z)\backslash e = e/R(x\backslash e, y, z)$ by Theorem 783. Hence we are done by Proposition 24. □

Theorem 964. $(R(x,y,z)\backslash e)\backslash e = R((x\backslash e)\backslash e, y, z)$.

Proof. We have $e/R((x\backslash e)\backslash e, y, z) = R(e/(x\backslash e), y, z)\backslash e$ by Theorem 963. Then

$$e/R((x\backslash e)\backslash e, y, z) = R(x,y,z)\backslash e \tag{467}$$

by Proposition 24. We have $(e/R((x\backslash e)\backslash e, y, z))\backslash e = R((x\backslash e)\backslash e, y, z)$ by Proposition 25. Hence we are done by (467). □

Theorem 965. $(x \cdot z) \cdot ((((x \cdot z)\backslash(y \cdot z))\backslash e)\backslash e) = (x \cdot (((x\backslash y)\backslash e)\backslash e)) \cdot z$.

Proof. We have $((((y \cdot z)/(x \cdot z))\backslash e)\backslash e) \cdot (x \cdot z) = (x \cdot z) \cdot ((((x \cdot z)\backslash(y \cdot z))\backslash e)\backslash e)$ by Theorem 921. Then $((R(y/x, x, z)\backslash e)\backslash e) \cdot (x \cdot z) = (x \cdot z) \cdot ((((x \cdot z)\backslash(y \cdot z))\backslash e)\backslash e)$ by Proposition 55. Then

$$R(((y/x)\backslash e)\backslash e, x, z) \cdot (x \cdot z) = (x \cdot z) \cdot ((((x \cdot z)\backslash(y \cdot z))\backslash e)\backslash e) \tag{468}$$

by Theorem 964. We have $R(((y/x)\backslash e)\backslash e, x, z) \cdot (x \cdot z) = ((((y/x)\backslash e)\backslash e) \cdot x) \cdot z$ by Proposition 54. Then $(x \cdot z) \cdot ((((x \cdot z)\backslash(y \cdot z))\backslash e)\backslash e) = ((((y/x)\backslash e)\backslash e) \cdot x) \cdot z$ by (468). Hence we are done by Theorem 921. □

Theorem 966. $K(z\backslash e, R(z,x,y)) = K(z\backslash e, z)$.

Proof. We have $(((((z \cdot x) \cdot y)/(x \cdot y))\backslash e)\backslash e)\backslash((z \cdot x) \cdot y) = ((z \cdot x) \cdot y) \cdot (((((z \cdot x) \cdot y)\backslash(x \cdot y))\backslash e)\backslash e)$ by Theorem 922. Then $((R(z,x,y)\backslash e)\backslash e)\backslash((z \cdot x) \cdot y) = ((z \cdot x) \cdot y) \cdot (((((z \cdot x) \cdot y)\backslash(x \cdot y))\backslash e)\backslash e)$ by Definition 5. Then $((z \cdot x) \cdot ((((z \cdot x)\backslash x)\backslash e)\backslash e)) \cdot y = ((R(z,x,y)\backslash e)\backslash e)\backslash((z \cdot x) \cdot y)$ by Theorem 965. Then

$$((R(z,x,y)\backslash e)\backslash e)\backslash((z \cdot x) \cdot y) = (K(z\backslash e, z) \cdot x) \cdot y \tag{469}$$

by Theorem 947. We have $K(R(z,x,y)\backslash e, R(z,x,y)) \cdot (R(z,x,y)\backslash((z \cdot x) \cdot y)) = T(R(z,x,y), R(z,x,y)\backslash e)\backslash((z \cdot x) \cdot y)$ by Theorem 915. Then $K(R(z,x,y)\backslash e, R(z,x,y)) \cdot (x \cdot y) = T(R(z,x,y), R(z,x,y)\backslash e)\backslash((z \cdot x) \cdot y)$ by Theorem 6. Then $T(R(z,x,y), R(z,x,y)\backslash e)\backslash((z \cdot x) \cdot y) = (K(z\backslash e, R(z,x,y)) \cdot x) \cdot y$ by Theorem 962. Then $((R(z,x,y)\backslash e)\backslash e)\backslash((z \cdot x) \cdot y) = (K(z\backslash e, R(z,x,y)) \cdot x) \cdot y$ by Proposition 49. Then $K(z\backslash e, z) \cdot x = K(z\backslash e, R(z,x,y)) \cdot x$ by (469) and Proposition 8. Hence we are done by Proposition 10. □

Theorem 967. $L(z, x, K(y\backslash e, y)) = z$.

Proof. We have $(R(y,x,z) \cdot (y \cdot T(y\backslash e, R(y,x,z))))/R(T(y, y \cdot T(y\backslash e, R(y,x,z))), x, z) = y \cdot T(y\backslash e, R(y,x,z))$ by Theorem 456. Then $(R(y,x,z) \cdot (y \cdot T(y\backslash e, R(y,x,z))))/R(y,x,z) = y \cdot T(y\backslash e, R(y,x,z))$ by Theorem 203. Then $(y \cdot (R(y,x,z) \cdot T(y\backslash e, R(y,x,z))))/R(y,x,z) = y \cdot T(y\backslash e, R(y,x,z))$ by Theorem 227. Then $(y \cdot ((y\backslash e) \cdot R(y,x,z)))/R(y,x,z) = y \cdot T(y\backslash e, R(y,x,z))$ by Proposition 46. Then $L(R(y,x,z), y\backslash e, y)/R(y,x,z) = y \cdot T(y\backslash e, R(y,x,z))$ by Proposition 56. Then

$$R(e/(e/y), x, z)/R(y,x,z) = y \cdot T(y\backslash e, R(y,x,z)) \tag{470}$$

by Theorem 851. We have $(((e/y) \cdot R(y,x,z))/(e/y))/R(y,x,z) = R(y,x,z)\backslash(((e/y) \cdot R(y,x,z))/(e/y))$ by Theorem 940. Then $R(e/(e/y), x, z)/R(y,x,z) = R(y,x,z)\backslash(((e/y) \cdot R(y,x,z))/(e/y))$ by Theorem 813. Then $R(y,x,z)\backslash R(e/(e/y), x, z) = R(e/(e/y), x, z)/R(y,x,z)$ by Theorem 813. Then $R(y,x,z)\backslash R(e/(e/y), x, z) = y \cdot T(y\backslash e, R(y,x,z))$ by (470). Then

$$R(y,x,z)\backslash R(e/(e/y), x, z) = K(y\backslash e, R(y,x,z)) \tag{471}$$

by Theorem 961. We have $((R(y,x,z)\backslash e)/(R(y,x,z)\backslash L(R(y,x,z),y\backslash e,y)))\backslash(R(y,x,z)\backslash e) = R(y,x,z)\backslash L(R(y,x,z),y\backslash e,y)$ by Proposition 25. Then $(L(R(y,x,z),y\backslash e,y)\backslash e)\backslash(R(y,x,z)\backslash e) = R(y,x,z)\backslash L(R(y,x,z),y\backslash e,y)$ by Theorem 906. Then $(R(e/(e/y),x,z)\backslash e)\backslash(R(y,x,z)\backslash e) = R(y,x,z)\backslash L(R(y,x,z),y\backslash e,y)$ by Theorem 851. Then $R(y,x,z)\backslash R(e/(e/y),x,z) = (R(e/(e/y),x,z)\backslash e)\backslash(R(y,x,z)\backslash e)$ by Theorem 851. Then $(R(e/(e/y),x,z)\backslash e)\backslash(R(y,x,z)\backslash e) = K(y\backslash e,R(y,x,z))$ by (471). Then

$$(e/R(y,x,z))\backslash(R(y,x,z)\backslash e) = K(y\backslash e,R(y,x,z)) \quad (472)$$

by Theorem 783. We have $(e/R(y,x,z))\backslash(R(y,x,z)\backslash e) = K(R(y,x,z)\backslash e,R(y,x,z))$ by Proposition 90. Then

$$K(y\backslash e,R(y,x,z)) = K(R(y,x,z)\backslash e,R(y,x,z)) \quad (473)$$

by (472). We have $K(R(y,x,z)\backslash e,R(y,x,z))\cdot(x\cdot z) = (K(y\backslash e,R(y,x,z))\cdot x)\cdot z$ by Theorem 962. Then $K(y\backslash e,R(y,x,z))\cdot(x\cdot z) = (K(y\backslash e,R(y,x,z))\cdot x)\cdot z$ by (473). Then $L(z,x,K(y\backslash e,R(y,x,z))) = z$ by Theorem 54. Hence we are done by Theorem 966. □

Theorem 968. $(K(x\backslash e,x)\cdot y)\cdot z = K(x\backslash e,x)\cdot(y\cdot z)$.

Proof. We have $(K(x\backslash e,x)\cdot y)\cdot L(z,y,K(x\backslash e,x)) = K(x\backslash e,x)\cdot(y\cdot z)$ by Proposition 52. Hence we are done by Theorem 967. □

Theorem 969. $(K(x\backslash e,x)\cdot y)/z = K(x\backslash e,x)\cdot(y/z)$.

Proof. We have $(K(x\backslash e,x)\cdot y)/L(z,y/z,K(x\backslash e,x)) = K(x\backslash e,x)\cdot(y/z)$ by Theorem 472. Hence we are done by Theorem 967. □

Theorem 970. $R(K(x\backslash e,x)\cdot y,z,w) = K(x\backslash e,x)\cdot R(y,z,w)$.

Proof. We have $((K(x\backslash e,x)\cdot(y\cdot z))\cdot w)/(L(z,y,K(x\backslash e,x))\cdot w) = R(K(x\backslash e,x)\cdot y,L(z,y,K(x\backslash e,x)),w)$ by Theorem 797. Then $((K(x\backslash e,x)\cdot(y\cdot z))\cdot w)/(z\cdot w) = R(K(x\backslash e,x)\cdot y,L(z,y,K(x\backslash e,x)),w)$ by Theorem 967. Then

$$R(K(x\backslash e,x)\cdot y,L(z,y,K(x\backslash e,x)),w) = (K(x\backslash e,x)\cdot((y\cdot z)\cdot w))/(z\cdot w) \quad (474)$$

by Theorem 968. We have $(K(x\backslash e,x)\cdot((y\cdot z)\cdot w))/(z\cdot w) = K(x\backslash e,x)\cdot(((y\cdot z)\cdot w)/(z\cdot w))$ by Theorem 969. Then $R(K(x\backslash e,x)\cdot y,L(z,y,K(x\backslash e,x)),w) = K(x\backslash e,x)\cdot(((y\cdot z)\cdot w)/(z\cdot w))$ by (474). Then $R(K(x\backslash e,x)\cdot y,L(z,y,K(x\backslash e,x)),w) = K(x\backslash e,x)\cdot R(y,z,w)$ by Definition 5. Hence we are done by Theorem 967. □

Theorem 971. $R(x\cdot w,y,z) = (w\backslash R(w\cdot x,y,z))\cdot w$.

Proof. We have $(x\cdot T(w,w\backslash e))\cdot K(w\backslash e,w) = x\cdot w$ by Theorem 909. Then

$$K(w\backslash e,w)\cdot(x\cdot T(w,w\backslash e)) = x\cdot w \quad (475)$$

by Theorem 923. We have $K(w\backslash e,w)\cdot R(x\cdot T(w,w\backslash e),y,z) = (w\backslash R(w\cdot x,y,z))\cdot w$ by Theorem 927. Then $R(K(w\backslash e,w)\cdot(x\cdot T(w,w\backslash e)),y,z) = (w\backslash R(w\cdot x,y,z))\cdot w$ by Theorem 970. Hence we are done by (475). □

Theorem 972. $R(y\cdot x,z,w)/x = x\backslash R(x\cdot y,z,w)$.

Proof. We have $(x\backslash R(x\cdot y,z,w))\cdot x = R(y\cdot x,z,w)$ by Theorem 971. Hence we are done by Proposition 1. □

Theorem 973. $(y\backslash R(x,z,w))\cdot y = R((y\backslash x)\cdot y,z,w)$.

Proof. We have $(y\backslash R(y\cdot(y\backslash x),z,w))\cdot y = R((y\backslash x)\cdot y,z,w)$ by Theorem 971. Hence we are done by Axiom 4. □

Theorem 974. $R((z\backslash(x/(e/y)))\cdot z,y\backslash e,y) = (z\backslash(x\cdot y))\cdot z$.

Proof. We have $R((z\backslash(x/(e/y)))\cdot z, y\backslash e, y) = (z\backslash R(x/(e/y), y\backslash e, y))\cdot z$ by Theorem 973. Hence we are done by Theorem 912. □

Theorem 975. $y\cdot(R(x,z,w)/y) = R(y\cdot(x/y), z, w)$.

Proof. We have $R((x/y)\cdot y, z, w)/y = y\backslash R(y\cdot(x/y), z, w)$ by Theorem 972. Then

$$R(x,z,w)/y = y\backslash R(y\cdot(x/y), z, w) \tag{476}$$

by Axiom 6. We have $y\cdot(y\backslash R(y\cdot(x/y), z, w)) = R(y\cdot(x/y), z, w)$ by Axiom 4. Hence we are done by (476). □

Theorem 976. $y\cdot x = (x\cdot(y\backslash T(y,x)))\cdot y$.

Proof. We have $T((e/(e/(x\backslash(x/(y\backslash e)))))\cdot x, K((x\backslash(x/(y\backslash e)))\backslash e, x\backslash(x/(y\backslash e)))) = ((x\backslash(x/(y\backslash e)))\cdot x)\cdot K((x\backslash(x/(y\backslash e)))\backslash e, x\backslash(x/(y\backslash e)))$ by Theorem 916. Then $T((e/(e/(x\backslash(x/(y\backslash e)))))\cdot x, K((x\backslash(x/(y\backslash e)))\backslash e, x\backslash(x/(y\backslash e)))) = ((x\backslash(x/(y\backslash e)))\cdot x)\cdot K((x/(y\backslash e))\backslash x, x\backslash(x/(y\backslash e)))$ by Theorem 666. Then $(e/(e/(x\backslash(x/(y\backslash e)))))\cdot x = ((x\backslash(x/(y\backslash e)))\cdot x)\cdot K((x/(y\backslash e))\backslash x, x\backslash(x/(y\backslash e)))$ by Theorem 717. Then $((x\backslash(x/(y\backslash e)))\cdot x)\cdot K(y\backslash e, x\backslash(x/(y\backslash e))) = (e/(e/(x\backslash(x/(y\backslash e)))))\cdot x$ by Proposition 25. Then $((x\backslash(x/(y\backslash e)))\cdot x)\cdot K(y\backslash e, (y\backslash e)\backslash e) = (e/(e/(x\backslash(x/(y\backslash e)))))\cdot x$ by Theorem 895. Then $((x\backslash(x/(y\backslash e)))\cdot x)\cdot K(y\backslash e, y) = (e/(e/(x\backslash(x/(y\backslash e)))))\cdot x$ by Theorem 779. Then $((x\backslash(x/(y\backslash e)))\cdot x)\cdot K(y\backslash e, y) = (e/((x\backslash(x/(e/y)))\backslash e))\cdot x$ by Theorem 419. Then

$$((x\backslash(x/(y\backslash e)))\cdot x)\cdot K(y\backslash e, y) = (x\backslash(x/(e/y)))\cdot x \tag{477}$$

by Proposition 24. We have $((((x\backslash(x/(y\backslash e)))\cdot x)\cdot K(y\backslash e, y))\cdot(e/y))\cdot y = R(((x\backslash(x/(y\backslash e)))\cdot x)\cdot K(y\backslash e, y), y\backslash e, y)$ by Theorem 911. Then $(((x\backslash(x/(y\backslash e)))\cdot x)\cdot(y\backslash e))\cdot y = R(((x\backslash(x/(y\backslash e)))\cdot x)\cdot K(y\backslash e, y), y\backslash e, y)$ by Theorem 411. Then $R(((x\backslash(x/(y\backslash e)))\cdot x)\cdot K(y\backslash e, y), y\backslash e, y) = (x\cdot((y\backslash e)\cdot T((y\backslash e)\backslash e, x)))\cdot y$ by Theorem 840. Then $R((x\backslash(x/(e/y)))\cdot x, y\backslash e, y) = (x\cdot((y\backslash e)\cdot T((y\backslash e)\backslash e, x)))\cdot y$ by (477). Then $R((x\backslash(x/(e/y)))\cdot x, y\backslash e, y) = (x\cdot(y\backslash T(y,x)))\cdot y$ by Theorem 145. Then $(x\backslash(x\cdot y))\cdot x = (x\cdot(y\backslash T(y,x)))\cdot y$ by Theorem 974. Hence we are done by Axiom 3. □

Theorem 977. $(x\cdot y)/x = y\cdot(x\backslash T(x,y))$.

Proof. We have $(y\cdot(x\backslash T(x,y)))\cdot x = x\cdot y$ by Theorem 976. Hence we are done by Proposition 1. □

Theorem 978. $T(y,x)\cdot(x\backslash T(x, T(y,x))) = y$.

Proof. We have

$$(T(y,x)\cdot(x\backslash T(x, T(y,x))))\cdot x = x\cdot T(y,x) \tag{478}$$

by Theorem 976. We have $y\cdot x = x\cdot T(y,x)$ by Proposition 46. Hence we are done by (478) and Proposition 8. □

Theorem 979. $y\backslash((x\cdot y)/x) = x\backslash T(x,y)$.

Proof. We have $y\cdot(x\backslash T(x,y)) = (x\cdot y)/x$ by Theorem 977. Hence we are done by Proposition 2. □

Theorem 980. $y\backslash T(y, x\backslash e) = x\cdot((y\cdot(x\backslash e))/y)$.

Proof. We have $(x\backslash e)\cdot(((y\cdot(x\backslash e))/y)/(x\backslash e)) = (y\cdot(x\backslash e))/y$ by Theorem 942. Then

$$(x\backslash e)\cdot(x\cdot((y\cdot(x\backslash e))/y)) = (y\cdot(x\backslash e))/y \tag{479}$$

by Theorem 859. We have $(x\backslash e)\cdot(y\backslash T(y, x\backslash e)) = (y\cdot(x\backslash e))/y$ by Theorem 977. Hence we are done by (479) and Proposition 7. □

Theorem 981. $x\backslash T(x,y) = ((x\cdot y)/x)/y$.

Proof. We have
$$y \cdot (x\backslash T(x,y)) = (x \cdot y)/x \tag{480}$$
by Theorem 977. We have $y \cdot (((x \cdot y)/x)/y) = (x \cdot y)/x$ by Theorem 942. Hence we are done by (480) and Proposition 7. ☐

Theorem 982. $x \cdot (y \cdot T(y\backslash e, x)) = ((y\backslash e) \cdot x)/(y\backslash e)$.

Proof. We have $x \cdot ((y\backslash e)\backslash T(y\backslash e, x)) = ((y\backslash e) \cdot x)/(y\backslash e)$ by Theorem 977. Hence we are done by Theorem 729. ☐

Theorem 983. $((x \cdot y)/x) \cdot (y\backslash e) = x\backslash T(x,y)$.

Proof. We have $((L(x,e/x,y)\cdot y)/L(x,e/x,y))\cdot(y\backslash e) = y\backslash((L(x,e/x,y)\cdot y)/L(x,e/x,y))$ by Theorem 195. Then $(y\backslash((L(x,e/x,y) \cdot y)/L(x,e/x,y)))/(y\backslash e) = (L(x,e/x,y) \cdot y)/L(x,e/x,y)$ by Proposition 1. Then $(y\backslash((L(x,e/x,y)\cdot y)/L(x,e/x,y)))/(y\backslash e) = y\cdot(T(x,y)/x)$ by Theorem 883. Then $(y\backslash(y\cdot(T(x,y)/x)))/(y\backslash e) = y \cdot (T(x,y)/x)$ by Theorem 883. Then $(T(x,y)/x)/(y\backslash e) = y \cdot (T(x,y)/x)$ by Axiom 3. Then $(x\backslash T(x,y))/(y\backslash e) = y \cdot (T(x,y)/x)$ by Theorem 758. Then
$$(x\backslash T(x,y))/(y\backslash e) = y \cdot (x\backslash T(x,y)) \tag{481}$$
by Theorem 758. We have $((x\backslash T(x,y))/(y\backslash e)) \cdot (y\backslash e) = x\backslash T(x,y)$ by Axiom 6. Then $(y \cdot (x\backslash T(x,y))) \cdot (y\backslash e) = x\backslash T(x,y)$ by (481). Hence we are done by Theorem 977. ☐

Theorem 984. $((y \cdot x)/y)\backslash x = T(y,x)\backslash y$.

Proof. We have $(x \cdot (y\backslash T(y,x)))\backslash x = T(y,x)\backslash y$ by Theorem 952. Hence we are done by Theorem 977. ☐

Theorem 985. $T(x,y)\backslash x = y\backslash T(y, T(x,y))$.

Proof. We have $T(x,y) \cdot (y\backslash T(y, T(x,y))) = x$ by Theorem 978. Hence we are done by Proposition 2. ☐

Theorem 986. $x\backslash T(x,y) = T(y, T(x,y))\backslash y$.

Proof. We have $(T(x,y) \cdot (y\backslash T(y, T(x,y))))\backslash T(x,y) = T(y, T(x,y))\backslash y$ by Theorem 952. Hence we are done by Theorem 978. ☐

Theorem 987. $y\backslash(x/(x/y)) = (x/y)\backslash T(x/y, y)$.

Proof. We have $y\backslash(((x/y) \cdot y)/(x/y)) = (x/y)\backslash T(x/y, y)$ by Theorem 979. Hence we are done by Axiom 6. ☐

Theorem 988. $L(y\backslash T(y,x), x, y) = K(y, (y \cdot x)/y)$.

Proof. We have $L(x\backslash((y \cdot x)/y), x, y) = K(y, (y \cdot x)/y)$ by Theorem 799. Hence we are done by Theorem 979. ☐

Theorem 989. $(y\backslash T(y,x)) \cdot y = T(y, R(x, x\backslash e, y))$.

Proof. We have $(x\backslash((y \cdot x)/y)) \cdot y = T(y, R(x, x\backslash e, y))$ by Theorem 589. Hence we are done by Theorem 979. ☐

Theorem 990. $(x/y)\backslash(y\backslash x) = T(y, y\backslash x)\backslash y$.

Proof. We have $((y \cdot (y\backslash x))/y)\backslash(y\backslash x) = T(y, y\backslash x)\backslash y$ by Theorem 984. Hence we are done by Axiom 4. ☐

Theorem 991. $y \cdot (T(x,y)\backslash x) = T(y, T(x,y))$.

Proof. We have $y \cdot (y\backslash T(y, T(x, y))) = T(y, T(x, y))$ by Axiom 4. Hence we are done by Theorem 985. □

Theorem 992. $T(y, T(x, y)) \cdot (x\backslash T(x, y)) = y$.

Proof. We have $T(y, T(x, y)) \cdot (T(y, T(x, y))\backslash y) = y$ by Axiom 4. Hence we are done by Theorem 986. □

Theorem 993. $(x\backslash T(x, y))\backslash y = T(y, T(x, y))$.

Proof. We have $(T(y, T(x, y))\backslash y)\backslash y = T(y, T(x, y))$ by Theorem 930. Hence we are done by Theorem 986. □

Theorem 994. $T(y, T(L(x\backslash e, x, y)\backslash e, y)) = K(x, y)\backslash y$.

Proof. We have $y \cdot (T(L(x\backslash e, x, y)\backslash e, y)\backslash(L(x\backslash e, x, y)\backslash e)) = K(x, y)\backslash y$ by Theorem 959. Hence we are done by Theorem 991. □

Theorem 995. $T(x, y)\backslash((x \cdot y)/x) = x\backslash y$.

Proof. We have $(x\backslash T(x, y)) \cdot (x \cdot (T(x, y)\backslash y)) = L(y, x\backslash T(x, y), x)$ by Theorem 955. Then

$$(x\backslash T(x, y)) \cdot T(y, T(x, y)) = L(y, x\backslash T(x, y), x) \tag{482}$$

by Theorem 177. We have $(x\backslash T(x, y)) \cdot T(T(y, x\backslash T(x, y)), T(x, y)) = T(y, T(x, y)) \cdot (x\backslash T(x, y))$ by Proposition 50. Then $(x\backslash T(x, y)) \cdot T(y, T(x, y)) = T(y, T(x, y)) \cdot (x\backslash T(x, y))$ by Theorem 936. Then

$$T(y, T(x, y)) \cdot (x\backslash T(x, y)) = L(y, x\backslash T(x, y), x) \tag{483}$$

by (482). We have

$$x \cdot (T(x, y)\backslash((x\backslash T(x, y)) \cdot y)) = L(y, x\backslash T(x, y), x) \tag{484}$$

by Theorem 832.

$$\begin{array}{lll} & x \cdot (T(x, y)\backslash(y \cdot (x\backslash T(x, y)))) & \\ = & L(y, x\backslash T(x, y), x) & \text{by (484), Theorem 937} \\ = & y & \text{by (483), Theorem 992.} \end{array}$$

Then $x \cdot (T(x, y)\backslash(y \cdot (x\backslash T(x, y)))) = y$. Then $x\backslash y = T(x, y)\backslash(y \cdot (x\backslash T(x, y)))$ by Proposition 2. Hence we are done by Theorem 977. □

Theorem 996. $T(y, x) \cdot (y\backslash x) = (y \cdot x)/y$.

Proof. We have $T(y, x)\cdot(T(y, x)\backslash((y\cdot x)/y)) = (y\cdot x)/y$ by Axiom 4. Hence we are done by Theorem 995. □

Theorem 997. $T(x, y) = T(x, x/(x/y))$.

Proof. We have

$$T(x, y) \cdot (x\backslash y) = (x \cdot y)/x \tag{485}$$

by Theorem 996. We have $T(x, x/(x/y)) \cdot (x\backslash y) = (x \cdot y)/x$ by Theorem 861. Hence we are done by (485) and Proposition 8. □

Theorem 998. $T(x, T(y, x)) \cdot (y \cdot x) = x \cdot (x \cdot y)$.

Proof. We have $T(x, x/(x/T(y, x))) \cdot (y \cdot x) = x \cdot (x \cdot y)$ by Theorem 848. Hence we are done by Theorem 997. □

Theorem 999. $T(x, R(y, y\backslash e, x)) = T(x, y)$.

Proof. We have $(x\backslash T(x,y)) \cdot x = T(x,y)$ by Theorem 759. Hence we are done by Theorem 989. ☐

Theorem 1000. $T(x, R(y\backslash e, y, x)) = T(x, y\backslash e)$.

Proof. We have $T(x, R(y\backslash e, (y\backslash e)\backslash e, x)) = T(x, y\backslash e)$ by Theorem 999. Hence we are done by Theorem 704. ☐

Theorem 1001. $T(y, (x\backslash y)\backslash e) = y \cdot K(y, x/y)$.

Proof. We have $T(y, R((x\backslash y)\backslash e, ((x\backslash y)\backslash e)\backslash e, y)) = y \cdot K(y, x/y)$ by Theorem 954. Hence we are done by Theorem 999. ☐

Theorem 1002. $T(y, x\backslash e) = T(y, y/(x \cdot y))$.

Proof. We have $T(y, ((y/(x \cdot y))\backslash e)\backslash e) = T(y, y/(x \cdot y))$ by Theorem 721. Then $T(y, R(x\backslash e, x, y)) = T(y, y/(x \cdot y))$ by Theorem 670. Hence we are done by Theorem 1000. ☐

Theorem 1003. $T(x, y\backslash e) = T(x, x/(x \cdot y))$.

Proof. We have $T(x, R(e/y, y, x)) = T(x, R(y\backslash e, y, x))$ by Theorem 725. Then $T(x, x/(x \cdot y)) = T(x, R(y\backslash e, y, x))$ by Theorem 944. Hence we are done by Theorem 1000. ☐

Theorem 1004. $y\backslash T(y, x\backslash e) = K(y, (y/x)/y)$.

Proof. We have $T(y, R(e/x, x, y)) = T(y, y/(y \cdot x))$ by Theorem 944. Then

$$T(y, y/(x \cdot y)) = T(y, y/(y \cdot x)) \tag{486}$$

by Proposition 79. We have $y\backslash T(y, y/(y \cdot x)) = x \cdot ((y \cdot (x\backslash e))/y)$ by Theorem 846. Then $y\backslash T(y, y/(y \cdot x)) = K(y, (y/x)/y)$ by Theorem 333. Then $y\backslash T(y, y/(x \cdot y)) = K(y, (y/x)/y)$ by (486). Hence we are done by Theorem 1002. ☐

Theorem 1005. $(x \cdot ((x/y)/x)) \cdot y = T(x, y\backslash e)$.

Proof. We have $(x \cdot ((x/y)/x)) \cdot y = T(x, x/(x \cdot y))$ by Theorem 881. Hence we are done by Theorem 1003. ☐

Theorem 1006. $L(x\backslash T(x,y), y, x) = K(T(x,y), y)$.

Proof. We have $L((x\backslash(T(x,y) \cdot y))/y, y, x) = (y \cdot ((x \cdot y)\backslash(T(x,y) \cdot y)))/y$ by Proposition 75. Then $L((x\backslash(T(x,y) \cdot y))/y, y, x) = (y \cdot K(T(x,y), y))/y$ by Theorem 12. Then $L((T(x,y) \cdot (x\backslash y))/y, y, x) = (y \cdot K(T(x,y), y))/y$ by Theorem 173. Then $L((T(x,y) \cdot (x\backslash y))/y, y, x) = K(T(x,y), y)$ by Theorem 934. Then $L(((x \cdot y)/x)/y, y, x) = K(T(x,y), y)$ by Theorem 996. Hence we are done by Theorem 981. ☐

Theorem 1007. $L(T(x\backslash T(x,y), z), y, x) = T(K(T(x,y), y), z)$.

Proof. We have $L(T(x\backslash T(x,y), z), y, x) = T(L(x\backslash T(x,y), y, x), z)$ by Axiom 8. Hence we are done by Theorem 1006. ☐

Theorem 1008. $K(T(y,x), x) = K(y, (y \cdot x)/y)$.

Proof. We have $L(y\backslash T(y,x), x, y) = K(y, (y \cdot x)/y)$ by Theorem 988. Hence we are done by Theorem 1006. ☐

Theorem 1009. $(x/y)\backslash(y\backslash x) = y\backslash(x/(x/y))$.

Proof. We have $(x/y)\backslash T(x/y, y) = y\backslash(x/(x/y))$ by Theorem 987. Hence we are done by Proposition 47. ☐

Theorem 1010. $T(y, y\backslash x)\backslash y = y\backslash(x/(x/y))$.

Proof. We have $T(y, y\backslash x)\backslash y = (x/y)\backslash(y\backslash x)$ by Theorem 990. Hence we are done by Theorem 1009. ☐

Theorem 1011. $(y \cdot x)/((y \cdot x)/y) = y \cdot (T(y, x)\backslash y)$.

Proof. We have $y \cdot (y\backslash((y \cdot x)/((y \cdot x)/y))) = (y \cdot x)/((y \cdot x)/y)$ by Axiom 4. Then $y \cdot (((y \cdot x)/y)\backslash(y\backslash(y \cdot x))) = (y \cdot x)/((y \cdot x)/y)$ by Theorem 1009. Then $y \cdot (T(y, y\backslash(y \cdot x))\backslash y) = (y \cdot x)/((y \cdot x)/y)$ by Theorem 990. Hence we are done by Axiom 3. □

Theorem 1012. $T(y, ((y \cdot x)\backslash y)\backslash K(x, y)) = K(x, y)\backslash y$.

Proof. We have $T(y, T(L(x\backslash e, x, y)\backslash e, y)) = K(x, y)\backslash y$ by Theorem 994. Hence we are done by Theorem 540. □

Theorem 1013. $((L(x\backslash e, x, y)\backslash e) \cdot y)/(L(x\backslash e, x, y)\backslash e) = y \cdot K(x, y)$.

Proof. We have $((L(x\backslash e, x, y)\backslash e) \cdot y)/(L(x\backslash e, x, y)\backslash e) = y \cdot (L(x\backslash e, x, y) \cdot T(L(x\backslash e, x, y)\backslash e, y))$ by Theorem 982. Hence we are done by Theorem 542. □

Theorem 1014. $T(x\backslash T(x, y\backslash e), z) = x\backslash T(x, T(y, z)\backslash e)$.

Proof. We have $T(y \cdot ((x \cdot (y\backslash e))/x), z) = K(x, (x/T(y, z))/x)$ by Theorem 344. Then $T(x\backslash T(x, x/(x \cdot y)), z) = K(x, (x/T(y, z))/x)$ by Theorem 846. Then

$$T(x\backslash T(x, y\backslash e), z) = K(x, (x/T(y, z))/x) \tag{487}$$

by Theorem 1003. We have $K(x, (x/T(y, z))/x) = x\backslash T(x, T(y, z)\backslash e)$ by Theorem 1004. Hence we are done by (487). □

Theorem 1015. $T(y, x) \cdot T(z, y) = (T(y, x) \cdot (z\backslash T(z, y))) \cdot z$.

Proof. We have $R(y, z\backslash T(z, y), z) = ((y \cdot (z\backslash T(z, y))) \cdot z)/((z\backslash T(z, y)) \cdot z)$ by Definition 5. Then $R(y, z\backslash T(z, y), z) = (z \cdot y)/((z\backslash T(z, y)) \cdot z)$ by Theorem 976. Then

$$(z \cdot y)/T(z, y) = R(y, z\backslash T(z, y), z) \tag{488}$$

by Theorem 759. We have $(z \cdot y)/T(z, y) = y$ by Theorem 5. Then

$$R(y, z\backslash T(z, y), z) = y \tag{489}$$

by (488). We have $R(T(y, x), z\backslash T(z, y), z) = T(R(y, z\backslash T(z, y), z), x)$ by Axiom 9. Then

$$R(T(y, x), z\backslash T(z, y), z) = T(y, x) \tag{490}$$

by (489). We have $R(T(y, x), z\backslash T(z, y), z) \cdot ((z\backslash T(z, y)) \cdot z) = (T(y, x) \cdot (z\backslash T(z, y))) \cdot z$ by Proposition 54. Then $R(T(y, x), z\backslash T(z, y), z) \cdot T(z, y) = (T(y, x) \cdot (z\backslash T(z, y))) \cdot z$ by Theorem 759. Hence we are done by (490). □

Theorem 1016. $K(T(y, x), x) \cdot z = K(y, (y \cdot x)/y) \cdot z$.

Proof. We have $(z \cdot L(T(y\backslash T(y, x), z), x, y)) \cdot K(L(T(y\backslash T(y, x), z), x, y), z) = L(T(y\backslash T(y, x), z), x, y) \cdot z$ by Proposition 82. Then $(L(y\backslash T(y, x), x, y) \cdot z) \cdot K(L(T(y\backslash T(y, x), z), x, y), z) = L(T(y\backslash T(y, x), z), x, y) \cdot z$ by Proposition 58. Then $(K(y, (y \cdot x)/y) \cdot z) \cdot K(L(T(y\backslash T(y, x), z), x, y), z) = L(T(y\backslash T(y, x), z), x, y) \cdot z$ by Theorem 988. Then

$$(K(y, (y \cdot x)/y) \cdot z) \cdot K(T(K(T(y, x), x), z), z) = L(T(y\backslash T(y, x), z), x, y) \cdot z \tag{491}$$

by Theorem 1007. We have

$$(z \cdot T(K(T(y, x), x), z)) \cdot K(T(K(T(y, x), x), z), z) = T(K(T(y, x), x), z) \cdot z \tag{492}$$

by Proposition 82.

$$\begin{aligned} & (K(T(y,x),x)\cdot z)\cdot K(T(K(T(y,x),x),z),z) \\ = & \quad T(K(T(y,x),x),z)\cdot z \quad \text{by (492), Proposition 46} \\ = & \quad (K(y,(y\cdot x)/y)\cdot z)\cdot K(T(K(T(y,x),x),z),z) \quad \text{by (491), Theorem 1007.} \end{aligned}$$

Then $(K(T(y,x),x)\cdot z)\cdot K(T(K(T(y,x),x),z),z) = (K(y,(y\cdot x)/y)\cdot z)\cdot K(T(K(T(y,x),x),z),z)$. Hence we are done by Proposition 10. □

Theorem 1017. $T(x,y)\cdot z = x\cdot((x\backslash T(x,y))\cdot z)$.

Proof. We have

$$x\cdot(((y\backslash e)\cdot(x\backslash(x\cdot z)))\cdot((x\cdot(((y\backslash e)\cdot(x\backslash(x\cdot z)))\backslash(x\backslash(x\cdot z))))/x)) = (x\cdot(((x\cdot z)/((y\backslash e)\cdot(x\backslash(x\cdot z))))/x))\cdot((y\backslash e)\cdot(x\backslash(x\cdot z))) \tag{493}$$

by Theorem 882. Then $x\cdot(((y\backslash e)\cdot(x\backslash(x\cdot z)))\cdot((x\cdot(((y\backslash e)\cdot(x\backslash(x\cdot z)))\backslash(x\backslash(x\cdot z))))/x)) = (x\cdot(R(x/(y\backslash e),y\backslash e,x\backslash(x\cdot z))/x))\cdot((y\backslash e)\cdot(x\backslash(x\cdot z)))$ by Theorem 794. Then $(x\cdot(R(x/(y\backslash e),y\backslash e,x\backslash(x\cdot z))/x))\cdot((y\backslash e)\cdot(x\backslash(x\cdot z))) = (x\cdot(((x\cdot z)/((y\backslash e)\cdot(x\backslash(x\cdot z))))/x))\cdot((y\backslash e)\cdot(x\backslash(x\cdot z)))$ by (493). Then

$$R(x\cdot((x/(y\backslash e))/x),y\backslash e,x\backslash(x\cdot z))\cdot((y\backslash e)\cdot(x\backslash(x\cdot z))) = (x\cdot(((x\cdot z)/((y\backslash e)\cdot(x\backslash(x\cdot z))))/x))\cdot((y\backslash e)\cdot(x\backslash(x\cdot z))) \tag{494}$$

by Theorem 975. We have $R(x\cdot((x/(y\backslash e))/x),y\backslash e,x\backslash(x\cdot z))\cdot((y\backslash e)\cdot(x\backslash(x\cdot z))) = ((x\cdot((x/(y\backslash e))/x))\cdot(y\backslash e))\cdot(x\backslash(x\cdot z))$ by Proposition 54. Then

$$(x\cdot(((x\cdot z)/((y\backslash e)\cdot(x\backslash(x\cdot z))))/x))\cdot((y\backslash e)\cdot(x\backslash(x\cdot z))) = ((x\cdot((x/(y\backslash e))/x))\cdot(y\backslash e))\cdot(x\backslash(x\cdot z)) \tag{495}$$

by (494). We have $x\cdot(((y\backslash e)\cdot((x\cdot((y\backslash e)\backslash e))/x))\cdot(x\backslash(x\cdot z))) = (x\cdot(((x\cdot z)/((y\backslash e)\cdot(x\backslash(x\cdot z))))/x))\cdot((y\backslash e)\cdot(x\backslash(x\cdot z)))$ by (493) and Theorem 343. Then $(x\cdot(((x\cdot z)/((y\backslash e)\cdot(x\backslash(x\cdot z))))/x))\cdot((y\backslash e)\cdot(x\backslash(x\cdot z))) = x\cdot((x\backslash T(x,(y\backslash e)\backslash e))\cdot(x\backslash(x\cdot z)))$ by Theorem 980. Then $((x\cdot((x/(y\backslash e))/x))\cdot(y\backslash e))\cdot(x\backslash(x\cdot z)) = x\cdot((x\backslash T(x,(y\backslash e)\backslash e))\cdot(x\backslash(x\cdot z)))$ by (495). Then $T(x,(y\backslash e)\backslash e)\cdot(x\backslash(x\cdot z)) = x\cdot((x\backslash T(x,(y\backslash e)\backslash e))\cdot(x\backslash(x\cdot z)))$ by Theorem 1005. Then $x\cdot((x\backslash T(x,y))\cdot(x\backslash(x\cdot z))) = T(x,(y\backslash e)\backslash e)\cdot(x\backslash(x\cdot z))$ by Theorem 721. Then $T(x,y)\cdot(x\backslash(x\cdot z)) = x\cdot((x\backslash T(x,y))\cdot(x\backslash(x\cdot z)))$ by Theorem 721. Then $x\cdot((x\backslash T(x,y))\cdot z) = T(x,y)\cdot(x\backslash(x\cdot z))$ by Axiom 3. Hence we are done by Axiom 3. □

Theorem 1018. $(x\backslash T(x,y))\cdot(x\cdot z) = T(x,y)\cdot z$.

Proof. We have

$$x\cdot((x\backslash T(x,y))\cdot(x\cdot z)) = T(x,y)\cdot(x\cdot z) \tag{496}$$

by Theorem 1017. We have $x\cdot(T(x,y)\cdot z) = T(x,y)\cdot(x\cdot z)$ by Theorem 172. Hence we are done by (496) and Proposition 7. □

Theorem 1019. $R(z,K(x,z)\backslash e,y) = z$.

Proof. We have $z\cdot((z\backslash T(z,T(L(x\backslash e,x,z)\backslash e,z)))\cdot y) = T(z,T(L(x\backslash e,x,z)\backslash e,z))\cdot y$ by Theorem 1017. Then

$$(T(z,T(L(x\backslash e,x,z)\backslash e,z))\cdot y)/((z\backslash T(z,T(L(x\backslash e,x,z)\backslash e,z)))\cdot y) = z \tag{497}$$

by Proposition 1. We have $R(z,z\backslash T(z,T(L(x\backslash e,x,z)\backslash e,z)),y) = (T(z,T(L(x\backslash e,x,z)\backslash e,z))\cdot y)/((z\backslash T(z,T(L(x\backslash e,x,z)\backslash e,z)))\cdot y)$ by Theorem 785. Then $R(z,z\backslash T(z,T(L(x\backslash e,x,z)\backslash e,z)),y) = z$ by (497). Then $R(z,z\backslash(K(x,z)\backslash z),y) = z$ by Theorem 994. Hence we are done by Theorem 958. □

Theorem 1020. $(T(y,z)\cdot x)/(y\cdot x) = y\backslash T(y,z)$.

Proof. We have $(y\backslash T(y,z))\cdot(y\cdot x) = T(y,z)\cdot x$ by Theorem 1018. Hence we are done by Proposition 1. □

Theorem 1021. $L(y\backslash T(y,T(x,y)),T(x,y),y) = R(K(y,x),y,z)$.

Proof. We have $R(y\backslash T(y,T(x,y)),y,z) = (T(y,T(x,y))\cdot z)/(y\cdot z)$ by Theorem 943. Then

$$R(T(x,y)\backslash x,y,z) = (T(y,T(x,y))\cdot z)/(y\cdot z) \tag{498}$$

by Theorem 985. We have $L(R(T(x,y)\backslash x,y,z),T(x,y),y) = R((x\cdot y)\backslash(y\cdot x),y,z)$ by Theorem 817. Then $L((T(y,T(x,y))\cdot z)/(y\cdot z),T(x,y),y) = R((x\cdot y)\backslash(y\cdot x),y,z)$ by (498). Then $L((T(y,T(x,y))\cdot z)/(y\cdot z),T(x,y),y) = R(K(y,x),y,z)$ by Definition 2. Hence we are done by Theorem 1020. □

Theorem 1022. $(T(y,x)\backslash y)\cdot(y\cdot x) = x\cdot y$.

Proof. We have $x\cdot((x\backslash T(x,T(y,x)))\cdot(y\cdot x)) = T(x,T(y,x))\cdot(y\cdot x)$ by Theorem 1017. Then

$$x\cdot((T(y,x)\backslash y)\cdot(y\cdot x)) = T(x,T(y,x))\cdot(y\cdot x) \tag{499}$$

by Theorem 985. We have $x\cdot(x\cdot y) = T(x,T(y,x))\cdot(y\cdot x)$ by Theorem 998. Hence we are done by (499) and Proposition 7. □

Theorem 1023. $(x\cdot y)/(y\cdot x) = T(y,x)\backslash y$.

Proof. We have $(T(y,x)\backslash y)\cdot(y\cdot x) = x\cdot y$ by Theorem 1022. Hence we are done by Proposition 1. □

Theorem 1024. $(x\cdot(y/x))/y = x\backslash((x\backslash y)\backslash y)$.

Proof. We have $(y/x)\backslash((y/x)\cdot x) = x$ by Axiom 3. Then $(y/x)\backslash(x\cdot(x\backslash((y/x)\cdot x))) = x$ by Axiom 4. Then

$$(y/x)\backslash(x\cdot(x\backslash y)) = x \tag{500}$$

by Axiom 6. We have $(y/x)\cdot((y/x)\backslash(x\cdot(x\backslash y))) = x\cdot(x\backslash y)$ by Axiom 4. Then

$$(y/x)\cdot x = x\cdot(x\backslash y) \tag{501}$$

by (500). We have $((x\backslash y)\cdot(x\backslash T(x,x\backslash y)))\cdot x = x\cdot(x\backslash y)$ by Theorem 976. Then $(x\backslash y)\cdot(x\backslash T(x,x\backslash y)) = y/x$ by (501) and Proposition 8. Then $(x\backslash y)\cdot(x\backslash((x\backslash y)\backslash y)) = y/x$ by Proposition 49. Then

$$(x\backslash y)\backslash(y/x) = x\backslash((x\backslash y)\backslash y) \tag{502}$$

by Proposition 2. We have $(x\cdot(y/x))/((y/x)\cdot x) = T(y/x,x)\backslash(y/x)$ by Theorem 1023. Then $(x\cdot(y/x))/y = T(y/x,x)\backslash(y/x)$ by Axiom 6. Then $(x\backslash y)\backslash(y/x) = (x\cdot(y/x))/y$ by Proposition 47. Hence we are done by (502). □

Theorem 1025. $K(y,x) = R(K(y,x),y,z)$.

Proof. We have $L(y\backslash T(y,T(x,y)),T(x,y),y) = K(y,(y\cdot T(x,y))/y)$ by Theorem 988. Then $L(y\backslash T(y,T(x,y)),T(x,y),y) = K(y,x)$ by Proposition 48. Hence we are done by Theorem 1021. □

Theorem 1026. $((y\backslash x)/x)\backslash T((y\backslash x)/x,z) = y\cdot T(y\backslash e,z)$.

Proof. We have $R(T((y\backslash x)/x,z)/((y\backslash x)/x),(y\backslash x)/x,x) = (T((y\backslash x)/x,z)\cdot x)/(y\backslash x)$ by Theorem 67. Then $R(((y\backslash x)/x)\backslash T((y\backslash x)/x,z),(y\backslash x)/x,x) = (T((y\backslash x)/x,z)\cdot x)/(y\backslash x)$ by Theorem 758. Then

$$R(((y\backslash x)/x)\backslash T((y\backslash x)/x,z),(y\backslash x)/x,x) = (x\cdot T(x\backslash(y\backslash x),z))/(y\backslash x) \tag{503}$$

by Proposition 61. We have $R(((y\backslash x)/x)\backslash T((y\backslash x)/x,z),(y\backslash x)/x,x) = (T((y\backslash x)/x,z)\cdot x)/(((y\backslash x)/x)\cdot x)$ by Theorem 943. Then $R(((y\backslash x)/x)\backslash T((y\backslash x)/x,z),(y\backslash x)/x,x) = ((y\backslash x)/x)\backslash T((y\backslash x)/x,z)$ by Theorem 1020. Then

$$((y\backslash x)/x)\backslash T((y\backslash x)/x,z) = (x\cdot T(x\backslash(y\backslash x),z))/(y\backslash x) \tag{504}$$

by (503). We have $(x\cdot T(x\backslash(y\backslash x),z))/(y\backslash x) = y\cdot T(y\backslash e,z)$ by Theorem 843. Hence we are done by (504). □

Theorem 1027. $T(x, x\backslash(y\backslash x)) = T(x, y\backslash e)$.

Proof. We have $(x \cdot R(e/y, y, y\backslash x))/(y\backslash x) = ((x \cdot (e/y))/x) \cdot y$ by Theorem 828. Then

$$(x \cdot ((y\backslash x)/x))/(y\backslash x) = ((x \cdot (e/y))/x) \cdot y \tag{505}$$

by Theorem 37. We have $((x \cdot (e/y))/x) \cdot y = y \cdot ((x \cdot (y\backslash e))/x)$ by Theorem 125. Then $(x \cdot ((y\backslash x)/x))/(y\backslash x) = y \cdot ((x \cdot (y\backslash e))/x)$ by (505). Then $(x \cdot ((y\backslash x)/x))/(y\backslash x) = x\backslash T(x, y\backslash e)$ by Theorem 980. Then

$$x\backslash((x\backslash(y\backslash x))\backslash(y\backslash x)) = x\backslash T(x, y\backslash e) \tag{506}$$

by Theorem 1024. We have $(x\backslash T(x, x\backslash(y\backslash x))) \cdot x = T(x, x\backslash(y\backslash x))$ by Theorem 759. Then $(x\backslash((x\backslash(y\backslash x))\backslash(y\backslash x))) \cdot x = T(x, x\backslash(y\backslash x))$ by Proposition 49. Then

$$(x\backslash T(x, y\backslash e)) \cdot x = T(x, x\backslash(y\backslash x)) \tag{507}$$

by (506). We have $(x\backslash T(x, y\backslash e)) \cdot x = T(x, y\backslash e)$ by Theorem 759. Hence we are done by (507). □

Theorem 1028. $x \cdot T(x\backslash e, y) = T(y, x\backslash e)\backslash y$.

Proof. We have

$$((x\backslash y)/y)\backslash T((x\backslash y)/y, y) = y\backslash((x\backslash y)/((x\backslash y)/y)) \tag{508}$$

by Theorem 987. We have

$$T(y, y\backslash(x\backslash y))\backslash y = y\backslash((x\backslash y)/((x\backslash y)/y)) \tag{509}$$

by Theorem 1010.

$$\begin{array}{rcll} & & x \cdot T(x\backslash e, y) & \\ & = & y\backslash((x\backslash y)/((x\backslash y)/y)) & \text{by (508), Theorem 1026} \\ & = & T(y, x\backslash e)\backslash y & \text{by (509), Theorem 1027.} \end{array}$$

Hence we are done. □

Theorem 1029. $K(y\backslash(y/x), y) = T(y, x\backslash e)\backslash y$.

Proof. We have $K(y\backslash(y/x), y) = x \cdot T(x\backslash e, y)$ by Theorem 196. Hence we are done by Theorem 1028. □

Theorem 1030. $T(y, x)\backslash y = x\backslash T(x, y)$.

Proof. We have $(x\backslash e) \cdot T((x\backslash e)\backslash e, y) = x\backslash T(x, y)$ by Theorem 145. Then $T(y, (x\backslash e)\backslash e)\backslash y = x\backslash T(x, y)$ by Theorem 1028. Hence we are done by Theorem 721. □

Theorem 1031. $T(y, x) \cdot (x\backslash T(x, y)) = y$.

Proof. We have $T(y, x) \cdot (T(y, x)\backslash y) = y$ by Axiom 4. Hence we are done by Theorem 1030. □

Theorem 1032. $T(x, T(y, x)) = T(x, y)$.

Proof.

$$\begin{array}{rcll} & & T(x, T(y, x)) \cdot (y\backslash T(y, x)) & \\ & = & x & \text{by Theorem 992} \\ & = & T(x, y) \cdot (y\backslash T(y, x)) & \text{by Theorem 1031.} \end{array}$$

Then $T(x, T(y, x)) \cdot (y\backslash T(y, x)) = T(x, y) \cdot (y\backslash T(y, x))$. Hence we are done by Proposition 10. □

Theorem 1033. $y \cdot x = T(y, x) \cdot T(x, y)$.

Proof. We have $(T(y,x)\cdot(x\backslash T(x,y)))\cdot x = T(y,x)\cdot T(x,y)$ by Theorem 1015. Hence we are done by Theorem 1031. □

Theorem 1034. $T(x,z/y) = T(x,(y/z)\backslash e)$.

Proof. We have $((((y/z)\backslash y)/y)\backslash T(((y/z)\backslash y)/y,x))\backslash x = T(x,T(((y/z)\backslash y)/y,x))$ by Theorem 993. Then

$$((y/z)\cdot T((y/z)\backslash e,x))\backslash x = T(x,T(((y/z)\backslash y)/y,x)) \tag{510}$$

by Theorem 1026. We have $(((y/z)\backslash e)\backslash T((y/z)\backslash e,x))\backslash x = T(x,T((y/z)\backslash e,x))$ by Theorem 993. Then $((y/z)\cdot T((y/z)\backslash e,x))\backslash x = T(x,T((y/z)\backslash e,x))$ by Theorem 729. Then $T(x,T(((y/z)\backslash y)/y,x)) = T(x,T((y/z)\backslash e,x))$ by (510). Then

$$T(x,((y/z)\backslash y)/y) = T(x,T((y/z)\backslash e,x)) \tag{511}$$

by Theorem 1032. We have $T(x,T((y/z)\backslash e,x)) = T(x,(y/z)\backslash e)$ by Theorem 1032. Then $T(x,((y/z)\backslash y)/y) = T(x,(y/z)\backslash e)$ by (511). Hence we are done by Proposition 25. □

Theorem 1035. $K(x,y)\backslash e = K(y,(y\cdot x)/y)$.

Proof. We have $(T(y,((y\cdot x)\backslash y)\backslash e)\backslash y)\backslash y = T(y,((y\cdot x)\backslash y)\backslash e)$ by Theorem 930. Then $K(y\backslash(y/((y\cdot x)\backslash y)),y)\backslash y = T(y,((y\cdot x)\backslash y)\backslash e)$ by Theorem 1029. Then

$$K(y\backslash(y\cdot x),y)\backslash y = T(y,((y\cdot x)\backslash y)\backslash e) \tag{512}$$

by Proposition 24. We have $T(y,((y\cdot x)\backslash y)\backslash e) = y\cdot K(y,(y\cdot x)/y)$ by Theorem 1001. Then

$$K(y\backslash(y\cdot x),y)\backslash y = y\cdot K(y,(y\cdot x)/y) \tag{513}$$

by (512). We have $y\cdot(K(y\backslash(y\cdot x),y)\backslash e) = K(y\backslash(y\cdot x),y)\backslash y$ by Theorem 777. Then $y\cdot(K(y\backslash(y\cdot x),y)\backslash e) = y\cdot K(y,(y\cdot x)/y)$ by (513). Then $K(y\backslash(y\cdot x),y)\backslash e = K(y,(y\cdot x)/y)$ by Proposition 9. Hence we are done by Axiom 3. □

Theorem 1036. $K(y,x) = K(T(x,y),y)\backslash e$.

Proof. We have $K(y,(y\cdot T(x,y))/y) = K(T(x,y),y)\backslash e$ by Theorem 1035. Hence we are done by Proposition 48. □

Theorem 1037. $K(T(y,x),x) = K(x,y)\backslash e$.

Proof. We have $K(T(y,x),x) = K(y,(y\cdot x)/y)$ by Theorem 1008. Hence we are done by Theorem 1035. □

Theorem 1038. $y\cdot K(y,x) = K(T(x,y),y)\backslash y$.

Proof. We have $y\cdot(K(T(x,y),y)\backslash e) = K(T(x,y),y)\backslash y$ by Theorem 777. Hence we are done by Theorem 1036. □

Theorem 1039. $R(z,K(z,x),y) = z$.

Proof. We have $R(z,K(T(x,z),z)\backslash e,y) = z$ by Theorem 1019. Hence we are done by Theorem 1036. □

Theorem 1040. $x\cdot(y\cdot K(x,y)) = (x\cdot K(x,y))\cdot y$.

Proof. We have $R(x,K(x,y),y)\cdot(y\cdot K(x,y)) = (x\cdot K(x,y))\cdot y$ by Theorem 938. Hence we are done by Theorem 1039. □

Theorem 1041. $R(y\backslash T(y,z),z,x) = y\backslash T(y,z)$.

Proof. We have $K(y,(y/R(z\backslash e,z,x))/y) = R(z\backslash e,z,x)\cdot((y\cdot(R(z\backslash e,z,x)\backslash e))/y)$ by Theorem 333. Then $K(y,(y/R(z\backslash e,z,x))/y) = R((z\backslash e)\cdot((y\cdot((z\backslash e)\backslash e))/y),z,x)$ by Theorem 642. Then

$$R(y\backslash T(y,(z\backslash e)\backslash e),z,x) = K(y,(y/R(z\backslash e,z,x))/y) \tag{514}$$

by Theorem 980. We have $K(y,(y/R(z\backslash e,z,x))/y) = y\backslash T(y,R(z\backslash e,z,x)\backslash e)$ by Theorem 1004. Then

$$R(y\backslash T(y,(z\backslash e)\backslash e),z,x) = y\backslash T(y,R(z\backslash e,z,x)\backslash e) \tag{515}$$

by (514). We have $y\backslash T(y,T(x/(z\cdot x),T(z,e))\backslash e) = T(y\backslash T(y,(x/(z\cdot x))\backslash e),T(z,e))$ by Theorem 1014. Then $y\backslash T(y,R(z\backslash e,z,x)\backslash e) = T(y\backslash T(y,(x/(z\cdot x))\backslash e),T(z,e))$ by Theorem 945. Then $R(y\backslash T(y,(z\backslash e)\backslash e),z,x) = T(y\backslash T(y,(x/(z\cdot x))\backslash e),T(z,e))$ by (515). Then $T(y\backslash T(y,(x/(z\cdot x))\backslash e),T(z,e)) = R(y\backslash T(y,z),z,x)$ by Theorem 721. Then $T(y\backslash T(y,(z\cdot x)/x),T(z,e)) = R(y\backslash T(y,z),z,x)$ by Theorem 1034. Then

$$T(y\backslash T(y,z),T(z,e)) = R(y\backslash T(y,z),z,x) \tag{516}$$

by Axiom 5. We have $T(K(z\backslash(z/(y\backslash e)),z),T(z,e)) = K(z\backslash(z/(y\backslash e)),z)$ by Theorem 939. Then $K(K(z\backslash(z/(y\backslash e)),z),T(z,e)) = e$ by Proposition 22. Then $K(y\backslash T(y,z),T(z,e)) = e$ by Theorem 198. Then $T(y\backslash T(y,z),T(z,e)) = y\backslash T(y,z)$ by Proposition 23. Hence we are done by (516). □

Theorem 1042. $K(x,y)\cdot(T(y,z)\cdot w) = (K(x,y)\cdot T(y,z))\cdot w$.

Proof. We have $R((((L(x\backslash e,x,y)\backslash e)\cdot y)/(L(x\backslash e,x,y)\backslash e))/y,y,(y\backslash e)\cdot((y\backslash e)\backslash w))\backslash((((L(x\backslash e,x,y)\backslash e)\cdot y)/(L(x\backslash e,x,y)\backslash e))\cdot((y\backslash e)\cdot((y\backslash e)\backslash w))) = y\cdot((y\backslash e)\cdot((y\backslash e)\backslash w))$ by Theorem 800. Then $R((L(x\backslash e,x,y)\backslash e)\backslash T(L(x\backslash e,x,y)\backslash e,y),y,(y$ $((y\backslash e)\backslash w))\backslash((((L(x\backslash e,x,y)\backslash e)\cdot y)/(L(x\backslash e,x,y)\backslash e))\cdot((y\backslash e)\cdot((y\backslash e)\backslash w))) = y\cdot((y\backslash e)\cdot((y\backslash e)\backslash w))$ by Theorem 981. Then

$$((L(x\backslash e,x,y)\backslash e)\backslash T(L(x\backslash e,x,y)\backslash e,y))\backslash((((L(x\backslash e,x,y)\backslash e)\cdot y)/(L(x\backslash e,x,y)\backslash e))\cdot((y\backslash e)\cdot((y\backslash e)\backslash w))) = y\cdot((y\backslash e)\cdot((y\backslash e)\backslash w)) \tag{517}$$

by Theorem 1041. We have $L((y\backslash e)\backslash w,y\backslash e,((L(x\backslash e,x,y)\backslash e)\cdot y)/(L(x\backslash e,x,y)\backslash e)) = ((((L(x\backslash e,x,y)\backslash e)\cdot y)/(L(x\backslash e,x,y)\backslash e))\cdot(y\backslash e))\backslash((((L(x\backslash e,x,y)\backslash e)\cdot y)/(L(x\backslash e,x,y)\backslash e))\cdot((y\backslash e)\cdot((y\backslash e)\backslash w)))$ by Definition 4. Then $L((y\backslash e)\backslash w,y\backslash e,((L(x\backslash e,x,y)\backslash e)\cdot y)/(L(x\backslash e,x,y)\backslash e)) = ((L(x\backslash e,x,y)\backslash e)\backslash T(L(x\backslash e,x,y)\backslash e,y))\backslash((((L(x\backslash e,x,y)\backslash e)\cdot y)/(L(x\backslash e,x,y)\backslash e))\cdot((y\backslash e)\cdot((y\backslash e)\backslash w)))$ by Theorem 983. Then

$$y\cdot((y\backslash e)\cdot((y\backslash e)\backslash w)) = L((y\backslash e)\backslash w,y\backslash e,((L(x\backslash e,x,y)\backslash e)\cdot y)/(L(x\backslash e,x,y)\backslash e)) \tag{518}$$

by (517). We have $y\cdot((y\backslash e)\cdot((y\backslash e)\backslash w)) = L((y\backslash e)\backslash w,y\backslash e,y)$ by Proposition 56. Then $L((y\backslash e)\backslash w,y\backslash e,((L(x\backslash e,x,y)\backslash e)\cdot y)/(L(x\backslash e,x,y)\backslash e)) = L((y\backslash e)\backslash w,y\backslash e,y)$ by (518). Then

$$L((y\backslash e)\backslash w,y\backslash e,y\cdot K(x,y)) = L((y\backslash e)\backslash w,y\backslash e,y) \tag{519}$$

by Theorem 1013. We have $L((y\backslash e)\backslash w,y\backslash e,y) = y\cdot w$ by Theorem 62. Then

$$L((y\backslash e)\backslash w,y\backslash e,y\cdot K(x,y)) = y\cdot w \tag{520}$$

by (519). We have $(K(x,y)/(y\backslash e))\cdot((y\backslash e)\cdot T((y\backslash e)\backslash e,z)) = K(x,y)\cdot T(K(x,y)\backslash(K(x,y)/(y\backslash e)),z)$ by Theorem 139. Then $(y\cdot K(x,y))\cdot((y\backslash e)\cdot T((y\backslash e)\backslash e,z)) = K(x,y)\cdot T(K(x,y)\backslash(K(x,y)/(y\backslash e)),z)$ by Theorem 898. Then $K(x,y)\cdot T(K(x,y)\backslash(K(x,y)/(y\backslash e)),z) = (y\cdot K(x,y))\cdot(y\backslash T(y,z))$ by Theorem 145. Then $K(x,y)\cdot T(K(x,y)\backslash(y\cdot K(x,y)),z) = (y\cdot K(x,y))\cdot(y\backslash T(y,z))$ by Theorem 898. Then $K(x,y)\cdot T(T(y,K(x,y)),z) = (y\cdot K(x,y))\cdot(y\backslash T(y,z))$ by Definition 3. Then

$$K(x,y)\cdot T(y,z) = (y\cdot K(x,y))\cdot(y\backslash T(y,z)) \tag{521}$$

by Theorem 755. We have $((y\cdot K(x,y))\cdot(y\backslash e))\cdot(w\cdot T(w\backslash L((y\backslash e)\backslash w,y\backslash e,y\cdot K(x,y)),z)) = ((y\cdot K(x,y))\cdot((y\backslash e)\cdot T((y\backslash e)\backslash e,z)))\cdot w$ by Theorem 884. Then $K(x,y)\cdot(w\cdot T(w\backslash L((y\backslash e)\backslash w,y\backslash e,y\cdot K(x,y)),z)) = ((y\cdot K(x,y))\cdot((y\backslash e)\cdot T((y\backslash e)\backslash e,z)))\cdot w$ by Theorem 897. Then $K(x,y)\cdot(w\cdot T(w\backslash L((y\backslash e)\backslash w,y\backslash e,y\cdot K(x,y)),z)) = ((y\cdot K(x,y))\cdot(y\backslash T(y,z)))\cdot w$ by Theorem 145. Then $K(x,y)\cdot(w\cdot T(w\backslash L((y\backslash e)\backslash w,y\backslash e,y\cdot K(x,y)),z)) = (K(x,y)\cdot T(y,z))\cdot w$ by (521). Then $K(x,y)\cdot(w\cdot T(w\backslash(y\cdot w),z)) = (K(x,y)\cdot T(y,z))\cdot w$ by (520). Hence we are done by Theorem 466. □

Theorem 1043. $L(w, T(z,x), K(y,z)) = w$.

Proof. We have $(K(y,z) \cdot T(z,x)) \cdot w = K(y,z) \cdot (T(z,x) \cdot w)$ by Theorem 1042. Hence we are done by Theorem 54. ☐

Theorem 1044. $L(z, y, K(x,y)) = z$.

Proof. We have $L(z, T(y,y), K(x,y)) = z$ by Theorem 1043. Hence we are done by Theorem 450. ☐

Theorem 1045. $L(z, K(x,y), y) = z$.

Proof. We have $((y \cdot (((y \cdot x)\backslash y)\backslash e))/((y \cdot (((y \cdot x)\backslash y)\backslash e))/y))\backslash(y \cdot ((T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y) \cdot z)) = y \cdot (((y \cdot (((y \cdot x)\backslash y)\backslash e))/((y \cdot (((y \cdot x)\backslash y)\backslash e))/y))\backslash((T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y) \cdot z))$ by Theorem 857. Then

$$(y \cdot (T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y))\backslash(y \cdot ((T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y) \cdot z)) = y \cdot (((y \cdot (((y \cdot x)\backslash y)\backslash e))/((y \cdot (((y \cdot x)\backslash y)\backslash e))/y))\backslash((T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y) \cdot z)) \tag{522}$$

by Theorem 1011.

$$\begin{aligned} & L(z, T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y, y) \\ = \quad & (y \cdot (T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y))\backslash(y \cdot ((T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y) \cdot z)) && \text{by Definition 4} \\ = \quad & y \cdot ((y \cdot (T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y))\backslash((T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y) \cdot z)) && \text{by (522), Theorem 1011.} \end{aligned}$$

Then

$$L(z, T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y, y) = y \cdot ((y \cdot (T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y))\backslash((T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y) \cdot z)). \tag{523}$$

We have $L(y\backslash z, y, T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y) = ((T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y) \cdot y)\backslash((T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y) \cdot z)$ by Proposition 53. Then $L(y\backslash z, y, T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y) = (y \cdot (T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y))\backslash((T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y) \cdot z)$ by Theorem 931. Then

$$y \cdot L(y\backslash z, y, T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y) = L(z, T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y, y) \tag{524}$$

by (523). We have $L(y\backslash z, y, K(y\backslash(y/(((((y \cdot x)\backslash y)\backslash e)\backslash y)/T(y, ((y \cdot x)\backslash y)\backslash e))), y)) = y\backslash z$ by Theorem 1044. Then $L(y\backslash z, y, T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y) = y\backslash z$ by Theorem 932. Then

$$y \cdot (y\backslash z) = L(z, T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y, y) \tag{525}$$

by (524). We have $y \cdot (y\backslash z) = z$ by Axiom 4. Then

$$L(z, T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y, y) = z \tag{526}$$

by (525). We have $K(x,y) = ((y \cdot x)\backslash y) \cdot T(((y \cdot x)\backslash y)\backslash e, y)$ by Theorem 539. Then $K(x,y) = T(y, ((y \cdot x)\backslash y)\backslash e)\backslash y$ by Theorem 1028. Hence we are done by (526). ☐

Theorem 1046. $(y \cdot K(x,y)) \cdot z = y \cdot (K(x,y) \cdot z)$.

Proof. We have $(y \cdot K(x,y)) \cdot L(z, K(x,y), y) = y \cdot (K(x,y) \cdot z)$ by Proposition 52. Hence we are done by Theorem 1045. ☐

Theorem 1047. $R(z, K(x,z), y) = z$.

Proof. We have $(z \cdot K(x,z)) \cdot y = z \cdot (K(x,z) \cdot y)$ by Theorem 1046. Hence we are done by Theorem 64. ☐

Theorem 1048. $T(x,y) = T(x, y \cdot K(x,y))$.

Proof. We have $(K(x,y)\cdot T(y,y))\cdot T(x,y\cdot K(x,y)) = K(x,y)\cdot(T(y,y)\cdot T(x,y\cdot K(x,y)))$ by Theorem 1042. Then $R(K(x,y),T(y,y),T(x,y\cdot K(x,y))) = K(x,y)$ by Theorem 64. Then

$$R(K(x,y),y,T(x,y\cdot K(x,y))) = K(x,y) \tag{527}$$

by Theorem 450. We have $R((y\cdot K(x,y))/y,y,T(x,y\cdot K(x,y)))\backslash(x\cdot(y\cdot K(x,y))) = y\cdot T(x,y\cdot K(x,y))$ by Theorem 801. Then $R(K(x,y),y,T(x,y\cdot K(x,y)))\backslash(x\cdot(y\cdot K(x,y))) = y\cdot T(x,y\cdot K(x,y))$ by Theorem 934. Then

$$K(x,y)\backslash(x\cdot(y\cdot K(x,y))) = y\cdot T(x,y\cdot K(x,y)) \tag{528}$$

by (527). We have $R(K(x,y),x,y)\cdot(x\cdot y) = (K(x,y)\cdot x)\cdot y$ by Proposition 54. Then $K(x,y)\cdot(x\cdot y) = (K(x,y)\cdot x)\cdot y$ by Theorem 1025. Then $(x\cdot K(x,y))\cdot y = K(x,y)\cdot(x\cdot y)$ by Theorem 928. Then $x\cdot(y\cdot K(x,y)) = K(x,y)\cdot(x\cdot y)$ by Theorem 1040. Then $K(x,y)\backslash(x\cdot(y\cdot K(x,y))) = x\cdot y$ by Proposition 2. Then $y\cdot T(x,y\cdot K(x,y)) = x\cdot y$ by (528). Hence we are done by Theorem 11. □

Theorem 1049. *$K(x,y)\cdot T(x,y) = T(x\cdot K(x,y),y)$.*

Proof. We have $y\backslash(L(x,K(x,y),y)\cdot(y\cdot K(x,y))) = K(x,y)\cdot T(x,y\cdot K(x,y))$ by Theorem 805. Then

$$y\backslash(x\cdot(y\cdot K(x,y))) = K(x,y)\cdot T(x,y\cdot K(x,y)) \tag{529}$$

by Theorem 1045. We have

$$T(x\cdot K(x,y),y) = y\backslash((x\cdot K(x,y))\cdot y) \tag{530}$$

by Definition 3.

$$\begin{aligned} & K(x,y)\cdot T(x,y) \\ = \;& y\backslash(x\cdot(y\cdot K(x,y))) && \text{by (529), Theorem 1048} \\ = \;& T(x\cdot K(x,y),y) && \text{by (530), Theorem 1040.} \end{aligned}$$

Hence we are done. □

Theorem 1050. *$K(x,y)\backslash y = (y\cdot z)/(K(x,y)\cdot z)$.*

Proof. We have $R(T(y,((y\cdot x)\backslash y)\backslash K(x,y)),K(x,y),z) = T(R(y,K(x,y),z),((y\cdot x)\backslash y)\backslash K(x,y))$ by Axiom 9. Then

$$R(T(y,((y\cdot x)\backslash y)\backslash K(x,y)),K(x,y),z) = T(y,((y\cdot x)\backslash y)\backslash K(x,y)) \tag{531}$$

by Theorem 1047. We have $T(y,((y\cdot x)\backslash y)\backslash K(x,y)) = K(x,y)\backslash y$ by Theorem 1012. Then $R(T(y,((y\cdot x)\backslash y)\backslash K(x,y)),K(x,y),z) = K(x,y)\backslash y$ by (531). Then

$$R(K(x,y)\backslash y,K(x,y),z) = K(x,y)\backslash y \tag{532}$$

by Theorem 1012. We have $R(K(x,y)\backslash y,K(x,y),z) = (((K(x,y)\backslash y)\cdot K(x,y))\cdot z)/(K(x,y)\cdot z)$ by Definition 5. Then $R(K(x,y)\backslash y,K(x,y),z) = (y\cdot z)/(K(x,y)\cdot z)$ by Theorem 765. Hence we are done by (532). □

Theorem 1051. *$(K(x,y)\backslash e)\cdot(K(x,y)\cdot z) = z$.*

Proof. We have $R(y,K(x,y)\backslash e,K(x,y)\cdot z)\backslash((y\cdot(K(x,y)\backslash e))\cdot(K(x,y)\cdot z)) = (K(x,y)\backslash e)\cdot(K(x,y)\cdot z)$ by Theorem 6. Then $R(y,K(x,y)\backslash e,K(x,y)\cdot z)\backslash((K(x,y)\backslash y)\cdot(K(x,y)\cdot z)) = (K(x,y)\backslash e)\cdot(K(x,y)\cdot z)$ by Theorem 777. Then

$$y\backslash((K(x,y)\backslash y)\cdot(K(x,y)\cdot z)) = (K(x,y)\backslash e)\cdot(K(x,y)\cdot z) \tag{533}$$

by Theorem 1019. We have $y\cdot(y\backslash((K(x,y)\backslash y)\cdot(K(x,y)\cdot z)))=(K(x,y)\backslash y)\cdot(K(x,y)\cdot z)$ by Axiom 4. Then

$$y\cdot((K(x,y)\backslash e)\cdot(K(x,y)\cdot z))=(K(x,y)\backslash y)\cdot(K(x,y)\cdot z) \tag{534}$$

by (533). We have $((y\cdot z)/(K(x,y)\cdot z))\cdot(K(x,y)\cdot z)=y\cdot z$ by Axiom 6. Then $(K(x,y)\backslash y)\cdot(K(x,y)\cdot z)=y\cdot z$ by Theorem 1050. Then $y\cdot((K(x,y)\backslash e)\cdot(K(x,y)\cdot z))=y\cdot z$ by (534). Hence we are done by Proposition 9. □

Theorem 1052. $K(y,x)\backslash z=K(T(x,y),y)\cdot z$.

Proof. We have $L(((K(y,x)\backslash z)\cdot(e/K(T(x,y),y)))\cdot K(T(x,y),y),K(T(x,y),y)\backslash e,K(T(x,y),y))/K(T(x,y),y)=(K(T(x,y),y)\backslash e)\cdot(K(T(x,y),y)\cdot((K(y,x)\backslash z)\cdot(e/K(T(x,y),y))))$ by Theorem 894. Then

$$L(R(K(y,x)\backslash z,e/K(T(x,y),y),K(T(x,y),y)),K(T(x,y),y)\backslash e,K(T(x,y),y))/K(T(x,y),y)=(K(T(x,y),y)\backslash e)\cdot(K(T(x,y),y)\cdot(\tag{535}$$

by Proposition 69. We have $L(R(K(y,x)\backslash z,e/K(T(x,y),y),K(T(x,y),y)),K(T(x,y),y)\backslash e,K(T(x,y),y))/K(T(x,y),y)=L(K(y,x)\backslash z,K(T(x,y),y)\backslash e,K(T(x,y),y))\cdot(e/K(T(x,y),y))$ by Theorem 84. Then

$$(K(T(x,y),y)\backslash e)\cdot(K(T(x,y),y)\cdot((K(y,x)\backslash z)\cdot(e/K(T(x,y),y))))=L(K(y,x)\backslash z,K(T(x,y),y)\backslash e,K(T(x,y),y))\cdot(e/K(T(x,y),y) \tag{536}$$

by (535). We have $(K(T(x,y),y)\backslash e)\cdot(K(T(x,y),y)\cdot((K(y,x)\backslash z)\cdot(e/K(T(x,y),y))))=(K(y,x)\backslash z)\cdot(e/K(T(x,y),y))$ by Theorem 1051. Then $L(K(y,x)\backslash z,K(T(x,y),y)\backslash e,K(T(x,y),y))\cdot(e/K(T(x,y),y))=(K(y,x)\backslash z)\cdot(e/K(T(x,y),y))$ by (536). Then $L(K(y,x)\backslash z,K(T(x,y),y)\backslash e,K(T(x,y),y))=K(y,x)\backslash z$ by Proposition 10. Then

$$L(K(y,x)\backslash z,K(y,x),K(T(x,y),y))=K(y,x)\backslash z \tag{537}$$

by Theorem 1036. We have $(e/K(y,x))\backslash L(K(y,x)\backslash z,K(y,x),K(y,x)\backslash e)=K(y,x)\cdot(K(y,x)\backslash z)$ by Theorem 913. Then $(K(y,x)\backslash e)\backslash L(K(y,x)\backslash z,K(y,x),K(y,x)\backslash e)=K(y,x)\cdot(K(y,x)\backslash z)$ by Theorem 901. Then

$$(K(y,x)\backslash e)\backslash L(K(y,x)\backslash z,K(y,x),K(T(x,y),y))=K(y,x)\cdot(K(y,x)\backslash z) \tag{538}$$

by Theorem 1037. We have $K(x,(x\cdot y)/x)\cdot(K(x,(x\cdot y)/x)\backslash L(K(y,x)\backslash z,K(y,x),K(T(x,y),y)))=L(K(y,x)\backslash z,K(y,x),K(T(x,y),y))$ by Axiom 4. Then $K(T(x,y),y)\cdot(K(x,(x\cdot y)/x)\backslash L(K(y,x)\backslash z,K(y,x),K(T(x,y),y)))=L(K(y,x)\backslash z,K(y,x),K(T(x,y),y))$ by Theorem 1016. Then $K(T(x,y),y)\cdot((K(y,x)\backslash e)\backslash L(K(y,x)\backslash z,K(y,x),K(T(x,y),y)))=L(K(y,x)\backslash z,K(y,x),K(T(x,y),y))$ by Theorem 1035. Then $K(T(x,y),y)\cdot(K(y,x)\cdot(K(y,x)\backslash z))=L(K(y,x)\backslash z,K(y,x),K(T(x,y),y))$ by (538). Then $K(T(x,y),y)\cdot z=L(K(y,x)\backslash z,K(y,x),K(T(x,y),y))$ by Axiom 4. Hence we are done by (537). □

Theorem 1053. $K(x,y)=K(T(x,y),y)$.

Proof. We have $T(x,y)\cdot(K(T(x,y),y)\cdot T(K(T(x,y),y)\backslash y,x))=(T(y,x)\cdot T(x,y))\cdot K(T(x,y),y)$ by Theorem 634. Then $T(x,y)\cdot(K(T(x,y),y)\cdot T(K(T(x,y),y)\backslash y,x))=(y\cdot x)\cdot K(T(x,y),y)$ by Theorem 1033. Then $T(x,y)\cdot(K(T(x,y),y)\cdot T(y\cdot K(y,x),x))=(y\cdot x)\cdot K(T(x,y),y)$ by Theorem 1038. Then

$$T(x,y)\cdot(K(y,x)\backslash T(y\cdot K(y,x),x))=(y\cdot x)\cdot K(T(x,y),y) \tag{539}$$

by Theorem 1052. We have $K(y,x)\cdot T(y,x)=T(y\cdot K(y,x),x)$ by Theorem 1049. Then $K(y,x)\backslash T(y\cdot K(y,x),x)=T(y,x)$ by Proposition 2. Then

$$T(x,y)\cdot T(y,x)=(y\cdot x)\cdot K(T(x,y),y) \tag{540}$$

by (539). We have $T(x,y)\cdot T(y,x)=x\cdot y$ by Theorem 1033. Then $(y\cdot x)\cdot K(T(x,y),y)=x\cdot y$ by (540). Hence we are done by Theorem 460. □

The following was first conjectured in [9].

Theorem 1054. $K(y,x)\cdot K(x,y)=e$.

Proof. We have $K(T(y,x),x) \cdot (K(T(y,x),x)\backslash e) = e$ by Axiom 4. Then $K(T(y,x),x) \cdot K(x,y) = e$ by Theorem 1036. Hence we are done by Theorem 1053. ☐

Theorem 1055. $K(x, x\backslash y) = ((x\backslash y) \cdot x)\backslash y$.

Proof. We have $K(x, x\backslash y) = ((x\backslash y) \cdot x)\backslash(x \cdot (x\backslash y))$ by Definition 2. Hence we are done by Axiom 4. ☐

Theorem 1056. $((x \cdot y) \cdot z)/a(x,y,z) = x \cdot (y \cdot z)$.

Proof. We have $((x \cdot y) \cdot z)/((x \cdot (y \cdot z))\backslash((x \cdot y) \cdot z)) = x \cdot (y \cdot z)$ by Proposition 24. Hence we are done by Definition 1. ☐

Theorem 1057. $T(R(x,y,z), y \cdot z) = (y \cdot z)\backslash((x \cdot y) \cdot z)$.

Proof. We have $T(((x \cdot y) \cdot z)/(y \cdot z), y \cdot z) = (y \cdot z)\backslash((x \cdot y) \cdot z)$ by Proposition 47. Hence we are done by Definition 5. ☐

Theorem 1058. $L(K(y,z), z \cdot y, x) = (x \cdot (z \cdot y))\backslash(x \cdot (y \cdot z))$.

Proof. We have $L(K(y,z), z \cdot y, x) = (x \cdot (z \cdot y))\backslash(x \cdot ((z \cdot y) \cdot K(y,z)))$ by Definition 4. Hence we are done by Proposition 82. ☐

Theorem 1059. $T(y \cdot x, K(x,y)) = K(x,y)\backslash(x \cdot y)$.

Proof. We have $T(y \cdot x, K(x,y)) = K(x,y)\backslash((y \cdot x) \cdot K(x,y))$ by Definition 3. Hence we are done by Proposition 82. ☐

Theorem 1060. $L(K(z\backslash y, z), y, x) = (x \cdot y)\backslash(x \cdot ((z\backslash y) \cdot z))$.

Proof. We have $L(K(z\backslash y, z), y, x) = (x \cdot y)\backslash(x \cdot (y \cdot K(z\backslash y, z)))$ by Definition 4. Hence we are done by Theorem 17. ☐

Theorem 1061. $L(L(w,z,y), y \cdot z, x) = (x \cdot (y \cdot z))\backslash(x \cdot (y \cdot (z \cdot w)))$.

Proof. We have $L(L(w,z,y), y \cdot z, x) = (x \cdot (y \cdot z))\backslash(x \cdot ((y \cdot z) \cdot L(w,z,y)))$ by Definition 4. Hence we are done by Proposition 52. ☐

Theorem 1062. $y\backslash e = (x \cdot y)\backslash R(x,y,y\backslash e)$.

Proof. We have $R(x,y,y\backslash e)\backslash(R(x,y,y\backslash e) \cdot e) = e$ by Axiom 3. Then $R(x,y,y\backslash e)\backslash((x \cdot y) \cdot ((x \cdot y)\backslash(R(x,y,y\backslash e) \cdot e))) = e$ by Axiom 4. Then

$$R(x,y,y\backslash e)\backslash((x \cdot y) \cdot ((x \cdot y)\backslash R(x,y,y\backslash e))) = e \tag{541}$$

by Axiom 2. We have $R(x,y,y\backslash e) \cdot (R(x,y,y\backslash e)\backslash((x \cdot y) \cdot ((x \cdot y)\backslash R(x,y,y\backslash e)))) = (x \cdot y) \cdot ((x \cdot y)\backslash R(x,y,y\backslash e))$ by Axiom 4. Then $R(x,y,y\backslash e) \cdot e = (x \cdot y) \cdot ((x \cdot y)\backslash R(x,y,y\backslash e))$ by (541). Then

$$(x \cdot y) \cdot ((x \cdot y)\backslash R(x,y,y\backslash e)) = R(x,y,y\backslash e) \cdot (y \cdot (y\backslash e)) \tag{542}$$

by Axiom 4. We have $(x \cdot y) \cdot (y\backslash e) = R(x,y,y\backslash e) \cdot (y \cdot (y\backslash e))$ by Proposition 54. Hence we are done by (542) and Proposition 7. ☐

Theorem 1063. $R(x,z,T(y,z)) \cdot (y \cdot z) = (x \cdot z) \cdot T(y,z)$.

Proof. We have $R(x,z,T(y,z)) \cdot (z \cdot T(y,z)) = (x \cdot z) \cdot T(y,z)$ by Proposition 54. Hence we are done by Proposition 46. ☐

Theorem 1064. $((x \cdot y) \cdot z) \cdot L(w, y \cdot z, R(x,y,z)) = R(x,y,z) \cdot ((y \cdot z) \cdot w)$.

Proof. We have $(R(x,y,z) \cdot (y \cdot z)) \cdot L(w, y \cdot z, R(x,y,z)) = R(x,y,z) \cdot ((y \cdot z) \cdot w)$ by Proposition 52. Hence we are done by Proposition 54. ☐

Theorem 1065. $R(y/x, x, y\backslash z)\backslash z = x \cdot (y\backslash z)$.

Proof. We have $y \cdot (y\backslash z) = z$ by Axiom 4. Then $((y \cdot (y\backslash z))/(x \cdot (y\backslash z))) \cdot (x \cdot (y\backslash z)) = z$ by Axiom 6. Then $((y \cdot (y\backslash z))/(x \cdot (y\backslash z)))\backslash z = x \cdot (y\backslash z)$ by Proposition 2. Hence we are done by Proposition 55. □

Theorem 1066. $(y \cdot (x \cdot z))/L(T(x, z), z, y) = y \cdot z$.

Proof. We have $(y \cdot (z \cdot T(x, z)))/L(T(x, z), z, y) = y \cdot z$ by Theorem 453. Hence we are done by Proposition 46. □

Theorem 1067. $R(x, x\backslash y, z)\backslash(y \cdot z) = (x\backslash y) \cdot z$.

Proof. We have $R(x, x\backslash y, z)\backslash((x \cdot (x\backslash y)) \cdot z) = (x\backslash y) \cdot z$ by Theorem 6. Hence we are done by Axiom 4. □

Theorem 1068. $R(y \cdot x, K(x, y), z)\backslash((x \cdot y) \cdot z) = K(x, y) \cdot z$.

Proof. We have $R(y \cdot x, K(x, y), z)\backslash(((y \cdot x) \cdot K(x, y)) \cdot z) = K(x, y) \cdot z$ by Theorem 6. Hence we are done by Proposition 82. □

Theorem 1069. $R(x, x\backslash e, x) = x/K(x\backslash e, x)$.

Proof. We have $R(x, x\backslash e, x) = x/((x\backslash e) \cdot x)$ by Proposition 66. Hence we are done by Proposition 76. □

Theorem 1070. $L((u \cdot w)\backslash x, y, z) = L(L(w\backslash(u\backslash x), y, z), w, u)$.

Proof. We have $L(L(w\backslash(u\backslash x), w, u), y, z) = L(L(w\backslash(u\backslash x), y, z), w, u)$ by Axiom 11. Hence we are done by Proposition 70. □

Theorem 1071. $R(T(w, w \cdot z), x, y) = L(R(T(w, z), x, y), z, w)$.

Proof. We have $R(L(T(w, z), z, w), x, y) = L(R(T(w, z), x, y), z, w)$ by Axiom 10. Hence we are done by Proposition 63. □

Theorem 1072. $L(T(x\backslash y, z), x, e/x) = T((e/x) \cdot y, z)$.

Proof. We have $L(T(x\backslash y, z), x, e/x) = T(L(x\backslash y, x, e/x), z)$ by Axiom 8. Hence we are done by Theorem 63. □

Theorem 1073. $y \cdot ((y\backslash z) \cdot T(x, z)) = L(x, y\backslash z, y) \cdot z$.

Proof. We have $z \cdot L(T(x, z), y\backslash z, y) = L(x, y\backslash z, y) \cdot z$ by Proposition 58. Hence we are done by Theorem 52. □

Theorem 1074. $y \cdot ((e/y) \cdot (x \cdot y)) = L(x, y, e/y) \cdot y$.

Proof. We have $y \cdot L(T(x, y), y, e/y) = L(x, y, e/y) \cdot y$ by Proposition 58. Hence we are done by Theorem 72. □

Theorem 1075. $(x\backslash e) \cdot L(y, K(x\backslash e, x), e/x) = (e/x) \cdot (K(x\backslash e, x) \cdot y)$.

Proof. We have $((e/x) \cdot K(x\backslash e, x)) \cdot L(y, K(x\backslash e, x), e/x) = (e/x) \cdot (K(x\backslash e, x) \cdot y)$ by Proposition 52. Hence we are done by Theorem 109. □

Theorem 1076. $((z \cdot y) \cdot R(x, z, y))/y = (((z \cdot y) \cdot x)/(z \cdot y)) \cdot z$.

Proof. We have $(((((z\cdot y)\cdot x)/(z\cdot y))\cdot z)\cdot y)/y = (((z\cdot y)\cdot x)/(z\cdot y))\cdot z$ by Axiom 5. Then $((((((z\cdot y)\cdot x)/(z\cdot y))\cdot z)\cdot y)/(z\cdot y))\cdot(z\cdot y))/y = (((z\cdot y)\cdot x)/(z\cdot y))\cdot z$ by Axiom 6. Then $(R(((z\cdot y)\cdot x)/(z\cdot y), z, y)\cdot(z\cdot y))/y = (((z \cdot y) \cdot x)/(z \cdot y)) \cdot z$ by Definition 5. Hence we are done by Theorem 87. □

Theorem 1077. $T(R(x, y, T(x, y)), x \cdot y) = T(x, y)$.

Proof. We have $T(((x \cdot y) \cdot T(x, y))/(x \cdot y), x \cdot y) = T(x, y)$ by Theorem 7. Hence we are done by Theorem 787. □

Theorem 1078. $R(T(x, x \cdot y), y, T(x, y)) = T(x, y)$.

Proof. We have $R(T(x, x \cdot y), y, T(x, y)) = T(R(x, y, T(x, y)), x \cdot y)$ by Axiom 9. Hence we are done by Theorem 1077. □

Theorem 1079. $T(x, y) \cdot (x \cdot y) = (T(x, x \cdot y) \cdot y) \cdot T(x, y)$.

Proof. We have $R(T(x, x \cdot y), y, T(x, y)) \cdot (x \cdot y) = (T(x, x \cdot y) \cdot y) \cdot T(x, y)$ by Theorem 1063. Hence we are done by Theorem 1078. □

Theorem 1080. $(e/x)\backslash T((e/x) \cdot y, z) = x \cdot T(x\backslash y, z)$.

Proof. We have $(e/x)\backslash L(T(x\backslash y, z), x, e/x) = x \cdot T(x\backslash y, z)$ by Theorem 71. Hence we are done by Theorem 1072. □

Theorem 1081. $x\backslash(L(y, x\backslash z, x) \cdot z) = (x\backslash z) \cdot T(y, z)$.

Proof. We have $x \cdot ((x\backslash z) \cdot T(y, z)) = L(y, x\backslash z, x) \cdot z$ by Theorem 1073. Hence we are done by Proposition 2. □

Theorem 1082. $x\backslash((y \cdot (x \cdot y))/y) = T(y, x \cdot y)$.

Proof. We have $((x \cdot T(y, x \cdot y)) \cdot y)/y = x \cdot T(y, x \cdot y)$ by Axiom 5. Then $((((x \cdot T(y, x \cdot y)) \cdot y)/(x \cdot y)) \cdot (x \cdot y))/y = x \cdot T(y, x \cdot y)$ by Axiom 6. Then $(y \cdot (x \cdot y))/y = x \cdot T(y, x \cdot y)$ by Theorem 492. Hence we are done by Proposition 2. □

Theorem 1083. $T(x/y, T(y, x)) = T(y, x)\backslash((y \cdot x)/y)$.

Proof. We have $T(x/y, T(y, x)) = T(y, x)\backslash((x/y) \cdot T(y, x))$ by Definition 3. Hence we are done by Theorem 818. □

Theorem 1084. $(T(x, y) \cdot z)\backslash((x \cdot y) \cdot z) = R(T(y, T(x, y) \cdot z), T(x, y), z)$.

Proof. We have $(T(x, y) \cdot z)\backslash((y \cdot T(x, y)) \cdot z) = R(T(y, T(x, y) \cdot z), T(x, y), z)$ by Proposition 81. Hence we are done by Proposition 46. □

Theorem 1085. $x\backslash((z\backslash(x \cdot y)) \cdot z) = (x\backslash z) \cdot T((x\backslash z)\backslash y, z)$.

Proof. We have $x\backslash(L((x\backslash z)\backslash y, x\backslash z, x) \cdot z) = (x\backslash z) \cdot T((x\backslash z)\backslash y, z)$ by Theorem 1081. Hence we are done by Theorem 60. □

Theorem 1086. $(x \cdot (y\backslash x))/x = y\backslash T(x, x/y)$.

Proof. We have $(x \cdot T(y\backslash T(x, x/y), x))/x = y\backslash T(x, x/y)$ by Proposition 48. Hence we are done by Theorem 513. □

Theorem 1087. $(y/x)\backslash T(y, y/(y/x)) = (y \cdot x)/y$.

Proof. We have $(y/x)\backslash T(y, y/(y/x)) = (y \cdot ((y/x)\backslash y))/y$ by Theorem 1086. Hence we are done by Proposition 25. □

Theorem 1088. $(y \cdot (T(y, z) \cdot x))/(y \cdot x) = T(y, z)$.

Proof. We have $T(y, z) \cdot (y \cdot x) = y \cdot (T(y, z) \cdot x)$ by Theorem 172. Hence we are done by Proposition 1. □

Theorem 1089. $(y \cdot x)/(y \cdot (T(y, z)\backslash x)) = T(y, z)$.

Proof. We have $(y \cdot (T(y, z) \cdot (T(y, z)\backslash x)))/(y \cdot (T(y, z)\backslash x)) = T(y, z)$ by Theorem 1088. Hence we are done by Axiom 4. □

Theorem 1090. $T(x, x\backslash y)\backslash(x \cdot z) = x \cdot (((x\backslash y)\backslash y)\backslash z)$.

Proof. We have $T(x, x\backslash y)\backslash(x \cdot z) = x \cdot (T(x, x\backslash y)\backslash z)$ by Theorem 174. Hence we are done by Proposition 49. $\square$

Theorem 1091. $T(y, x)/(x\backslash y) = (y \cdot x)/y$.

Proof. We have $((y \cdot x)/y) \cdot (x\backslash y) = T(y, x)$ by Theorem 179. Hence we are done by Proposition 1. $\square$

Theorem 1092. $T(y, x) \cdot K(x\backslash y, (y \cdot x)/y) = (x\backslash y) \cdot ((y \cdot x)/y)$.

Proof. We have $(((y \cdot x)/y) \cdot (x\backslash y)) \cdot K(x\backslash y, (y \cdot x)/y) = (x\backslash y) \cdot ((y \cdot x)/y)$ by Proposition 82. Hence we are done by Theorem 179. $\square$

Theorem 1093. $(y/(y/x)) \cdot (x\backslash(y/x)) = x\backslash y$.

Proof. We have $T(y/x, x) = (((y/x) \cdot x)/(y/x)) \cdot (x\backslash(y/x))$ by Theorem 179. Then

$$T(y/x, x) = (y/(y/x)) \cdot (x\backslash(y/x)) \tag{543}$$

by Axiom 6. We have $T(y/x, x) = x\backslash y$ by Proposition 47. Hence we are done by (543). $\square$

Theorem 1094. $(z/x) \cdot (x \cdot T(x\backslash z, y)) = T(z/x, y) \cdot z$.

Proof. We have $T(z/x, y) \cdot ((z/x) \cdot x) = (z/x) \cdot (T(z/x, y) \cdot x)$ by Theorem 172. Then $T(z/x, y) \cdot z = (z/x) \cdot (T(z/x, y) \cdot x)$ by Axiom 6. Hence we are done by Proposition 61. $\square$

Theorem 1095. $((x\backslash y)\backslash y)\backslash(x \cdot z) = x \cdot (((x\backslash y)\backslash y)\backslash z)$.

Proof. We have $T(x, x\backslash y)\backslash(x \cdot z) = x \cdot (((x\backslash y)\backslash y)\backslash z)$ by Theorem 1090. Hence we are done by Proposition 49. $\square$

Theorem 1096. $(z\backslash(x \cdot (y\backslash z))) \cdot z = (y \cdot T(y\backslash x, z)) \cdot (y\backslash z)$.

Proof. We have $R(T(x/y, z), y, y\backslash z) \cdot z = (T(x/y, z) \cdot y) \cdot (y\backslash z)$ by Theorem 25. Then $(z\backslash(x \cdot (y\backslash z))) \cdot z = (T(x/y, z) \cdot y) \cdot (y\backslash z)$ by Theorem 142. Hence we are done by Proposition 61. $\square$

Theorem 1097. $x\backslash L(x, x\backslash e, R(x, y, z)) = (x\backslash e) \cdot ((x\backslash R(x, y, z))\backslash R(x, y, z))$.

Proof. We have $(x\backslash e) \cdot L((x\backslash e)\backslash e, x\backslash e, R(x, y, z)) = x\backslash L(x, x\backslash e, R(x, y, z))$ by Theorem 836. Then $(x\backslash e) \cdot ((R(x, y, z) \cdot (x\backslash e))\backslash R(x, y, z)) = x\backslash L(x, x\backslash e, R(x, y, z))$ by Proposition 78. Hence we are done by Proposition 89. $\square$

Theorem 1098. $(y \cdot (y \cdot K(x, y)))/y = y \cdot K(x, y)$.

Proof. We have $(y \cdot (y \cdot K(x, y)))/T(y, y \cdot K(x, y)) = y \cdot K(x, y)$ by Theorem 5. Hence we are done by Theorem 541. $\square$

Theorem 1099. $(y\backslash(x\backslash y)) \cdot y = (x \cdot T(x\backslash e, y)) \cdot (x\backslash y)$.

Proof. We have $(y\backslash(e \cdot (x\backslash y))) \cdot y = (x \cdot T(x\backslash e, y)) \cdot (x\backslash y)$ by Theorem 1096. Hence we are done by Axiom 1. $\square$

Theorem 1100. $R(T(x, y), x\backslash e, x) = T(T(x, x\backslash e), y)$.

Proof. We have $R(T(T(x, (x\backslash e) \cdot x), y), x\backslash e, x) = T(R(T(x, (x\backslash e) \cdot x), x\backslash e, x), y)$ by Axiom 9. Then $R(T(T(x, (x\backslash e) \cdot x), y), x\backslash e, x) = T(T(x, x\backslash e), y)$ by Theorem 113. Hence we are done by Theorem 204. $\square$

Theorem 1101. $L(T(z, z\backslash e), x, y)/z = L(z, x, y) \cdot (e/z)$.

Proof. We have $L(T(z, e/z), x, y)/z = L(z, x, y) \cdot (e/z)$ by Theorem 164. Hence we are done by Theorem 211. ☐

Theorem 1102. $(x \cdot y) \cdot ((w \cdot ((x \cdot y)\backslash(x \cdot z)))/w) = x \cdot (y \cdot ((w \cdot (y\backslash z))/w))$.

Proof. We have $(x \cdot y) \cdot ((w \cdot L(y\backslash z, y, x))/w) = x \cdot (y \cdot ((w \cdot (y\backslash z))/w))$ by Theorem 815. Hence we are done by Proposition 53. ☐

Theorem 1103. $x\backslash((z \cdot (x \cdot y))/z) = (x\backslash e) \cdot ((z \cdot ((x\backslash e)\backslash y))/z)$.

Proof. We have $L((z \cdot ((x\backslash e)\backslash y))/z, x\backslash e, x) = (z \cdot L((x\backslash e)\backslash y, x\backslash e, x))/z$ by Theorem 122. Then

$$L((z \cdot ((x\backslash e)\backslash y))/z, x\backslash e, x) = (z \cdot (x \cdot y))/z \tag{544}$$

by Theorem 62. We have $x\backslash L((z \cdot ((x\backslash e)\backslash y))/z, x\backslash e, x) = (x\backslash e) \cdot ((z \cdot ((x\backslash e)\backslash y))/z)$ by Proposition 57. Hence we are done by (544). ☐

Theorem 1104. $(e/x)\backslash((z \cdot ((e/x) \cdot y))/z) = x \cdot ((z \cdot (x\backslash y))/z)$.

Proof. We have $L((z \cdot (x\backslash y))/z, x, e/x) = (z \cdot L(x\backslash y, x, e/x))/z$ by Theorem 122. Then

$$L((z \cdot (x\backslash y))/z, x, e/x) = (z \cdot ((e/x) \cdot y))/z \tag{545}$$

by Theorem 63. We have $(e/x)\backslash L((z \cdot (x\backslash y))/z, x, e/x) = x \cdot ((z \cdot (x\backslash y))/z)$ by Theorem 71. Hence we are done by (545). ☐

Theorem 1105. $(x \cdot T(z, y))\backslash((x \cdot z) \cdot T(z, y)) = z$.

Proof. We have $(x \cdot T(z, y)) \cdot z = (x \cdot z) \cdot T(z, y)$ by Theorem 238. Hence we are done by Proposition 2. ☐

Theorem 1106. $(x \cdot (y\backslash z))/(z/y) = (x/(z/y)) \cdot (y\backslash z)$.

Proof. We have $(x \cdot (y\backslash z))/(y\backslash z) = x$ by Axiom 5. Then

$$(((x \cdot (y\backslash z))/(z/y)) \cdot (z/y))/(y\backslash z) = x \tag{546}$$

by Axiom 6. We have $((((x \cdot (y\backslash z))/(z/y)) \cdot (z/y))/(y\backslash z)) \cdot (y\backslash z) = ((x \cdot (y\backslash z))/(z/y)) \cdot (z/y)$ by Axiom 6. Then $((((((x \cdot (y\backslash z))/(z/y)) \cdot (z/y))/(y\backslash z))/(z/y)) \cdot (z/y)) \cdot (y\backslash z) = ((x \cdot (y\backslash z))/(z/y)) \cdot (z/y)$ by Axiom 6. Then

$$((x/(z/y)) \cdot (z/y)) \cdot (y\backslash z) = ((x \cdot (y\backslash z))/(z/y)) \cdot (z/y) \tag{547}$$

by (546). We have $((x/(z/y)) \cdot (z/y)) \cdot (y\backslash z) = ((x/(z/y)) \cdot (y\backslash z)) \cdot (z/y)$ by Theorem 241. Then $((x \cdot (y\backslash z))/(z/y)) \cdot (z/y) = ((x/(z/y)) \cdot (y\backslash z)) \cdot (z/y)$ by (547). Hence we are done by Proposition 10. ☐

Theorem 1107. $x\backslash T(e/(e/x), y) = (x\backslash e) \cdot T(x, y)$.

Proof. We have $x\backslash L(T(x, y), x\backslash e, x) = (x\backslash e) \cdot T(x, y)$ by Proposition 57. Then $x\backslash T(L(x, x\backslash e, x), y) = (x\backslash e) \cdot T(x, y)$ by Axiom 8. Then $x\backslash T(x \cdot K(x\backslash e, x), y) = (x\backslash e) \cdot T(x, y)$ by Theorem 70. Hence we are done by Theorem 262. ☐

Theorem 1108. $K(x\backslash e, x) \cdot x = e/(e/x)$.

Proof. We have $K(x\backslash e, x) \cdot x = x \cdot K(x\backslash e, x)$ by Theorem 933. Hence we are done by Theorem 262. ☐

Theorem 1109. $(e/(e/x))/x = K(x\backslash e, x)$.

Proof. We have $K(x\backslash e, x) \cdot x = e/(e/x)$ by Theorem 1108. Hence we are done by Proposition 1. ☐

Theorem 1110. $(x\backslash e)\backslash T(e/x, y) = ((x\backslash e)\backslash e) \cdot T(x\backslash e, y)$.

Proof. We have $(x\backslash e)\backslash T(e/(e/(x\backslash e)), y) = ((x\backslash e)\backslash e) \cdot T(x\backslash e, y)$ by Theorem 1107. Hence we are done by Proposition 24. ☐

Theorem 1111. $(T(y/(x/z), w) \cdot (x/z)) \cdot z = x \cdot T(x\backslash(y \cdot z), w)$.

Proof. We have $T((y \cdot z)/x, w) \cdot x = x \cdot T(x\backslash(y \cdot z), w)$ by Proposition 61. Hence we are done by Theorem 169. □

Theorem 1112. $((x \cdot (x \cdot y))/x)/y = T(x, x/(e/y))$.

Proof. We have $((x \cdot (x/(e/y)))/x) \cdot (e/y) = T(x, x/(e/y))$ by Theorem 512. Hence we are done by Theorem 232. □

Theorem 1113. $y \cdot (x \cdot T(x\backslash T(x, y), z)) = (y \cdot x) \cdot T(K(x, y), z)$.

Proof. We have $(y \cdot x) \cdot T((y \cdot x)\backslash(x \cdot y), z) = y \cdot (x \cdot T(x\backslash(y\backslash(x \cdot y)), z))$ by Theorem 136. Then $(y \cdot x) \cdot T(K(x, y), z) = y \cdot (x \cdot T(x\backslash(y\backslash(x \cdot y)), z))$ by Definition 2. Hence we are done by Definition 3. □

Theorem 1114. $((x \cdot y)/x)/(x\backslash y) = T(x, x/(x/y))$.

Proof. We have $T(x, x/(x/y)) \cdot (x\backslash y) = (x \cdot y)/x$ by Theorem 861. Hence we are done by Proposition 1. □

Theorem 1115. $z \cdot K(y\backslash(y/T(x, z)), y) = (x \cdot T(x\backslash e, y)) \cdot z$.

Proof. We have $z \cdot (z\backslash((x \cdot T(x\backslash e, y)) \cdot z)) = (x \cdot T(x\backslash e, y)) \cdot z$ by Axiom 4. Hence we are done by Theorem 295. □

Theorem 1116. $(x \cdot T(x\backslash(z \cdot y), w))/y = (x/y) \cdot T((x/y)\backslash z, w)$.

Proof. We have $(T(z/(x/y), w) \cdot (x/y)) \cdot y = x \cdot T(x\backslash(z \cdot y), w)$ by Theorem 1111. Then

$$(x \cdot T(x\backslash(z \cdot y), w))/y = T(z/(x/y), w) \cdot (x/y) \tag{548}$$

by Proposition 1. We have $T(z/(x/y), w) \cdot (x/y) = (x/y) \cdot T((x/y)\backslash z, w)$ by Proposition 61. Hence we are done by (548). □

Theorem 1117. $(z \cdot (x \cdot T(x\backslash y, w)))/y = ((z \cdot x)/y) \cdot T(((z \cdot x)/y)\backslash z, w)$.

Proof. We have $((z \cdot x) \cdot T((z \cdot x)\backslash(z \cdot y), w))/y = ((z \cdot x)/y) \cdot T(((z \cdot x)/y)\backslash z, w)$ by Theorem 1116. Hence we are done by Theorem 486. □

Theorem 1118. $L(K(x, x\backslash e)\backslash y, K(x, x\backslash e), x) = x \cdot (((x\backslash e)\backslash e)\backslash y)$.

Proof. We have $L(K(x, x\backslash e)\backslash y, K(x, x\backslash e), x) = (x \cdot K(x, x\backslash e))\backslash(x \cdot y)$ by Proposition 53. Then

$$L(K(x, x\backslash e)\backslash y, K(x, x\backslash e), x) = T(x, x\backslash e)\backslash(x \cdot y) \tag{549}$$

by Theorem 562. We have $T(x, x\backslash e)\backslash(x \cdot y) = x \cdot (((x\backslash e)\backslash e)\backslash y)$ by Theorem 1090. Hence we are done by (549). □

Theorem 1119. $T(y, R(y, z, w) \cdot x) = L(T(y, x), x, R(y, z, w))$.

Proof. We have $(R(y, z, w) \cdot x)\backslash(R(y, z, w) \cdot (y \cdot x)) = L(T(y, x), x, R(y, z, w))$ by Proposition 62. Then

$$(R(y, z, w) \cdot x)\backslash(y \cdot (R(y, z, w) \cdot x)) = L(T(y, x), x, R(y, z, w)) \tag{550}$$

by Theorem 227. We have $T(y, R(y, z, w) \cdot x) = (R(y, z, w) \cdot x)\backslash(y \cdot (R(y, z, w) \cdot x))$ by Definition 3. Hence we are done by (550). □

Theorem 1120. $(x \cdot L(x\backslash y, w, u)) \cdot z = x \cdot (z \cdot L(z\backslash(x\backslash(y \cdot z)), w, u))$.

Proof. We have $(x \cdot z) \cdot L((x \cdot z)\backslash(y \cdot z), w, u) = x \cdot (z \cdot L(z\backslash(x\backslash(y \cdot z)), w, u))$ by Theorem 155. Hence we are done by Theorem 332. □

Theorem 1121. $L(x, x\backslash e, R(x, y, z)) = T(e/(e/x), x\backslash R(x, y, z))$.

Proof. We have $L(T(x, x\backslash e), x\backslash e, R(x, y, z))/x = L(x, x\backslash e, R(x, y, z)) \cdot (e/x)$ by Theorem 1101. Then $T(x, R(x, y, z) \cdot (x\backslash e))/x = L(x, x\backslash e, R(x, y, z)) \cdot (e/x)$ by Theorem 1119. Then

$$L(x, x\backslash e, R(x, y, z)) \cdot (e/x) = T(x, x\backslash R(x, y, z))/x \tag{551}$$

by Proposition 89. We have $T(e/(e/x), x\backslash R(x, y, z)) \cdot (e/x) = T(x, x\backslash R(x, y, z))/x$ by Theorem 147. Hence we are done by (551) and Proposition 8. □

Theorem 1122. $x\backslash L(x, x\backslash e, R(x, y, z)) = (x\backslash e) \cdot T(x, x\backslash R(x, y, z))$.

Proof. We have $x\backslash T(e/(e/x), x\backslash R(x, y, z)) = (x\backslash e) \cdot T(x, x\backslash R(x, y, z))$ by Theorem 1107. Hence we are done by Theorem 1121. □

Theorem 1123. $T(x, y\backslash x) = T(x, x \cdot (y\backslash e))$.

Proof. We have $T(T(((x/y) \cdot y) \cdot (y\backslash e), (x/y) \cdot y)/(y\backslash e), ((x/y) \cdot y) \cdot (y\backslash e)) = (x/y) \cdot y$ by Theorem 509. Then $T(T(R(x/y, y, y\backslash e), (x/y) \cdot y)/(y\backslash e), ((x/y) \cdot y) \cdot (y\backslash e)) = (x/y) \cdot y$ by Proposition 68. Then $T(R(T(x/y, (x/y) \cdot y), y, y\backslash e)/(y\backslash e), ((x/y) \cdot y) \cdot (y\backslash e)) = (x/y) \cdot y$ by Axiom 9. Then $T(T(x/y, (x/y) \cdot y) \cdot y, ((x/y) \cdot y) \cdot (y\backslash e)) = (x/y) \cdot y$ by Theorem 73. Then

$$T(T(x/y, (x/y) \cdot y) \cdot y, R(x/y, y, y\backslash e)) = (x/y) \cdot y \tag{552}$$

by Proposition 68. We have $T(T(T(x/y, (x/y) \cdot y) \cdot y, R(x/y, y, y\backslash e)), T(x/y, y)) = T(T(T(x/y, (x/y) \cdot y) \cdot y, T(x/y, y)), R(x/y, y, y\backslash e))$ by Axiom 7. Then

$$T((x/y) \cdot y, T(x/y, y)) = T(T(T(x/y, (x/y) \cdot y) \cdot y, T(x/y, y)), R(x/y, y, y\backslash e)) \tag{553}$$

by (552). We have $T(x/y, y) \cdot ((x/y) \cdot y) = (T(x/y, (x/y) \cdot y) \cdot y) \cdot T(x/y, y)$ by Theorem 1079. Then $T(T(x/y, (x/y) \cdot y) \cdot y, T(x/y, y)) = (x/y) \cdot y$ by Theorem 11. Then $T((x/y) \cdot y, R(x/y, y, y\backslash e)) = T((x/y) \cdot y, T(x/y, y))$ by (553). Then $T(x, R(x/y, y, y\backslash e)) = T((x/y) \cdot y, T(x/y, y))$ by Axiom 6. Then $T((x/y) \cdot y, T(x/y, y)) = T(x, x \cdot (y\backslash e))$ by Proposition 71. Then $T(x, T(x/y, y)) = T(x, x \cdot (y\backslash e))$ by Axiom 6. Hence we are done by Proposition 47. □

Theorem 1124. $T(y \cdot T(x, x\backslash e), z)/((x\backslash e)\backslash e) = x\backslash T(x \cdot y, z)$.

Proof. We have $T(y \cdot ((x\backslash e)\backslash e), z)/((x\backslash e)\backslash e) = (x\backslash e) \cdot T((x\backslash e)\backslash y, z)$ by Theorem 497. Then

$$T(y \cdot T(x, x\backslash e), z)/((x\backslash e)\backslash e) = (x\backslash e) \cdot T((x\backslash e)\backslash y, z) \tag{554}$$

by Proposition 49. We have $(x\backslash e) \cdot T((x\backslash e)\backslash y, z) = x\backslash T(x \cdot y, z)$ by Theorem 140. Hence we are done by (554). □

Theorem 1125. $T(y \cdot T(x, x\backslash e), z)/T(x, x\backslash e) = x\backslash T(x \cdot y, z)$.

Proof. We have $T(y \cdot T(x, x\backslash e), z)/((x\backslash e)\backslash e) = x\backslash T(x \cdot y, z)$ by Theorem 1124. Hence we are done by Proposition 49. □

Theorem 1126. $T(y \cdot ((x\backslash e)\backslash e), z)/T(x, x\backslash e) = x\backslash T(x \cdot y, z)$.

Proof. We have $T(y \cdot T(x, x\backslash e), z)/T(x, x\backslash e) = x\backslash T(x \cdot y, z)$ by Theorem 1125. Hence we are done by Proposition 49. □

Theorem 1127. $(y \cdot x)\backslash((x \cdot y) \cdot T(K(y, x), z)) = K(x, y) \cdot T(K(x, y)\backslash e, z)$.

Proof. We have $(x \cdot y) \cdot T((x \cdot y)\backslash(y \cdot x), z) = (y \cdot x) \cdot K(z\backslash(z/((y \cdot x)\backslash(x \cdot y))), z)$ by Theorem 284. Then $(x \cdot y) \cdot T((x \cdot y)\backslash(y \cdot x), z) = (y \cdot x) \cdot K(z\backslash(z/K(x, y)), z)$ by Definition 2. Then $(x \cdot y) \cdot T(K(y, x), z) = (y \cdot x) \cdot K(z\backslash(z/K(x, y)), z)$ by Definition 2. Then $(y \cdot x) \cdot (K(x, y) \cdot T(K(x, y)\backslash e, z)) = (x \cdot y) \cdot T(K(y, x), z)$ by Theorem 196. Hence we are done by Proposition 2. □

Theorem 1128. $y \cdot ((y \cdot ((x \cdot (y\backslash e))/x)) \cdot z) = (y \cdot ((x \cdot (y\backslash e))/x)) \cdot (y \cdot z)$.

Proof. We have $(y\cdot(y\cdot z))\cdot((x\cdot((y\cdot(y\cdot z))\backslash(y\cdot z)))/x) = y\cdot((y\cdot z)\cdot((x\cdot((y\cdot z)\backslash z))/x))$ by Theorem 1102. Then $(y \cdot ((x \cdot (y\backslash e))/x)) \cdot (y \cdot z) = y \cdot ((y \cdot z) \cdot ((x \cdot ((y \cdot z)\backslash z))/x))$ by Theorem 343. Hence we are done by Theorem 343. ☐

Theorem 1129. $(y\backslash e) \cdot T(y, x) = T((y\backslash e) \cdot T(y, x), y\backslash e)$.

Proof. We have $T((y\backslash e)\cdot T((y\backslash e)\backslash K(y\backslash e, y), x), y\backslash e) = (y\backslash e)\cdot T((y\backslash e)\backslash T(K(y\backslash e, y), y\backslash e), x)$ by Theorem 309. Then $T((y\backslash e) \cdot T(y, x), y\backslash e) = (y\backslash e) \cdot T((y\backslash e)\backslash T(K(y\backslash e, y), y\backslash e), x)$ by Theorem 21. Then $(y\backslash e) \cdot T((y\backslash e)\backslash K(y\backslash e, y), x) = T((y\backslash e)\cdot T(y, x), y\backslash e)$ by Theorem 734. Hence we are done by Theorem 21. ☐

Theorem 1130. $T(y\backslash e, (y\backslash e) \cdot T(y, x)) = y\backslash e$.

Proof. We have $T((y\backslash e) \cdot T(y, x), y\backslash e) = (y\backslash e) \cdot T(y, x)$ by Theorem 1129. Hence we are done by Proposition 21. ☐

Theorem 1131. $T(y, y \cdot T(e/y, x)) = y$.

Proof. We have $(e/y)\backslash e = y$ by Proposition 25. Then $T((e/y)\backslash e, ((e/y)\backslash e) \cdot T(e/y, x)) = y$ by Theorem 1130. Then $T(y, ((e/y)\backslash e) \cdot T(e/y, x)) = y$ by Proposition 25. Hence we are done by Proposition 25. ☐

Theorem 1132. $T(x \cdot T(e/x, y), x) = x \cdot T(e/x, y)$.

Proof. We have $T(x, x \cdot T(e/x, y)) = x$ by Theorem 1131. Hence we are done by Proposition 21. ☐

Theorem 1133. $R(e/y, y, (x \cdot y)\backslash x) = y\backslash e$.

Proof. We have $(y\backslash e) \cdot (y \cdot ((x \cdot y)\backslash x)) = (x \cdot y)\backslash x$ by Theorem 379. Then

$$((x \cdot y)\backslash x)/(y \cdot ((x \cdot y)\backslash x)) = y\backslash e \tag{555}$$

by Proposition 1. We have $R(e/y, y, (x \cdot y)\backslash x) = ((x \cdot y)\backslash x)/(y \cdot ((x \cdot y)\backslash x))$ by Proposition 79. Hence we are done by (555). ☐

Theorem 1134. $R(e/y, y, x\backslash(x/y)) = y\backslash e$.

Proof. We have $R(e/y, y, ((x/y)\cdot y)\backslash(x/y)) = y\backslash e$ by Theorem 1133. Hence we are done by Axiom 6. ☐

Theorem 1135. $K(y, y\backslash e) \cdot T(x, y) = T(K(y, y\backslash e) \cdot x, y)$.

Proof. We have $K(y, y\backslash e)\cdot T(K(y, y\backslash e)\backslash(K(y, y\backslash e)\cdot x), y) = T(K(y, y\backslash e)\cdot x, y)$ by Theorem 620. Hence we are done by Axiom 3. ☐

Theorem 1136. $K(y, y\backslash e) \cdot (x \cdot y) = (K(y, y\backslash e) \cdot x) \cdot y$.

Proof. We have $y \cdot (K(y, y\backslash e) \cdot (y\backslash(x \cdot y))) = K(y, y\backslash e) \cdot (x \cdot y)$ by Theorem 327. Then $y \cdot (K(y, y\backslash e) \cdot T(x, y)) = K(y, y\backslash e) \cdot (x \cdot y)$ by Definition 3. Then

$$y \cdot T(K(y, y\backslash e) \cdot x, y) = K(y, y\backslash e) \cdot (x \cdot y) \tag{556}$$

by Theorem 1135. We have $y \cdot T(K(y, y\backslash e) \cdot x, y) = (K(y, y\backslash e) \cdot x) \cdot y$ by Proposition 46. Hence we are done by (556). ☐

Theorem 1137. $L(y, x, K(y, y\backslash e)) = y$.

Proof. We have $(K(y, y\backslash e) \cdot x) \cdot y = K(y, y\backslash e) \cdot (x \cdot y)$ by Theorem 1136. Hence we are done by Theorem 54. ☐

Theorem 1138. $(e/y) \cdot (x \cdot K(y\backslash e, y)) = ((e/y) \cdot x) \cdot K(y\backslash e, y)$.

Proof. We have $R(e/y, x, K(e/y, (e/y)\backslash e)) = e/y$ by Theorem 624. Then

$$R(e/y, x, K(y\backslash e, y)) = e/y \tag{557}$$

by Theorem 272. We have $R(e/y, x, K(y\backslash e, y))\cdot(x\cdot K(y\backslash e, y)) = ((e/y)\cdot x)\cdot K(y\backslash e, y)$ by Proposition 54. Hence we are done by (557). □

Theorem 1139. $R(x \cdot T(x\backslash e, w), y, z) = R(x, y, z) \cdot T(R(x, y, z)\backslash e, w)$.

Proof. We have $K(w\backslash(w/R(x, y, z)), w) = R(x, y, z) \cdot T(R(x, y, z)\backslash e, w)$ by Theorem 196. Hence we are done by Theorem 636. □

Theorem 1140. $T(T(x, y), z) \cdot (x \cdot w) = x \cdot (T(T(x, y), z) \cdot w)$.

Proof. We have $((x \cdot w)/(x \cdot (T(x, y)\backslash w))) \cdot ((x \cdot (T(x, y)\backslash w)) \cdot T((x \cdot (T(x, y)\backslash w))\backslash(x \cdot w), z)) = T((x \cdot w)/(x\cdot(T(x, y)\backslash w)), z)\cdot(x\cdot w)$ by Theorem 1094. Then $T(x, y)\cdot((x\cdot(T(x, y)\backslash w))\cdot T((x\cdot(T(x, y)\backslash w))\backslash(x\cdot w), z)) = T((x \cdot w)/(x \cdot (T(x, y)\backslash w)), z) \cdot (x \cdot w)$ by Theorem 1089. Then $T(x, y) \cdot (x \cdot ((T(x, y)\backslash w) \cdot T((T(x, y)\backslash w)\backslash w, z))) = T((x \cdot w)/(x \cdot (T(x, y)\backslash w)), z) \cdot (x \cdot w)$ by Theorem 486. Then

$$T(x, y) \cdot (x \cdot (T(T(x, y), z) \cdot (T(x, y)\backslash w))) = T((x \cdot w)/(x \cdot (T(x, y)\backslash w)), z) \cdot (x \cdot w) \tag{558}$$

by Theorem 48. We have $T(x, y) \cdot (x \cdot (T(T(x, y), z) \cdot (T(x, y)\backslash w))) = x \cdot (T(x, y) \cdot (T(T(x, y), z) \cdot (T(x, y)\backslash w)))$ by Theorem 172. Then $T((x\cdot w)/(x\cdot(T(x, y)\backslash w)), z)\cdot(x\cdot w) = x\cdot(T(x, y)\cdot(T(T(x, y), z)\cdot (T(x, y)\backslash w)))$ by (558). Then $T((x\cdot w)/(x\cdot(T(x, y)\backslash w)), z)\cdot(x\cdot w) = x\cdot(T(T(x, y), z)\cdot w)$ by Theorem 830. Hence we are done by Theorem 1089. □

Theorem 1141. $T(T(z, x), y) \cdot w = z \cdot (T(T(z, x), y) \cdot (z\backslash w))$.

Proof. We have $T(T(z, x), y) \cdot (z \cdot (z\backslash w)) = z \cdot (T(T(z, x), y) \cdot (z\backslash w))$ by Theorem 1140. Hence we are done by Axiom 4. □

Theorem 1142. $x \cdot K(y, y\backslash e) = (x/y) \cdot T(y, y\backslash e)$.

Proof. We have $((x/y) \cdot y) \cdot K(y, y\backslash e) = (x/y) \cdot T(y, y\backslash e)$ by Theorem 406. Hence we are done by Axiom 6. □

Theorem 1143. $x \cdot K(y, y\backslash e) = (x/y) \cdot ((y\backslash e)\backslash e)$.

Proof. We have $((x/y) \cdot y) \cdot K(y, y\backslash e) = (x/y) \cdot ((y\backslash e)\backslash e)$ by Theorem 407. Hence we are done by Axiom 6. □

Theorem 1144. $(x \cdot (y\backslash e))/(e/y) = x \cdot K(y\backslash e, y)$.

Proof. We have $(x \cdot (K(y\backslash e, y) \cdot (e/y)))/L(e/y, K(y\backslash e, y), x) = x \cdot K(y\backslash e, y)$ by Theorem 453. Then $(x\cdot(y\backslash e))/L(e/y, K(y\backslash e, y), x) = x\cdot K(y\backslash e, y)$ by Theorem 255. Hence we are done by Theorem 644. □

Theorem 1145. $(x \cdot K(y\backslash e, y))/(y\backslash e) = x/(e/y)$.

Proof. We have $(x/(e/y))\cdot(y\backslash e) = (x\cdot(y\backslash e))/(e/y)$ by Theorem 1106. Then $((x\cdot(y\backslash e))/(e/y))/(y\backslash e) = x/(e/y)$ by Proposition 1. Hence we are done by Theorem 1144. □

Theorem 1146. $x \cdot T(x\backslash(x/y), y) = x/T(y, y\backslash e)$.

Proof. We have $x\cdot T(x\backslash(x/y), y) = x/((y\backslash e)\backslash e)$ by Theorem 422. Hence we are done by Proposition 49. □

Theorem 1147. $(x/y) \cdot K(y\backslash e, y) = x/T(y, y\backslash e)$.

Proof. We have $(x/y) \cdot K(y\backslash e, y) = x \cdot T(x\backslash(x/y), y)$ by Theorem 154. Hence we are done by Theorem 1146. □

Theorem 1148. $(x/T(y, y\backslash e)) \cdot y = x \cdot K(y\backslash e, y)$.

Proof. We have $((x/y) \cdot K(y\backslash e, y)) \cdot y = x \cdot K(y\backslash e, y)$ by Theorem 315. Hence we are done by Theorem 1147. ☐

Theorem 1149. $(z\backslash T(z \cdot x, y)) \cdot z = T(x \cdot ((z\backslash e)\backslash e), y) \cdot K(z\backslash e, z)$.

Proof. We have $(T(x \cdot ((z\backslash e)\backslash e), y)/T(z, z\backslash e)) \cdot z = T(x \cdot ((z\backslash e)\backslash e), y) \cdot K(z\backslash e, z)$ by Theorem 1148. Hence we are done by Theorem 1126. ☐

Theorem 1150. $((x\backslash y)\backslash e)\backslash K(y\backslash x, x\backslash y) = x\backslash y$.

Proof. We have $((x\backslash y)\backslash e)\backslash K((x\backslash y)\backslash e, x\backslash y) = x\backslash y$ by Theorem 21. Hence we are done by Theorem 666. ☐

Theorem 1151. $R(x, y\backslash e, y) = R(x, e/y, y)$.

Proof. We have $(x \cdot (e/y)) \cdot y = R(x, e/y, y)$ by Proposition 69. Hence we are done by Theorem 911. ☐

Theorem 1152. $((x\backslash e)\backslash e)\backslash(K(x, x\backslash e) \cdot y) = x\backslash y$.

Proof. We have

$$T(x, x\backslash e) \cdot (x\backslash y) = K(x, x\backslash e) \cdot y \tag{559}$$

by Theorem 431. We have $T(x, x\backslash e) \cdot (x\backslash y) = ((x\backslash e)\backslash e) \cdot (x\backslash y)$ by Theorem 464. Then $((x\backslash e)\backslash e) \cdot (x\backslash y) = K(x, x\backslash e) \cdot y$ by (559). Hence we are done by Proposition 2. ☐

Theorem 1153. $x \cdot (K(x, x\backslash e) \cdot y) = T(x, x\backslash e) \cdot y$.

Proof. We have $x \cdot (T(x, x\backslash e) \cdot (x\backslash y)) = T(x, x\backslash e) \cdot y$ by Theorem 830. Hence we are done by Theorem 431. ☐

Theorem 1154. $L(y, K(x, x\backslash e), x) = y$.

Proof. We have $L(y, K(x, x\backslash e), x) = L(y, x, K(x, x\backslash e))$ by Theorem 587. Hence we are done by Theorem 432. ☐

Theorem 1155. $x = L(x, K(y\backslash e, y), e/y)$.

Proof. We have $L(x, e/y, K(e/y, (e/y)\backslash e)) = L(x, K(y\backslash e, y), e/y)$ by Theorem 588. Hence we are done by Theorem 432. ☐

Theorem 1156. $K(x, x\backslash e)\backslash y = x \cdot (((x\backslash e)\backslash e)\backslash y)$.

Proof. We have $L(K(x, x\backslash e)\backslash y, K(x, x\backslash e), x) = x \cdot (((x\backslash e)\backslash e)\backslash y)$ by Theorem 1118. Hence we are done by Theorem 1154. ☐

Theorem 1157. $(x\backslash e) \cdot y = (e/x) \cdot (K(x\backslash e, x) \cdot y)$.

Proof. We have $(x\backslash e) \cdot L(y, K(x\backslash e, x), e/x) = (e/x) \cdot (K(x\backslash e, x) \cdot y)$ by Theorem 1075. Hence we are done by Theorem 1155. ☐

Theorem 1158. $L(y, x, x\backslash e) = (e/x) \cdot (x \cdot y)$.

Proof. We have $L(L(y, x, x\backslash e), K(x\backslash e, x), e/x) = (e/x) \cdot (x \cdot y)$ by Theorem 618. Hence we are done by Theorem 1155. ☐

Theorem 1159. $T(x, (e/x) \cdot y) = T(x, (x\backslash e) \cdot y)$.

Proof. We have $T(x, K(x\backslash e, x) \cdot L(x\backslash y, x, x\backslash e)) = T(x, L(x\backslash y, x, x\backslash e))$ by Theorem 630. Then $T(x, (x\backslash e) \cdot y) = T(x, L(x\backslash y, x, x\backslash e))$ by Theorem 156. Hence we are done by Theorem 434. ☐

Theorem 1160. $R(e/y, K(y\backslash e, y), x) = e/y$.

Proof. We have $R(e/y, K(e/y, (e/y)\backslash e), x) = e/y$ by Theorem 1039. Hence we are done by Theorem 272. $\square$

Theorem 1161. $K(x\backslash e, x) \cdot y = (x\backslash e) \cdot ((e/x)\backslash y)$.

Proof. We have $K(x\backslash e, x) \cdot ((e/x) \cdot ((e/x)\backslash y)) = (x\backslash e) \cdot ((e/x)\backslash y)$ by Theorem 682. Hence we are done by Axiom 4. $\square$

Theorem 1162. $(e/y) \cdot (x \cdot K(y\backslash e, y)) = (y\backslash e) \cdot T(x, K(y\backslash e, y))$.

Proof. We have $(e/y) \cdot (K(y\backslash e, y) \cdot T(x, K(y\backslash e, y))) = (y\backslash e) \cdot T(x, K(y\backslash e, y))$ by Theorem 1157. Hence we are done by Proposition 46. $\square$

Theorem 1163. $(e/(e/x))\backslash(x \cdot y) = K(x, x\backslash e) \cdot y$.

Proof. We have $(e/(e/x)) \cdot (K(x, x\backslash e) \cdot y) = x \cdot y$ by Theorem 684. Hence we are done by Proposition 2. $\square$

Theorem 1164. $K(x, x\backslash e) \cdot ((y\backslash(e/(e/x))) \cdot y) = x \cdot K(y\backslash(e/(e/x)), y)$.

Proof. We have $K(x, x\backslash e) \cdot ((e/(e/x)) \cdot K(y\backslash(e/(e/x)), y)) = x \cdot K(y\backslash(e/(e/x)), y)$ by Theorem 685. Hence we are done by Theorem 17. $\square$

Theorem 1165. $(x \cdot (y\backslash(x\backslash y)))/x = y\backslash((e/(e/x))\backslash y)$.

Proof. We have $T(y\backslash(K(x, x\backslash e) \cdot (x\backslash y)), x) = y\backslash(x\backslash y)$ by Theorem 324. Then

$$T(y\backslash((e/(e/x))\backslash y), x) = y\backslash(x\backslash y) \tag{560}$$

by Theorem 690. We have $(x \cdot T(y\backslash((e/(e/x))\backslash y), x))/x = y\backslash((e/(e/x))\backslash y)$ by Proposition 48. Hence we are done by (560). $\square$

Theorem 1166. $K(x, x\backslash e) \cdot y = (x\backslash e)\backslash((e/x) \cdot y)$.

Proof. We have $K(x, x\backslash e) \cdot ((e/x)\backslash((e/x) \cdot y)) = (x\backslash e)\backslash((e/x) \cdot y)$ by Theorem 440. Hence we are done by Axiom 3. $\square$

Theorem 1167. $(e/x) \cdot y = (x\backslash e) \cdot (K(x, x\backslash e) \cdot y)$.

Proof. We have $(e/x) \cdot ((e/x)\backslash((x\backslash e) \cdot (K(x, x\backslash e) \cdot y))) = (x\backslash e) \cdot (K(x, x\backslash e) \cdot y)$ by Axiom 4. Hence we are done by Theorem 441. $\square$

Theorem 1168. $L(y, K(x\backslash e, x), (x\backslash e)\backslash e) = y$.

Proof. We have $L(y, K(x\backslash e, (x\backslash e)\backslash e), (x\backslash e)\backslash e) = y$ by Theorem 1045. Hence we are done by Theorem 779. $\square$

Theorem 1169. $x \cdot y = ((x\backslash e)\backslash e) \cdot (K(x\backslash e, x) \cdot y)$.

Proof. We have $T(x, x\backslash e) = (x\backslash e)\backslash e$ by Proposition 49. Then

$$R(x, x\backslash e, x) = (x\backslash e)\backslash e \tag{561}$$

by Theorem 207. We have $x/K(x\backslash e, x) = R(x, x\backslash e, x)$ by Theorem 1069. Then

$$x/K(x\backslash e, x) = (x\backslash e)\backslash e \tag{562}$$

by (561). We have $(x/K(x\backslash e, x)) \cdot K(x\backslash e, x) = x$ by Axiom 6. Then

$$((x\backslash e)\backslash e) \cdot K(x\backslash e, x) = x \tag{563}$$

by (562). We have $(((x\backslash e)\backslash e) \cdot K(x\backslash e, x)) \cdot L(y, K(x\backslash e, x), (x\backslash e)\backslash e) = ((x\backslash e)\backslash e) \cdot (K(x\backslash e, x) \cdot y)$ by Proposition 52. Then $x \cdot L(y, K(x\backslash e, x), (x\backslash e)\backslash e) = ((x\backslash e)\backslash e) \cdot (K(x\backslash e, x) \cdot y)$ by (563). Hence we are done by Theorem 1168. $\square$

Theorem 1170. $x \cdot y = T(x, x\backslash e) \cdot (K(x\backslash e, x) \cdot y)$.

Proof. We have $x/K(x\backslash e, x) = R(x, x\backslash e, x)$ by Theorem 1069. Then

$$x/K(x\backslash e, x) = T(x, x\backslash e) \tag{564}$$

by Theorem 207. We have $(x/K(x\backslash e, x)) \cdot K(x\backslash e, x) = x$ by Axiom 6. Then

$$T(x, x\backslash e) \cdot K(x\backslash e, x) = x \tag{565}$$

by (564). We have $(T(x, x\backslash e) \cdot K(x\backslash e, x)) \cdot L(y, K(x\backslash e, x), T(x, x\backslash e)) = T(x, x\backslash e) \cdot (K(x\backslash e, x) \cdot y)$ by Proposition 52. Then

$$x \cdot L(y, K(x\backslash e, x), T(x, x\backslash e)) = T(x, x\backslash e) \cdot (K(x\backslash e, x) \cdot y) \tag{566}$$

by (565). We have $L(y, K(x\backslash e, x), (x\backslash e)\backslash e) = y$ by Theorem 1168. Then $L(y, K(x\backslash e, x), T(x, x\backslash e)) = y$ by Proposition 49. Hence we are done by (566). ☐

Theorem 1171. $K(x\backslash e, x) \cdot (T(x, x\backslash e) \cdot y) = x \cdot y$.

Proof. We have

$$K(x, x\backslash e) \cdot (K(x\backslash e, x) \cdot (T(x, x\backslash e) \cdot y)) = T(x, x\backslash e) \cdot y \tag{567}$$

by Theorem 442. We have $K(x, x\backslash e) \cdot (x \cdot y) = T(x, x\backslash e) \cdot y$ by Theorem 676. Hence we are done by (567) and Proposition 7. ☐

Theorem 1172. $T(x, x\backslash e)\backslash y = x\backslash(K(x\backslash e, x) \cdot y)$.

Proof. We have $T(x, x\backslash e) \cdot (x\backslash(K(x\backslash e, x) \cdot y)) = K(x, x\backslash e) \cdot (K(x\backslash e, x) \cdot y)$ by Theorem 431. Then $T(x, x\backslash e)\backslash(K(x, x\backslash e) \cdot (K(x\backslash e, x) \cdot y)) = x\backslash(K(x\backslash e, x) \cdot y)$ by Proposition 2. Hence we are done by Theorem 442. ☐

Theorem 1173. $(e/(e/x)) \cdot y = x \cdot (K(x\backslash e, x) \cdot y)$.

Proof. We have $(e/(e/x)) \cdot (K(x, x\backslash e) \cdot (K(x\backslash e, x) \cdot y)) = x \cdot (K(x\backslash e, x) \cdot y)$ by Theorem 684. Hence we are done by Theorem 442. ☐

Theorem 1174. $(x\backslash e) \cdot (x \cdot y) = (e/x) \cdot ((e/(e/x)) \cdot y)$.

Proof. We have $(x\backslash e) \cdot (K(x, x\backslash e) \cdot ((e/(e/x)) \cdot y)) = (e/x) \cdot ((e/(e/x)) \cdot y)$ by Theorem 1167. Hence we are done by Theorem 685. ☐

Theorem 1175. $x\backslash L(y, e/x, x) = (x\backslash e) \cdot y$.

Proof. We have

$$x \cdot (K(x, x\backslash e) \cdot (x\backslash L(y, e/x, x))) = (x \cdot (e/x)) \cdot L(y, e/x, x) \tag{568}$$

by Theorem 326. We have $x \cdot ((e/x) \cdot y) = (x \cdot (e/x)) \cdot L(y, e/x, x)$ by Proposition 52. Then

$$K(x, x\backslash e) \cdot (x\backslash L(y, e/x, x)) = (e/x) \cdot y \tag{569}$$

by (568) and Proposition 7. We have $K(x, x\backslash e) \cdot ((x\backslash e) \cdot y) = (e/x) \cdot y$ by Theorem 697. Then $K(x, x\backslash e) \cdot (x\backslash L(y, e/x, x)) = K(x, x\backslash e) \cdot ((x\backslash e) \cdot y)$ by (569). Hence we are done by Proposition 9. ☐

Theorem 1176. $L(x, e/y, y) = L(x, y\backslash e, y)$.

Proof. We have

$$y\backslash L(L(x, y\backslash e, y), e/y, y) = (y\backslash e) \cdot L(x, y\backslash e, y) \tag{570}$$

by Theorem 1175. We have $y\backslash L(L(x, y\backslash e, y), e/y, y) = (y\backslash e) \cdot L(x, e/y, y)$ by Theorem 803. Then $(y\backslash e) \cdot L(x, e/y, y) = (y\backslash e) \cdot L(x, y\backslash e, y)$ by (570). Hence we are done by Proposition 9. ☐

Theorem 1177. $L(x, y, y\backslash e) = L(x, (y\backslash e)\backslash e, y\backslash e)$.

Proof. We have $L(x, e/(y\backslash e), y\backslash e) = L(x, (y\backslash e)\backslash e, y\backslash e)$ by Theorem 1176. Hence we are done by Proposition 24. ☐

Theorem 1178. $K(y, y\backslash e) \cdot x = T(x, K(y\backslash e, y)) \cdot K(y, y\backslash e)$.

Proof. We have

$$T(T(x, K(y\backslash e, y)), K(y, y\backslash e)) \cdot K(y\backslash e, y) = K(y\backslash e, y) \cdot T(x, K(y\backslash e, y)) \tag{571}$$

by Theorem 699. We have $x \cdot K(y\backslash e, y) = K(y\backslash e, y) \cdot T(x, K(y\backslash e, y))$ by Proposition 46. Then

$$T(T(x, K(y\backslash e, y)), K(y, y\backslash e)) = x \tag{572}$$

by (571) and Proposition 8. We have $K(y, y\backslash e) \cdot T(T(x, K(y\backslash e, y)), K(y, y\backslash e)) = T(x, K(y\backslash e, y)) \cdot K(y, y\backslash e)$ by Proposition 46. Hence we are done by (572). ☐

Theorem 1179. $L(z, K(x\backslash e, x), T(x, y)) = z$.

Proof. We have $T(R(x, K(x\backslash e, x), z), y) \cdot (K(x\backslash e, x) \cdot z) = (T(x, y) \cdot K(x\backslash e, x)) \cdot z$ by Proposition 65. Then $T(x, y) \cdot (K(x\backslash e, x) \cdot z) = (T(x, y) \cdot K(x\backslash e, x)) \cdot z$ by Theorem 1047. Hence we are done by Theorem 54. ☐

Theorem 1180. $(((y\backslash e)\backslash e) \cdot x)\backslash x = T((y \cdot x)\backslash x, y)$.

Proof. We have $T(x/(((y\backslash e)\backslash e) \cdot x), ((y\backslash e)\backslash e) \cdot x) = (((y\backslash e)\backslash e) \cdot x)\backslash x$ by Proposition 47. Then

$$T(R(y\backslash e, y, x), ((y\backslash e)\backslash e) \cdot x) = (((y\backslash e)\backslash e) \cdot x)\backslash x \tag{573}$$

by Theorem 703. We have $R(T(y\backslash e, ((y\backslash e)\backslash e) \cdot x), y, x) = T(R(y\backslash e, y, x), ((y\backslash e)\backslash e) \cdot x)$ by Axiom 9. Then

$$R(T(y\backslash e, ((y\backslash e)\backslash e) \cdot x), y, x) = (((y\backslash e)\backslash e) \cdot x)\backslash x \tag{574}$$

by (573). We have $T(y\backslash e, (e/(y\backslash e)) \cdot x) = T(y\backslash e, ((y\backslash e)\backslash e) \cdot x)$ by Theorem 1159. Then $T(y\backslash e, y \cdot x) = T(y\backslash e, ((y\backslash e)\backslash e) \cdot x)$ by Proposition 24. Then

$$R(T(y\backslash e, y \cdot x), y, x) = (((y\backslash e)\backslash e) \cdot x)\backslash x \tag{575}$$

by (574). We have $T(T(x/(y \cdot x), y), y \cdot x) = T((y \cdot x)\backslash x, y)$ by Proposition 51. Then

$$T(R(y\backslash e, y, x), y \cdot x) = T((y \cdot x)\backslash x, y) \tag{576}$$

by Theorem 89. We have $R(T(y\backslash e, y \cdot x), y, x) = T(R(y\backslash e, y, x), y \cdot x)$ by Axiom 9. Then $R(T(y\backslash e, y \cdot x), y, x) = T((y \cdot x)\backslash x, y)$ by (576). Hence we are done by (575). ☐

Theorem 1181. $e/(e/((y \cdot x)\backslash x)) = (y \cdot ((y \cdot x)\backslash x))/y$.

Proof. We have $(y \cdot x)\backslash((e/(e/y))\backslash(y \cdot x)) = (y \cdot ((y \cdot x)\backslash(y\backslash(y \cdot x))))/y$ by Theorem 1165. Then $(y \cdot x)\backslash((e/(e/y))\backslash(y \cdot x)) = (y \cdot ((y \cdot x)\backslash x))/y$ by Axiom 3. Then $(y \cdot x)\backslash(K(y, y\backslash e) \cdot x) = (y \cdot ((y \cdot x)\backslash x))/y$ by Theorem 1163. Then $T(e/(e/(x/(y \cdot x))), y \cdot x) = (y \cdot ((y \cdot x)\backslash x))/y$ by Theorem 710. Hence we are done by Theorem 713. ☐

Theorem 1182. $K(x\backslash e, x) \cdot (y \cdot K(x, x\backslash e)) = y$.

Proof. We have

$$T((K(x\backslash e, x) \cdot (y \cdot K(x, x\backslash e))) \cdot K(x, x\backslash e), K(x\backslash e, x)) = (K(x\backslash e, x) \cdot (y \cdot K(x, x\backslash e))) \cdot K(x, x\backslash e) \tag{577}$$

by Theorem 717. We have $T((K(x\backslash e, x) \cdot (y \cdot K(x, x\backslash e))) \cdot K(x, x\backslash e), K(x\backslash e, x)) = y \cdot K(x, x\backslash e)$ by Theorem 658. Then $(K(x\backslash e, x) \cdot (y \cdot K(x, x\backslash e))) \cdot K(x, x\backslash e) = y \cdot K(x, x\backslash e)$ by (577). Hence we are done by Proposition 10. ☐

Theorem 1183. $y \cdot K(x, x\backslash e) = K(x, x\backslash e) \cdot y$.

Proof. We have $T(y, K(x\backslash e, x)) \cdot K(x, x\backslash e) = K(x, x\backslash e) \cdot y$ by Theorem 1178. Hence we are done by Theorem 717. ☐

Theorem 1184. $((y\backslash e) \cdot x)/(y\backslash e) = ((e/y) \cdot x)/(e/y)$.

Proof. We have $((e/y) \cdot x) \cdot K(y\backslash e, y) = (e/y) \cdot (x \cdot K(y\backslash e, y))$ by Theorem 1138. Then

$$((e/y) \cdot x) \cdot K(y\backslash e, y) = (y\backslash e) \cdot T(x, K(y\backslash e, y)) \tag{578}$$

by Theorem 1162. We have $(((e/y) \cdot x) \cdot K(y\backslash e, y))/(y\backslash e) = ((e/y) \cdot x)/(e/y)$ by Theorem 1145. Then $((y\backslash e) \cdot T(x, K(y\backslash e, y)))/(y\backslash e) = ((e/y) \cdot x)/(e/y)$ by (578). Hence we are done by Theorem 717. ☐

Theorem 1185. $(e/y) \cdot x = (y\backslash e) \cdot (x \cdot K(y, y\backslash e))$.

Proof. We have $(y\backslash e) \cdot (K(y, y\backslash e) \cdot T(x, K(y, y\backslash e))) = (e/y) \cdot T(x, K(y, y\backslash e))$ by Theorem 1167. Then $(y\backslash e) \cdot (x \cdot K(y, y\backslash e)) = (e/y) \cdot T(x, K(y, y\backslash e))$ by Proposition 46. Hence we are done by Theorem 718. ☐

Theorem 1186. $(e/y) \cdot x = ((y\backslash e) \cdot x) \cdot K(y, y\backslash e)$.

Proof. We have $T((e/y) \cdot x, K(y, y\backslash e)) = ((y\backslash e) \cdot x) \cdot K(y, y\backslash e)$ by Theorem 700. Hence we are done by Theorem 718. ☐

Theorem 1187. $(y\backslash x) \cdot y = (((y\backslash e)\backslash e)\backslash x) \cdot ((y\backslash e)\backslash e)$.

Proof. We have

$$((((y\backslash e)\backslash e)\backslash x) \cdot ((y\backslash e)\backslash e)) \cdot (y\backslash e) = T(x \cdot (y\backslash e), (y\backslash e)\backslash e) \tag{579}$$

by Theorem 887. We have $(y \cdot T(y\backslash x, (y\backslash e)\backslash e)) \cdot (y\backslash e) = T(x \cdot (y\backslash e), (y\backslash e)\backslash e)$ by Theorem 506. Then $(((y\backslash e)\backslash e)\backslash x) \cdot ((y\backslash e)\backslash e) = y \cdot T(y\backslash x, (y\backslash e)\backslash e)$ by (579) and Proposition 8. Then $(((y\backslash e)\backslash e)\backslash x) \cdot ((y\backslash e)\backslash e) = T((y\backslash x) \cdot y, K(y, y\backslash e))$ by Theorem 705. Hence we are done by Theorem 718. ☐

Theorem 1188. $T(x, T((y\backslash e)\backslash e, z)) = T(x, T(y, z))$.

Proof. We have $T(x, (T(y, z)\backslash e)\backslash e) = T(x, T(y, z))$ by Theorem 721. Hence we are done by Theorem 919. ☐

Theorem 1189. $R(x, x\backslash y, K(z\backslash e, z))\backslash(K(z\backslash e, z) \cdot y) = (x\backslash y) \cdot K(z\backslash e, z)$.

Proof. We have $R(x, x\backslash y, K(z\backslash e, z))\backslash(y \cdot K(z\backslash e, z)) = (x\backslash y) \cdot K(z\backslash e, z)$ by Theorem 1067. Hence we are done by Theorem 923. ☐

Theorem 1190. $(y\backslash e) \cdot ((e/y)\backslash x) = x \cdot K(y\backslash e, y)$.

Proof. We have $(y\backslash e) \cdot ((e/y)\backslash x) = K(y\backslash e, y) \cdot x$ by Theorem 1161. Hence we are done by Theorem 923. ☐

Theorem 1191. $x \cdot (e/y) = (K(y, y\backslash e) \cdot x) \cdot (y\backslash e)$.

Proof. We have $((K(y, y\backslash e) \cdot x) \cdot K(y\backslash e, y)) \cdot (e/y) = (K(y, y\backslash e) \cdot x) \cdot (y\backslash e)$ by Theorem 411. Hence we are done by Theorem 924. ☐

Theorem 1192. $L((x \cdot K(y, y\backslash e))\backslash z, x \cdot K(y, y\backslash e), K(y\backslash e, y)) = x\backslash(K(y\backslash e, y) \cdot z)$.

Proof. We have $L((x \cdot K(y, y\backslash e))\backslash z, x \cdot K(y, y\backslash e), K(y\backslash e, y)) = (K(y\backslash e, y) \cdot (x \cdot K(y, y\backslash e)))\backslash(K(y\backslash e, y) \cdot z)$ by Proposition 53. Hence we are done by Theorem 1182. ☐

Theorem 1193. $((y\backslash e)\backslash e) \cdot x = y \cdot (x \cdot K(y, y\backslash e))$.

Proof. We have $((y\backslash e)\backslash e) \cdot (K(y\backslash e, y) \cdot (x \cdot K(y, y\backslash e))) = y \cdot (x \cdot K(y, y\backslash e))$ by Theorem 1169. Hence we are done by Theorem 1182. ☐

Theorem 1194. $T(x, T(y\backslash e, z)) = T(x, T(e/y, z))$.

Proof. We have $T(x, T(((e/y)\backslash e)\backslash e, z)) = T(x, T(e/y, z))$ by Theorem 1188. Hence we are done by Proposition 25. □

Theorem 1195. $L(y \cdot K(x\backslash e, x), x, x\backslash e) = (x\backslash e) \cdot (x \cdot y)$.

Proof. We have

$$((x\backslash e)\backslash e)\backslash(x \cdot y) = x \cdot (((x\backslash e)\backslash e)\backslash y) \tag{580}$$

by Theorem 1095. We have

$$T(y, K(x, x\backslash e)) \cdot K(x\backslash e, x) = K(x, x\backslash e)\backslash y \tag{581}$$

by Theorem 659.

$$\begin{array}{lll} & ((x\backslash e)\backslash e)\backslash(x \cdot y) & \\ = & K(x, x\backslash e)\backslash y & \text{by (580), Theorem 1156} \\ = & y \cdot K(x\backslash e, x) & \text{by (581), Theorem 718.} \end{array}$$

Then

$$((x\backslash e)\backslash e)\backslash(x \cdot y) = y \cdot K(x\backslash e, x). \tag{582}$$

We have $L(((x\backslash e)\backslash e)\backslash(x \cdot y), (x\backslash e)\backslash e, x\backslash e) = (x\backslash e) \cdot (x \cdot y)$ by Theorem 62. Then $L(((x\backslash e)\backslash e)\backslash(x \cdot y), x, x\backslash e) = (x\backslash e) \cdot (x \cdot y)$ by Theorem 1177. Hence we are done by (582). □

Theorem 1196. $(x \cdot y)/x = T((x \cdot y)/x, K(y, z))$.

Proof. We have $(x \cdot T(y, K(y, z)))/x = T((x \cdot y)/x, K(y, z))$ by Theorem 50. Hence we are done by Theorem 736. □

Theorem 1197. $(T(x, y)\backslash x)\backslash e = x\backslash T(x, y)$.

Proof. We have $T(y\backslash T(y, R(T(x, y), T(x, y)\backslash e, y)), K(y\backslash(y/(x\backslash e)), y)) = y\backslash T(y, R(T(x, y), T(x, y)\backslash e, y))$ by Theorem 619. Then $K(y\backslash T(y, R(T(x, y), T(x, y)\backslash e, y)), K(y\backslash(y/(x\backslash e)), y)) = e$ by Proposition 22. Then $K(y\backslash((T(x, y)\backslash x) \cdot y), K(y\backslash(y/(x\backslash e)), y)) = e$ by Theorem 591. Then $K(T(T(x, y)\backslash x, y), K(y\backslash(y/(x\backslash e)), y)) = e$ by Definition 3. Then $K(T(x, y)\backslash x, K(y\backslash(y/(x\backslash e)), y)) = e$ by Theorem 738. Then

$$K(T(x, y)\backslash x, x\backslash T(x, y)) = e \tag{583}$$

by Theorem 198. We have $((x\backslash T(x, y))\backslash e)\backslash K(T(x, y)\backslash x, x\backslash T(x, y)) = x\backslash T(x, y)$ by Theorem 1150. Then $((x\backslash T(x, y))\backslash e)\backslash e = x\backslash T(x, y)$ by (583). Hence we are done by Theorem 749. □

Theorem 1198. $x \cdot ((T(x, y)\backslash x)\backslash e) = T(x, y)$.

Proof. We have $x \cdot (x\backslash T(x, y)) = T(x, y)$ by Axiom 4. Hence we are done by Theorem 1197. □

Theorem 1199. $(e/x) \cdot T((e/x)\backslash e, y) = x\backslash T(x, y)$.

Proof. We have $(e/x) \cdot T((e/x)\backslash e, y) = T(x, y)/x$ by Theorem 149. Hence we are done by Theorem 758. □

Theorem 1200. $T(y, T(y\backslash e, x)) = (y\backslash e)\backslash e$.

Proof. We have $T(e/(y\backslash e), T(y\backslash e, x)) = (y\backslash e)\backslash e$ by Theorem 762. Hence we are done by Proposition 24. □

Theorem 1201. $T(x\backslash e, x) = T(x\backslash e, T(x, y))$.

Proof. We have $T(T(e/x,T(x,y)),x) = T(x\backslash e,T(x,y))$ by Proposition 51. Hence we are done by Theorem 762. □

Theorem 1202. $K(x\backslash e,T(x,y)) = x \cdot T(x\backslash e,T(x,y))$.

Proof. We have $K(T(x,y)\backslash(T(x,y)/x),T(x,y)) = x{\cdot}T(x\backslash e,T(x,y))$ by Theorem 196. Then $K(T(e/x,T(x,y)),T(x,y)) = x \cdot T(x\backslash e,T(x,y))$ by Theorem 501. Hence we are done by Theorem 762. □

Theorem 1203. $T(x,x\backslash e) = T(x,T(x\backslash e,y))$.

Proof. We have $T(x,x\backslash e) = (x\backslash e)\backslash e$ by Proposition 49. Hence we are done by Theorem 1200. □

Theorem 1204. $(x\backslash e)\backslash T(e/x,y) = x \cdot T(e/x,y)$.

Proof. We have $(e/x)\cdot((e/(e/x))\cdot T(e/x,y)) = (x\backslash e)\cdot(x \cdot T(e/x,y))$ by Theorem 1174. Then $(e/x)\cdot(T(e/x,y)/(e/x)) = (x\backslash e)\cdot(x\cdot T(e/x,y))$ by Theorem 150. Then $T(e/x,y) = (x\backslash e)\cdot(x\cdot T(e/x,y))$ by Theorem 757. Hence we are done by Proposition 2. □

Theorem 1205. $K(x\backslash e,x) = K(x\backslash e,T(x,y))$.

Proof. We have $x \cdot T(x\backslash e,T(x,y)) = K(x\backslash e,T(x,y))$ by Theorem 1202. Then

$$x \cdot T(x\backslash e,x) = K(x\backslash e,T(x,y)) \tag{584}$$

by Theorem 1201. We have $x \cdot T(x\backslash e,x) = (x\backslash e)\cdot x$ by Proposition 46. Then

$$K(x\backslash e,T(x,y)) = (x\backslash e)\cdot x \tag{585}$$

by (584). We have $K(x\backslash e,x) = (x\backslash e)\cdot x$ by Proposition 76. Hence we are done by (585). □

Theorem 1206. $((x\backslash e)\backslash e)\cdot T(x\backslash e,y) = x \cdot T(e/x,y)$.

Proof. We have $((x\backslash e)\backslash e)\cdot T(x\backslash e,y) = (x\backslash e)\backslash T(e/x,y)$ by Theorem 1110. Hence we are done by Theorem 1204. □

Theorem 1207. $K(x,x\backslash e)\cdot T(x\backslash e,y) = T(e/x,y)$.

Proof. We have $R(x,T(e/x,y),x)\cdot T(T(e/x,y),x) = T(x\cdot T(e/x,y),x)$ by Theorem 230. Then $T(x,T(e/x,y)){\cdot}T(T(e/x,y),x) = T(x{\cdot}T(e/x,y),x)$ by Theorem 865. Then $T(x,T(e/x,y)){\cdot}T(x\backslash e,y) = T(x\cdot T(e/x,y),x)$ by Proposition 51. Then

$$T(x,T(e/x,y))\cdot T(x\backslash e,y) = x\cdot T(e/x,y) \tag{586}$$

by Theorem 1132. We have $T(x,T(e/x,y)) = T(x,T(x\backslash e,y))$ by Theorem 1194. Then $T(x,T(e/x,y)) = T(x,x\backslash e)$ by Theorem 1203. Then

$$T(x,x\backslash e)\cdot T(x\backslash e,y) = x\cdot T(e/x,y) \tag{587}$$

by (586). We have $x\cdot(K(x,x\backslash e)\cdot T(x\backslash e,y)) = T(x,x\backslash e)\cdot T(x\backslash e,y)$ by Theorem 1153. Then $x\cdot(K(x,x\backslash e)\cdot T(x\backslash e,y)) = x\cdot T(e/x,y)$ by (587). Hence we are done by Proposition 9. □

Theorem 1208. $T(e/y,x) = T(y\backslash e,x)\cdot K(y,y\backslash e)$.

Proof. We have $K(y,y\backslash e)\cdot T(y\backslash e,x) = T(y\backslash e,x)\cdot K(y,y\backslash e)$ by Theorem 1183. Hence we are done by Theorem 1207. □

Theorem 1209. $T(x\backslash e,y)\cdot(K(x,x\backslash e)\cdot z) = T(e/x,y)\cdot z$.

Proof. We have $R(T(x\backslash e,y),K(x,x\backslash e),z) = T(R(x\backslash e,K(x,x\backslash e),z),y)$ by Axiom 9. Then

$$R(T(x\backslash e,y),K(x,x\backslash e),z) = T(x\backslash e,y) \tag{588}$$

by Theorem 1047. We have $R(T(x\backslash e,y),K(x,x\backslash e),z)\backslash((T(x\backslash e,y)\cdot K(x,x\backslash e))\cdot z) = K(x,x\backslash e)\cdot z$ by Theorem 6. Then $T(x\backslash e,y)\backslash((T(x\backslash e,y)\cdot K(x,x\backslash e))\cdot z) = K(x,x\backslash e)\cdot z$ by (588). Then

$$T(x\backslash e,y)\backslash(T(e/x,y)\cdot z) = K(x,x\backslash e)\cdot z \tag{589}$$

by Theorem 1208. We have $T(x\backslash e,y)\cdot(T(x\backslash e,y)\backslash(T(e/x,y)\cdot z)) = T(e/x,y)\cdot z$ by Axiom 4. Hence we are done by (589). ☐

Theorem 1210. $T(y\backslash e,x)\cdot((y\backslash e)\backslash z) = T(e/y,x)\cdot((e/y)\backslash z)$.

Proof. We have $T(y\backslash e,x)\cdot(K(y,y\backslash e)\cdot((e/y)\backslash z)) = T(e/y,x)\cdot((e/y)\backslash z)$ by Theorem 1209. Hence we are done by Theorem 440. ☐

Theorem 1211. $(y\cdot(x\backslash T(x,y)))\cdot(T(x,y)\backslash x) = y$.

Proof. We have $(y\cdot(x\backslash T(x,y)))\cdot((y\cdot(x\backslash T(x,y)))\backslash y) = y$ by Axiom 4. Hence we are done by Theorem 952. ☐

Theorem 1212. $((((e/x)\cdot y)\backslash y)\backslash e)\backslash e = ((x\backslash e)\cdot y)\backslash y$.

Proof. We have $(e/(e/(((e/x)\cdot y)\backslash y)))\backslash e = e/(((e/x)\cdot y)\backslash y)$ by Proposition 25. Then

$$(((e/x)\cdot(((e/x)\cdot y)\backslash y))/(e/x))\backslash e = e/(((e/x)\cdot y)\backslash y) \tag{590}$$

by Theorem 1181. We have $(((((e/x)\cdot(((e/x)\cdot y)\backslash y))/(e/x))\backslash e)\backslash e)\cdot(e/x) = (e/x)\cdot((((e/x)\backslash((e/x)\cdot(((e/x)\cdot y)\backslash y)))\backslash e)\backslash e)$ by Theorem 921. Then $((e/(((e/x)\cdot y)\backslash y))\backslash e)\cdot(e/x) = (e/x)\cdot((((e/x)\backslash((e/x)\cdot(((e/x)\cdot y)\backslash y)))\backslash e)\backslash e)$ by (590). Then $(e/x)\cdot((((e/x)\backslash((e/x)\cdot(((e/x)\cdot y)\backslash y)))\backslash e)\backslash e) = (((e/x)\cdot y)\backslash y)\cdot(e/x)$ by Proposition 25. Then

$$(e/x)\cdot(((((e/x)\cdot y)\backslash y)\backslash e)\backslash e) = (((e/x)\cdot y)\backslash y)\cdot(e/x) \tag{591}$$

by Axiom 3. We have $T(((e/x)\cdot y)\backslash y,e/x) = ((((e/x)\backslash e)\backslash e)\cdot y)\backslash y$ by Theorem 1180. Then

$$T(((e/x)\cdot y)\backslash y,e/x) = ((x\backslash e)\cdot y)\backslash y \tag{592}$$

by Proposition 25. We have $(e/x)\cdot T(((e/x)\cdot y)\backslash y,e/x) = (((e/x)\cdot y)\backslash y)\cdot(e/x)$ by Proposition 46. Then $(e/x)\cdot(((x\backslash e)\cdot y)\backslash y) = (((e/x)\cdot y)\backslash y)\cdot(e/x)$ by (592). Hence we are done by (591) and Proposition 7. ☐

Theorem 1213. $K(((x\backslash e)\cdot y)\backslash y,z) = K(((e/x)\cdot y)\backslash y,z)$.

Proof. We have $K(((((e/x)\cdot y)\backslash y)\backslash e)\backslash e,z) = K(((e/x)\cdot y)\backslash y,z)$ by Theorem 733. Hence we are done by Theorem 1212. ☐

Theorem 1214. $(e/x)\cdot(T(x\backslash e,y)\cdot z) = (x\backslash e)\cdot(T(e/x,y)\cdot z)$.

Proof. We have $((T(e/x,y)\cdot((e/x)\backslash e))\cdot(e/x))\cdot L(z,((e/x)\backslash e)\cdot(e/x),R(T(e/x,y),(e/x)\backslash e,e/x)) = R(T(e/x,y),(e/x)\backslash e,e/x)\cdot((((e/x)\backslash e)\cdot(e/x))\cdot z)$ by Theorem 1064. Then $((T(e/x,y)\cdot((e/x)\backslash e))\cdot(e/x))\cdot L(z,((e/x)\backslash e)\cdot(e/x),T(T(e/x,(e/x)\backslash e),y)) = R(T(e/x,y),(e/x)\backslash e,e/x)\cdot((((e/x)\backslash e)\cdot(e/x))\cdot z)$ by Theorem 1100. Then $(((e/x)\backslash T(e/x,y))\cdot(e/x))\cdot L(z,((e/x)\backslash e)\cdot(e/x),T(T(e/x,(e/x)\backslash e),y)) = R(T(e/x,y),(e/x)\backslash e,e/x)\cdot((((e/x)\backslash e)\cdot(e/x))\cdot z)$ by Theorem 146. Then

$$(((e/x)\backslash T(e/x,y))\cdot(e/x))\cdot L(z,K((e/x)\backslash e,e/x),T(T(e/x,(e/x)\backslash e),y)) = R(T(e/x,y),(e/x)\backslash e,e/x)\cdot((((e/x)\backslash e)\cdot(e/x))\cdot z) \tag{593}$$

by Proposition 76. We have $L(z,K(T(e/x,(e/x)\backslash e)\backslash e,T(e/x,(e/x)\backslash e)),T(T(e/x,(e/x)\backslash e),y)) = z$ by Theorem 1179. Then $L(z,K((e/x)\backslash e,e/x),T(T(e/x,(e/x)\backslash e),y)) = z$ by Theorem 929. Then

$(((e/x)\backslash T(e/x,y)) \cdot (e/x)) \cdot z = R(T(e/x,y),(e/x)\backslash e, e/x) \cdot ((((e/x)\backslash e) \cdot (e/x)) \cdot z)$ by (593). Then $T(T(e/x,(e/x)\backslash e),y) \cdot ((((e/x)\backslash e) \cdot (e/x)) \cdot z) = (((e/x)\backslash T(e/x,y)) \cdot (e/x)) \cdot z$ by Theorem 1100. Then

$$T(T(e/x,(e/x)\backslash e),y) \cdot (K((e/x)\backslash e, e/x) \cdot z) = (((e/x)\backslash T(e/x,y)) \cdot (e/x)) \cdot z \tag{594}$$

by Proposition 76. We have $K((e/x)\backslash e, e/x) \cdot (T(e/x,(e/x)\backslash e) \cdot (T(T(e/x,(e/x)\backslash e),y) \cdot (T(e/x,(e/x)\backslash e)\backslash z))) = (e/x) \cdot (T(T(e/x,(e/x)\backslash e),y) \cdot (T(e/x,(e/x)\backslash e)\backslash z))$ by Theorem 1171. Then

$$K((e/x)\backslash e, e/x) \cdot (T(T(e/x,(e/x)\backslash e),y) \cdot z) = (e/x) \cdot (T(T(e/x,(e/x)\backslash e),y) \cdot (T(e/x,(e/x)\backslash e)\backslash z)) \tag{595}$$

by Theorem 830. We have $(e/x) \cdot (T(T(e/x,(e/x)\backslash e),y) \cdot ((e/x)\backslash (K((e/x)\backslash e, e/x) \cdot z))) = T(T(e/x,(e/x)\backslash e),y) \cdot (K((e/x)\backslash e, e/x) \cdot z)$ by Theorem 1141. Then $(e/x) \cdot (T(T(e/x,(e/x)\backslash e),y) \cdot (T(e/x,(e/x)\backslash e)\backslash z)) = T(T(e/x,(e/x)\backslash e),y) \cdot (K((e/x)\backslash e, e/x) \cdot z)$ by Theorem 1172. Then $K((e/x)\backslash e, e/x) \cdot (T(T(e/x,(e/x)\backslash e),y) \cdot z) = T(T(e/x,(e/x)\backslash e),y) \cdot (K((e/x)\backslash e, e/x) \cdot z)$ by (595). Then

$$(((e/x)\backslash T(e/x,y)) \cdot (e/x)) \cdot z = K((e/x)\backslash e, e/x) \cdot (T(T(e/x,(e/x)\backslash e),y) \cdot z) \tag{596}$$

by (594). We have $T(e/x,(e/x)\backslash e) \cdot (K((e/x)\backslash e, e/x) \cdot (T(T(e/x,(e/x)\backslash e),y) \cdot z)) = (e/x) \cdot (T(T(e/x,(e/x)\backslash e),y) \cdot z)$ by Theorem 1170. Then $T(e/x,(e/x)\backslash e) \cdot ((((e/x)\backslash T(e/x,y)) \cdot (e/x)) \cdot z) = (e/x) \cdot (T(T(e/x,(e/x)\backslash e),y) \cdot z)$ by (596). Then $T(e/x,(e/x)\backslash e) \cdot (T(e/x,y) \cdot z) = (e/x) \cdot (T(T(e/x,(e/x)\backslash e),y) \cdot z)$ by Theorem 759. Then

$$T(e/x,(e/x)\backslash e) \cdot (T(e/x,y) \cdot z) = (e/x) \cdot (T(((e/x)\backslash e)\backslash e,y) \cdot z) \tag{597}$$

by Proposition 49. We have $T(e/x,(e/x)\backslash e) \cdot (T(e/x,y) \cdot z) = (((e/x)\backslash e)\backslash e) \cdot (T(e/x,y) \cdot z)$ by Theorem 464. Then $T(e/x,(e/x)\backslash e) \cdot (T(e/x,y) \cdot z) = (x\backslash e) \cdot (T(e/x,y) \cdot z)$ by Proposition 25. Then $(e/x) \cdot (T(((e/x)\backslash e)\backslash e,y) \cdot z) = (x\backslash e) \cdot (T(e/x,y) \cdot z)$ by (597). Hence we are done by Proposition 25. □

Theorem 1215. $(T(x\backslash e,y) \cdot z) \cdot K(x,x\backslash e) = T(e/x,y) \cdot z$.

Proof.

$$\begin{aligned} & (x\backslash e) \cdot ((T(x\backslash e,y) \cdot z) \cdot K(x,x\backslash e)) \\ = \ & (e/x) \cdot (T(x\backslash e,y) \cdot z) && \text{by Theorem 1185} \\ = \ & (x\backslash e) \cdot (T(e/x,y) \cdot z) && \text{by Theorem 1214.} \end{aligned}$$

Then $(x\backslash e) \cdot ((T(x\backslash e,y) \cdot z) \cdot K(x,x\backslash e)) = (x\backslash e) \cdot (T(e/x,y) \cdot z)$. Hence we are done by Proposition 9. □

Theorem 1216. $y \cdot (((y \cdot x)/y)\backslash x) = (x\backslash ((y \cdot x)/y))\backslash y$.

Proof. We have $(y\backslash T(y,R(x,x\backslash e,y)))\backslash y = y \cdot (T(y,R(x,x\backslash e,y))\backslash y)$ by Theorem 956. Then $(x\backslash ((y \cdot x)/y))\backslash y = y \cdot (T(y,R(x,x\backslash e,y))\backslash y)$ by Theorem 760. Hence we are done by Theorem 769. □

Theorem 1217. $L(x,T(e/z,y),z) = L(x,T(z\backslash e,y),(z\backslash e)\backslash e)$.

Proof. We have $z \cdot ((T(z\backslash e,y) \cdot x) \cdot K(z,z\backslash e)) = ((z\backslash e)\backslash e) \cdot (T(z\backslash e,y) \cdot x)$ by Theorem 1193. Then

$$z \cdot (T(e/z,y) \cdot x) = ((z\backslash e)\backslash e) \cdot (T(z\backslash e,y) \cdot x) \tag{598}$$

by Theorem 1215. We have $L(x,T(z\backslash e,y),(z\backslash e)\backslash e) = (((z\backslash e)\backslash e) \cdot T(z\backslash e,y))\backslash (((z\backslash e)\backslash e) \cdot (T(z\backslash e,y) \cdot x))$ by Definition 4. Then $L(x,T(z\backslash e,y),(z\backslash e)\backslash e) = (z \cdot T(e/z,y))\backslash (((z\backslash e)\backslash e) \cdot (T(z\backslash e,y) \cdot x))$ by Theorem 1206. Then

$$(z \cdot T(e/z,y))\backslash (z \cdot (T(e/z,y) \cdot x)) = L(x,T(z\backslash e,y),(z\backslash e)\backslash e) \tag{599}$$

by (598). We have $L(x,T(e/z,y),z) = (z \cdot T(e/z,y))\backslash (z \cdot (T(e/z,y) \cdot x))$ by Definition 4. Hence we are done by (599). □

Theorem 1218. $L(x,y,z\backslash e) = L(L(x,y,e/z),(e/z) \cdot y,K(z\backslash e,z))$.

Proof. We have $(K(z\backslash e,z)\cdot((e/z)\cdot y))\backslash(K(z\backslash e,z)\cdot((e/z)\cdot(y\cdot x))) = L(L(x,y,e/z),(e/z)\cdot y,K(z\backslash e,z))$ by Theorem 1061. Then $((z\backslash e)\cdot y)\backslash(K(z\backslash e,z)\cdot((e/z)\cdot(y\cdot x))) = L(L(x,y,e/z),(e/z)\cdot y,K(z\backslash e,z))$ by Theorem 682. Then

$$((z\backslash e)\cdot y)\backslash((z\backslash e)\cdot(y\cdot x)) = L(L(x,y,e/z),(e/z)\cdot y,K(z\backslash e,z)) \tag{600}$$

by Theorem 682. We have $L(x,y,z\backslash e) = ((z\backslash e)\cdot y)\backslash((z\backslash e)\cdot(y\cdot x))$ by Definition 4. Hence we are done by (600). □

Theorem 1219. $K(x,((y\backslash e)\cdot z)\backslash z) = K(x,((e/y)\cdot z)\backslash z)$.

Proof. We have $K(x,((((e/y)\cdot z)\backslash z)\backslash e)\backslash e) = K(x,((e/y)\cdot z)\backslash z)$ by Theorem 779. Hence we are done by Theorem 1212. □

Theorem 1220. $x\cdot K(x\backslash e,R(x,y,z)) = L(x,x\backslash e,R(x,y,z))$.

Proof. We have $R(e/(x\backslash e),x\backslash e,(x\backslash R(x,y,z))\backslash((x\backslash R(x,y,z))/(x\backslash e))) = ((x\backslash R(x,y,z))\backslash((x\backslash R(x,y,z))/(x\backslash e)))/((x\backslash e)\cdot((x\backslash R(x,y,z))\backslash((x\backslash R(x,y,z))/(x\backslash e))))$ by Proposition 79. Then $(x\backslash e)\backslash e = ((x\backslash R(x,y,z))\backslash((x\backslash R(x,y,z))/(x\backslash e)))/((x\backslash e)\cdot((x\backslash R(x,y,z))\backslash((x\backslash R(x,y,z))/(x\backslash e))))$ by Theorem 1134. Then $((x\backslash R(x,y,z))\backslash R(x,y,z))/((x\backslash e)\cdot((x\backslash R(x,y,z))\backslash((x\backslash R(x,y,z))/(x\backslash e)))) = (x\backslash e)\backslash e$ by Theorem 834. Then $T(x,x\backslash R(x,y,z))/((x\backslash e)\cdot((x\backslash R(x,y,z))\backslash((x\backslash R(x,y,z))/(x\backslash e)))) = (x\backslash e)\backslash e$ by Proposition 49. Then $T(x,x\backslash R(x,y,z))/((x\backslash e)\cdot((x\backslash R(x,y,z))\backslash R(x,y,z))) = (x\backslash e)\backslash e$ by Theorem 834. Then

$$T(x,x\backslash R(x,y,z))/(x\backslash L(x,x\backslash e,R(x,y,z))) = (x\backslash e)\backslash e \tag{601}$$

by Theorem 1097. We have $R(R(x,x\backslash e,T(x,x\backslash R(x,y,z))),y,z) = R(R(x,y,z),x\backslash e,T(x,x\backslash R(x,y,z)))$ by Axiom 12. Then $R(T(x,x\backslash R(x,y,z))/((x\backslash e)\cdot T(x,x\backslash R(x,y,z))),y,z) = R(R(x,y,z),x\backslash e,T(x,x\backslash R(x,y,z)))$ by Proposition 66. Then $R(R(x,y,z),x\backslash e,T(x,x\backslash R(x,y,z))) = R(T(x,x\backslash R(x,y,z))/(x\backslash L(x,x\backslash e,R(x,y,z))),y,z)$ by Theorem 1122. Then

$$R((x\backslash e)\backslash e,y,z) = R(R(x,y,z),x\backslash e,T(x,x\backslash R(x,y,z))) \tag{602}$$

by (601). We have $R((x\backslash R(x,y,z))/(x\backslash e),x\backslash e,T(x,x\backslash R(x,y,z)))\backslash(x\cdot(x\backslash R(x,y,z))) = (x\backslash e)\cdot T(x,x\backslash R(x,y,z))$ by Theorem 801. Then $R((x\backslash R(x,y,z))/(x\backslash e),x\backslash e,T(x,x\backslash R(x,y,z)))\backslash(x\cdot(x\backslash R(x,y,z))) = x\backslash L(x,x\backslash e,R(x,y,z))$ by Theorem 1122. Then $R(R(x,y,z),x\backslash e,T(x,x\backslash R(x,y,z)))\backslash(x\cdot(x\backslash R(x,y,z))) = x\backslash L(x,x\backslash e,R(x,y,z))$ by Theorem 834. Then $R((x\backslash e)\backslash e,y,z)\backslash(x\cdot(x\backslash R(x,y,z))) = x\backslash L(x,x\backslash e,R(x,y,z))$ by (602). Then $R((x\backslash e)\backslash e,y,z)\backslash R(x,y,z) = x\backslash L(x,x\backslash e,R(x,y,z))$ by Axiom 4. Then $R(T(x,x\backslash e),y,z)\backslash R(x,y,z) = x\backslash L(x,x\backslash e,R(x,y,z))$ by Proposition 49. Then

$$K(x\backslash e,R(x,y,z)) = x\backslash L(x,x\backslash e,R(x,y,z)) \tag{603}$$

by Theorem 960. We have $x\cdot(x\backslash L(x,x\backslash e,R(x,y,z))) = L(x,x\backslash e,R(x,y,z))$ by Axiom 4. Hence we are done by (603). □

Theorem 1221. $L(z,x,K(y,y\backslash e)) = z$.

Proof. We have $L(z,x,K((e/y)\backslash e,e/y)) = z$ by Theorem 967. Hence we are done by Theorem 253. □

Theorem 1222. $(y\cdot K(x\backslash e,x))\cdot z = K(x\backslash e,x)\cdot(y\cdot z)$.

Proof. We have $(K(x\backslash e,x)\cdot y)\cdot L(z,y,K(x\backslash e,x)) = K(x\backslash e,x)\cdot(y\cdot z)$ by Proposition 52. Then $(y\cdot K(x\backslash e,x))\cdot L(z,y,K(x\backslash e,x)) = K(x\backslash e,x)\cdot(y\cdot z)$ by Theorem 923. Hence we are done by Theorem 967. □

Theorem 1223. $(x\cdot K(y,y\backslash e))\backslash z = x\backslash(K(y\backslash e,y)\cdot z)$.

Proof. We have $L((x\cdot K(y,y\backslash e))\backslash z,x\cdot K(y,y\backslash e),K(y\backslash e,y)) = x\backslash(K(y\backslash e,y)\cdot z)$ by Theorem 1192. Hence we are done by Theorem 967. □

Theorem 1224. $L(x, y, e/z) = L(x, y, z\backslash e)$.

Proof. We have $L(L(x, y, e/z), (e/z) \cdot y, K(z\backslash e, z)) = L(x, y, z\backslash e)$ by Theorem 1218. Hence we are done by Theorem 967. ☐

Theorem 1225. $(K(x, x\backslash e) \cdot y) \cdot z = K(x, x\backslash e) \cdot (y \cdot z)$.

Proof. We have $(K(x, x\backslash e) \cdot y) \cdot L(z, y, K(x, x\backslash e)) = K(x, x\backslash e) \cdot (y \cdot z)$ by Proposition 52. Hence we are done by Theorem 1221. ☐

Theorem 1226. $(K(x, x\backslash e) \cdot y)/z = K(x, x\backslash e) \cdot (y/z)$.

Proof. We have $(K(x, x\backslash e) \cdot y)/L(z, y/z, K(x, x\backslash e)) = K(x, x\backslash e) \cdot (y/z)$ by Theorem 472. Hence we are done by Theorem 1221. ☐

Theorem 1227. $L(x, y, z) = L(x, y, (z\backslash e)\backslash e)$.

Proof. We have $L(x, y, e/(z\backslash e)) = L(x, y, (z\backslash e)\backslash e)$ by Theorem 1224. Hence we are done by Proposition 24. ☐

Theorem 1228. $L(x, T(z\backslash e, y), z) = L(x, T(e/z, y), z)$.

Proof. We have $L(x, T(z\backslash e, y), (z\backslash e)\backslash e) = L(x, T(e/z, y), z)$ by Theorem 1217. Hence we are done by Theorem 1227. ☐

Theorem 1229. $(e/y)/x = ((y\backslash e)/x) \cdot K(y, y\backslash e)$.

Proof. We have $(K(y, y\backslash e) \cdot (y\backslash e))/L(x, (y\backslash e)/x, K(y, y\backslash e)) = K(y, y\backslash e) \cdot ((y\backslash e)/x)$ by Theorem 472. Then $(e/y)/L(x, (y\backslash e)/x, K(y, y\backslash e)) = K(y, y\backslash e) \cdot ((y\backslash e)/x)$ by Theorem 849. Then

$$(e/y)/x = K(y, y\backslash e) \cdot ((y\backslash e)/x) \tag{604}$$

by Theorem 1221. We have $K(y, y\backslash e) \cdot ((y\backslash e)/x) = ((y\backslash e)/x) \cdot K(y, y\backslash e)$ by Theorem 1183. Hence we are done by (604). ☐

Theorem 1230. $R(K(z\backslash e, z), x, y) = K(z\backslash e, z)$.

Proof. We have $(K(z\backslash e, z) \cdot x) \cdot y = K(z\backslash e, z) \cdot (x \cdot y)$ by Theorem 968. Hence we are done by Theorem 64. ☐

Theorem 1231. $(y \cdot K(x, x\backslash e)) \cdot z = K(x, x\backslash e) \cdot (y \cdot z)$.

Proof. We have $(K(x, x\backslash e) \cdot y) \cdot z = K(x, x\backslash e) \cdot (y \cdot z)$ by Theorem 1225. Hence we are done by Theorem 1183. ☐

Theorem 1232. $a((y \cdot x)\backslash e, y, x)/x = (e/(y \cdot x)) \cdot y$.

Proof. We have $a((y \cdot x)\backslash e, y, x) = (((y \cdot x)\backslash e) \cdot (y \cdot x))\backslash((((y \cdot x)\backslash e) \cdot y) \cdot x)$ by Definition 1. Then $a((y \cdot x)\backslash e, y, x) = K((y \cdot x)\backslash e, y \cdot x)\backslash((((y \cdot x)\backslash e) \cdot y) \cdot x)$ by Proposition 76. Then

$$K(y \cdot x, (y \cdot x)\backslash e) \cdot ((((y \cdot x)\backslash e) \cdot y) \cdot x) = a((y \cdot x)\backslash e, y, x) \tag{605}$$

by Theorem 696. We have $(K(y \cdot x, (y \cdot x)\backslash e) \cdot (((y \cdot x)\backslash e) \cdot y)) \cdot x = K(y \cdot x, (y \cdot x)\backslash e) \cdot ((((y \cdot x)\backslash e) \cdot y) \cdot x)$ by Theorem 1225. Then $((e/(y \cdot x)) \cdot y) \cdot x = K(y \cdot x, (y \cdot x)\backslash e) \cdot ((((y \cdot x)\backslash e) \cdot y) \cdot x)$ by Theorem 697. Then $((e/(y \cdot x)) \cdot y) \cdot x = a((y \cdot x)\backslash e, y, x)$ by (605). Hence we are done by Proposition 1. ☐

Theorem 1233. $T(y \cdot (T(y, T(x, y))\backslash y), x) = y$.

Proof. We have $(y \cdot (x\backslash T(x, y))) \cdot x = x \cdot y$ by Theorem 976. Then $(y \cdot (T(y, T(x, y))\backslash y)) \cdot x = x \cdot y$ by Theorem 986. Hence we are done by Theorem 11. ☐

Theorem 1234. $x\backslash y = T(y/x, T(x, y))$.

Proof. We have $T(x,y)\backslash((x\cdot y)/x) = T(y/x, T(x,y))$ by Theorem 1083. Hence we are done by Theorem 995. □

Theorem 1235. $T(x\backslash y, x) = T(x\backslash y, T(x,y))$.

Proof. We have $T(T(y/x, T(x,y)), x) = T(x\backslash y, T(x,y))$ by Proposition 51. Hence we are done by Theorem 1234. □

Theorem 1236. $x\cdot((x\cdot y)/x) = T(x,y)\cdot y$.

Proof. We have $x\cdot(T(x,y)\cdot(x\backslash y)) = T(x,y)\cdot y$ by Theorem 830. Hence we are done by Theorem 996. □

Theorem 1237. $T(y, y/x) = T(y, x\backslash y)$.

Proof. We have $T(y, y/(y/(x\backslash y))) = T(y, x\backslash y)$ by Theorem 997. Hence we are done by Proposition 24. □

Theorem 1238. $(y/x)\backslash T(y,x) = (y\cdot x)/y$.

Proof. We have $(y/x)\backslash T(y, y/(y/x)) = (y\cdot x)/y$ by Theorem 1087. Hence we are done by Theorem 997. □

Theorem 1239. $T(x,y)/((x\cdot y)/x) = x/y$.

Proof. We have $T(x, x/(x/y))/((x/y)\backslash T(x, x/(x/y))) = x/y$ by Proposition 24. Then $T(x, x/(x/y))/((x\cdot y)/x) = x/y$ by Theorem 1087. Hence we are done by Theorem 997. □

Theorem 1240. $T(x, x\cdot y) = T(x, x/(e/y))$.

Proof. We have $((x\cdot(x\cdot(x\backslash(x\cdot y))))/x)/(x\backslash(x\cdot y)) = T(x, x/(e/(x\backslash(x\cdot y))))$ by Theorem 1112. Then $((x\cdot(x\cdot y))/x)/(x\backslash(x\cdot y)) = T(x, x/(e/(x\backslash(x\cdot y))))$ by Axiom 4. Then $T(x, x/(e/(x\backslash(x\cdot y)))) = T(x, x/(x/(x\cdot y)))$ by Theorem 1114. Then $T(x, x/(e/y)) = T(x, x/(x/(x\cdot y)))$ by Axiom 3. Hence we are done by Theorem 997. □

Theorem 1241. $T(y,x) = T(y, (x\backslash y)\backslash y)$.

Proof. We have $T(y, y/(x\backslash y)) = T(y, (x\backslash y)\backslash y)$ by Theorem 1237. Hence we are done by Proposition 24. □

Theorem 1242. $T(y, (x\backslash e)\backslash y) = T(y, x\cdot y)$.

Proof. We have $(x\backslash e)\cdot((y\cdot((x\backslash e)\backslash y))/y) = T(y, y/(x\backslash e))$ by Theorem 515. Then

$$x\backslash((y\cdot(x\cdot y))/y) = T(y, y/(x\backslash e)) \tag{606}$$

by Theorem 1103. We have $x\backslash((y\cdot(x\cdot y))/y) = T(y, x\cdot y)$ by Theorem 1082. Then $T(y, y/(x\backslash e)) = T(y, x\cdot y)$ by (606). Hence we are done by Theorem 1237. □

Theorem 1243. $T(x, T(y, y\backslash x)) = T(x,y)$.

Proof. We have $T(x, (y\backslash x)\backslash x) = T(x,y)$ by Theorem 1241. Hence we are done by Proposition 49. □

Theorem 1244. $T(y\cdot x, T(y,x)) = T(y\cdot x, y)$.

Proof. We have $T(y\cdot x, T(y, y\backslash(y\cdot x))) = T(y\cdot x, y)$ by Theorem 1243. Hence we are done by Axiom 3. □

Theorem 1245. $T(y, y\backslash x)\cdot(y\backslash x) = y\cdot(x/y)$.

Proof. We have $T(y, y\backslash x)\cdot(y\backslash x) = y\cdot((y\cdot(y\backslash x))/y)$ by Theorem 1236. Hence we are done by Axiom 4. □

Theorem 1246. $x\backslash T(x\cdot y,y)=((x\cdot y)\cdot y)/(x\cdot y)$.

Proof. We have $((x\cdot y)/y)\backslash T(x\cdot y,y)=((x\cdot y)\cdot y)/(x\cdot y)$ by Theorem 1238. Hence we are done by Axiom 5. □

Theorem 1247. $T(x\cdot y,x\backslash e)=x\cdot((x\cdot y)\cdot(y/(x\cdot y)))$.

Proof. We have $(x\cdot y)\cdot(((x\cdot y)\cdot((x\cdot y)\backslash y))/(x\cdot y))=(x\cdot(((x\cdot y)\cdot(x\backslash e))/(x\cdot y)))\cdot y$ by Theorem 343. Then

$$(x\cdot y)\cdot(y/(x\cdot y))=(x\cdot(((x\cdot y)\cdot(x\backslash e))/(x\cdot y)))\cdot y \tag{607}$$

by Axiom 4. We have $(x\cdot(((x\cdot y)\cdot(x\backslash e))/(x\cdot y)))\cdot(x\cdot y)=x\cdot((x\cdot(((x\cdot y)\cdot(x\backslash e))/(x\cdot y)))\cdot y)$ by Theorem 1128. Then $T(x\cdot y,R(x\backslash e,x,x\cdot y))=x\cdot((x\cdot(((x\cdot y)\cdot(x\backslash e))/(x\cdot y)))\cdot y)$ by Theorem 926. Then $x\cdot((x\cdot y)\cdot(y/(x\cdot y)))=T(x\cdot y,R(x\backslash e,x,x\cdot y))$ by (607). Hence we are done by Theorem 1000. □

Theorem 1248. $x\backslash T(y,x\backslash e)=y\cdot((x\backslash y)/y)$.

Proof. We have $(x\cdot y)\cdot((y\cdot((x\cdot y)\backslash y))/y)=T(y,y/(x\cdot y))$ by Theorem 515. Then

$$(x\cdot y)\cdot L((x\backslash y)/y,y,x)=T(y,y/(x\cdot y)) \tag{608}$$

by Proposition 75. We have $x\backslash((x\cdot y)\cdot L((x\backslash y)/y,y,x))=y\cdot((x\backslash y)/y)$ by Theorem 790. Then $x\backslash T(y,y/(x\cdot y))=y\cdot((x\backslash y)/y)$ by (608). Hence we are done by Theorem 1002. □

Theorem 1249. $y\backslash(x\backslash T(y,x\backslash e))=(x\backslash y)/y$.

Proof. We have $y\cdot((x\backslash y)/y)=x\backslash T(y,x\backslash e)$ by Theorem 1248. Hence we are done by Proposition 2. □

Theorem 1250. $K(T(y,y\backslash x),y\backslash x)=K(y,x/y)$.

Proof. We have $K(T(y,y\backslash x),y\backslash x)=K(y,(y\cdot(y\backslash x))/y)$ by Theorem 1008. Hence we are done by Axiom 4. □

Theorem 1251. $K((y\backslash x)\backslash x,y\backslash x)=K(y,x/y)$.

Proof. We have $K(T(y,y\backslash x),y\backslash x)=K(y,x/y)$ by Theorem 1250. Hence we are done by Proposition 49. □

Theorem 1252. $R(T(y,y\cdot x),x,y)=T(y,T(x,y))$.

Proof. We have $T(y,y/(y/T(x,y)))\cdot(x\cdot y)=y\cdot(y\cdot x)$ by Theorem 848. Then

$$(y\cdot(y\cdot x))/(x\cdot y)=T(y,y/(y/T(x,y))) \tag{609}$$

by Proposition 1. We have

$$T(y,y/(y/T(x,y)))=(y/T(x,y))\cdot x \tag{610}$$

by Theorem 825. Then

$$(y\cdot(y\cdot x))/(x\cdot y)=(y/T(x,y))\cdot x \tag{611}$$

by (609). We have $R(((y\cdot(y\cdot x))/y)/x,x,y)=(y\cdot(y\cdot x))/(x\cdot y)$ by Proposition 74. Then $R(T(y,y/(e/x)),x,y)=(y\cdot(y\cdot x))/(x\cdot y)$ by Theorem 1112. Then

$$R(T(y,y/(e/x)),x,y)=(y/T(x,y))\cdot x \tag{612}$$

by (611).

$$\begin{aligned}
&R(T(y,y\cdot x),x,y)\\
=\;&(y/T(x,y))\cdot x && \text{by (612), Theorem 1240}\\
=\;&T(y,T(x,y)) && \text{by (610), Theorem 997.}
\end{aligned}$$

Hence we are done. □

Theorem 1253. $y \cdot T(x, T(y, x)) = L(R(x, y, x), y, x) \cdot y$.

Proof. We have $y \cdot L(T(R(x, y, x), y), y, x) = L(R(x, y, x), y, x) \cdot y$ by Proposition 58. Then $y \cdot L(R(T(x, y), y, x), y, x) = L(R(x, y, x), y, x) \cdot y$ by Axiom 9. Then $y \cdot R(T(x, x \cdot y), y, x) = L(R(x, y, x), y, x) \cdot y$ by Theorem 1071. Hence we are done by Theorem 1252. □

Theorem 1254. $L(R(y, x, y), x, y) = T((x \cdot y)/x, T(x, y))$.

Proof. We have

$$L(R(y, x, y), x, y) \cdot x = x \cdot T(y, T(x, y)) \tag{613}$$

by Theorem 1253. We have $T((x \cdot y)/x, T(x, y)) \cdot x = x \cdot T(y, T(x, y))$ by Theorem 47. Hence we are done by (613) and Proposition 8. □

Theorem 1255. $y \cdot (T(y, x)\backslash y) = x/(y\backslash((y \cdot x)/y))$.

Proof. We have $((y \cdot x)/((y \cdot x)/y)) \cdot (y\backslash((y \cdot x)/y)) = y\backslash(y \cdot x)$ by Theorem 1093. Then $(y\backslash(y \cdot x))/(y\backslash((y \cdot x)/y)) = (y \cdot x)/((y \cdot x)/y)$ by Proposition 1. Then $x/(y\backslash((y \cdot x)/y)) = (y \cdot x)/((y \cdot x)/y)$ by Axiom 3. Hence we are done by Theorem 1011. □

Theorem 1256. $K(y\backslash x, T(y, x)) = K(y\backslash x, y)$.

Proof. We have $(x\backslash y)\backslash K(T(y, x)\backslash(T(y, x)/(x\backslash y)), T(y, x)) = T((x\backslash y)\backslash e, T(y, x))$ by Theorem 197. Then $(x\backslash y)\backslash K(T(y, x)\backslash((y \cdot x)/y), T(y, x)) = T((x\backslash y)\backslash e, T(y, x))$ by Theorem 1091. Then

$$(x\backslash y)\backslash K(T(x/y, T(y, x)), T(y, x)) = T((x\backslash y)\backslash e, T(y, x)) \tag{614}$$

by Theorem 1083. We have $(x\backslash y) \cdot ((x\backslash y)\backslash K(T(x/y, T(y, x)), T(y, x))) = K(T(x/y, T(y, x)), T(y, x))$ by Axiom 4. Then

$$(x\backslash y) \cdot T((x\backslash y)\backslash e, T(y, x)) = K(T(x/y, T(y, x)), T(y, x)) \tag{615}$$

by (614). We have $(x\backslash y) \cdot T((x\backslash y)\backslash e, T(y, x)) = x\backslash(y \cdot T(y\backslash x, T(y, x)))$ by Theorem 568. Then $K(T(x/y, T(y, x)), T(y, x)) = x\backslash(y \cdot T(y\backslash x, T(y, x)))$ by (615). Then

$$K(y\backslash x, T(y, x)) = x\backslash(y \cdot T(y\backslash x, T(y, x))) \tag{616}$$

by Theorem 1234. We have $x\backslash(y \cdot T(y\backslash x, y)) = K(y\backslash x, y)$ by Theorem 46. Then $x\backslash(y \cdot T(y\backslash x, T(y, x))) = K(y\backslash x, y)$ by Theorem 1235. Hence we are done by (616). □

Theorem 1257. $K(y, x) = K(y, T(x, x \cdot y))$.

Proof. We have $K(x\backslash(x \cdot y), T(x, x \cdot y)) = K(x\backslash(x \cdot y), x)$ by Theorem 1256. Then $K(y, T(x, x \cdot y)) = K(x\backslash(x \cdot y), x)$ by Axiom 3. Hence we are done by Axiom 3. □

Theorem 1258. $K(x\backslash e, T(y, x\backslash y)) = K(x\backslash e, y)$.

Proof. We have $K(x\backslash e, T(y, y \cdot (x\backslash e))) = K(x\backslash e, y)$ by Theorem 1257. Hence we are done by Theorem 1123. □

Theorem 1259. $K(x, T(y, x \cdot y)) = K(x, y)$.

Proof. We have $K((x\backslash e)\backslash e, T(y, (x\backslash e)\backslash y)) = K((x\backslash e)\backslash e, y)$ by Theorem 1258. Then

$$K((x\backslash e)\backslash e, T(y, x \cdot y)) = K((x\backslash e)\backslash e, y) \tag{617}$$

by Theorem 1242. We have $K((x\backslash e)\backslash e, T(y, x \cdot y)) = K(x, T(y, x \cdot y))$ by Theorem 733. Then

$$K((x\backslash e)\backslash e, y) = K(x, T(y, x \cdot y)) \tag{618}$$

by (617). We have $K((x\backslash e)\backslash e, y) = K(x, y)$ by Theorem 733. Hence we are done by (618). □

Theorem 1260. $K(x,T(x\backslash y,y)) = K(x,x\backslash y)$.

Proof. We have $K(x,T(x\backslash y,x\cdot(x\backslash y))) = K(x,x\backslash y)$ by Theorem 1259. Hence we are done by Axiom 4. □

Theorem 1261. $T(K(y,z),y\cdot K(x,y)) = K(y,z)$.

Proof. We have $T(((L(x\backslash e,x,y)\backslash e)\cdot y)/(L(x\backslash e,x,y)\backslash e),K(y,z)) = ((L(x\backslash e,x,y)\backslash e)\cdot y)/(L(x\backslash e,x,y)\backslash e)$ by Theorem 1196. Then $T(K(y,z),((L(x\backslash e,x,y)\backslash e)\cdot y)/(L(x\backslash e,x,y)\backslash e)) = K(y,z)$ by Proposition 21. Hence we are done by Theorem 1013. □

Theorem 1262. $y/(y\cdot K(x,y)) = (y\cdot K(x,y))\backslash y$.

Proof. We have $(y\cdot x)\backslash((x\cdot y)\cdot T(K(y,x),y\cdot K(x,y))) = K(x,y)\cdot T(K(x,y)\backslash e,y\cdot K(x,y))$ by Theorem 1127. Then $(y\cdot x)\backslash((x\cdot y)\cdot K(y,x)) = K(x,y)\cdot T(K(x,y)\backslash e,y\cdot K(x,y))$ by Theorem 1261. Then

$$(y\cdot x)\backslash(y\cdot x) = K(x,y)\cdot T(K(x,y)\backslash e,y\cdot K(x,y)) \tag{619}$$

by Proposition 82. We have $(y\cdot x)\backslash(y\cdot x) = e$ by Proposition 28. Then

$$K(x,y)\cdot T(K(x,y)\backslash e,y\cdot K(x,y)) = e \tag{620}$$

by (619). We have $K((y\cdot K(x,y))\backslash y,y\cdot K(x,y)) = K(x,y)\cdot T(K(x,y)\backslash e,y\cdot K(x,y))$ by Theorem 184. Then

$$K((y\cdot K(x,y))\backslash y,y\cdot K(x,y)) = e \tag{621}$$

by (620). We have $T(y,y\cdot K(x,y))\cdot K((y\cdot K(x,y))\backslash y,(y\cdot(y\cdot K(x,y)))/y) = ((y\cdot K(x,y))\backslash y)\cdot((y\cdot(y\cdot K(x,y)))/y)$ by Theorem 1092. Then $T(y,y\cdot K(x,y))\cdot K((y\cdot K(x,y))\backslash y,y\cdot K(x,y)) = ((y\cdot K(x,y))\backslash y)\cdot((y\cdot(y\cdot K(x,y)))/y)$ by Theorem 1098. Then $y\cdot K((y\cdot K(x,y))\backslash y,y\cdot K(x,y)) = ((y\cdot K(x,y))\backslash y)\cdot((y\cdot(y\cdot K(x,y)))/y)$ by Theorem 541. Then

$$((y\cdot K(x,y))\backslash y)\cdot((y\cdot(y\cdot K(x,y)))/y) = y\cdot e \tag{622}$$

by (621). We have $(((y\cdot K(x,y))\backslash y)\cdot((y\cdot(y\cdot K(x,y)))/y))\cdot e = ((y\cdot K(x,y))\backslash y)\cdot((y\cdot(y\cdot K(x,y)))/y)$ by Axiom 2. Then $y\cdot e = (((y\cdot K(x,y))\backslash y)\cdot((y\cdot(y\cdot K(x,y)))/y))\cdot e$ by (622). Then $y = ((y\cdot K(x,y))\backslash y)\cdot((y\cdot(y\cdot K(x,y)))/y)$ by Proposition 10. Then $((y\cdot K(x,y))\backslash y)\cdot(y\cdot K(x,y)) = y$ by Theorem 1098. Hence we are done by Proposition 1. □

Theorem 1263. $(K(x,y)\backslash e)\backslash y = y\cdot K(x,y)$.

Proof. We have $(y\cdot K(x,y))\backslash R(y,K(x,y),K(x,y)\backslash e) = K(x,y)\backslash e$ by Theorem 1062. Then $(y\cdot K(x,y))\backslash y = K(x,y)\backslash e$ by Theorem 1047. Then

$$y/(y\cdot K(x,y)) = K(x,y)\backslash e \tag{623}$$

by Theorem 1262. We have $(y/(y\cdot K(x,y)))\backslash y = y\cdot K(x,y)$ by Proposition 25. Hence we are done by (623). □

Theorem 1264. $(x\backslash((y\backslash e)\backslash e))\cdot K(y\backslash e,y) = ((x\backslash y)/y)\cdot y$.

Proof. We have $(K(y,y\backslash e)\cdot y)/L((y/x)\backslash y,y/x,K(y,y\backslash e)) = K(y,y\backslash e)\cdot(y/x)$ by Theorem 61. Then $((y\backslash e)\backslash e)/L((y/x)\backslash y,y/x,K(y,y\backslash e)) = K(y,y\backslash e)\cdot(y/x)$ by Theorem 552. Then

$$((y\backslash e)\backslash e)/((y/x)\backslash y) = K(y,y\backslash e)\cdot(y/x) \tag{624}$$

by Theorem 1221. We have $K(y,y\backslash e)\cdot(y/x) = (y/x)\cdot K(y,y\backslash e)$ by Theorem 1183. Then $((y\backslash e)\backslash e)/((y/x)\backslash y) = (y/x)\cdot K(y,y\backslash e)$ by (624). Then

$$((y\backslash e)\backslash e)/x = (y/x)\cdot K(y,y\backslash e) \tag{625}$$

by Proposition 25. We have $T(y \cdot ((y/x)/y), x) = x\backslash((y \cdot ((y/x)/y)) \cdot x)$ by Definition 3. Then $T(y \cdot ((y/x)/y), x) = x\backslash T(y, y/(y \cdot x))$ by Theorem 881. Then

$$x\backslash T(y, x\backslash e) = T(y \cdot ((y/x)/y), x) \tag{626}$$

by Theorem 1003. We have $T(((y/x)/y) \cdot ((y\backslash e)\backslash e), x) \cdot K(y\backslash e, y) = (y\backslash T(y \cdot ((y/x)/y), x)) \cdot y$ by Theorem 1149. Then $T(((y/x)/y) \cdot ((y\backslash e)\backslash e), x) \cdot K(y\backslash e, y) = (y\backslash(x\backslash T(y, x\backslash e))) \cdot y$ by (626). Then $T((y/x) \cdot K(y, y\backslash e), x) \cdot K(y\backslash e, y) = (y\backslash(x\backslash T(y, x\backslash e))) \cdot y$ by Theorem 1143. Then $T(((y\backslash e)\backslash e)/x, x) \cdot K(y\backslash e, y) = (y\backslash(x\backslash T(y, x\backslash e))) \cdot y$ by (625). Then $(x\backslash((y\backslash e)\backslash e)) \cdot K(y\backslash e, y) = (y\backslash(x\backslash T(y, x\backslash e))) \cdot y$ by Proposition 47. Hence we are done by Theorem 1249. ☐

Theorem 1265. $(x\backslash((y\backslash e)\backslash e)) \cdot z = K(y, y\backslash e) \cdot ((x\backslash y) \cdot z)$.

Proof. We have $(((x\backslash y) \cdot K(y, y\backslash e))/K(y, y\backslash e)) \cdot K(y, y\backslash e) = (x\backslash y) \cdot K(y, y\backslash e)$ by Axiom 6. Then $(((((x\backslash y) \cdot K(y, y\backslash e))/K(y, y\backslash e))/y) \cdot y) \cdot K(y, y\backslash e) = (x\backslash y) \cdot K(y, y\backslash e)$ by Axiom 6. Then $(((x\backslash y)/y) \cdot y) \cdot K(y, y\backslash e) = (x\backslash y) \cdot K(y, y\backslash e)$ by Axiom 5. Then $((x\backslash y) \cdot K(y, y\backslash e))/K(y, y\backslash e) = ((x\backslash y)/y) \cdot y$ by Proposition 1. Then

$$((x\backslash y) \cdot K(y, y\backslash e)) \cdot K(y\backslash e, y) = ((x\backslash y)/y) \cdot y \tag{627}$$

by Theorem 425. We have $(x\backslash((y\backslash e)\backslash e)) \cdot K(y\backslash e, y) = ((x\backslash y)/y) \cdot y$ by Theorem 1264. Then

$$x\backslash((y\backslash e)\backslash e) = (x\backslash y) \cdot K(y, y\backslash e) \tag{628}$$

by (627) and Proposition 8. We have $((x\backslash y) \cdot K(y, y\backslash e)) \cdot z = K(y, y\backslash e) \cdot ((x\backslash y) \cdot z)$ by Theorem 1231. Hence we are done by (628). ☐

Theorem 1266. $(e/(e/y))/T(x, x\backslash y) = x\backslash(e/(e/y))$.

Proof. We have $x \cdot (x\backslash T(e/(e/y), (e/(e/y))\backslash e)) = ((e/(e/y))\backslash e)\backslash e$ by Theorem 795. Then

$$x\backslash(((e/(e/y))\backslash e)\backslash e) = x\backslash T(e/(e/y), (e/(e/y))\backslash e) \tag{629}$$

by Proposition 2. We have $(((e/(e/y))\backslash e)\backslash e)/T(x, x\backslash(((e/(e/y))\backslash e)\backslash e)) = x\backslash(((e/(e/y))\backslash e)\backslash e)$ by Theorem 15. Then

$$(((e/(e/y))\backslash e)\backslash e)/T(x, x\backslash T(e/(e/y), (e/(e/y))\backslash e)) = x\backslash(((e/(e/y))\backslash e)\backslash e) \tag{630}$$

by (629). We have $((((e/(e/y))\backslash e)\backslash e)/T(x, x\backslash T(e/(e/y), (e/(e/y))\backslash e))) \cdot K((e/(e/y))\backslash e, ((e/(e/y))\backslash e)\backslash e) = (e/((e/(e/y))\backslash e))/T(x, x\backslash T(e/(e/y), (e/(e/y))\backslash e))$ by Theorem 1229. Then $(x\backslash(((e/(e/y))\backslash e)\backslash e)) \cdot K((e/(e/y))\backslash e, ((e/(e/y))\backslash e)\backslash e) = (e/((e/(e/y))\backslash e))/T(x, x\backslash T(e/(e/y), (e/(e/y))\backslash e))$ by (630). Then $(e/((e/(e/y))\backslash e))/T(x, x\backslash T(e/(e/y), (e/(e/y))\backslash e)) = (x\backslash(((e/(e/y))\backslash e)\backslash e)) \cdot K((e/(e/y))\backslash e, e/(e/y))$ by Theorem 779. Then

$$(e/(e/y))/T(x, x\backslash T(e/(e/y), (e/(e/y))\backslash e)) = (x\backslash(((e/(e/y))\backslash e)\backslash e)) \cdot K((e/(e/y))\backslash e, e/(e/y)) \tag{631}$$

by Proposition 24. We have $((x\backslash(e/(e/y)))/(e/(e/y))) \cdot (e/(e/y)) = x\backslash(e/(e/y))$ by Axiom 6. Then $(x\backslash(((e/(e/y))\backslash e)\backslash e)) \cdot K((e/(e/y))\backslash e, e/(e/y)) = x\backslash(e/(e/y))$ by Theorem 1264. Then $(e/(e/y))/T(x, x\backslash T(e/(e/y), (e/(e/y))\backslash e))$ $x\backslash(e/(e/y))$ by (631). Then $(e/(e/y))/T(x, x\backslash T(e/(e/y), e/y)) = x\backslash(e/(e/y))$ by Proposition 25. Hence we are done by Theorem 14. ☐

Theorem 1267. $((y\backslash(e/x)) \cdot y) \cdot x = x \cdot ((y\backslash(x\backslash e)) \cdot y)$.

Proof. We have $((y\backslash(y/(x \cdot y))) \cdot y) \cdot (x \cdot y) = y \cdot ((x \cdot y) \cdot T((x \cdot y)\backslash e, y))$ by Theorem 840. Then $((y\backslash R(e/x, x, y)) \cdot y) \cdot (x \cdot y) = y \cdot ((x \cdot y) \cdot T((x \cdot y)\backslash e, y))$ by Proposition 79. Then

$$R((y\backslash(e/x)) \cdot y, x, y) \cdot (x \cdot y) = y \cdot ((x \cdot y) \cdot T((x \cdot y)\backslash e, y)) \tag{632}$$

by Theorem 973. We have $R((y\backslash(e/x)) \cdot y, x, y) \cdot (x \cdot y) = (((y\backslash(e/x)) \cdot y) \cdot x) \cdot y$ by Proposition 54. Then $y \cdot ((x \cdot y) \cdot T((x \cdot y)\backslash e, y)) = (((y\backslash(e/x)) \cdot y) \cdot x) \cdot y$ by (632). Then

$$y \cdot (x \cdot (y \cdot T(y\backslash(x\backslash e), y))) = (((y\backslash(e/x)) \cdot y) \cdot x) \cdot y \tag{633}$$

by Theorem 136. We have $((e/(y\cdot x))\backslash T(e/(y\cdot x),y))\cdot y = y\cdot((e/(y\cdot x))\backslash T(e/(y\cdot x),y))$ by Theorem 937. Then $((e/(y\cdot x))\backslash T(e/(y\cdot x),y))\cdot y = ((e/(y\cdot x))\cdot y)/(e/(y\cdot x))$ by Theorem 977. Then $((e/(y\cdot x))\backslash T(e/(y\cdot x),y))\cdot y = (a((y\cdot x)\backslash e,y,x)/x)/(e/(y\cdot x))$ by Theorem 1232. Then

$$(a((y\cdot x)\backslash e,y,x)/x)/(e/(y\cdot x)) = ((y\cdot x)\cdot T((y\cdot x)\backslash e,y))\cdot y \tag{634}$$

by Theorem 102.

$$\begin{aligned}
& (a((y\cdot x)\backslash e,y,x)/x)/(e/(y\cdot x)) \\
= \quad & (((y\backslash(e/x))\cdot y)\cdot x)\cdot y && \text{by (634), Theorem 530} \\
= \quad & y\cdot(x\cdot((y\backslash(x\backslash e))\cdot y)) && \text{by (633), Proposition 46.}
\end{aligned}$$

Then

$$(a((y\cdot x)\backslash e,y,x)/x)/(e/(y\cdot x)) = y\cdot(x\cdot((y\backslash(x\backslash e))\cdot y)). \tag{635}$$

We have $(((y\cdot x)\backslash e)\cdot y)/((y\cdot x)\backslash e) = y\cdot((y\cdot x)\cdot T((y\cdot x)\backslash e,y))$ by Theorem 982. Then

$$(((y\cdot x)\backslash e)\cdot y)/((y\cdot x)\backslash e) = y\cdot(((y\backslash(e/x))\cdot y)\cdot x) \tag{636}$$

by Theorem 530. We have $((e/(y\cdot x))\cdot y)/(e/(y\cdot x)) = (((y\cdot x)\backslash e)\cdot y)/((y\cdot x)\backslash e)$ by Theorem 1184. Then $((e/(y\cdot x))\cdot y)/(e/(y\cdot x)) = y\cdot(((y\backslash(e/x))\cdot y)\cdot x)$ by (636). Then $(a((y\cdot x)\backslash e,y,x)/x)/(e/(y\cdot x)) = y\cdot(((y\backslash(e/x))\cdot y)\cdot x)$ by Theorem 1232. Then $y\cdot(((y\backslash(e/x))\cdot y)\cdot x) = y\cdot(x\cdot((y\backslash(x\backslash e))\cdot y))$ by (635). Hence we are done by Proposition 9. □

Theorem 1268. $((y\backslash x)\cdot y)\cdot(x\backslash e) = (e/x)\cdot((y\backslash x)\cdot y)$.

Proof. We have $((y\backslash(e/(x\backslash e)))\cdot y)\cdot(x\backslash e) = (x\backslash e)\cdot((y\backslash((x\backslash e)\backslash e))\cdot y)$ by Theorem 1267. Then

$$((y\backslash x)\cdot y)\cdot(x\backslash e) = (x\backslash e)\cdot((y\backslash((x\backslash e)\backslash e))\cdot y) \tag{637}$$

by Proposition 24. We have $(x\backslash e)\cdot(K(x,x\backslash e)\cdot((y\backslash x)\cdot y)) = (e/x)\cdot((y\backslash x)\cdot y)$ by Theorem 1167. Then $(x\backslash e)\cdot((y\backslash((x\backslash e)\backslash e))\cdot y) = (e/x)\cdot((y\backslash x)\cdot y)$ by Theorem 1265. Hence we are done by (637). □

Theorem 1269. $L(z,y\backslash T(y,x),y) = z$.

Proof. We have

$$y\cdot((y\backslash T(y,x))\cdot z) = T(y,x)\cdot z \tag{638}$$

by Theorem 1017. We have $T(y,x)\cdot L(z,y\backslash T(y,x),y) = y\cdot((y\backslash T(y,x))\cdot z)$ by Theorem 52. Then $T(y,x)\cdot L(z,y\backslash T(y,x),y) = T(y,x)\cdot z$ by (638). Hence we are done by Proposition 9. □

Theorem 1270. $(y\backslash T(y,x))\cdot z = T(y,x)\cdot(y\backslash z)$.

Proof. We have

$$y\cdot((y\backslash T(y,x))\cdot z) = T(y,x)\cdot z \tag{639}$$

by Theorem 1017. We have $y\cdot(T(y,x)\cdot(y\backslash z)) = T(y,x)\cdot z$ by Theorem 830. Hence we are done by (639) and Proposition 7. □

Theorem 1271. $L(z,T(x,T(y,x))\backslash x,y) = z$.

Proof. We have $L(z,y\backslash T(y,x),y) = z$ by Theorem 1269. Hence we are done by Theorem 986. □

Theorem 1272. $(x\backslash T(x,y))\backslash z = x\cdot(T(x,y)\backslash z)$.

Proof. We have $L((x\backslash T(x,y))\backslash z,x\backslash T(x,y),x) = T(x,y)\backslash(x\cdot z)$ by Theorem 60. Then

$$L((x\backslash T(x,y))\backslash z,T(y,T(x,y))\backslash y,x) = T(x,y)\backslash(x\cdot z) \tag{640}$$

by Theorem 986. We have $T(x,y)\backslash(x\cdot z) = x\cdot(T(x,y)\backslash z)$ by Theorem 174. Then $L((x\backslash T(x,y))\backslash z,T(y,T(x,y))\backslash y,x) = x\cdot(T(x,y)\backslash z)$ by (640). Hence we are done by Theorem 1271. □

Theorem 1273. $K(y,x) = T(T(x,y)\backslash x, x \cdot y)$.

Proof. We have
$$T(T(x,y)\backslash x, x \cdot y) = (x \cdot y)\backslash((T(x,y)\backslash x) \cdot (x \cdot y)) \quad (641)$$
by Definition 3.

$$\begin{aligned} & K(y,x) \\ = \quad & (x \cdot y)\backslash(y \cdot x) && \text{by Definition 2} \\ = \quad & T(T(x,y)\backslash x, x \cdot y) && \text{by (641), Theorem 1022.} \end{aligned}$$

Hence we are done. □

Theorem 1274. $T(x,y) \cdot K(y,x) = y\backslash(K(y,x) \cdot (x \cdot y))$.

Proof. We have $(y\backslash(x \cdot y)) \cdot T((y\backslash(x \cdot y))\backslash x, x \cdot y) = y\backslash(((x \cdot y)\backslash(y \cdot x)) \cdot (x \cdot y))$ by Theorem 1085. Then $(y\backslash(x \cdot y)) \cdot T((y\backslash(x \cdot y))\backslash x, x \cdot y) = y\backslash(K(y,x) \cdot (x \cdot y))$ by Definition 2. Then $T(x,y) \cdot T((y\backslash(x \cdot y))\backslash x, x \cdot y) = y\backslash(K(y,x) \cdot (x \cdot y))$ by Definition 3. Then $T(x,y) \cdot T(T(x,y)\backslash x, x \cdot y) = y\backslash(K(y,x) \cdot (x \cdot y))$ by Definition 3. Hence we are done by Theorem 1273. □

Theorem 1275. $(T(x,y)\backslash x) \cdot z = T(y,x) \cdot (y\backslash z)$.

Proof. We have $(y\backslash T(y,x)) \cdot z = T(y,x) \cdot (y\backslash z)$ by Theorem 1270. Hence we are done by Theorem 1030. □

Theorem 1276. $T(x\backslash y, x) = T(x, x\backslash y)\backslash y$.

Proof. We have $T(x\backslash y, T(x, x\backslash y)) = T(x, x\backslash y)\backslash y$ by Theorem 463. Hence we are done by Theorem 1032. □

Theorem 1277. $y = L(R(y,x,y), x, y)$.

Proof. We have $y \cdot (x\backslash T(x,y)) = (x \cdot y)/x$ by Theorem 977. Then
$$y \cdot (T(y, T(x,y))\backslash y) = (x \cdot y)/x \quad (642)$$
by Theorem 986. We have $T((x \cdot y)/x, T(x,y)) = L(R(y,x,y), x, y)$ by Theorem 1254. Then
$$T(y \cdot (T(y, T(x,y))\backslash y), T(x,y)) = L(R(y,x,y), x, y) \quad (643)$$
by (642). We have $T(y \cdot (T(y, T(T(x,y),y))\backslash y), T(x,y)) = y$ by Theorem 1233. Then $T(y \cdot (T(y, T(x,y))\backslash y), T(x,y)) = y$ by Theorem 1032. Hence we are done by (643). □

Theorem 1278. $T(x\backslash T(x \cdot y, y), T(x \cdot y, y)) = y$.

Proof. We have $T(((x \cdot y) \cdot y)/(x \cdot y), T(x \cdot y, y)) = L(R(y, x \cdot y, y), x \cdot y, y)$ by Theorem 1254. Then $T(x\backslash T(x \cdot y, y), T(x \cdot y, y)) = L(R(y, x \cdot y, y), x \cdot y, y)$ by Theorem 1246. Hence we are done by Theorem 1277. □

Theorem 1279. $((x \cdot y)/x)\backslash x = y\backslash((y \cdot x)/y)$.

Proof. We have $y \cdot (x\backslash T(x,y)) = (x \cdot y)/x$ by Theorem 977. Then
$$y \cdot (T(y,x)\backslash y) = (x \cdot y)/x \quad (644)$$
by Theorem 1030. We have $(x/(y\backslash((y \cdot x)/y)))\backslash x = y\backslash((y \cdot x)/y)$ by Proposition 25. Then $(y \cdot (T(y,x)\backslash y))\backslash x = y\backslash((y \cdot x)/y)$ by Theorem 1255. Hence we are done by (644). □

Theorem 1280. $K(x,y) \cdot (y \cdot x) = y \cdot (x \cdot K(x,y))$.

Proof. We have $y \cdot (x \cdot T(x\backslash T(x,y), y \cdot x)) = (y \cdot x) \cdot T(K(x,y), y \cdot x)$ by Theorem 1113. Then $y \cdot (x \cdot T(T(y,T(x,y))\backslash y, y \cdot x)) = (y \cdot x) \cdot T(K(x,y), y \cdot x)$ by Theorem 986. Then $L(T(T(y,T(x,y))\backslash y, y \cdot x), x, y) = T(K(x,y), y \cdot x)$ by Theorem 54. Then $L(T(T(y,x)\backslash y, y \cdot x), x, y) = T(K(x,y), y \cdot x)$ by Theorem 1032. Then

$$L(K(x,y), x, y) = T(K(x,y), y \cdot x) \tag{645}$$

by Theorem 1273. We have $(y \cdot x) \cdot T(K(x,y), y \cdot x) = K(x,y) \cdot (y \cdot x)$ by Proposition 46. Then

$$(y \cdot x) \cdot L(K(x,y), x, y) = K(x,y) \cdot (y \cdot x) \tag{646}$$

by (645). We have $(y \cdot x) \cdot L(K(x,y), x, y) = y \cdot (x \cdot K(x,y))$ by Proposition 52. Hence we are done by (646). □

Theorem 1281. *$(y\backslash T(y,x))\backslash z = (T(y,x)\backslash y) \cdot z$.*

Proof. We have $((y \cdot x)/y) \cdot (x \cdot (x\backslash((((y \cdot x)/y)\backslash x) \cdot z))) = x \cdot (((y \cdot x)/y) \cdot (x\backslash((((y \cdot x)/y)\backslash x) \cdot z)))$ by Theorem 854. Then

$$((y \cdot x)/y) \cdot ((((y \cdot x)/y)\backslash x) \cdot z) = x \cdot (((y \cdot x)/y) \cdot (x\backslash((((y \cdot x)/y)\backslash x) \cdot z))) \tag{647}$$

by Axiom 4. We have $x \cdot L(z, ((y \cdot x)/y)\backslash x, (y \cdot x)/y) = ((y \cdot x)/y) \cdot ((((y \cdot x)/y)\backslash x) \cdot z)$ by Theorem 52. Then $((y \cdot x)/y) \cdot (x\backslash((((y \cdot x)/y)\backslash x) \cdot z)) = L(z, ((y \cdot x)/y)\backslash x, (y \cdot x)/y)$ by (647) and Proposition 7. Then $((y \cdot x)/y) \cdot (x\backslash((T(y,x)\backslash y) \cdot z)) = L(z, ((y \cdot x)/y)\backslash x, (y \cdot x)/y)$ by Theorem 984. Then $((y \cdot x)/y) \cdot (x\backslash((T(y,x)\backslash y) \cdot z)) = L(z, T(y,x)\backslash y, (y \cdot x)/y)$ by Theorem 984. Then

$$((y \cdot x)/y)\backslash L(z, T(y,x)\backslash y, (y \cdot x)/y) = x\backslash((T(y,x)\backslash y) \cdot z) \tag{648}$$

by Proposition 2. We have $L(z, T(y, T((y \cdot x)/y, y))\backslash y, (y \cdot x)/y) = z$ by Theorem 1271. Then $L(z, T(y,x)\backslash y, (y \cdot x)/y) = z$ by Theorem 7. Then

$$((y \cdot x)/y)\backslash z = x\backslash((T(y,x)\backslash y) \cdot z) \tag{649}$$

by (648). We have $x \cdot (x\backslash((T(y,x)\backslash y) \cdot z)) = (T(y,x)\backslash y) \cdot z$ by Axiom 4. Then

$$x \cdot (((y \cdot x)/y)\backslash z) = (T(y,x)\backslash y) \cdot z \tag{650}$$

by (649). We have $R(((y \cdot x)/y)/x, x, ((y \cdot x)/y)\backslash z)\backslash z = x \cdot (((y \cdot x)/y)\backslash z)$ by Theorem 1065. Then $R(y\backslash T(y,x), x, ((y \cdot x)/y)\backslash z)\backslash z = x \cdot (((y \cdot x)/y)\backslash z)$ by Theorem 981. Then $R(y\backslash T(y,x), x, ((y \cdot x)/y)\backslash z)\backslash z = (T(y,x)\backslash y) \cdot z$ by (650). Hence we are done by Theorem 1041. □

Theorem 1282. *$(T(y,x)\backslash y) \cdot (T(y,x) \cdot z) = y \cdot z$.*

Proof. We have $(y\backslash T(y,x)) \cdot (y \cdot z) = T(y,x) \cdot z$ by Theorem 1018. Then $(y\backslash T(y,x))\backslash(T(y,x) \cdot z) = y \cdot z$ by Proposition 2. Hence we are done by Theorem 1281. □

Theorem 1283. *$K(y,x)\backslash x = x \cdot K(x,y)$.*

Proof. We have $K(T(y,x), x)\backslash x = x \cdot K(x,y)$ by Theorem 1038. Hence we are done by Theorem 1053. □

Theorem 1284. *$T(x \cdot y, K(y,x)) = y \cdot (x \cdot K(x,y))$.*

Proof. We have $(T(y, R(x, y, (x \cdot y)\backslash y))\backslash y) \cdot y = y \cdot (T(y, R(x, y, (x \cdot y)\backslash y))\backslash y)$ by Theorem 931. Then $T(y, T(y, R(x, y, (x \cdot y)\backslash y))\backslash y) = y$ by Theorem 11. Then $T(y, K(y,x)\backslash e) = y$ by Theorem 949. Then

$$T(K(y,x)\backslash e, y) = K(y,x)\backslash e \tag{651}$$

by Proposition 21. We have $(y\backslash(K(y,x)\backslash y)) \cdot y = (K(y,x) \cdot T(K(y,x)\backslash e, y)) \cdot (K(y,x)\backslash y)$ by Theorem 1099. Then

$$(y\backslash(K(y,x)\backslash y)) \cdot y = (K(y,x) \cdot (K(y,x)\backslash e)) \cdot (K(y,x)\backslash y) \tag{652}$$

by (651). We have $K(y,x) \cdot (K(y,x)\backslash e) = e$ by Axiom 4. Then

$$e \cdot (K(y,x)\backslash y) = (y\backslash(K(y,x)\backslash y)) \cdot y \tag{653}$$

by (652). We have $e \cdot ((y\backslash(K(y,x)\backslash y)) \cdot y) = (y\backslash(K(y,x)\backslash y)) \cdot y$ by Axiom 1. Then $e \cdot (K(y,x)\backslash y) = e \cdot ((y\backslash(K(y,x)\backslash y)) \cdot y)$ by (653). Then

$$K(y,x)\backslash y = (y\backslash(K(y,x)\backslash y)) \cdot y \tag{654}$$

by Proposition 9. We have $(((y\backslash(K(y,x)\backslash y)) \cdot y) \cdot x)/a(y\backslash(K(y,x)\backslash y), y, x) = (y\backslash(K(y,x)\backslash y)) \cdot (y \cdot x)$ by Theorem 1056. Then $((K(y,x)\backslash y) \cdot x)/a(y\backslash(K(y,x)\backslash y), y, x) = (y\backslash(K(y,x)\backslash y)) \cdot (y \cdot x)$ by (654). Then

$$((K(y,x)\backslash y) \cdot x)/a(K(y,x)\backslash e, y, x) = (y\backslash(K(y,x)\backslash y)) \cdot (y \cdot x) \tag{655}$$

by Theorem 957. We have $L(x,y,K(T(x,y),y)) = x$ by Theorem 1044. Then $a(K(T(x,y),y),y,x) = e$ by Proposition 20. Then $a(K(y,x)\backslash e, y, x) = e$ by Theorem 1037. Then

$$((K(y,x)\backslash y) \cdot x)/e = (y\backslash(K(y,x)\backslash y)) \cdot (y \cdot x) \tag{656}$$

by (655). We have $((K(y,x)\backslash y) \cdot x)/e = (K(y,x)\backslash y) \cdot x$ by Proposition 27. Then $(y\backslash(K(y,x)\backslash y)) \cdot (y \cdot x) = (K(y,x)\backslash y) \cdot x$ by (656). Then

$$(K(y,x)\backslash e) \cdot (y \cdot x) = (K(y,x)\backslash y) \cdot x \tag{657}$$

by Theorem 957. We have $(K(y,x)\backslash e) \cdot (K(y,x) \cdot (K(y,x)\backslash(y \cdot x))) = K(y,x)\backslash(y \cdot x)$ by Theorem 1051. Then $(K(y,x)\backslash e) \cdot (y \cdot x) = K(y,x)\backslash(y \cdot x)$ by Axiom 4. Then

$$K(y,x)\backslash(y \cdot x) = (K(y,x)\backslash y) \cdot x \tag{658}$$

by (657). We have $(K(T(x,y),y)\backslash e)\backslash y = y \cdot K(T(x,y),y)$ by Theorem 1263. Then

$$K(y,x)\backslash y = y \cdot K(T(x,y),y) \tag{659}$$

by Theorem 1036. We have $(y \cdot K(T(x,y),y)) \cdot x = y \cdot (K(T(x,y),y) \cdot x)$ by Theorem 1046. Then $(K(y,x)\backslash y) \cdot x = y \cdot (K(T(x,y),y) \cdot x)$ by (659). Then $y \cdot (K(y,x)\backslash x) = (K(y,x)\backslash y) \cdot x$ by Theorem 1052. Then

$$K(y,x)\backslash(y \cdot x) = y \cdot (K(y,x)\backslash x) \tag{660}$$

by (658). We have $K(y,x)\backslash(y \cdot x) = T(x \cdot y, K(y,x))$ by Theorem 1059. Then $y \cdot (K(y,x)\backslash x) = T(x \cdot y, K(y,x))$ by (660). Hence we are done by Theorem 1283. □

Theorem 1285. *$T(x \cdot y, x)/R(x,y,x) = y$.*

Proof. We have $(x \cdot y) \cdot R(T(x, x \cdot y), y, x) = R(x,y,x) \cdot (x \cdot y)$ by Proposition 59. Then

$$(x \cdot y) \cdot T(x, T(y,x)) = R(x,y,x) \cdot (x \cdot y) \tag{661}$$

by Theorem 1252. We have $R(x,y,x) \cdot (x \cdot y) = x \cdot (R(x,y,x) \cdot y)$ by Theorem 227. Then

$$(x \cdot y) \cdot T(x, T(y,x)) = x \cdot (R(x,y,x) \cdot y) \tag{662}$$

by (661). We have $T(x \cdot y, T(x,y)) = T(x \cdot y, x)$ by Theorem 1244. Then

$$T(x \cdot y, T(x, T(y,x))) = T(x \cdot y, x) \tag{663}$$

by Theorem 1032. We have $T(x \cdot y, T(x, T(y,x)))/(((x \cdot y) \cdot T(x, T(y,x)))/(x \cdot y)) = (x \cdot y)/T(x, T(y,x))$ by Theorem 1239. Then $T(x \cdot y, x)/(((x \cdot y) \cdot T(x, T(y,x)))/(x \cdot y)) = (x \cdot y)/T(x, T(y,x))$ by (663). Then

$$T(x \cdot y, x)/((x \cdot (R(x,y,x) \cdot y))/(x \cdot y)) = (x \cdot y)/T(x, T(y,x)) \tag{664}$$

by (662). We have $R((x\cdot(x\cdot y))/(x\cdot y),y,x)\cdot(x\cdot y) = (x\cdot y)\cdot R((x\cdot y)\backslash(x\cdot(x\cdot y)),y,x)$ by Proposition 83. Then $R((x\cdot(x\cdot y))/(x\cdot y),y,x)\cdot(x\cdot y) = x\cdot(y\cdot R(y\backslash(x\cdot y),y,x))$ by Theorem 516. Then $R((x\cdot(x\cdot y))/(x\cdot y),y,x)\cdot(x\cdot y) = x\cdot(R((x\cdot y)/y,y,x)\cdot y)$ by Proposition 83. Then $R((x\cdot(x\cdot y))/(x\cdot y),y,x)\cdot(x\cdot y) = x\cdot(R(x,y,x)\cdot y)$ by Axiom 5. Then $(x\cdot(R(x,y,x)\cdot y))/(x\cdot y) = R((x\cdot(x\cdot y))/(x\cdot y),y,x)$ by Proposition 1. Then $T(x\cdot y,x)/R((x\cdot(x\cdot y))/(x\cdot y),y,x) = (x\cdot y)/T(x,T(y,x))$ by (664). Then

$$T(x\cdot y,x)/R(x,y,x) = (x\cdot y)/T(x,T(y,x)) \quad (665)$$

by Axiom 5. We have $T((T(y,x)\cdot T(x,y))\backslash((T(y,x)\cdot x)\cdot T(x,y)),T(y,x)) = R(T(x,T(y,x)\cdot T(x,y)),T(y,x),T(x,y))$ by Theorem 519. Then $T(x,T(y,x)) = R(T(x,T(y,x)\cdot T(x,y)),T(y,x),T(x,y))$ by Theorem 1105. Then $(T(y,x)\cdot T(x,y))\backslash((y\cdot x)\cdot T(x,y)) = T(x,T(y,x))$ by Theorem 1084. Then

$$(T(y,x)\cdot T(x,y))\backslash((x\cdot y)\cdot x) = T(x,T(y,x)) \quad (666)$$

by Theorem 114. We have $((x\cdot y)\cdot x)/((T(y,x)\cdot T(x,y))\backslash((x\cdot y)\cdot x)) = T(y,x)\cdot T(x,y)$ by Proposition 24. Then

$$((x\cdot y)\cdot x)/T(x,T(y,x)) = T(y,x)\cdot T(x,y) \quad (667)$$

by (666). We have $((x\cdot y)\cdot x)/T(x,T(y,x)) = ((x\cdot y)/T(x,T(y,x)))\cdot x$ by Theorem 239. Then

$$T(y,x)\cdot T(x,y) = ((x\cdot y)/T(x,T(y,x)))\cdot x \quad (668)$$

by (667). We have $(T(y,x)\cdot(x\backslash T(x,y)))\cdot x = T(y,x)\cdot T(x,y)$ by Theorem 1015. Then $T(y,x)\cdot(x\backslash T(x,y)) = (x\cdot y)/T(x,T(y,x))$ by (668) and Proposition 8. Then

$$T(x\cdot y,x)/R(x,y,x) = T(y,x)\cdot(x\backslash T(x,y)) \quad (669)$$

by (665). We have $T(y,x)\cdot(x\backslash T(x,y)) = y$ by Theorem 1031. Hence we are done by (669). ☐

Theorem 1286. *$x\cdot R(y,x,y) = T(y\cdot x,y)$.*

Proof. We have $(T(y\cdot x,y)/R(y,x,y))\cdot R(y,x,y) = T(y\cdot x,y)$ by Axiom 6. Hence we are done by Theorem 1285. ☐

Theorem 1287. *$K(y\backslash T(x,y),y) = K(y\backslash x,y)$.*

Proof. We have $((((y\backslash x)\cdot y)\cdot y)/((y\backslash x)\cdot y))\cdot(y\backslash x) = (y\backslash x)\cdot R(y,y\backslash x,y)$ by Theorem 482. Then

$$((y\backslash x)\backslash T((y\backslash x)\cdot y,y))\cdot(y\backslash x) = (y\backslash x)\cdot R(y,y\backslash x,y) \quad (670)$$

by Theorem 1246. We have $(((y\backslash x)\backslash T((y\backslash x)\cdot y,y))\cdot(y\backslash x))\backslash T((y\backslash x)\cdot y,y) = K(y\backslash x,(y\backslash x)\backslash T((y\backslash x)\cdot y,y))$ by Theorem 1055. Then

$$((y\backslash x)\cdot R(y,y\backslash x,y))\backslash T((y\backslash x)\cdot y,y) = K(y\backslash x,(y\backslash x)\backslash T((y\backslash x)\cdot y,y)) \quad (671)$$

by (670). We have $K(y\backslash x,T((y\backslash x)\backslash T((y\backslash x)\cdot y,y),T((y\backslash x)\cdot y,y))) = K(y\backslash x,(y\backslash x)\backslash T((y\backslash x)\cdot y,y))$ by Theorem 1260. Then $K(y\backslash x,y) = K(y\backslash x,(y\backslash x)\backslash T((y\backslash x)\cdot y,y))$ by Theorem 1278. Then $((y\backslash x)\cdot R(y,y\backslash x,y))\backslash T((y\backslash x)\cdot y,y) = K(y\backslash x,y)$ by (671). Then

$$K(y\backslash x,y) = T(y\cdot(y\backslash x),y)\backslash T((y\backslash x)\cdot y,y) \quad (672)$$

by Theorem 1286. We have $T(y\cdot(y\backslash x),y)\backslash((y\backslash T(y\cdot(y\backslash x),y))\cdot y) = K(y\backslash T(y\cdot(y\backslash x),y),y)$ by Theorem 2. Then $T(y\cdot(y\backslash x),y)\backslash T((y\backslash(y\cdot(y\backslash x)))\cdot y,y) = K(y\backslash T(y\cdot(y\backslash x),y),y)$ by Theorem 310. Then $T(y\cdot(y\backslash x),y)\backslash T((y\backslash x)\cdot y,y) = K(y\backslash T(y\cdot(y\backslash x),y),y)$ by Axiom 3. Then $K(y\backslash x,y) = K(y\backslash T(y\cdot(y\backslash x),y),y)$ by (672). Hence we are done by Axiom 4. ☐

Theorem 1288. *$K(((x\cdot y)/x)\backslash x,y) = K(y\backslash x,y)$.*

Proof. We have $K(y\backslash T((y\cdot x)/y,y),y) = K(y\backslash((y\cdot x)/y),y)$ by Theorem 1287. Then $K(y\backslash x,y) = K(y\backslash((y\cdot x)/y),y)$ by Theorem 7. Hence we are done by Theorem 1279. ☐

Theorem 1289. $T(y,(x\backslash y)\cdot x)=y$.

Proof. We have $(y/((y\cdot x)/y))\cdot T((y\cdot x)/y,y)=(((y\cdot x)/y)\cdot y)/((y\cdot x)/y)$ by Theorem 818. Then $(y/((y\cdot x)/y))\cdot T((y\cdot x)/y,y)=(y\cdot x)/((y\cdot x)/y)$ by Axiom 6. Then $(y/((y\cdot x)/y))\cdot x=(y\cdot x)/((y\cdot x)/y)$ by Theorem 7. Then

$$y\cdot(T(y,x)\backslash y)=(y/((y\cdot x)/y))\cdot x \tag{673}$$

by Theorem 1011. We have $T(y/((y\cdot x)/y),x)=x\backslash((y/((y\cdot x)/y))\cdot x)$ by Definition 3. Then

$$T(y/((y\cdot x)/y),x)=x\backslash(y\cdot(T(y,x)\backslash y)) \tag{674}$$

by (673). We have $T(y/((y\cdot x)/y),T((y\cdot x)/y,y))=((y\cdot x)/y)\backslash y$ by Theorem 1234. Then $T(y/((y\cdot x)/y),x)=((y\cdot x)/y)\backslash y$ by Theorem 7. Then

$$x\backslash(y\cdot(T(y,x)\backslash y))=((y\cdot x)/y)\backslash y \tag{675}$$

by (674). We have $((y\cdot x)/y)\cdot T(((y\cdot x)/y)\backslash y,x)=y\cdot K(x\backslash(x/(y\backslash((y\cdot x)/y))),x)$ by Theorem 284. Then

$$((y\cdot x)/y)\cdot T(((y\cdot x)/y)\backslash y,x)=y\cdot K(x\backslash(y\cdot(T(y,x)\backslash y)),x) \tag{676}$$

by Theorem 1255. We have $((y\cdot x)/y)\cdot T(((y\cdot x)/y)\backslash y,x)=(y\cdot(x\cdot T(x\backslash y,x)))/y$ by Theorem 1117. Then $y\cdot K(x\backslash(y\cdot(T(y,x)\backslash y)),x)=(y\cdot(x\cdot T(x\backslash y,x)))/y$ by (676). Then $y\cdot K(x\backslash(y\cdot(T(y,x)\backslash y)),x)=(y\cdot((x\backslash y)\cdot x))/y$ by Proposition 46. Then

$$y\cdot K(((y\cdot x)/y)\backslash y,x)=(y\cdot((x\backslash y)\cdot x))/y \tag{677}$$

by (675). We have

$$T(y,(x\backslash y)\cdot x)\cdot(y\backslash((x\backslash y)\cdot x))=(y\cdot((x\backslash y)\cdot x))/y \tag{678}$$

by Theorem 996.

$$\begin{aligned} & T(y,(x\backslash y)\cdot x)\cdot K(x\backslash y,x) \\ = \;& (y\cdot((x\backslash y)\cdot x))/y && \text{by (678), Theorem 2} \\ = \;& y\cdot K(x\backslash y,x) && \text{by (677), Theorem 1288.} \end{aligned}$$

Then $T(y,(x\backslash y)\cdot x)\cdot K(x\backslash y,x)=y\cdot K(x\backslash y,x)$. Hence we are done by Proposition 10. ☐

Theorem 1290. $T((y\backslash x)\cdot y,x)=(y\backslash x)\cdot y$.

Proof. We have $T(x,(y\backslash x)\cdot y)=x$ by Theorem 1289. Hence we are done by Proposition 21. ☐

Theorem 1291. $T(y\cdot(x/y),x)=y\cdot(x/y)$.

Proof. We have $(x/y)\cdot y=x$ by Axiom 6. Then

$$(y\backslash(y\cdot(x/y)))\cdot y=x \tag{679}$$

by Axiom 3. We have $T(y\cdot(x/y),(y\backslash(y\cdot(x/y)))\cdot y)=y\cdot(x/y)$ by Theorem 1289. Hence we are done by (679). ☐

Theorem 1292. $R(z,K(x\backslash e,x),y)=z$.

Proof. We have $R(e/(x\backslash e),K((x\backslash e)\backslash e,x\backslash e),(x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))\cdot T(R(e/(x\backslash e),K((x\backslash e)\backslash e,x\backslash e),(x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))\backslash e,z)=R((e/(x\backslash e))\cdot T((e/(x\backslash e))\backslash e,z),K((x\backslash e)\backslash e,x\backslash e),(x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))$ by Theorem 1139. Then $(e/(x\backslash e))\cdot T(R(e/(x\backslash e),K((x\backslash e)\backslash e,x\backslash e),(x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))\backslash e,z)=R((e/(x\backslash e))\cdot T((e/(x\backslash e))\backslash e,z),K((x\backslash e)\backslash e,x\backslash e),(x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))$ by Theorem 1160. Then

$$R((e/(x\backslash e))\cdot T((e/(x\backslash e))\backslash e,z),K((x\backslash e)\backslash e,x\backslash e),(x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))=(e/(x\backslash e))\cdot T((e/(x\backslash e))\backslash e,z) \tag{680}$$

by Theorem 1160. We have $(e/(x\backslash e))\cdot T((e/(x\backslash e))\backslash e,z)=(x\backslash e)\backslash T(x\backslash e,z)$ by Theorem 1199. Then $R((e/(x\backslash e))\cdot T((e/(x\backslash e))\backslash e,z),K((x\backslash e)\backslash e,x\backslash e),(x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))=(x\backslash e)\backslash T(x\backslash e,z)$ by (680). Then

$$R((x\backslash e)\backslash T(x\backslash e,z),K((x\backslash e)\backslash e,x\backslash e),(x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))=(x\backslash e)\backslash T(x\backslash e,z) \quad (681)$$

by Theorem 1199. We have $R(T(x\backslash e,z)\cdot((x\backslash e)\backslash e),K((x\backslash e)\backslash e,T(x\backslash e,z)),(x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))\backslash((((x\backslash e)\backslash e)\cdot T(x\backslash e,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))=K((x\backslash e)\backslash e,T(x\backslash e,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))$ by Theorem 1068. Then $R((x\backslash e)\backslash T(x\backslash e,z),K((x\backslash e)\backslash e,T(x\backslash e,z)),(x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))\backslash((((x\backslash e)\backslash e)\cdot T(x\backslash e,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))=K((x\backslash e)\backslash e,T(x\backslash e,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))$ by Theorem 146. Then $R((x\backslash e)\backslash T(x\backslash e,z),K((x\backslash e)\backslash e,x\backslash e),(x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))\backslash((((x\backslash e)\backslash e)\cdot T(x\backslash e,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))=K((x\backslash e)\backslash e,T(x\backslash e,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))$ by Theorem 1205. Then $((x\backslash e)\backslash T(x\backslash e,z))\backslash((((x\backslash e)\backslash e)\cdot T(x\backslash e,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))=K((x\backslash e)\backslash e,T(x\backslash e,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))$ by (681). Then

$$((x\backslash e)\backslash T(x\backslash e,z))\backslash((((x\backslash e)\backslash e)\cdot T(x\backslash e,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))=K((x\backslash e)\backslash e,x\backslash e)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))) \quad (682)$$

by Theorem 1205. We have $((x\backslash e)\backslash T(x\backslash e,z))\cdot(((x\backslash e)\backslash T(x\backslash e,z))\backslash((((x\backslash e)\backslash e)\cdot T(x\backslash e,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))=(((x\backslash e)\backslash e)\cdot T(x\backslash e,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))$ by Axiom 4. Then $((x\backslash e)\backslash T(x\backslash e,z))\cdot(K((x\backslash e)\backslash e,x\backslash e)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))=(((x\backslash e)\backslash e)\cdot T(x\backslash e,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))$ by (682). Then $((x\backslash e)\backslash T(x\backslash e,z))\cdot(((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))\cdot K((x\backslash e)\backslash e,x\backslash e))=(((x\backslash e)\backslash e)\cdot T(x\backslash e,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))$ by Theorem 923. Then

$$T(x\backslash e,z)\cdot((x\backslash e)\backslash(((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))\cdot K((x\backslash e)\backslash e,x\backslash e)))=(((x\backslash e)\backslash e)\cdot T(x\backslash e,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))) \quad (683)$$

by Theorem 1270. We have $(x\backslash e)\backslash(K((x\backslash e)\backslash e,x\backslash e)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))=T(x\backslash e,(x\backslash e)\backslash e)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))$ by Theorem 1172. Then $(x\backslash e)\backslash(((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))\cdot K((x\backslash e)\backslash e,x\backslash e))=T(x\backslash e,(x\backslash e)\backslash e)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))$ by Theorem 923. Then

$$T(x\backslash e,z)\cdot(T(x\backslash e,(x\backslash e)\backslash e)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))=(((x\backslash e)\backslash e)\cdot T(x\backslash e,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))) \quad (684)$$

by (683). We have $(((x\backslash e)\backslash e)\backslash e)\backslash(K(x\backslash e,(x\backslash e)\backslash e)\cdot(K((x\backslash e)\backslash e,x\backslash e)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))=(x\backslash e)\backslash(K((x\backslash e)\backslash e,x\backslash e)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))$ by Theorem 1152. Then $(((x\backslash e)\backslash e)\backslash e)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))=(x\backslash e)\backslash(K((x\backslash e)\backslash e,x\backslash e)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))$ by Theorem 442. Then

$$(x\backslash e)\backslash(K(x,x\backslash e)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))=(((x\backslash e)\backslash e)\backslash e)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))) \quad (685)$$

by Theorem 733. We have $T(x\backslash e,z)\cdot(K(x,x\backslash e)\cdot((x\backslash e)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))=T(e/x,z)\cdot((x\backslash e)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))$ by Theorem 1209. Then $T(x\backslash e,z)\cdot((x\backslash e)\backslash(K(x,x\backslash e)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))=T(e/x,z)\cdot((x\backslash e)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))$ by Theorem 600. Then $T(x\backslash e,z)\cdot((((x\backslash e)\backslash e)\backslash e)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))=T(e/x,z)\cdot((x\backslash e)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))$ by (685). Then $T(x\backslash e,z)\cdot(T(x\backslash e,(x\backslash e)\backslash e)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))=T(e/x,z)\cdot((x\backslash e)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))$ by Proposition 49. Then $(((x\backslash e)\backslash e)\cdot T(x\backslash e,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))=T(e/x,z)\cdot((x\backslash e)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))$ by (684). Then

$$(x\cdot T(e/x,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))=T(e/x,z)\cdot((x\backslash e)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))) \quad (686)$$

by Theorem 1206. We have $(e/x)\cdot(T(e/x,z)\cdot((e/x)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))=T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))$ by Theorem 830. Then $(e/x)\cdot(T(x\backslash e,z)\cdot((x\backslash e)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))=T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))$ by Theorem 1210. Then $(x\backslash e)\cdot(T(e/x,z)\cdot((x\backslash e)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))=T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))$ by Theorem 1214. Then $(T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))/(T(e/x,z)\cdot((x\backslash e)\backslash((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))=x\backslash e$ by Proposition 1. Then

$$(T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))/((x\cdot T(e/x,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))=x\backslash e \quad (687)$$

by (686). We have $((T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))/((x\cdot T(e/x,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))\cdot(((T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))/((x\cdot T(e/x,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))\backslash(((T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))/((x\cdot T(e/x,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))\cdot(x\cdot(x\backslash((e/x)\backslash y))))) = ((T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))/((x\cdot T(e/x,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))\cdot(x\cdot(x\backslash((e/x)\backslash y)))$ by Axiom 4. Then $((T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))/((x\cdot T(e/x,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))\cdot((x\cdot T(e/x,z))\cdot((x\cdot T(e/x,z))\backslash(((T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))/((x\cdot T(e/x,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))\backslash(((T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))/((x\cdot T(e/x,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))\cdot(x\cdot(x\backslash((e/x)\backslash y))))))) = ((T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))/((x\cdot T(e/x,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))\cdot(x\cdot(x\backslash((e/x)\backslash y)))$ by Axiom 4. Then

$$((T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))/((x\cdot T(e/x,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))\cdot((x\cdot T(e/x,z))\cdot((x\cdot T(e/x,z))\backslash(x \tag{688}$$

by Axiom 3. We have $((T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))/((x\cdot T(e/x,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))\cdot((x\cdot T(e/x,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))) = T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))$ by Axiom 6. Then $((T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))))/((x\cdot T(e/x,z))\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))))\cdot(x\cdot(x\backslash((e/x)\backslash y))) = T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))$ by (688). Then

$$(x\backslash e)\cdot(x\cdot(x\backslash((e/x)\backslash y))) = T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))) \tag{689}$$

by (687). We have $T(z,x\backslash e)\cdot(z\backslash(x\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))))) = (T(x\backslash e,z)\backslash(x\backslash e))\cdot(x\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))))$ by Theorem 1275. Then $T(z,x\backslash e)\cdot(z\backslash(x\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))))) = ((x\backslash e)\backslash T(x\backslash e,z))\backslash(x\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))))$ by Theorem 1281. Then

$$(T(z,x\backslash e)\backslash z)\backslash(x\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y))))) = T(z,x\backslash e)\cdot(z\backslash(x\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))))) \tag{690}$$

by Theorem 1030. We have $L(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)),T(x\backslash e,z),x) = (x\cdot T(x\backslash e,z))\backslash(x\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))))$ by Definition 4. Then $L(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)),T(x\backslash e,z),x) = (T(z,x\backslash e)\backslash z)\backslash(x\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))))$ by Theorem 1028. Then

$$L(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)),T(x\backslash e,z),x) = T(z,x\backslash e)\cdot(z\backslash(x\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))))) \tag{691}$$

by (690). We have $(x\cdot((z\cdot(x\backslash e))/z))\cdot(x\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y))))) = x\cdot((x\cdot((z\cdot(x\backslash e))/z))\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))))$ by Theorem 1128. Then $K(z,(z/x)/z)\cdot(x\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y))))) = x\cdot((x\cdot((z\cdot(x\backslash e))/z))\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))))$ by Theorem 333. Then $K(z,(z/x)/z)\cdot(x\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y))))) = x\cdot((z\backslash T(z,x\backslash e))\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))))$ by Theorem 980. Then $(z\backslash T(z,x\backslash e))\cdot(x\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y))))) = x\cdot((z\backslash T(z,x\backslash e))\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))))$ by Theorem 1004. Then $T(z,x\backslash e)\cdot(z\backslash(x\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))))) = x\cdot((z\backslash T(z,x\backslash e))\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))))$ by Theorem 1270. Then $T(z,x\backslash e)\cdot(z\backslash(x\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))))) = x\cdot(T(z,x\backslash e)\cdot(z\backslash(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y))))))$ by Theorem 1270. Then

$$L(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)),T(x\backslash e,z),x) = x\cdot(T(z,x\backslash e)\cdot(z\backslash(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))))) \tag{692}$$

by (691). We have $T(z,x\backslash e)\cdot(z\backslash(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y))))) = (T(x\backslash e,z)\backslash(x\backslash e))\cdot(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y))))$ by Theorem 1275. Then $T(z,x\backslash e)\cdot(z\backslash(T(x\backslash e,z)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y))))) = (x\backslash e)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))$ by Theorem 1282. Then

$$x\cdot((x\backslash e)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))) = L(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)),T(x\backslash e,z),x) \tag{693}$$

by (692). We have $x\cdot((x\backslash e)\cdot(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)))) = L(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)),x\backslash e,x)$ by Proposition 56. Then

$$L(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)),T(x\backslash e,z),x) = L(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)),x\backslash e,x) \tag{694}$$

by (693). We have $L(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)),T(e/x,z),x) = (x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))$ by Proposition 53. Then $L(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)),T(x\backslash e,z),x) = (x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y)))$ by Theorem 1228. Then

$$L(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)),x\backslash e,x) = (x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))) \quad (695)$$

by (694). We have $L(x\backslash L((e/x)\backslash((z\cdot(x\backslash((e/x)\backslash y)))/z),x\backslash e,x),x,x\backslash e) = (e/x)\cdot L((e/x)\backslash((z\cdot(x\backslash((e/x)\backslash y)))/z),x\backslash e,x)$ by Theorem 434. Then

$$L((x\backslash e)\cdot((e/x)\backslash((z\cdot(x\backslash((e/x)\backslash y)))/z)),x,x\backslash e) = (e/x)\cdot L((e/x)\backslash((z\cdot(x\backslash((e/x)\backslash y)))/z),x\backslash e,x) \quad (696)$$

by Proposition 57. We have $T((e/x)\cdot L((e/x)\backslash((z\cdot(x\backslash((e/x)\backslash y)))/z),x\backslash e,x),z) = T(e/x,z)\cdot L(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)),x\backslash e,x)$ by Theorem 879. Then

$$T(L((x\backslash e)\cdot((e/x)\backslash((z\cdot(x\backslash((e/x)\backslash y)))/z)),x,x\backslash e),z) = T(e/x,z)\cdot L(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)),x\backslash e,x) \quad (697)$$

by (696). We have $L(((z\cdot(x\backslash((e/x)\backslash y)))/z)\cdot K(x\backslash e,x),x,x\backslash e) = (x\backslash e)\cdot(x\cdot((z\cdot(x\backslash((e/x)\backslash y)))/z))$ by Theorem 1195. Then $L((x\backslash e)\cdot((e/x)\backslash((z\cdot(x\backslash((e/x)\backslash y)))/z)),x,x\backslash e) = (x\backslash e)\cdot(x\cdot((z\cdot(x\backslash((e/x)\backslash y)))/z))$ by Theorem 1190. Then $T((x\backslash e)\cdot(x\cdot((z\cdot(x\backslash((e/x)\backslash y)))/z)),z) = T(e/x,z)\cdot L(T(e/x,z)\backslash(x\backslash((e/x)\backslash y)),x\backslash e,x)$ by (697). Then $T((x\backslash e)\cdot(x\cdot((z\cdot(x\backslash((e/x)\backslash y)))/z)),z) = T(e/x,z)\cdot((x\cdot T(e/x,z))\backslash(x\cdot(x\backslash((e/x)\backslash y))))$ by (695). Then

$$T((x\backslash e)\cdot(x\cdot((z\cdot(x\backslash((e/x)\backslash y)))/z)),z) = (x\backslash e)\cdot(x\cdot(x\backslash((e/x)\backslash y))) \quad (698)$$

by (689). We have $(x\backslash e)\cdot((e/x)\backslash((z\cdot((e/x)\cdot((e/x)\backslash y)))/z)) = K(x\backslash e,x)\cdot((z\cdot((e/x)\cdot((e/x)\backslash y)))/z)$ by Theorem 1161. Then

$$(x\backslash e)\cdot(x\cdot((z\cdot(x\backslash((e/x)\backslash y)))/z)) = K(x\backslash e,x)\cdot((z\cdot((e/x)\cdot((e/x)\backslash y)))/z) \quad (699)$$

by Theorem 1104. We have $T((K(x\backslash e,x)\cdot(z\cdot((e/x)\cdot((e/x)\backslash y))))/z,z) = z\backslash(K(x\backslash e,x)\cdot(z\cdot((e/x)\cdot((e/x)\backslash y))))$ by Proposition 47. Then $T(K(x\backslash e,x)\cdot((z\cdot((e/x)\cdot((e/x)\backslash y)))/z),z) = z\backslash(K(x\backslash e,x)\cdot(z\cdot((e/x)\cdot((e/x)\backslash y))))$ by Theorem 969. Then

$$T((x\backslash e)\cdot(x\cdot((z\cdot(x\backslash((e/x)\backslash y)))/z)),z) = z\backslash(K(x\backslash e,x)\cdot(z\cdot((e/x)\cdot((e/x)\backslash y)))) \quad (700)$$

by (699). We have $L((x\backslash e)\cdot((e/x)\backslash y),K(x,x\backslash e),z) = (z\cdot K(x,x\backslash e))\backslash(z\cdot(K(x,x\backslash e)\cdot((x\backslash e)\cdot((e/x)\backslash y))))$ by Definition 4. Then $L((x\backslash e)\cdot((e/x)\backslash y),K(x,x\backslash e),z) = (z\cdot K(x,x\backslash e))\backslash(z\cdot((e/x)\cdot((e/x)\backslash y)))$ by Theorem 697. Then $z\backslash(K(x\backslash e,x)\cdot(z\cdot((e/x)\cdot((e/x)\backslash y)))) = L((x\backslash e)\cdot((e/x)\backslash y),K(x,x\backslash e),z)$ by Theorem 1223. Then $T((x\backslash e)\cdot(x\cdot((z\cdot(x\backslash((e/x)\backslash y)))/z)),z) = L((x\backslash e)\cdot((e/x)\backslash y),K(x,x\backslash e),z)$ by (700). Then $(x\backslash e)\cdot(x\cdot(x\backslash((e/x)\backslash y))) = L((x\backslash e)\cdot((e/x)\backslash y),K(x,x\backslash e),z)$ by (698). Then

$$L((x\backslash e)\cdot((e/x)\backslash y),K(x,x\backslash e),z) = (x\backslash e)\cdot((e/x)\backslash y) \quad (701)$$

by Axiom 4. We have $(z\cdot(K(x,x\backslash e)\cdot((x\backslash e)\cdot((e/x)\backslash y))))/L((x\backslash e)\cdot((e/x)\backslash y),K(x,x\backslash e),z) = z\cdot K(x,x\backslash e)$ by Theorem 453. Then $(z\cdot((e/x)\cdot((e/x)\backslash y)))/L((x\backslash e)\cdot((e/x)\backslash y),K(x,x\backslash e),z) = z\cdot K(x,x\backslash e)$ by Theorem 697. Then

$$(z\cdot((e/x)\cdot((e/x)\backslash y)))/((x\backslash e)\cdot((e/x)\backslash y)) = z\cdot K(x,x\backslash e) \quad (702)$$

by (701). We have $R(z,K(x\backslash e,x),(e/x)\cdot((e/x)\backslash y)) = ((z\cdot K(x\backslash e,x))\cdot((e/x)\cdot((e/x)\backslash y)))/(K(x\backslash e,x)\cdot((e/x)\cdot((e/x)\backslash y)))$ by Definition 5. Then $R(z,K(x\backslash e,x),(e/x)\cdot((e/x)\backslash y)) = ((z\cdot K(x\backslash e,x))\cdot((e/x)\cdot((e/x)\backslash y)))/((x\backslash e)\cdot((e/x)\backslash y))$ by Theorem 682. Then

$$(K(x\backslash e,x)\cdot(z\cdot((e/x)\cdot((e/x)\backslash y))))/((x\backslash e)\cdot((e/x)\backslash y)) = R(z,K(x\backslash e,x),(e/x)\cdot((e/x)\backslash y)) \quad (703)$$

by Theorem 1222. We have $(K(x\backslash e,x)\cdot(z\cdot((e/x)\cdot((e/x)\backslash y))))/((x\backslash e)\cdot((e/x)\backslash y)) = K(x\backslash e,x)\cdot((z\cdot((e/x)\cdot((e/x)\backslash y)))/((x\backslash e)\cdot((e/x)\backslash y)))$ by Theorem 969. Then $R(z,K(x\backslash e,x),(e/x)\cdot((e/x)\backslash y)) = K(x\backslash e,x)\cdot((z\cdot((e/x)\cdot((e/x)\backslash y)))/((x\backslash e)\cdot((e/x)\backslash y)))$ by (703). Then

$$K(x\backslash e,x)\cdot(z\cdot K(x,x\backslash e)) = R(z,K(x\backslash e,x),(e/x)\cdot((e/x)\backslash y)) \quad (704)$$

by (702). We have $K(x\backslash e,x)\cdot(z\cdot K(x,x\backslash e)) = z$ by Theorem 1182. Then $R(z,K(x\backslash e,x),(e/x)\cdot((e/x)\backslash y)) = z$ by (704). Hence we are done by Axiom 4. □

Theorem 1293. $x = R(x, y, K(z\backslash e, z))$.

Proof. We have $R(x, K(z\backslash e,(z\backslash e)\backslash e), y) = ((x \cdot K(z\backslash e,(z\backslash e)\backslash e)) \cdot y)/(K(z\backslash e,(z\backslash e)\backslash e) \cdot y)$ by Definition 5. Then

$$R(x, K(z\backslash e,(z\backslash e)\backslash e), y) = (K(z\backslash e,(z\backslash e)\backslash e) \cdot (x \cdot y))/(K(z\backslash e,(z\backslash e)\backslash e) \cdot y) \tag{705}$$

by Theorem 1231. We have

$$(K(z\backslash e,(z\backslash e)\backslash e) \cdot (x \cdot y))/(K(z\backslash e,(z\backslash e)\backslash e) \cdot y) = K(z\backslash e,(z\backslash e)\backslash e) \cdot ((x \cdot y)/(K(z\backslash e,(z\backslash e)\backslash e) \cdot y)) \tag{706}$$

by Theorem 1226. Then

$$((x \cdot y) \cdot K(z\backslash e,(z\backslash e)\backslash e))/(K(z\backslash e,(z\backslash e)\backslash e) \cdot y) = K(z\backslash e,(z\backslash e)\backslash e) \cdot ((x \cdot y)/(K(z\backslash e,(z\backslash e)\backslash e) \cdot y)) \tag{707}$$

by Theorem 1183. We have $((x \cdot y) \cdot T(K(z\backslash e,(z\backslash e)\backslash e), y))/(K(z\backslash e,(z\backslash e)\backslash e) \cdot y) = R(x, y, T(K(z\backslash e,(z\backslash e)\backslash e), y))$ by Theorem 787. Then $((x \cdot y) \cdot K(z\backslash e,(z\backslash e)\backslash e))/(K(z\backslash e,(z\backslash e)\backslash e) \cdot y) = R(x, y, T(K(z\backslash e,(z\backslash e)\backslash e), y))$ by Theorem 723. Then

$$K(z\backslash e,(z\backslash e)\backslash e) \cdot ((x \cdot y)/(K(z\backslash e,(z\backslash e)\backslash e) \cdot y)) = R(x, y, T(K(z\backslash e,(z\backslash e)\backslash e), y)) \tag{708}$$

by (707). We have $R(x, K(z\backslash e,(z\backslash e)\backslash e), y) = K(z\backslash e,(z\backslash e)\backslash e) \cdot ((x \cdot y)/(K(z\backslash e,(z\backslash e)\backslash e) \cdot y))$ by (705) and (706). Then $R(x, y, T(K(z\backslash e,(z\backslash e)\backslash e), y)) = R(x, K(z\backslash e,(z\backslash e)\backslash e), y)$ by (708). Then $R(x, y, K(z\backslash e,(z\backslash e)\backslash e)) = R(x, K(z\backslash e,(z\backslash e)\backslash e), y)$ by Theorem 723. Then $R(x, K(z\backslash e, z), y) = R(x, y, K(z\backslash e,(z\backslash e)\backslash e))$ by Theorem 779. Then $R(x, y, K(z\backslash e, z)) = R(x, K(z\backslash e, z), y)$ by Theorem 779. Hence we are done by Theorem 1292. ☐

Theorem 1294. $x\backslash(K(z\backslash e, z) \cdot y) = (x\backslash y) \cdot K(z\backslash e, z)$.

Proof. We have $R(x, x\backslash y, K(z\backslash e, z))\backslash(K(z\backslash e, z) \cdot y) = (x\backslash y) \cdot K(z\backslash e, z)$ by Theorem 1189. Hence we are done by Theorem 1293. ☐

Theorem 1295. $y \cdot (K(x\backslash e, x) \cdot z) = K(x\backslash e, x) \cdot (y \cdot z)$.

Proof. We have $R(y, K(x\backslash e, x), z) \cdot (K(x\backslash e, x) \cdot z) = (y \cdot K(x\backslash e, x)) \cdot z$ by Proposition 54. Then

$$y \cdot (K(x\backslash e, x) \cdot z) = (y \cdot K(x\backslash e, x)) \cdot z \tag{709}$$

by Theorem 1292. We have $(y \cdot K(x\backslash e, x)) \cdot z = K(x\backslash e, x) \cdot (y \cdot z)$ by Theorem 1222. Hence we are done by (709). ☐

Theorem 1296. $L(y, z, w) \cdot K(x\backslash e, x) = L(K(x\backslash e, x) \cdot y, z, w)$.

Proof. We have $(w \cdot z)\backslash(K(x\backslash e, x) \cdot (w \cdot (z \cdot y))) = ((w \cdot z)\backslash(w \cdot (z \cdot y))) \cdot K(x\backslash e, x)$ by Theorem 1294. Then $(w \cdot z)\backslash(w \cdot (K(x\backslash e, x) \cdot (z \cdot y))) = ((w \cdot z)\backslash(w \cdot (z \cdot y))) \cdot K(x\backslash e, x)$ by Theorem 1295. Then

$$(w \cdot z)\backslash(w \cdot (K(x\backslash e, x) \cdot (z \cdot y))) = L(y, z, w) \cdot K(x\backslash e, x) \tag{710}$$

by Definition 4. We have $L(K(x\backslash e, x) \cdot y, z, w) = (w \cdot z)\backslash(w \cdot (z \cdot (K(x\backslash e, x) \cdot y)))$ by Definition 4. Then $L(K(x\backslash e, x) \cdot y, z, w) = (w \cdot z)\backslash(w \cdot (K(x\backslash e, x) \cdot (z \cdot y)))$ by Theorem 1295. Hence we are done by (710). ☐

Theorem 1297. $K(x\backslash e, x) \cdot L(y, z, w) = L(K(x\backslash e, x) \cdot y, z, w)$.

Proof. We have $K(x\backslash e, x) \cdot L(y, z, w) = L(y, z, w) \cdot K(x\backslash e, x)$ by Theorem 923. Hence we are done by Theorem 1296. ☐

Theorem 1298. $(y\backslash L(y \cdot x, z, w)) \cdot y = L(x \cdot y, z, w)$.

Proof. We have $K(y\backslash e, y) \cdot L(L(y\backslash((y\backslash e)\backslash(((x \cdot y)/y) \cdot y)), z, w), y, y\backslash e) = (y\backslash e) \cdot (y \cdot L(y\backslash((y\backslash e)\backslash(((x \cdot y)/y) \cdot y)), z, w))$ by Theorem 59. Then $K(y\backslash e, y) \cdot L(((y\backslash e) \cdot y)\backslash(((x \cdot y)/y) \cdot y), z, w) = (y\backslash e) \cdot (y \cdot L(y\backslash((y\backslash e)\backslash(((x \cdot y)/y) \cdot y)), z, w))$ by Theorem 1070. Then

$$K(y\backslash e, y) \cdot L(K(y\backslash e, y)\backslash(((x \cdot y)/y) \cdot y), z, w) = (y\backslash e) \cdot (y \cdot L(y\backslash((y\backslash e)\backslash(((x \cdot y)/y) \cdot y)), z, w)) \quad (711)$$

by Proposition 76. We have $(y\backslash e) \cdot (y \cdot L(y\backslash((y\backslash e)\backslash(((x \cdot y)/y) \cdot y)), z, w)) = ((y\backslash e) \cdot L((y\backslash e)\backslash((x \cdot y)/y), z, w)) \cdot y$ by Theorem 1120. Then $K(y\backslash e, y) \cdot L(K(y\backslash e, y)\backslash(((x \cdot y)/y) \cdot y), z, w) = ((y\backslash e) \cdot L((y\backslash e)\backslash((x \cdot y)/y), z, w)) \cdot y$ by (711). Then $K(y\backslash e, y) \cdot L(K(y\backslash e, y)\backslash(((x \cdot y)/y) \cdot y), z, w) = (y\backslash L(y \cdot ((x \cdot y)/y), z, w)) \cdot y$ by Theorem 827. Then $K(y\backslash e, y) \cdot L(K(y, y\backslash e) \cdot (((x \cdot y)/y) \cdot y), z, w) = (y\backslash L(y \cdot ((x \cdot y)/y), z, w)) \cdot y$ by Theorem 696. Then $K(y\backslash e, y) \cdot L(((x \cdot y)/y) \cdot T(y, y\backslash e), z, w) = (y\backslash L(y \cdot ((x \cdot y)/y), z, w)) \cdot y$ by Theorem 925. Then

$$K(y\backslash e, y) \cdot L((x \cdot y) \cdot K(y, y\backslash e), z, w) = (y\backslash L(y \cdot ((x \cdot y)/y), z, w)) \cdot y \quad (712)$$

by Theorem 1142. We have $L(K(y\backslash e, y) \cdot ((x \cdot y) \cdot K(y, y\backslash e)), z, w) = K(y\backslash e, y) \cdot L((x \cdot y) \cdot K(y, y\backslash e), z, w)$ by Theorem 1297. Then $L(x \cdot y, z, w) = K(y\backslash e, y) \cdot L((x \cdot y) \cdot K(y, y\backslash e), z, w)$ by Theorem 1182. Then $L(x \cdot y, z, w) = (y\backslash L(y \cdot ((x \cdot y)/y), z, w)) \cdot y$ by (712). Hence we are done by Axiom 5. □

Theorem 1299. $(y\backslash L(x, z, w)) \cdot y = L((y\backslash x) \cdot y, z, w)$.

Proof. We have $(y\backslash L(y \cdot (y\backslash x), z, w)) \cdot y = L((y\backslash x) \cdot y, z, w)$ by Theorem 1298. Hence we are done by Axiom 4. □

Theorem 1300. $((y\backslash e) \cdot x) \cdot (x/((y\backslash e) \cdot x)) = (y\backslash x) \cdot y$.

Proof. We have $((((((e/y) \cdot x)\backslash x)\backslash e)\backslash e)\backslash x) \cdot (((((e/y) \cdot x)\backslash x)\backslash e)\backslash e) = ((((e/y) \cdot x)\backslash x)\backslash x) \cdot (((e/y) \cdot x)\backslash x)$ by Theorem 1187. Then $((((y\backslash e) \cdot x)\backslash x)\backslash x) \cdot (((((e/y) \cdot x)\backslash x)\backslash e)\backslash e) = ((((e/y) \cdot x)\backslash x)\backslash x) \cdot (((e/y) \cdot x)\backslash x)$ by Theorem 1212. Then

$$((((y\backslash e) \cdot x)\backslash x)\backslash x) \cdot (((y\backslash e) \cdot x)\backslash x) = ((((e/y) \cdot x)\backslash x)\backslash x) \cdot (((e/y) \cdot x)\backslash x) \quad (713)$$

by Theorem 1212. We have $x\backslash(((((e/y) \cdot x)\backslash x)\backslash x) \cdot (((e/y) \cdot x)\backslash x)) = K((((e/y) \cdot x)\backslash x)\backslash x, ((e/y) \cdot x)\backslash x)$ by Theorem 2. Then $x\backslash(((((e/y) \cdot x)\backslash x)\backslash x) \cdot (((e/y) \cdot x)\backslash x)) = K((((e/y) \cdot x)\backslash x)\backslash x, ((y\backslash e) \cdot x)\backslash x)$ by Theorem 1219. Then

$$x\backslash(((((y\backslash e) \cdot x)\backslash x)\backslash x) \cdot (((y\backslash e) \cdot x)\backslash x)) = K((((e/y) \cdot x)\backslash x)\backslash x, ((y\backslash e) \cdot x)\backslash x) \quad (714)$$

by (713). We have $x\backslash(((((y\backslash e) \cdot x)\backslash x)\backslash x) \cdot (((y\backslash e) \cdot x)\backslash x)) = K((((y\backslash e) \cdot x)\backslash x)\backslash x, ((y\backslash e) \cdot x)\backslash x)$ by Theorem 2. Then

$$K((((e/y) \cdot x)\backslash x)\backslash x, ((y\backslash e) \cdot x)\backslash x) = K((((y\backslash e) \cdot x)\backslash x)\backslash x, ((y\backslash e) \cdot x)\backslash x) \quad (715)$$

by (714). We have $K((((e/y) \cdot x)\backslash x)\backslash x, ((e/y) \cdot x)\backslash x) = K((((e/y) \cdot x)\backslash x)\backslash x, ((y\backslash e) \cdot x)\backslash x)$ by Theorem 1219. Then $K((e/y) \cdot x, x/((e/y) \cdot x)) = K((((e/y) \cdot x)\backslash x)\backslash x, ((y\backslash e) \cdot x)\backslash x)$ by Theorem 1251. Then

$$K((((y\backslash e) \cdot x)\backslash x)\backslash x, ((y\backslash e) \cdot x)\backslash x) = K((e/y) \cdot x, x/((e/y) \cdot x)) \quad (716)$$

by (715). We have $K((((y\backslash e) \cdot x)\backslash x)\backslash x, ((y\backslash e) \cdot x)\backslash x) = K((y\backslash e) \cdot x, x/((y\backslash e) \cdot x))$ by Theorem 1251. Then

$$K((e/y) \cdot x, x/((e/y) \cdot x)) = K((y\backslash e) \cdot x, x/((y\backslash e) \cdot x)) \quad (717)$$

by (716). We have $x \cdot K((e/y) \cdot x, x/((e/y) \cdot x)) = ((e/y) \cdot x) \cdot (x/((e/y) \cdot x))$ by Theorem 459. Then

$$x \cdot K((y\backslash e) \cdot x, x/((y\backslash e) \cdot x)) = ((e/y) \cdot x) \cdot (x/((e/y) \cdot x)) \quad (718)$$

by (717). We have $x \cdot K((y\backslash e) \cdot x, x/((y\backslash e) \cdot x)) = ((y\backslash e) \cdot x) \cdot (x/((y\backslash e) \cdot x))$ by Theorem 459. Then

$$((e/y) \cdot x) \cdot (x/((e/y) \cdot x)) = ((y\backslash e) \cdot x) \cdot (x/((y\backslash e) \cdot x)) \quad (719)$$

by (718). We have $(e/y)\backslash T((e/y)\cdot x,(e/y)\backslash e) = y\cdot T(y\backslash x,(e/y)\backslash e)$ by Theorem 1080. Then

$$(e/y)\backslash((e/y)\cdot(((e/y)\cdot x)\cdot(x/((e/y)\cdot x)))) = y\cdot T(y\backslash x,(e/y)\backslash e) \tag{720}$$

by Theorem 1247. We have $(e/y)\backslash((e/y)\cdot(((e/y)\cdot x)\cdot(x/((e/y)\cdot x)))) = ((e/y)\cdot x)\cdot(x/((e/y)\cdot x))$ by Axiom 3. Then $y\cdot T(y\backslash x,(e/y)\backslash e) = ((e/y)\cdot x)\cdot(x/((e/y)\cdot x))$ by (720). Then $((y\backslash e)\cdot x)\cdot(x/((y\backslash e)\cdot x)) = y\cdot T(y\backslash x,(e/y)\backslash e)$ by (719). Then

$$((y\backslash e)\cdot x)\cdot(x/((y\backslash e)\cdot x)) = y\cdot T(y\backslash x, y) \tag{721}$$

by Proposition 25. We have $y\cdot T(y\backslash x,y) = (y\backslash x)\cdot y$ by Proposition 46. Hence we are done by (721). □

Theorem 1301. $((y\backslash e)\cdot x)\cdot R(y, y\backslash e, x) = (y\backslash x)\cdot y$.

Proof. We have $((y\backslash e)\cdot x)\cdot(x/((y\backslash e)\cdot x)) = (y\backslash x)\cdot y$ by Theorem 1300. Hence we are done by Proposition 66. □

Theorem 1302. $T(x, x\backslash y) = T(x,(x\backslash e)\cdot y)$.

Proof. We have $x\cdot((e/x)\cdot((x\backslash y)\cdot x)) = L(x\backslash y, x, e/x)\cdot x$ by Theorem 1074. Then

$$x\cdot((e/x)\cdot((x\backslash y)\cdot x)) = L(x\backslash y, x, x\backslash e)\cdot x \tag{722}$$

by Theorem 1224. We have $(((x\cdot((e/x)\cdot((x\backslash y)\cdot x)))/x)\backslash((e/x)\cdot((x\backslash y)\cdot x)))\cdot((((x\cdot((e/x)\cdot((x\backslash y)\cdot x)))/x)\backslash((e/x)\cdot((x\backslash y)\cdot x)))\backslash((e/x)\cdot((x\backslash y)\cdot x))) = (e/x)\cdot((x\backslash y)\cdot x)$ by Axiom 4. Then $(((x\cdot((e/x)\cdot((x\backslash y)\cdot x)))/x)\backslash((e/x)\cdot((x\backslash y)\cdot x)))\cdot((x\cdot((e/x)\cdot((x\backslash y)\cdot x)))/x) = (e/x)\cdot((x\backslash y)\cdot x)$ by Theorem 946. Then

$$(((L(x\backslash y,x,x\backslash e)\cdot x)/x)\backslash((e/x)\cdot((x\backslash y)\cdot x)))\cdot((x\cdot((e/x)\cdot((x\backslash y)\cdot x)))/x) = (e/x)\cdot((x\backslash y)\cdot x) \tag{723}$$

by (722). We have $(T(x,(e/x)\cdot((x\backslash y)\cdot x))\backslash((((e/x)\cdot((x\backslash y)\cdot x))\cdot T(x,(e/x)\cdot((x\backslash y)\cdot x)))/((e/x)\cdot((x\backslash y)\cdot x))))\backslash((e/x)\cdot((x\backslash y)\cdot x)) = ((e/x)\cdot((x\backslash y)\cdot x))\cdot(((((e/x)\cdot((x\backslash y)\cdot x))\cdot T(x,(e/x)\cdot((x\backslash y)\cdot x)))/((e/x)\cdot((x\backslash y)\cdot x)))\backslash T(x,(e/x)\cdot((x\backslash y)\cdot x)))$ by Theorem 1216. Then $(T(x,(e/x)\cdot((x\backslash y)\cdot x))\backslash x)\backslash((e/x)\cdot((x\backslash y)\cdot x)) = ((e/x)\cdot((x\backslash y)\cdot x))\cdot(((((e/x)\cdot((x\backslash y)\cdot x))\cdot T(x,(e/x)\cdot((x\backslash y)\cdot x)))/((e/x)\cdot((x\backslash y)\cdot x)))\backslash T(x,(e/x)\cdot((x\backslash y)\cdot x)))$ by Proposition 48. Then

$$((e/x)\cdot((x\backslash y)\cdot x))\cdot(x\backslash T(x,(e/x)\cdot((x\backslash y)\cdot x))) = (T(x,(e/x)\cdot((x\backslash y)\cdot x))\backslash x)\backslash((e/x)\cdot((x\backslash y)\cdot x)) \tag{724}$$

by Proposition 48. We have $(T(x,(e/x)\cdot((x\backslash y)\cdot x))\backslash x)\cdot((T(x,(e/x)\cdot((x\backslash y)\cdot x))\backslash x)\backslash((e/x)\cdot((x\backslash y)\cdot x))) = (e/x)\cdot((x\backslash y)\cdot x)$ by Axiom 4. Then

$$(T(x,(e/x)\cdot((x\backslash y)\cdot x))\backslash x)\cdot(((e/x)\cdot((x\backslash y)\cdot x))\cdot(x\backslash T(x,(e/x)\cdot((x\backslash y)\cdot x)))) = (e/x)\cdot((x\backslash y)\cdot x) \tag{725}$$

by (724).

$$\begin{aligned} & (L(x\backslash y,x,x\backslash e)\backslash((e/x)\cdot((x\backslash y)\cdot x)))\cdot((x\cdot((e/x)\cdot((x\backslash y)\cdot x)))/x) \\ = {} & (e/x)\cdot((x\backslash y)\cdot x) && \text{by (723), Axiom 5} \\ = {} & (T(x,(e/x)\cdot((x\backslash y)\cdot x))\backslash x)\cdot((x\cdot((e/x)\cdot((x\backslash y)\cdot x)))/x) && \text{by (725), Theorem 977.} \end{aligned}$$

Then $(L(x\backslash y,x,x\backslash e)\backslash((e/x)\cdot((x\backslash y)\cdot x)))\cdot((x\cdot((e/x)\cdot((x\backslash y)\cdot x)))/x) = (T(x,(e/x)\cdot((x\backslash y)\cdot x))\backslash x)\cdot((x\cdot((e/x)\cdot((x\backslash y)\cdot x)))/x)$. Then

$$L(x\backslash y,x,x\backslash e)\backslash((e/x)\cdot((x\backslash y)\cdot x)) = T(x,(e/x)\cdot((x\backslash y)\cdot x))\backslash x \tag{726}$$

by Proposition 10. We have $((e/x)\cdot(x\cdot(x\backslash y)))\backslash((e/x)\cdot((x\backslash y)\cdot x)) = L(K(x\backslash y,x),x\cdot(x\backslash y),e/x)$ by Theorem 1058. Then $L(x\backslash y,x,x\backslash e)\backslash((e/x)\cdot((x\backslash y)\cdot x)) = L(K(x\backslash y,x),x\cdot(x\backslash y),e/x)$ by Theorem 1158.

Then $L(x\backslash y, x, x\backslash e)\backslash((e/x)\cdot((x\backslash y)\cdot x)) = L(K(x\backslash y, x), x\cdot(x\backslash y), x\backslash e)$ by Theorem 1224. Then $T(x, (e/x)\cdot((x\backslash y)\cdot x))\backslash x = L(K(x\backslash y, x), x\cdot(x\backslash y), x\backslash e)$ by (726). Then

$$L(K(x\backslash y, x), x\cdot(x\backslash y), x\backslash e) = T(x, (x\backslash e)\cdot((x\backslash y)\cdot x))\backslash x \tag{727}$$

by Theorem 1159. We have $(((x\backslash e)\cdot(x\cdot(x\backslash y)))\cdot R(x, x\backslash e, x\cdot(x\backslash y)))/(x\cdot(x\backslash y)) = ((((x\backslash e)\cdot(x\cdot(x\backslash y)))\cdot x)/((x\backslash e)\cdot(x\cdot(x\backslash y))))\cdot(x\backslash e)$ by Theorem 1076. Then

$$((x\backslash(x\cdot(x\backslash y)))\cdot x)/(x\cdot(x\backslash y)) = ((((x\backslash e)\cdot(x\cdot(x\backslash y)))\cdot x)/((x\backslash e)\cdot(x\cdot(x\backslash y))))\cdot(x\backslash e) \tag{728}$$

by Theorem 1301. We have $((((x\backslash e)\cdot(x\cdot(x\backslash y)))\cdot x)/((x\backslash e)\cdot(x\cdot(x\backslash y))))\cdot(x\backslash e) = ((x\backslash e)\cdot(x\cdot(x\backslash y)))\backslash T((x\backslash e)\cdot(x\cdot(x\backslash y)), x)$ by Theorem 983. Then

$$((x\backslash(x\cdot(x\backslash y)))\cdot x)/(x\cdot(x\backslash y)) = ((x\backslash e)\cdot(x\cdot(x\backslash y)))\backslash T((x\backslash e)\cdot(x\cdot(x\backslash y)), x) \tag{729}$$

by (728). We have $((x\backslash e)\cdot(x\cdot(x\backslash y)))\backslash((x\backslash e)\cdot((x\backslash(x\cdot(x\backslash y)))\cdot x)) = L(K(x\backslash(x\cdot(x\backslash y)), x), x\cdot(x\backslash y), x\backslash e)$ by Theorem 1060. Then $((x\backslash e)\cdot(x\cdot(x\backslash y)))\backslash T((x\backslash e)\cdot(x\cdot(x\backslash y)), x) = L(K(x\backslash(x\cdot(x\backslash y)), x), x\cdot(x\backslash y), x\backslash e)$ by Theorem 891. Then $((x\backslash(x\cdot(x\backslash y)))\cdot x)/(x\cdot(x\backslash y)) = L(K(x\backslash(x\cdot(x\backslash y)), x), x\cdot(x\backslash y), x\backslash e)$ by (729). Then $L(K(x\backslash y, x), x\cdot(x\backslash y), x\backslash e) = ((x\backslash(x\cdot(x\backslash y)))\cdot x)/(x\cdot(x\backslash y))$ by Axiom 3. Then $((x\backslash y)\cdot x)/(x\cdot(x\backslash y)) = L(K(x\backslash y, x), x\cdot(x\backslash y), x\backslash e)$ by Axiom 3. Then

$$((x\backslash y)\cdot x)/(x\cdot(x\backslash y)) = T(x, (x\backslash e)\cdot((x\backslash y)\cdot x))\backslash x \tag{730}$$

by (727). We have $x\cdot((T(x, (x\backslash e)\cdot((x\backslash y)\cdot x))\backslash x)\backslash e) = T(x, (x\backslash e)\cdot((x\backslash y)\cdot x))$ by Theorem 1198. Then

$$x\cdot((((x\backslash y)\cdot x)/(x\cdot(x\backslash y)))\backslash e) = T(x, (x\backslash e)\cdot((x\backslash y)\cdot x)) \tag{731}$$

by (730). We have $x\cdot((T(x, x\backslash y)\backslash x)\backslash e) = T(x, x\backslash y)$ by Theorem 1198. Then $x\cdot((((x\backslash y)\cdot x)/(x\cdot(x\backslash y)))\backslash e) = T(x, x\backslash y)$ by Theorem 1023. Then $T(x, x\backslash y) = T(x, (x\backslash e)\cdot((x\backslash y)\cdot x))$ by (731). Then

$$T(x, T((x\backslash e)\cdot y, x)) = T(x, x\backslash y) \tag{732}$$

by Theorem 891. We have $T(x, T((x\backslash e)\cdot y, x)) = T(x, (x\backslash e)\cdot y)$ by Theorem 1032. Hence we are done by (732). □

Theorem 1303. $(((y\backslash e)\cdot x)\backslash x)\cdot((y\backslash e)\cdot x) = y\cdot(x/y)$.

Proof. We have $(K(y\backslash e, y)\cdot((e/y)\cdot x))/L(T(e/y, x), x, K(y\backslash e, y)) = K(y\backslash e, y)\cdot x$ by Theorem 1066. Then $((y\backslash e)\cdot x)/L(T(e/y, x), x, K(y\backslash e, y)) = K(y\backslash e, y)\cdot x$ by Theorem 682. Then

$$((y\backslash e)\cdot x)/T(e/y, x) = K(y\backslash e, y)\cdot x \tag{733}$$

by Theorem 967. We have $((e/y)\cdot T((e/y)\backslash e, (y\backslash e)\cdot x))\cdot x = x\cdot K(((y\backslash e)\cdot x)\backslash(((y\backslash e)\cdot x)/T(e/y, x)), (y\backslash e)\cdot x)$ by Theorem 1115. Then $((e/y)\cdot T((e/y)\backslash e, (y\backslash e)\cdot x))\cdot x = x\cdot K(((y\backslash e)\cdot x)\backslash(K(y\backslash e, y)\cdot x), (y\backslash e)\cdot x)$ by (733). Then

$$x\cdot K(((y\backslash e)\cdot x)\backslash(K(y\backslash e, y)\cdot x), (y\backslash e)\cdot x) = (y\backslash T(y, (y\backslash e)\cdot x))\cdot x \tag{734}$$

by Theorem 1199. We have $(((y\backslash e)\cdot x)\cdot K(y, y\backslash e))\backslash x = ((y\backslash e)\cdot x)\backslash(K(y\backslash e, y)\cdot x)$ by Theorem 1223. Then $((e/y)\cdot x)\backslash x = ((y\backslash e)\cdot x)\backslash(K(y\backslash e, y)\cdot x)$ by Theorem 1186. Then $x\cdot K(((e/y)\cdot x)\backslash x, (y\backslash e)\cdot x) = (y\backslash T(y, (y\backslash e)\cdot x))\cdot x$ by (734). Then

$$(y\backslash T(y, (y\backslash e)\cdot x))\cdot x = x\cdot K(((y\backslash e)\cdot x)\backslash x, (y\backslash e)\cdot x) \tag{735}$$

by Theorem 1213. We have $x\cdot K(((y\backslash e)\cdot x)\backslash x, (y\backslash e)\cdot x) = (((y\backslash e)\cdot x)\backslash x)\cdot((y\backslash e)\cdot x)$ by Theorem 17. Then $(y\backslash T(y, (y\backslash e)\cdot x))\cdot x = (((y\backslash e)\cdot x)\backslash x)\cdot((y\backslash e)\cdot x)$ by (735). Then

$$(((y\backslash e)\cdot x)\backslash x)\cdot((y\backslash e)\cdot x) = (y\backslash T(y, y\backslash x))\cdot x \tag{736}$$

by Theorem 1302. We have $T(y, y\backslash x)\cdot(y\backslash x) = y\cdot(x/y)$ by Theorem 1245. Then $(y\backslash T(y, y\backslash x))\cdot x = y\cdot(x/y)$ by Theorem 1270. Hence we are done by (736). □

Theorem 1304. $y\backslash(x/(x/y)) = K(y\backslash x, y)$.

Proof. We have $K(x\backslash e, R(x, y\backslash x, y)) = K(x\backslash e, x)$ by Theorem 966. Then

$$K(x\backslash e, R(x, y\backslash x, y)) = R(K(x\backslash e, x), y\backslash x, (y\backslash x)\backslash x) \tag{737}$$

by Theorem 1230. We have $(x \cdot R(K(x\backslash e, x), y\backslash x, (y\backslash x)\backslash x))/((y\backslash x)\backslash x) = ((x \cdot K(x\backslash e, x))/x) \cdot (y\backslash x)$ by Theorem 828. Then $(x \cdot R(K(x\backslash e, x), y\backslash x, (y\backslash x)\backslash x))/((y\backslash x)\backslash x) = ((e/(e/x))/x) \cdot (y\backslash x)$ by Theorem 262. Then $(x \cdot K(x\backslash e, R(x, y\backslash x, y)))/((y\backslash x)\backslash x) = ((e/(e/x))/x) \cdot (y\backslash x)$ by (737). Then $L(x, x\backslash e, R(x, y\backslash x, y))/((y\backslash x)\backslash x) = ((e/(e/x))/x) \cdot (y\backslash x)$ by Theorem 1220. Then

$$L(x, x\backslash e, R(x, y\backslash x, y))/((y\backslash x)\backslash x) = K(x\backslash e, x) \cdot (y\backslash x) \tag{738}$$

by Theorem 1109. We have $x \cdot K(x\backslash e, R(x, y\backslash x, y)) = L(x, x\backslash e, R(x, y\backslash x, y))$ by Theorem 1220. Then

$$x \cdot K(x\backslash e, x) = L(x, x\backslash e, R(x, y\backslash x, y)) \tag{739}$$

by Theorem 966. We have $x \cdot K(x\backslash e, x) = e/(e/x)$ by Theorem 262. Then $L(x, x\backslash e, R(x, y\backslash x, y)) = e/(e/x)$ by (739). Then $(e/(e/x))/((y\backslash x)\backslash x) = K(x\backslash e, x) \cdot (y\backslash x)$ by (738). Then $(e/(e/x))/T(y, y\backslash x) = K(x\backslash e, x) \cdot (y\backslash x)$ by Proposition 49. Then

$$y\backslash(e/(e/x)) = K(x\backslash e, x) \cdot (y\backslash x) \tag{740}$$

by Theorem 1266.

$$\begin{array}{lll} & K(x\backslash e, x) \cdot (K(x, x\backslash e) \cdot ((K(x\backslash e, x) \cdot (y\backslash x)) \cdot y)) & \\ = & (K(x\backslash e, x) \cdot (y\backslash x)) \cdot y & \text{by Theorem 693} \\ = & K(x\backslash e, x) \cdot ((y\backslash x) \cdot y) & \text{by Theorem 968.} \end{array}$$

Then $K(x\backslash e, x) \cdot (K(x, x\backslash e) \cdot ((K(x\backslash e, x) \cdot (y\backslash x)) \cdot y)) = K(x\backslash e, x) \cdot ((y\backslash x) \cdot y)$. Then $K(x, x\backslash e) \cdot ((K(x\backslash e, x) \cdot (y\backslash x)) \cdot y) = (y\backslash x) \cdot y$ by Proposition 9. Then

$$K(x, x\backslash e) \cdot ((y\backslash(e/(e/x))) \cdot y) = (y\backslash x) \cdot y \tag{741}$$

by (740). We have $K(x, x\backslash e) \cdot ((y\backslash(e/(e/x))) \cdot y) = x \cdot K(y\backslash(e/(e/x)), y)$ by Theorem 1164. Then

$$(y\backslash x) \cdot y = x \cdot K(y\backslash(e/(e/x)), y) \tag{742}$$

by (741). We have

$$(K(x, x\backslash e) \cdot ((y\backslash(e/(e/x))) \cdot y)) \cdot (x\backslash e) = ((y\backslash(e/(e/x))) \cdot y) \cdot (e/x) \tag{743}$$

by Theorem 1191.

$$\begin{array}{lll} & R((y\backslash(e/(e/x))) \cdot y, e/x, (e/x)\backslash e)/((e/x)\backslash e) & \\ = & ((y\backslash(e/(e/x))) \cdot y) \cdot (e/x) & \text{by Theorem 73} \\ = & (x \cdot K(y\backslash(e/(e/x)), y)) \cdot (x\backslash e) & \text{by (743), Theorem 1164.} \end{array}$$

Then $R((y\backslash(e/(e/x))) \cdot y, e/x, (e/x)\backslash e)/((e/x)\backslash e) = (x \cdot K(y\backslash(e/(e/x)), y)) \cdot (x\backslash e)$. Then $R((y\backslash(e/(e/x))) \cdot y, e/x, x)/((e/x)\backslash e) = (x \cdot K(y\backslash(e/(e/x)), y)) \cdot (x\backslash e)$ by Proposition 25. Then $R((y\backslash(e/(e/x))) \cdot y, x\backslash e, x)/((e/x)\backslash e) = (x \cdot K(y\backslash(e/(e/x)), y)) \cdot (x\backslash e)$ by Theorem 1151. Then $((y\backslash(e \cdot x)) \cdot y)/((e/x)\backslash e) = (x \cdot K(y\backslash(e/(e/x)), y)) \cdot (x\backslash e)$ by Theorem 974. Then $(x \cdot K(y\backslash(e/(e/x)), y)) \cdot (x\backslash e) = ((y\backslash x) \cdot y)/((e/x)\backslash e)$ by Axiom 1. Then $(x \cdot K(y\backslash(e/(e/x)), y)) \cdot (x\backslash e) = ((y\backslash x) \cdot y)/x$ by Proposition 25. Then

$$((y\backslash x) \cdot y) \cdot (x\backslash e) = ((y\backslash x) \cdot y)/x \tag{744}$$

by (742). We have $((y\backslash x)\cdot y)\cdot(x\backslash e)=(e/x)\cdot((y\backslash x)\cdot y)$ by Theorem 1268. Then

$$((y\backslash x)\cdot y)/x=(e/x)\cdot((y\backslash x)\cdot y) \tag{745}$$

by (744). We have $(((y\backslash e)\cdot x)\backslash T((y\backslash e)\cdot x,((y\backslash e)\cdot x)\backslash x))\backslash x=((y\backslash e)\cdot x)\cdot(T((y\backslash e)\cdot x,((y\backslash e)\cdot x)\backslash x)\backslash x)$ by Theorem 1272. Then

$$(((y\backslash e)\cdot x)\backslash T((y\backslash e)\cdot x,((y\backslash e)\cdot x)\backslash x))\backslash x=((y\backslash e)\cdot x)\cdot T(((y\backslash e)\cdot x)\backslash x,(y\backslash e)\cdot x) \tag{746}$$

by Theorem 1276. We have $((y\backslash e)\cdot x)\cdot T(((y\backslash e)\cdot x)\backslash x,(y\backslash e)\cdot x)=(((y\backslash e)\cdot x)\backslash x)\cdot((y\backslash e)\cdot x)$ by Proposition 46. Then $(((y\backslash e)\cdot x)\backslash T((y\backslash e)\cdot x,((y\backslash e)\cdot x)\backslash x))\backslash x=(((y\backslash e)\cdot x)\backslash x)\cdot((y\backslash e)\cdot x)$ by (746). Then

$$(((y\backslash e)\cdot x)\backslash((((y\backslash e)\cdot x)\backslash x)\backslash x))\backslash x=(((y\backslash e)\cdot x)\backslash x)\cdot((y\backslash e)\cdot x) \tag{747}$$

by Proposition 49. We have $x/((((y\backslash e)\cdot x)\backslash((((y\backslash e)\cdot x)\backslash x)\backslash x))\backslash x)=((y\backslash e)\cdot x)\backslash((((y\backslash e)\cdot x)\backslash x)\backslash x)$ by Proposition 24. Then

$$x/((((y\backslash e)\cdot x)\backslash x)\cdot((y\backslash e)\cdot x))=((y\backslash e)\cdot x)\backslash((((y\backslash e)\cdot x)\backslash x)\backslash x) \tag{748}$$

by (747). We have $(((y\backslash e)\cdot x)\cdot(x/((y\backslash e)\cdot x)))/x=((y\backslash e)\cdot x)\backslash((((y\backslash e)\cdot x)\backslash x)\backslash x)$ by Theorem 1024. Then $(((y\backslash e)\cdot x)\cdot(x/((y\backslash e)\cdot x)))/x=x/((((y\backslash e)\cdot x)\backslash x)\cdot((y\backslash e)\cdot x))$ by (748). Then $T(((y\backslash e)\cdot x)\cdot(x/((y\backslash e)\cdot x)),x)/x=x/((((y\backslash e)\cdot x)\backslash x)\cdot((y\backslash e)\cdot x))$ by Theorem 1291. Then $T((y\backslash x)\cdot y,x)/x=x/((((y\backslash e)\cdot x)\backslash x)\cdot((y\backslash e)\cdot x))$ by Theorem 1300. Then $x/(y\cdot(x/y))=T((y\backslash x)\cdot y,x)/x$ by Theorem 1303. Then

$$((y\backslash x)\cdot y)/x=x/(y\cdot(x/y)) \tag{749}$$

by Theorem 1290. We have $(x/y)\backslash T(x/y,y)=y\backslash(x/(x/y))$ by Theorem 987. Then

$$T(y,x/y)\backslash y=y\backslash(x/(x/y)) \tag{750}$$

by Theorem 1030. We have $((x/y)\cdot y)/(y\cdot(x/y))=T(y,x/y)\backslash y$ by Theorem 1023. Then $x/(y\cdot(x/y))=T(y,x/y)\backslash y$ by Axiom 6. Then $y\backslash(x/(x/y))=x/(y\cdot(x/y))$ by (750). Then $((y\backslash x)\cdot y)/x=y\backslash(x/(x/y))$ by (749). Then

$$(e/x)\cdot((y\backslash x)\cdot y)=y\backslash(x/(x/y)) \tag{751}$$

by (745). We have $L((x\backslash e)\backslash((e/x)\cdot((y\backslash x)\cdot y)),x\backslash e,x)=x\cdot((e/x)\cdot((y\backslash x)\cdot y))$ by Theorem 62. Then $L(K(x,x\backslash e)\cdot((y\backslash x)\cdot y),x\backslash e,x)=x\cdot((e/x)\cdot((y\backslash x)\cdot y))$ by Theorem 1166. Then $L((y\backslash((x\backslash e)\backslash e))\cdot y,x\backslash e,x)=x\cdot((e/x)\cdot((y\backslash x)\cdot y))$ by Theorem 1265. Then

$$L((y\backslash((x\backslash e)\backslash e))\cdot y,x\backslash e,x)=x\cdot(y\backslash(x/(x/y))) \tag{752}$$

by (751). We have $L((y\backslash((x\backslash e)\backslash e))\cdot y,x\backslash e,x)=(y\backslash L((x\backslash e)\backslash e,x\backslash e,x))\cdot y$ by Theorem 1299. Then $L((y\backslash((x\backslash e)\backslash e))\cdot y,x\backslash e,x)=(y\backslash(e\backslash(x\cdot e)))\cdot y$ by Theorem 60. Then $x\cdot(y\backslash(x/(x/y)))=(y\backslash(e\backslash(x\cdot e)))\cdot y$ by (752). Then $(y\backslash(e\backslash x))\cdot y=x\cdot(y\backslash(x/(x/y)))$ by Axiom 2. Then

$$x\cdot(y\backslash(x/(x/y)))=(y\backslash x)\cdot y \tag{753}$$

by Proposition 26. We have $x\cdot K(y\backslash x,y)=(y\backslash x)\cdot y$ by Theorem 17. Hence we are done by (753) and Proposition 7. □

Theorem 1305. *$T(K(y\backslash x,y),x)=K(y\backslash x,y)$.*

Proof. We have $K(y\backslash x,y)=T(T(y,y\backslash x)\backslash y,y\cdot(y\backslash x))$ by Theorem 1273. Then $K(y\backslash x,y)=T(T(y,y\backslash x)\backslash y,x)$ by Axiom 4. Then $T(y\backslash(x/(x/y)),x)=K(y\backslash x,y)$ by Theorem 1010. Hence we are done by Theorem 1304. □

Theorem 1306. *$T(x\cdot y,K(y,x))=x\cdot y$.*

Proof. We have $T(K(x\backslash(x\cdot y),x),x\cdot y)=K(x\backslash(x\cdot y),x)$ by Theorem 1305. Then $T(x\cdot y,K(x\backslash(x\cdot y),x))=x\cdot y$ by Proposition 21. Hence we are done by Axiom 3. □

Theorem 1307. $T(y,x) \cdot K(x,y) = y$.

Proof. We have $x \cdot (x \backslash (K(x,y) \cdot (y \cdot x))) = K(x,y) \cdot (y \cdot x)$ by Axiom 4. Then $x \cdot (T(y,x) \cdot K(x,y)) = K(x,y) \cdot (y \cdot x)$ by Theorem 1274. Then $x \cdot (T(y,x) \cdot K(x,y)) = y \cdot (x \cdot K(x,y))$ by Theorem 1280. Then

$$x \cdot (T(y,x) \cdot K(x,y)) = T(x \cdot y, K(y,x)) \tag{754}$$

by Theorem 1284. We have $T(x \cdot y, K(y,x)) = x \cdot y$ by Theorem 1306. Then $x \cdot (T(y,x) \cdot K(x,y)) = x \cdot y$ by (754). Hence we are done by Proposition 9. □

Theorem 1308. $x \backslash T(x,y) = K(x,y)$.

Proof.

$$\begin{array}{cll} & T(y,x) \cdot (x \backslash T(x,y)) & \\ = & y & \text{by Theorem 1031} \\ = & T(y,x) \cdot K(x,y) & \text{by Theorem 1307.} \end{array}$$

Then $T(y,x) \cdot (x \backslash T(x,y)) = T(y,x) \cdot K(x,y)$. Hence we are done by Proposition 9. □

Theorem 1309. $T(x,y) = x \cdot K(x,y)$.

Proof. We have $x \backslash T(x,y) = K(x,y)$ by Theorem 1308. Hence we are done by Proposition 5. □

Theorem 1310. $(x \cdot K(x,y))/x = K(x,y)$.

Proof. We have $(x \cdot T(K(x,y),x))/x = K(x,y)$ by Proposition 48. Hence we are done by Theorem 734. □

Theorem 1311. $K(y,z) \cdot T(y,x) = T(y,x) \cdot K(y,z)$.

Proof. We have $K(y,z) \cdot T(T(y,K(y,z)),x) = T(y,x) \cdot K(y,z)$ by Proposition 50. Hence we are done by Theorem 736. □

Theorem 1312. $T(x,((y \cdot x) \backslash x) \backslash e) = x \cdot K(x,y)$.

Proof. We have $T(x,((y \cdot x) \backslash x) \backslash e) = x \cdot K(x,(y \cdot x)/x)$ by Theorem 1001. Hence we are done by Axiom 5. □

Theorem 1313. $K(T(x,T(y,x)),T(y,x)) = K(x,y)$.

Proof. We have $K(T(x,T(y,x)),T(y,x)) = K(x,(x \cdot T(y,x))/x)$ by Theorem 1008. Hence we are done by Proposition 48. □

Theorem 1314. $x \cdot ((T(y,x) \backslash y) \cdot z) = T(x,y) \cdot z$.

Proof. We have $x \cdot ((x \backslash T(x,y)) \cdot z) = T(x,y) \cdot z$ by Theorem 1017. Hence we are done by Theorem 1030. □

Theorem 1315. $R(T(y,x),x,K(x,y)) = K(x,y) \backslash y$.

Proof. We have $T(x,((y \cdot x) \backslash x) \backslash e) \backslash (x \cdot y) = x \cdot (T(x,((y \cdot x) \backslash x) \backslash e) \backslash y)$ by Theorem 174. Then $(x \cdot K(x,y)) \backslash (x \cdot y) = x \cdot (T(x,((y \cdot x) \backslash x) \backslash e) \backslash y)$ by Theorem 1312. Then

$$x \cdot ((x \cdot K(x,y)) \backslash y) = (x \cdot K(x,y)) \backslash (x \cdot y) \tag{755}$$

by Theorem 1312. We have $(x \cdot K(x,y)) \backslash ((y \cdot x) \cdot K(x,y)) = R(T(y,x \cdot K(x,y)),x,K(x,y))$ by Proposition 81. Then $(x \cdot K(x,y)) \backslash (x \cdot y) = R(T(y,x \cdot K(x,y)),x,K(x,y))$ by Proposition 82. Then

$$R(T(y,x \cdot K(x,y)),x,K(x,y)) = x \cdot ((x \cdot K(x,y)) \backslash y) \tag{756}$$

by (755). We have $y \cdot ((T(x,y)\backslash x) \cdot (T(x,y) \cdot K(x,y))) = T(y,x) \cdot (T(x,y) \cdot K(x,y))$ by Theorem 1314. Then $y \cdot (x \cdot K(x,y)) = T(y,x) \cdot (T(x,y) \cdot K(x,y))$ by Theorem 1282. Then

$$T(y,x) \cdot (K(x,y) \cdot T(x,y)) = y \cdot (x \cdot K(x,y)) \tag{757}$$

by Theorem 1311.

$$\begin{array}{lcl} & T(y, x \cdot K(x,y)) \cdot T(x \cdot K(x,y), y) & \\ = & y \cdot (x \cdot K(x,y)) & \text{by Theorem 1033} \\ = & T(y,x) \cdot T(x \cdot K(x,y), y) & \text{by (757), Theorem 1049.} \end{array}$$

Then $T(y, x \cdot K(x,y)) \cdot T(x \cdot K(x,y), y) = T(y,x) \cdot T(x \cdot K(x,y), y)$. Then $T(y, x \cdot K(x,y)) = T(y,x)$ by Proposition 10. Then

$$R(T(y,x), x, K(x,y)) = x \cdot ((x \cdot K(x,y))\backslash y) \tag{758}$$

by (756). We have $R((x \cdot K(x,y))/x, x, (x \cdot K(x,y))\backslash y)\backslash y = x \cdot ((x \cdot K(x,y))\backslash y)$ by Theorem 1065. Then $R(K(x,y), x, (x \cdot K(x,y))\backslash y)\backslash y = x \cdot ((x \cdot K(x,y))\backslash y)$ by Theorem 1310. Then $K(x,y)\backslash y = x \cdot ((x \cdot K(x,y))\backslash y)$ by Theorem 1025. Hence we are done by (758). □

Theorem 1316. $K(x, T(y,x)) = K(x,y)$.

Proof. We have $K(T(x, T(y,x)), T(y,x)) = K(x,y)$ by Theorem 1313. Hence we are done by Theorem 1053. □

Theorem 1317. $R(T(x,y), y, K(y,x)) = x \cdot K(x,y)$.

Proof. We have $R(T(x,y), y, K(y,x)) = K(y,x)\backslash x$ by Theorem 1315. Hence we are done by Theorem 1283. □

Theorem 1318. $R(y, x, K(x,y)) = y$.

Proof. We have $x \cdot R(T(y,x), x, K(x,y)) = R(y, x, K(x,y)) \cdot x$ by Proposition 59. Then

$$x \cdot (y \cdot K(y,x)) = R(y, x, K(x,y)) \cdot x \tag{759}$$

by Theorem 1317. We have $T(y \cdot x, K(x,y)) = x \cdot (y \cdot K(y,x))$ by Theorem 1284. Then

$$T(y \cdot x, K(x,y)) = R(y, x, K(x,y)) \cdot x \tag{760}$$

by (759). We have $T(y \cdot x, K(x,y)) = y \cdot x$ by Theorem 1306. Then $R(y, x, K(x,y)) \cdot x = y \cdot x$ by (760). Hence we are done by Proposition 10. □

Theorem 1319. $T(y,x) = K(x,y)\backslash y$.

Proof. We have $R(T(y,x), x, K(x,y)) = K(x,y)\backslash y$ by Theorem 1315. Then $R(T(y,x), x, K(x, T(y,x))) = K(x,y)\backslash y$ by Theorem 1316. Hence we are done by Theorem 1318. □

Theorem 1320. $y/T(y,x) = K(x,y)$.

Proof. We have $y/(K(x,y)\backslash y) = K(x,y)$ by Proposition 24. Hence we are done by Theorem 1319. □

Theorem 1321. $L(z, T(x,y), y) = (x \cdot y)\backslash(y \cdot (T(x,y) \cdot z))$.

Proof. We have $L(z, T(x,y), y) = (y \cdot T(x,y))\backslash(y \cdot (T(x,y) \cdot z))$ by Definition 4. Hence we are done by Proposition 46. □

Theorem 1322. $y \cdot T(y,x) = T(y,x) \cdot y$.

Proof. We have $y \cdot T(T(y,x),y) = T(y,x) \cdot y$ by Proposition 46. Hence we are done by Theorem 451. ☐

Theorem 1323. $R(z\backslash e, x, y) = R(R(e/z, x, y), z, z\backslash e)$.

Proof. We have $R(R(e/z, z, z\backslash e), x, y) = R(R(e/z, x, y), z, z\backslash e)$ by Axiom 12. Hence we are done by Theorem 41. ☐

Theorem 1324. $x/L(z\backslash y, z, x/y) = (x/y) \cdot z$.

Proof. We have $x/(((x/y) \cdot z)\backslash((x/y) \cdot y)) = (x/y) \cdot z$ by Theorem 24. Hence we are done by Proposition 53. ☐

Theorem 1325. $R(R(e/x, x, y), x \cdot y, z) = (y \cdot z)/((x \cdot y) \cdot z)$.

Proof. We have $R(y/(x \cdot y), x \cdot y, z) = (y \cdot z)/((x \cdot y) \cdot z)$ by Proposition 55. Hence we are done by Proposition 79. ☐

Theorem 1326. $T(x \cdot y, y\backslash e) = (y\backslash e)\backslash R(x, y, y\backslash e)$.

Proof. We have $T(x \cdot y, y\backslash e) = (y\backslash e)\backslash((x \cdot y) \cdot (y\backslash e))$ by Definition 3. Hence we are done by Proposition 68. ☐

Theorem 1327. $L((e/w) \cdot x, y, z) = L(L(w\backslash x, y, z), w, e/w)$.

Proof. We have $L(L(w\backslash x, w, e/w), y, z) = L(L(w\backslash x, y, z), w, e/w)$ by Axiom 11. Hence we are done by Theorem 63. ☐

Theorem 1328. $L(x, z, y) \cdot (y \cdot z) = y \cdot (z \cdot T(x, y \cdot z))$.

Proof. We have $(y \cdot z) \cdot L(T(x, y \cdot z), z, y) = y \cdot (z \cdot T(x, y \cdot z))$ by Proposition 52. Hence we are done by Proposition 58. ☐

Theorem 1329. $T(x \cdot (y\backslash e), y) = R(y\backslash x, y, y\backslash e)$.

Proof. We have $T(R(x/y, y, y\backslash e), y) = R(y\backslash x, y, y\backslash e)$ by Proposition 60. Hence we are done by Proposition 71. ☐

Theorem 1330. $(x \cdot (((x\backslash y)\backslash z)\backslash z))/x = T(y/x, (x\backslash y)\backslash z)$.

Proof. We have $(x \cdot T(x\backslash y, (x\backslash y)\backslash z))/x = T(y/x, (x\backslash y)\backslash z)$ by Theorem 49. Hence we are done by Proposition 49. ☐

Theorem 1331. $R(x, y, y\backslash e) = T((y \cdot x) \cdot (y\backslash e), y)$.

Proof. We have $R(y\backslash(y \cdot x), y, y\backslash e) = T((y \cdot x) \cdot (y\backslash e), y)$ by Theorem 1329. Hence we are done by Axiom 3. ☐

Theorem 1332. $R((z \cdot (x/y))/z, y, y\backslash e) = (z \cdot (x \cdot (y\backslash e)))/z$.

Proof. We have $R((z \cdot (x/y))/z, y, y\backslash e) = (z \cdot R(x/y, y, y\backslash e))/z$ by Theorem 123. Hence we are done by Proposition 71. ☐

Theorem 1333. $T((e/y) \cdot x, y) = (e/y) \cdot ((y\backslash x) \cdot y)$.

Proof. We have $L(T(y\backslash x, y), y, e/y) = (e/y) \cdot ((y\backslash x) \cdot y)$ by Theorem 72. Hence we are done by Theorem 1072. ☐

Theorem 1334. $((x \cdot y)/x)/T(y/x, T(x, y)) = T(x, y)$.

Proof. We have $((y/x) \cdot T(x, y))/T(y/x, T(x, y)) = T(x, y)$ by Theorem 5. Hence we are done by Theorem 818. ☐

Theorem 1335. $(e/x)\backslash T(y,z) = x \cdot T(x\backslash((e/x)\backslash y), z)$.

Proof. We have $(e/x)\backslash T((e/x) \cdot ((e/x)\backslash y), z) = x \cdot T(x\backslash((e/x)\backslash y), z)$ by Theorem 1080. Hence we are done by Axiom 4. □

Theorem 1336. $(e/y)\backslash T(x,y) = (y\backslash((e/y)\backslash x)) \cdot y$.

Proof. We have $y \cdot T(y\backslash((e/y)\backslash x), y) = (y\backslash((e/y)\backslash x)) \cdot y$ by Proposition 46. Hence we are done by Theorem 1335. □

Theorem 1337. $(y \cdot T(x,z)) \cdot (y\backslash e) = T((y \cdot x) \cdot (y\backslash e), z)$.

Proof. We have $(y \cdot T(y\backslash(y \cdot x), z)) \cdot (y\backslash e) = T((y \cdot x) \cdot (y\backslash e), z)$ by Theorem 506. Hence we are done by Axiom 3. □

Theorem 1338. $(e/x)\backslash L((e/x) \cdot y, z, w) = x \cdot L(x\backslash y, z, w)$.

Proof. We have $(e/x)\backslash L(L(x\backslash y, z, w), x, e/x) = x \cdot L(x\backslash y, z, w)$ by Theorem 71. Hence we are done by Theorem 1327. □

Theorem 1339. $x\backslash(T(x,y)\backslash x) = T(x,y)\backslash e$.

Proof. We have $x \cdot (T(x,y)\backslash e) = T(x,y)\backslash x$ by Theorem 175. Hence we are done by Proposition 2. □

Theorem 1340. $((y \cdot x)/y)\backslash T(y,x) = x\backslash y$.

Proof. We have $((y \cdot x)/y) \cdot (x\backslash y) = T(y,x)$ by Theorem 179. Hence we are done by Proposition 2. □

Theorem 1341. $L(y \cdot (x\backslash e), z, w)/(x\backslash e) = x \cdot L(x\backslash y, z, w)$.

Proof. We have $R(L(y/x, z, w), x, x\backslash e) = L(R(y/x, x, x\backslash e), z, w)$ by Axiom 10. Then

$$R(L(y/x, z, w), x, x\backslash e) = L(y \cdot (x\backslash e), z, w) \tag{761}$$

by Proposition 71. We have $R(L(y/x, z, w), x, x\backslash e)/(x\backslash e) = L(y/x, z, w) \cdot x$ by Theorem 73. Then

$$L(y \cdot (x\backslash e), z, w)/(x\backslash e) = L(y/x, z, w) \cdot x \tag{762}$$

by (761). We have $L(y/x, z, w) \cdot x = x \cdot L(x\backslash y, z, w)$ by Theorem 85. Hence we are done by (762). □

Theorem 1342. $K(x, T(y, y\backslash z)) = K(x, (y\backslash z)\backslash z)$.

Proof. We have $K(x, T(y, y\backslash z)) = (T(y, y\backslash z) \cdot x)\backslash(x \cdot T(y, y\backslash z))$ by Definition 2. Then

$$K(x, T(y, y\backslash z)) = (T(y, y\backslash z) \cdot x)\backslash(x \cdot ((y\backslash z)\backslash z)) \tag{763}$$

by Theorem 806. We have $(T(y, y\backslash z) \cdot x)\backslash(x \cdot ((y\backslash z)\backslash z)) = K(x, (y\backslash z)\backslash z)$ by Theorem 478. Hence we are done by (763). □

Theorem 1343. $(x/z) \cdot T(z,y) = (x \cdot T(z,y))/z$.

Proof. We have $(x \cdot T(z,y))/T(z,y) = x$ by Axiom 5. Then

$$(((x \cdot T(z,y))/z) \cdot z)/T(z,y) = x \tag{764}$$

by Axiom 6. We have $((((x \cdot T(z,y))/z) \cdot z)/T(z,y)) \cdot T(z,y) = ((x \cdot T(z,y))/z) \cdot z$ by Axiom 6. Then $((((((x \cdot T(z,y))/z) \cdot z)/T(z,y))/z) \cdot z) \cdot T(z,y) = ((x \cdot T(z,y))/z) \cdot z$ by Axiom 6. Then

$$((x/z) \cdot z) \cdot T(z,y) = ((x \cdot T(z,y))/z) \cdot z \tag{765}$$

by (764). We have $((x/z) \cdot T(z,y)) \cdot z = ((x/z) \cdot z) \cdot T(z,y)$ by Theorem 238. Hence we are done by (765) and Proposition 8. □

Theorem 1344. $((x/y) \cdot T(y,z)) \cdot y = x \cdot T(y,z)$.

Proof. We have $((x/y) \cdot T(y,z)) \cdot y = ((x/y) \cdot y) \cdot T(y,z)$ by Theorem 238. Hence we are done by Axiom 6. □

Theorem 1345. $(e/T(y,x))\backslash(y/T(y,x)) = y$.

Proof. We have $(e/T(y,x)) \cdot y = y/T(y,x)$ by Theorem 550. Hence we are done by Proposition 2. □

Theorem 1346. $((x/z) \cdot T(z,y))\backslash(x \cdot T(z,y)) = z$.

Proof. We have $((x/z) \cdot T(z,y))\backslash(((x/z) \cdot z) \cdot T(z,y)) = z$ by Theorem 1105. Hence we are done by Axiom 6. □

Theorem 1347. $x/y = ((x/T(y,z))/y) \cdot T(y,z)$.

Proof. We have $((x/T(y,z)) \cdot T(y,z))/y = ((x/T(y,z))/y) \cdot T(y,z)$ by Theorem 1343. Hence we are done by Axiom 6. □

Theorem 1348. $(x/(y\backslash z)) \cdot (z/y) = (x \cdot (z/y))/(y\backslash z)$.

Proof. We have $(x \cdot (z/y))/(z/y) = x$ by Axiom 5. Then

$$(((x \cdot (z/y))/(y\backslash z)) \cdot (y\backslash z))/(z/y) = x \tag{766}$$

by Axiom 6. We have $((((x\cdot(z/y))/(y\backslash z))\cdot(y\backslash z))/(z/y))\cdot(z/y) = ((x\cdot(z/y))/(y\backslash z))\cdot(y\backslash z)$ by Axiom 6. Then $((((((x\cdot(z/y))/(y\backslash z))\cdot(y\backslash z))/(z/y))/(y\backslash z))\cdot(y\backslash z))\cdot(z/y) = ((x\cdot(z/y))/(y\backslash z))\cdot(y\backslash z)$ by Axiom 6. Then

$$((x/(y\backslash z)) \cdot (y\backslash z)) \cdot (z/y) = ((x \cdot (z/y))/(y\backslash z)) \cdot (y\backslash z) \tag{767}$$

by (766). We have $((x/(y\backslash z)) \cdot (z/y)) \cdot (y\backslash z) = ((x/(y\backslash z)) \cdot (y\backslash z)) \cdot (z/y)$ by Theorem 241. Hence we are done by (767) and Proposition 8. □

Theorem 1349. $T(y,x) \cdot K(y, y\backslash e) = y \cdot (T(y,x) \cdot (e/y))$.

Proof. We have $T(y,x) \cdot K(y, e/y) = y \cdot (T(y,x) \cdot (e/y))$ by Theorem 176. Hence we are done by Theorem 780. □

Theorem 1350. $T(y, T(y,x)\backslash e)/y = T(y,x) \cdot ((y \cdot (T(y,x)\backslash e))/y)$.

Proof. We have $(x \cdot T(T(y,x), e/T(y,x)))/x = T((x \cdot T(y,x))/x, e/T(y,x))$ by Theorem 50. Then

$$(x \cdot T(T(y,x), T(y,x)\backslash e))/x = T((x \cdot T(y,x))/x, e/T(y,x)) \tag{768}$$

by Theorem 211. We have $(x \cdot T(T(y,x), T(y,x)\backslash e))/x = T((x \cdot T(y,x))/x, T(y,x)\backslash e)$ by Theorem 50. Then $T((x \cdot T(y,x))/x, e/T(y,x)) = T((x \cdot T(y,x))/x, T(y,x)\backslash e)$ by (768). Then $T(y, e/T(y,x)) = T((x \cdot T(y,x))/x, T(y,x)\backslash e)$ by Proposition 48. Then

$$T(y, T(y,x)\backslash e) = T(y, e/T(y,x)) \tag{769}$$

by Proposition 48. We have $((e/T(y,x))\backslash(y/T(y,x)))\cdot(e/T(y,x)) = T(y, e/T(y,x))/T(y,x)$ by Theorem 152. Then $y \cdot (e/T(y,x)) = T(y, e/T(y,x))/T(y,x)$ by Theorem 1345. Then

$$T(y, T(y,x)\backslash e)/T(y,x) = y \cdot (e/T(y,x)) \tag{770}$$

by (769). We have $((T(y, T(y,x)\backslash e)/T(y,x))/y) \cdot T(y,x) = T(y, T(y,x)\backslash e)/y$ by Theorem 1347. Then

$$((y \cdot (e/T(y,x)))/y) \cdot T(y,x) = T(y, T(y,x)\backslash e)/y \tag{771}$$

by (770). We have $((y \cdot (e/T(y,x)))/y) \cdot T(y,x) = T(y,x) \cdot ((y \cdot (T(y,x)\backslash e))/y)$ by Theorem 125. Hence we are done by (771). □

Theorem 1351. $T(y,x)\cdot((T(y,x)\backslash y)/y) = T(y,T(y,x)\backslash e)/y$.

Proof. We have $T(y,x)\cdot((y\cdot(T(y,x)\backslash e))/y) = T(y,T(y,x)\backslash e)/y$ by Theorem 1350. Hence we are done by Theorem 175. ☐

Theorem 1352. $K(e/x,y) = L(x\cdot T(x\backslash e,y),e/x,y)$.

Proof. We have
$$L((e/x)\backslash T(e/x,y),e/x,y) = (y\cdot(e/x))\backslash((e/x)\cdot y) \tag{772}$$
by Proposition 73.

$$\begin{aligned} & K(e/x,y) \\ = \quad & (y\cdot(e/x))\backslash((e/x)\cdot y) \quad && \text{by Definition 2} \\ = \quad & L(x\cdot T(x\backslash e,y),e/x,y) \quad && \text{by (772), Theorem 102.} \end{aligned}$$

Hence we are done. ☐

Theorem 1353. $T(x/y,y/x) = ((x/y)\backslash e)\backslash e$.

Proof. We have $T(x/y,(x/y)\backslash e) = ((x/y)\backslash e)\backslash e$ by Proposition 49. Then $T(x/y,y/((x/y)\cdot y)) = ((x/y)\backslash e)\backslash e$ by Theorem 362. Hence we are done by Axiom 6. ☐

Theorem 1354. $K(x,y)\cdot(e/y) = (e/y)\cdot K(x,y)$.

Proof. We have $(((e/y)\cdot K(x,y))/(e/y))\cdot(e/y) = (e/y)\cdot K(x,y)$ by Axiom 6. Hence we are done by Theorem 365. ☐

Theorem 1355. $L(x,y\backslash e,y)/y = (y\backslash e)\cdot(y\cdot(x/y))$.

Proof. We have $L((x/y)\cdot y,y\backslash e,y)/y = (y\backslash e)\cdot(y\cdot(x/y))$ by Theorem 894. Hence we are done by Axiom 6. ☐

Theorem 1356. $K(y,z)\cdot K(z,x) = K(z,x)\cdot K(y,z)$.

Proof. We have $K(y,z)\cdot T(K(z,x),K(y,z)) = K(z,x)\cdot K(y,z)$ by Proposition 46. Hence we are done by Theorem 899. ☐

Theorem 1357. $K(z,w)\cdot T(K(w,x),y) = T(K(w,x),y)\cdot K(z,w)$.

Proof. We have $K(z,w)\cdot T(T(K(w,x),K(z,w)),y) = T(K(w,x),y)\cdot K(z,w)$ by Proposition 50. Hence we are done by Theorem 899. ☐

Theorem 1358. $L(x\backslash T(x,w),y,z) = K(w\backslash(w/L(x\backslash e,y,z)),w)$.

Proof. We have $L((x\backslash e)\cdot T((x\backslash e)\backslash e,w),y,z) = K(w\backslash(w/L(x\backslash e,y,z)),w)$ by Theorem 635. Hence we are done by Theorem 145. ☐

Theorem 1359. $L(K(w\backslash(w/x),w),y,z) = K(w\backslash(w/L(x,y,z)),w)$.

Proof. We have $L(x\cdot T(x\backslash e,w),y,z) = K(w\backslash(w/L(x,y,z)),w)$ by Theorem 635. Hence we are done by Theorem 196. ☐

Theorem 1360. $((w\cdot x)/w)\cdot T(((w\cdot x)/w)\backslash y,z) = (w\cdot(x\cdot T(x\backslash T(y,w),z)))/w$.

Proof. We have $((w\cdot x)\cdot T((w\cdot x)\backslash(y\cdot w),z))/w = ((w\cdot x)/w)\cdot T(((w\cdot x)/w)\backslash y,z)$ by Theorem 1116. Then $((w\cdot x)\cdot T((w\cdot x)\backslash(y\cdot w),z))/w = ((x/w)\cdot T(w,x))\cdot T(((w\cdot x)/w)\backslash y,z)$ by Theorem 818. Then $((x/w)\cdot T(w,x))\cdot T(((w\cdot x)/w)\backslash y,z) = (w\cdot(x\cdot T(x\backslash(w\backslash(y\cdot w)),z)))/w$ by Theorem 136. Then $((x/w)\cdot T(w,x))\cdot T(((w\cdot x)/w)\backslash y,z) = (w\cdot(x\cdot T(x\backslash T(y,w),z)))/w$ by Definition 3. Hence we are done by Theorem 818. ☐

Theorem 1361. $(x/T(y, y\backslash e)) \cdot K(y, y\backslash e) = x/y$.

Proof. We have $((x/T(y, y\backslash e))/y) \cdot T(y, y\backslash e) = x/y$ by Theorem 1347. Hence we are done by Theorem 1142. □

Theorem 1362. $x/(e/y) = (x/(y\backslash e)) \cdot K(y\backslash e, y)$.

Proof. We have $x/L(K(y\backslash e, y)\backslash(y\backslash e), K(y\backslash e, y), x/(y\backslash e)) = (x/(y\backslash e)) \cdot K(y\backslash e, y)$ by Theorem 1324. Then $x/L(e/y, K(y\backslash e, y), x/(y\backslash e)) = (x/(y\backslash e)) \cdot K(y\backslash e, y)$ by Theorem 256. Hence we are done by Theorem 644. □

Theorem 1363. $(x \cdot (y\backslash e))/K(y\backslash e, y) = x \cdot (e/y)$.

Proof. We have $(x \cdot (e/y)) \cdot K(y\backslash e, y) = x \cdot (y\backslash e)$ by Theorem 412. Hence we are done by Proposition 1. □

Theorem 1364. $(x \cdot (y\backslash e)) \cdot K(y, y\backslash e) = x \cdot (e/y)$.

Proof. We have $(x \cdot (y\backslash e))/K(y\backslash e, y) = x \cdot (e/y)$ by Theorem 1363. Hence we are done by Theorem 656. □

Theorem 1365. $(x \cdot y) \cdot K(y\backslash e, y) = x \cdot (e/(e/y))$.

Proof. We have

$$((x \cdot y) \cdot K(y\backslash e, y)) \cdot K(y, y\backslash e) = x \cdot y \tag{773}$$

by Theorem 426. We have $(x \cdot (e/(e/y))) \cdot K(y, y\backslash e) = x \cdot y$ by Theorem 415. Hence we are done by (773) and Proposition 8. □

Theorem 1366. $T(y, x \cdot (y\backslash e)) = T(y, x \cdot (e/y))$.

Proof. We have $T(y, (x \cdot (e/y)) \cdot K(y\backslash e, y)) = T(y, x \cdot (e/y))$ by Theorem 660. Hence we are done by Theorem 412. □

Theorem 1367. $(x \cdot y) \cdot (y\backslash e) = R(x, y, e/y)$.

Proof. We have $R(x, y, e/y) \cdot (y \cdot (e/y)) = (x \cdot y) \cdot (e/y)$ by Proposition 54. Then

$$R(x, y, e/y) \cdot K(y, y\backslash e) = (x \cdot y) \cdot (e/y) \tag{774}$$

by Theorem 250. We have $((x \cdot y) \cdot (y\backslash e)) \cdot K(y, y\backslash e) = (x \cdot y) \cdot (e/y)$ by Theorem 1364. Hence we are done by (774) and Proposition 8. □

Theorem 1368. $T(x \cdot (y\backslash e), K(y, y\backslash e)) = K(y, y\backslash e)\backslash(x \cdot (e/y))$.

Proof. We have $T(x \cdot (y\backslash e), K(y, y\backslash e)) = K(y, y\backslash e)\backslash((x \cdot (y\backslash e)) \cdot K(y, y\backslash e))$ by Definition 3. Hence we are done by Theorem 1364. □

Theorem 1369. $R(x, y, e/y) = R(x, y, y\backslash e)$.

Proof. We have $(x \cdot y) \cdot (y\backslash e) = R(x, y, y\backslash e)$ by Proposition 68. Hence we are done by Theorem 1367. □

Theorem 1370. $R(x, y\backslash e, y) = R(x, y\backslash e, (y\backslash e)\backslash e)$.

Proof. We have $R(x, y\backslash e, e/(y\backslash e)) = R(x, y\backslash e, (y\backslash e)\backslash e)$ by Theorem 1369. Hence we are done by Proposition 24. □

Theorem 1371. $K(x\backslash(x/y), y) = K(y\backslash e, y)$.

Proof. We have $K(((x/y) \cdot y)\backslash(x/y), y) = K(y\backslash e, y)$ by Theorem 665. Hence we are done by Axiom 6. □

Theorem 1372. $L(x\backslash T(x,y),(x\backslash e)\backslash e,y) = K(x,y)$.

Proof. We have $L(x\backslash T(x,y),(x\backslash e)\backslash e,y) = K(y\backslash(y/L(x\backslash e,(x\backslash e)\backslash e,y)),y)$ by Theorem 1358. Then $L(x\backslash T(x,y),(x\backslash e)\backslash e,y) = K(e/(x\backslash e),y)$ by Theorem 650. Hence we are done by Proposition 24. □

Theorem 1373. $((e/(e/x))\backslash y)/y = e/(e/((x\backslash y)/y))$.

Proof. We have $((K(y/(x\backslash y),(x\backslash y)/y)\cdot(x\backslash y))/y)\cdot y = K(y/(x\backslash y),(x\backslash y)/y)\cdot(x\backslash y)$ by Axiom 6. Then $(e/(e/((x\backslash y)/y)))\cdot y = K(y/(x\backslash y),(x\backslash y)/y)\cdot(x\backslash y)$ by Theorem 640. Then $K(x,x\backslash e)\cdot(x\backslash y) = (e/(e/((x\backslash y)/y)))\cdot y$ by Theorem 403. Then $(e/(e/x))\backslash y = (e/(e/((x\backslash y)/y)))\cdot y$ by Theorem 690. Hence we are done by Proposition 1. □

Theorem 1374. $((x\backslash e)\backslash e)\backslash((x\backslash e)\backslash y) = x\backslash((e/x)\backslash y)$.

Proof. We have $((x\backslash e)\backslash e)\backslash(K(x,x\backslash e)\cdot((e/x)\backslash y)) = x\backslash((e/x)\backslash y)$ by Theorem 1152. Hence we are done by Theorem 440. □

Theorem 1375. $(e/x)\cdot((x\backslash e)\backslash y) = K(x,x\backslash e)\cdot y$.

Proof. We have $(e/x)\cdot((e/x)\backslash(K(x,x\backslash e)\cdot y)) = K(x,x\backslash e)\cdot y$ by Axiom 4. Hence we are done by Theorem 692. □

Theorem 1376. $K(x\backslash e,x)\cdot(((x\backslash e)\backslash e)\cdot y) = x\cdot y$.

Proof. We have

$$K(x,x\backslash e)\cdot(K(x\backslash e,x)\cdot(((x\backslash e)\backslash e)\cdot y)) = ((x\backslash e)\backslash e)\cdot y \tag{775}$$

by Theorem 442. We have $K(x,x\backslash e)\cdot(x\cdot y) = ((x\backslash e)\backslash e)\cdot y$ by Theorem 679. Hence we are done by (775) and Proposition 7. □

Theorem 1377. $K(y,y\backslash e)\cdot(x\cdot K(y\backslash e,y)) = T(x,K(y\backslash e,y))$.

Proof. We have $K(y,y\backslash e)\cdot(K(y\backslash e,y)\cdot T(x,K(y\backslash e,y))) = T(x,K(y\backslash e,y))$ by Theorem 442. Hence we are done by Proposition 46. □

Theorem 1378. $K(y\backslash e,y)\cdot(x\cdot((y\backslash e)\backslash e)) = y\cdot T(x,(y\backslash e)\backslash e)$.

Proof. We have $K(y\backslash e,y)\cdot(((y\backslash e)\backslash e)\cdot T(x,(y\backslash e)\backslash e)) = y\cdot T(x,(y\backslash e)\backslash e)$ by Theorem 1376. Hence we are done by Proposition 46. □

Theorem 1379. $(((e/x)\backslash y)/y)\backslash e = e/(((x\backslash e)\backslash y)/y)$.

Proof. We have $(e/(e/(((x\backslash e)\backslash y)/y)))\backslash e = e/(((x\backslash e)\backslash y)/y)$ by Proposition 25. Then $(((e/(e/(x\backslash e)))\backslash y)/y)\backslash e = e/(((x\backslash e)\backslash y)/y)$ by Theorem 1373. Hence we are done by Proposition 24. □

Theorem 1380. $((((e/x)\backslash y)/y)\backslash e)\backslash e = ((x\backslash e)\backslash y)/y$.

Proof. We have $(e/(((x\backslash e)\backslash y)/y))\backslash e = ((x\backslash e)\backslash y)/y$ by Proposition 25. Hence we are done by Theorem 1379. □

Theorem 1381. $R(x\backslash e,x,y) = y/(T(x,x\backslash e)\cdot y)$.

Proof. We have $R(x\backslash e,x,y) = y/(((x\backslash e)\backslash e)\cdot y)$ by Theorem 703. Hence we are done by Proposition 49. □

Theorem 1382. $R(x,e/x,y)\backslash y = (x\backslash e)\cdot y$.

Proof. We have $R((e/x)\backslash e,e/x,y)\backslash y = (((e/x)\backslash e)\backslash e)\cdot y$ by Theorem 702. Then $R(x,e/x,y)\backslash y = (((e/x)\backslash e)\backslash e)\cdot y$ by Proposition 25. Hence we are done by Proposition 25. □

Theorem 1383. $R(x,x\backslash e,y) = R(x,e/x,y)$.

Proof. We have $y/(R(x,e/x,y)\backslash y) = R(x,e/x,y)$ by Proposition 24. Then

$$y/((x\backslash e)\cdot y) = R(x,e/x,y) \tag{776}$$

by Theorem 1382. We have $R(x,x\backslash e,y) = y/((x\backslash e)\cdot y)$ by Proposition 66. Hence we are done by (776). □

Theorem 1384. $R(e/(e/x),e/x,y) = R(e/(e/x),x\backslash e,y)$.

Proof. We have $L(R(x,e/x,y),x\backslash e,x) = R(e/(e/x),e/x,y)$ by Theorem 851. Then

$$L(R(x,x\backslash e,y),x\backslash e,x) = R(e/(e/x),e/x,y) \tag{777}$$

by Theorem 1383. We have $L(R(x,x\backslash e,y),x\backslash e,x) = R(e/(e/x),x\backslash e,y)$ by Theorem 851. Hence we are done by (777). □

Theorem 1385. $T((y\backslash e)\cdot x,K(y\backslash e,y)) = (e/y)\cdot(x\cdot K(y\backslash e,y))$.

Proof. We have $T(K(y\backslash e,y)\cdot((e/y)\cdot x),K(y\backslash e,y)) = ((e/y)\cdot x)\cdot K(y\backslash e,y)$ by Theorem 903. Then

$$T((y\backslash e)\cdot x,K(y\backslash e,y)) = ((e/y)\cdot x)\cdot K(y\backslash e,y) \tag{778}$$

by Theorem 682. We have $((e/y)\cdot x)\cdot K(y\backslash e,y) = (e/y)\cdot(x\cdot K(y\backslash e,y))$ by Theorem 1138. Hence we are done by (778). □

Theorem 1386. $x\backslash K(e/x,y) = (x\backslash e)\cdot K(x\backslash e,y)$.

Proof. We have

$$L((e/(e/x))\cdot T((e/(e/x))\backslash e,y),e/(e/(e/x)),y) = K(e/(e/(e/x)),y) \tag{779}$$

by Theorem 1352. Then

$$L((e/(e/x))\cdot T(e/x,y),e/(e/(e/x)),y) = K(e/(e/(e/x)),y) \tag{780}$$

by Proposition 25. We have $(e/(e/x))\backslash L((e/(e/x))\cdot T(e/x,y),e/(e/(e/x)),y) = (e/x)\cdot L((e/x)\backslash T(e/x,y),e/(e/(e/x)),y)$ by Theorem 1338. Then

$$(e/(e/x))\backslash K(e/(e/(e/x)),y) = (e/x)\cdot L((e/x)\backslash T(e/x,y),e/(e/(e/x)),y) \tag{781}$$

by (780). We have

$$L(T(e/x,y)/(e/x),e/(e/(e/x)),y) = K(e/(e/(e/x)),y) \tag{782}$$

by (779) and Theorem 149. We have $L(T(e/x,y)/(e/x),e/(e/(e/x)),y)\cdot(e/x) = (e/x)\cdot L((e/x)\backslash T(e/x,y),e/(e/(e/x)),y)$ by Theorem 85. Then $K(e/(e/(e/x)),y)\cdot(e/x) = (e/x)\cdot L((e/x)\backslash T(e/x,y),e/(e/(e/x)),y)$ by (782). Then

$$(e/(e/x))\backslash K(e/(e/(e/x)),y) = K(e/(e/(e/x)),y)\cdot(e/x) \tag{783}$$

by (781). We have $(((e/x)\cdot K(e/x,y))/(e/x))\cdot(e/x) = (e/x)\cdot K(e/x,y)$ by Axiom 6. Then $K(e/(e/(e/x)),y)\cdot(e/x) = (e/x)\cdot K(e/x,y)$ by Theorem 672. Then

$$(e/(e/x))\backslash K(e/(e/(e/x)),y) = (e/x)\cdot K(e/x,y) \tag{784}$$

by (783). We have $((e/x)\backslash((e/(e/x))\backslash K(e/(e/(e/x)),y)))\cdot(e/x) = (e/(e/x))\backslash T(K(e/(e/(e/x)),y),e/x)$ by Theorem 1336. Then $((e/x)\backslash((e/(e/x))\backslash K(e/(e/(e/x)),y)))\cdot(e/x) = (e/(e/x))\backslash K(e/x,y)$ by Theorem 673. Then $((e/x)\backslash((e/x)\cdot K(e/x,y)))\cdot(e/x) = (e/(e/x))\backslash K(e/x,y)$ by (784). Then

$$K(e/x,y)\cdot(e/x) = (e/(e/x))\backslash K(e/x,y) \tag{785}$$

by Axiom 3. We have $K(x\backslash e, x) \cdot ((e/(e/x))\backslash K(e/x, y)) = x\backslash K(e/x, y)$ by Theorem 695. Then

$$K(x\backslash e, x) \cdot (K(e/x, y) \cdot (e/x)) = x\backslash K(e/x, y) \tag{786}$$

by (785). We have $T(K(e/x, y), ((e/x)\backslash e)\backslash e) = T(K(e/x, y), e/x)$ by Theorem 721. Then

$$T(K(e/x, y), x\backslash e) = T(K(e/x, y), e/x) \tag{787}$$

by Proposition 25. We have $(e/x) \cdot T(K(e/x, y), e/x) = K(e/x, y) \cdot (e/x)$ by Proposition 46. Then

$$(e/x) \cdot T(K(e/x, y), x\backslash e) = K(e/x, y) \cdot (e/x) \tag{788}$$

by (787). We have $T(K(e/(e/(x\backslash e)), y), x\backslash e) = K(x\backslash e, y)$ by Theorem 673. Then $T(K(e/x, y), x\backslash e) = K(x\backslash e, y)$ by Proposition 24. Then $(e/x) \cdot K(x\backslash e, y) = K(e/x, y) \cdot (e/x)$ by (788). Then

$$K(x\backslash e, x) \cdot ((e/x) \cdot K(x\backslash e, y)) = x\backslash K(e/x, y) \tag{789}$$

by (786). We have $K(x\backslash e, x) \cdot ((e/x) \cdot K(x\backslash e, y)) = (x\backslash e) \cdot K(x\backslash e, y)$ by Theorem 682. Hence we are done by (789). ☐

Theorem 1387. $(x\backslash K(e/x, y)) \cdot ((x\backslash e)\backslash e) = K(x\backslash e, y)$.

Proof. We have $K(x\backslash e, y) = L((x\backslash e)\backslash T(x\backslash e, y), ((x\backslash e)\backslash e)\backslash e, y)$ by Theorem 1372. Then

$$K(x\backslash e, y) = L(T(x\backslash e, y) \cdot ((x\backslash e)\backslash e), ((x\backslash e)\backslash e)\backslash e, y) \tag{790}$$

by Theorem 146. We have $L(T(x\backslash e, y) \cdot ((x\backslash e)\backslash e), ((x\backslash e)\backslash e)\backslash e, y)/((x\backslash e)\backslash e) = (x\backslash e) \cdot L((x\backslash e)\backslash T(x\backslash e, y), ((x\backslash e)\backslash e)\backslash e, y)$ by Theorem 1341. Then $L(T(x\backslash e, y) \cdot ((x\backslash e)\backslash e), ((x\backslash e)\backslash e)\backslash e, y)/((x\backslash e)\backslash e) = (x\backslash e) \cdot K(x\backslash e, y)$ by Theorem 1372. Then

$$K(x\backslash e, y)/((x\backslash e)\backslash e) = (x\backslash e) \cdot K(x\backslash e, y) \tag{791}$$

by (790). We have $(K(x\backslash e, y)/((x\backslash e)\backslash e)) \cdot ((x\backslash e)\backslash e) = K(x\backslash e, y)$ by Axiom 6. Then $((x\backslash e) \cdot K(x\backslash e, y)) \cdot ((x\backslash e)\backslash e) = K(x\backslash e, y)$ by (791). Hence we are done by Theorem 1386. ☐

Theorem 1388. $T(y \cdot x, K(y\backslash e, y)) = y \cdot T(x, K(y\backslash e, y))$.

Proof. We have $K(y, y\backslash e) \cdot ((y \cdot x) \cdot K(y\backslash e, y)) = T(y \cdot x, K(y\backslash e, y))$ by Theorem 1377. Then

$$K(y, y\backslash e) \cdot (((y \cdot x)/((y\backslash e)\backslash e)) \cdot y) = T(y \cdot x, K(y\backslash e, y)) \tag{792}$$

by Theorem 424. We have $(K(y, y\backslash e) \cdot ((y \cdot x)/((y\backslash e)\backslash e))) \cdot y = K(y, y\backslash e) \cdot (((y \cdot x)/((y\backslash e)\backslash e)) \cdot y)$ by Theorem 1225. Then $(K(y, y\backslash e) \cdot ((y \cdot x)/((y\backslash e)\backslash e))) \cdot y = T(y \cdot x, K(y\backslash e, y))$ by (792). Then $(K(y\backslash e, y)\backslash ((y \cdot x)/((y\backslash e)\backslash e))) \cdot y = T(y \cdot x, K(y\backslash e, y))$ by Theorem 696. Then

$$(K(y\backslash e, y)\backslash ((y \cdot x)/T(y, y\backslash e))) \cdot y = T(y \cdot x, K(y\backslash e, y)) \tag{793}$$

by Proposition 49. We have $T((y \cdot x)/y, K(y\backslash e, y)) = K(y\backslash e, y)\backslash (((y \cdot x)/y) \cdot K(y\backslash e, y))$ by Definition 3. Then $T((y \cdot x)/y, K(y\backslash e, y)) = K(y\backslash e, y)\backslash ((y \cdot x) \cdot T((y \cdot x)\backslash ((y \cdot x)/y), y))$ by Theorem 154. Then $K(y\backslash e, y)\backslash ((y \cdot x)/T(y, y\backslash e)) = T((y \cdot x)/y, K(y\backslash e, y))$ by Theorem 1146. Then

$$T((y \cdot x)/y, K(y\backslash e, y)) \cdot y = T(y \cdot x, K(y\backslash e, y)) \tag{794}$$

by (793). We have $T((y \cdot x)/y, K(y\backslash e, y)) \cdot y = y \cdot T(x, K(y\backslash e, y))$ by Theorem 47. Hence we are done by (794). ☐

Theorem 1389. $(y \cdot (((y\backslash x)\backslash e)\backslash e))/y = ((x/y)\backslash e)\backslash e$.

Proof. We have $(y \cdot T(((x/y)\backslash e)\backslash e, y))/y = ((x/y)\backslash e)\backslash e$ by Proposition 48. Hence we are done by Theorem 920. ☐

Theorem 1390. $T(x/y,(y\backslash x)\backslash e) = ((x/y)\backslash e)\backslash e$.

Proof. We have $T(x/y,(y\backslash x)\backslash e) = (y \cdot (((y\backslash x)\backslash e)\backslash e))/y$ by Theorem 1330. Hence we are done by Theorem 1389. □

Theorem 1391. $((x/y)\backslash e)\backslash e = T(x/y, x\backslash y)$.

Proof. We have $T(x/y,(y\backslash x)\backslash e) = (y \cdot (((y\backslash x)\backslash e)\backslash e))/y$ by Theorem 1330. Then

$$T(x/y,(y\backslash x)\backslash e) = (y \cdot T(y\backslash x, x\backslash y))/y \tag{795}$$

by Theorem 387. We have $(y \cdot T(y\backslash x, x\backslash y))/y = T(x/y, x\backslash y)$ by Theorem 49. Then $T(x/y,(y\backslash x)\backslash e) = T(x/y, x\backslash y)$ by (795). Hence we are done by Theorem 1390. □

Theorem 1392. $T(y,((x \cdot y)/x)\backslash e) = (y\backslash e)\backslash e$.

Proof. We have $T(T((x\cdot y)/x, x),((x\cdot y)/x)\backslash e) = T((((x\cdot y)/x)\backslash e)\backslash e, x)$ by Theorem 18. Then $T(y,((x\cdot y)/x)\backslash e) = T((((x \cdot y)/x)\backslash e)\backslash e, x)$ by Theorem 7. Then $((x\backslash(x \cdot y))\backslash e)\backslash e = T(y,((x \cdot y)/x)\backslash e)$ by Theorem 920. Hence we are done by Axiom 3. □

Theorem 1393. $((x/(x \cdot y))\backslash e)\backslash e = x/(x \cdot ((y\backslash e)\backslash e))$.

Proof. We have

$$(((x/(x \cdot y))\backslash e)\backslash e) \cdot (K((x/(x \cdot y))\backslash e, x/(x \cdot y)) \cdot (x \cdot y)) = (x/(x \cdot y)) \cdot (x \cdot y) \tag{796}$$

by Theorem 1169. We have $T(x/(x \cdot y),(x/(x \cdot y))\backslash e) \cdot (K((x/(x \cdot y))\backslash e, x/(x \cdot y)) \cdot (x \cdot y)) = (((x/(x \cdot y))\backslash e)\backslash e) \cdot (K((x/(x\cdot y))\backslash e, x/(x\cdot y)) \cdot (x\cdot y))$ by Theorem 464. Then $T(x/(x\cdot y),(x/(x\cdot y))\backslash e) \cdot (K((x/(x\cdot y))\backslash e, x/(x \cdot y)) \cdot (x \cdot y)) = (x/(x \cdot y)) \cdot (x \cdot y)$ by (796). Then

$$T(x/(x \cdot y),(x/(x \cdot y))\backslash e)\backslash((x/(x \cdot y)) \cdot (x \cdot y)) = K((x/(x \cdot y))\backslash e, x/(x \cdot y)) \cdot (x \cdot y) \tag{797}$$

by Proposition 2. We have $x/(T(x/(x\cdot y),(x/(x\cdot y))\backslash e)\backslash((x/(x\cdot y)) \cdot (x\cdot y))) = T(x/(x\cdot y),(x/(x\cdot y))\backslash e)$ by Theorem 24. Then $x/(K((x/(x\cdot y))\backslash e, x/(x\cdot y)) \cdot (x\cdot y)) = T(x/(x\cdot y),(x/(x\cdot y))\backslash e)$ by (797). Then

$$x/(K((x \cdot y)/x, x/(x \cdot y)) \cdot (x \cdot y)) = T(x/(x \cdot y),(x/(x \cdot y))\backslash e) \tag{798}$$

by Theorem 429. We have $T(x/(x \cdot y),(x/(x \cdot y))\backslash e) = ((x/(x \cdot y))\backslash e)\backslash e$ by Proposition 49. Then

$$x/(K((x \cdot y)/x, x/(x \cdot y)) \cdot (x \cdot y)) = ((x/(x \cdot y))\backslash e)\backslash e \tag{799}$$

by (798). We have $K((x \cdot y)/x, x/(x \cdot y)) \cdot (x \cdot y) = x \cdot T(x\backslash(x \cdot y), x/(x \cdot y))$ by Theorem 430. Then $K((x \cdot y)/x, x/(x \cdot y)) \cdot (x \cdot y) = x \cdot T(y, x/(x \cdot y))$ by Axiom 3. Then

$$x/(x \cdot T(y, x/(x \cdot y))) = ((x/(x \cdot y))\backslash e)\backslash e \tag{800}$$

by (799). We have $T(y, y\backslash e) = (y\backslash e)\backslash e$ by Proposition 49. Then

$$T(y, y\backslash e) = T(y,((x \cdot y)/x)\backslash e) \tag{801}$$

by Theorem 1392. We have $T(y,((x \cdot y)/x)\backslash e) = T(y, x/(x \cdot y))$ by Theorem 1034. Then $T(y, y\backslash e) = T(y, x/(x \cdot y))$ by (801). Then $x/(x \cdot T(y, y\backslash e)) = ((x/(x \cdot y))\backslash e)\backslash e$ by (800). Hence we are done by Proposition 49. □

Theorem 1394. $x/(x \cdot (y\backslash e)) = ((x/(x \cdot (e/y)))\backslash e)\backslash e$.

Proof. We have $x/(x \cdot (((e/y)\backslash e)\backslash e)) = ((x/(x \cdot (e/y)))\backslash e)\backslash e$ by Theorem 1393. Hence we are done by Proposition 25. □

Theorem 1395. $y \cdot T(x,(y\backslash e)\backslash e) = x \cdot y$.

Proof. We have $T(T(x{\cdot}y, K(y, y\backslash e)), K(y\backslash e, y)) = x{\cdot}y$ by Theorem 701. Then $T(y{\cdot}T(x, (y\backslash e)\backslash e), K(y\backslash e, y)) = x \cdot y$ by Theorem 705. Hence we are done by Theorem 717. ☐

Theorem 1396. $(e/x) \cdot (y \cdot K(x\backslash e, x)) = (x\backslash e) \cdot y$.

Proof. We have $(e/x) \cdot (y \cdot K(x\backslash e, x)) = (x\backslash e) \cdot T(y, K(x\backslash e, x))$ by Theorem 1162. Hence we are done by Theorem 717. ☐

Theorem 1397. $T(x, y\backslash e) = T(x, e/y)$.

Proof. We have $T(T(x, ((e/y)\backslash e)\backslash e), K((e/y)\backslash e, e/y)) = T(x, e/y)$ by Theorem 708. Then $T(T(x, y\backslash e), K((e/y)\backslash e, e/y)) = T(x, e/y)$ by Proposition 25. Hence we are done by Theorem 717. ☐

Theorem 1398. $x \cdot (e/y) = (e/y) \cdot T(x, y\backslash e)$.

Proof. We have $K(y, y\backslash e) \cdot ((x \cdot (e/y)) \cdot K(y\backslash e, y)) = T(x \cdot (e/y), K(y\backslash e, y))$ by Theorem 1377. Then

$$K(y, y\backslash e) \cdot (x \cdot (y\backslash e)) = T(x \cdot (e/y), K(y\backslash e, y)) \tag{802}$$

by Theorem 412. We have $(e/y) \cdot ((y\backslash e)\backslash(x \cdot (y\backslash e))) = K(y, y\backslash e) \cdot (x \cdot (y\backslash e))$ by Theorem 1375. Then $(e/y) \cdot T(x, y\backslash e) = K(y, y\backslash e) \cdot (x \cdot (y\backslash e))$ by Definition 3. Then $T(x \cdot (e/y), K(y\backslash e, y)) = (e/y) \cdot T(x, y\backslash e)$ by (802). Hence we are done by Theorem 717. ☐

Theorem 1399. $(y \cdot K(x\backslash e, x))/(x \cdot y) = R(x\backslash e, x, y)$.

Proof. We have $K(((y/K(x, x\backslash e))/(x{\cdot}(y/K(x, x\backslash e))))\backslash e, (y/K(x, x\backslash e))/(x{\cdot}(y/K(x, x\backslash e)))) = K(x, x\backslash e)$ by Theorem 687. Then $K(R(e/x, x, y/K(x, x\backslash e))\backslash e, (y/K(x, x\backslash e))/(x \cdot (y/K(x, x\backslash e)))) = K(x, x\backslash e)$ by Proposition 79. Then

$$K(R(e/x, x, y/K(x, x\backslash e))\backslash e, R(e/x, x, y/K(x, x\backslash e))) = K(x, x\backslash e) \tag{803}$$

by Proposition 79. We have $R(R(e/x, x, y/K(x, x\backslash e)), x{\cdot}(y/K(x, x\backslash e)), K(R(e/x, x, y/K(x, x\backslash e))\backslash e, R(e/x, x, y/K(x, x\backslash e)))) = R(e/x, x, y/K(x, x\backslash e))$ by Theorem 1293. Then

$$R(R(e/x, x, y/K(x, x\backslash e)), x \cdot (y/K(x, x\backslash e)), K(x, x\backslash e)) = R(e/x, x, y/K(x, x\backslash e)) \tag{804}$$

by (803). We have $R(R(e/x, x, y/K(x, x\backslash e)), x{\cdot}(y/K(x, x\backslash e)), K(x, x\backslash e)) = ((y/K(x, x\backslash e)){\cdot}K(x, x\backslash e))/((x{\cdot} (y/K(x, x\backslash e))) \cdot K(x, x\backslash e))$ by Theorem 1325. Then

$$R(e/x, x, y/K(x, x\backslash e)) = ((y/K(x, x\backslash e)) \cdot K(x, x\backslash e))/((x \cdot (y/K(x, x\backslash e))) \cdot K(x, x\backslash e)) \tag{805}$$

by (804). We have $R(e/x, x, (y/K(x, x\backslash e)){\cdot}K(x, x\backslash e)) = ((y/K(x, x\backslash e)){\cdot}K(x, x\backslash e))/(x{\cdot}((y/K(x, x\backslash e)){\cdot} K(x, x\backslash e)))$ by Proposition 79. Then $R(e/x, x, (y/K(x, x\backslash e)){\cdot}K(x, x\backslash e)) = ((y/K(x, x\backslash e)){\cdot}K(x, x\backslash e))/((x{\cdot} (y/K(x, x\backslash e))) \cdot K(x, x\backslash e))$ by Theorem 623. Then

$$R(e/x, x, y/K(x, x\backslash e)) = R(e/x, x, (y/K(x, x\backslash e)) \cdot K(x, x\backslash e)) \tag{806}$$

by (805). We have $R(R(e/x, x, (y/K(x, x\backslash e)){\cdot}K(x, x\backslash e)), x, x\backslash e) = R(x\backslash e, x, (y/K(x, x\backslash e)){\cdot}K(x, x\backslash e))$ by Theorem 1323. Then

$$R(R(e/x, x, y/K(x, x\backslash e)), x, x\backslash e) = R(x\backslash e, x, (y/K(x, x\backslash e)) \cdot K(x, x\backslash e)) \tag{807}$$

by (806). We have $R(R(e/x, x, y/K(x, x\backslash e)), x, x\backslash e) = R(x\backslash e, x, y/K(x, x\backslash e))$ by Theorem 1323. Then

$$R(x\backslash e, x, (y/K(x, x\backslash e)) \cdot K(x, x\backslash e)) = R(x\backslash e, x, y/K(x, x\backslash e)) \tag{808}$$

by (807). We have $(K(x\backslash e, x) \cdot (K(x, x\backslash e) \cdot (y \cdot K(x\backslash e, x))))/(x \cdot (K(x, x\backslash e) \cdot (y \cdot K(x\backslash e, x)))) = R(x\backslash e, x, K(x, x\backslash e) \cdot (y \cdot K(x\backslash e, x)))$ by Theorem 791. Then $(y \cdot K(x\backslash e, x))/(x \cdot (K(x, x\backslash e) \cdot (y \cdot K(x\backslash e, x)))) = R(x\backslash e, x, K(x, x\backslash e) \cdot (y \cdot K(x\backslash e, x)))$ by Theorem 693. Then

$$R(x\backslash e, x, K(x, x\backslash e) \cdot (y \cdot K(x\backslash e, x))) = (y \cdot K(x\backslash e, x))/(T(x, x\backslash e) \cdot (y \cdot K(x\backslash e, x))) \tag{809}$$

by Theorem 1153. We have $(y \cdot K(x\backslash e, x))/(T(x, x\backslash e) \cdot (y \cdot K(x\backslash e, x))) = R(x\backslash e, x, y \cdot K(x\backslash e, x))$ by Theorem 1381. Then $R(x\backslash e, x, K(x, x\backslash e) \cdot (y \cdot K(x\backslash e, x))) = R(x\backslash e, x, y \cdot K(x\backslash e, x))$ by (809). Then

$$R(x\backslash e, x, T(y, K(x\backslash e, x))) = R(x\backslash e, x, y \cdot K(x\backslash e, x)) \tag{810}$$

by Theorem 1377. We have $(K(x\backslash e, x) \cdot T(y, K(x\backslash e, x)))/(x \cdot T(y, K(x\backslash e, x))) = R(x\backslash e, x, T(y, K(x\backslash e, x)))$ by Theorem 791. Then

$$(y \cdot K(x\backslash e, x))/(x \cdot T(y, K(x\backslash e, x))) = R(x\backslash e, x, T(y, K(x\backslash e, x))) \tag{811}$$

by Proposition 46.

$$\begin{array}{lcl} & (y \cdot K(x\backslash e, x))/T(x \cdot y, K(x\backslash e, x)) & \\ = & R(x\backslash e, x, T(y, K(x\backslash e, x))) & \text{by (811), Theorem 1388} \\ = & R(x\backslash e, x, y/K(x, x\backslash e)) & \text{by (810), Theorem 425} \\ = & R(x\backslash e, x, y) & \text{by (808), Axiom 6.} \end{array}$$

Then $(y \cdot K(x\backslash e, x))/T(x \cdot y, K(x\backslash e, x)) = R(x\backslash e, x, y)$. Hence we are done by Theorem 717. □

Theorem 1400. $x \cdot (y\backslash e) = K(y\backslash e, y) \cdot (x \cdot (e/y))$.

Proof. We have $K(y, y\backslash e)\backslash(x \cdot (e/y)) = T(x \cdot (y\backslash e), K(y, y\backslash e))$ by Theorem 1368. Then $K(y\backslash e, y) \cdot (x \cdot (e/y)) = T(x \cdot (y\backslash e), K(y, y\backslash e))$ by Theorem 694. Hence we are done by Theorem 718. □

Theorem 1401. $T(x, T(y\backslash e, y)) = T(x, y\backslash e)$.

Proof. We have $T(x, ((y\backslash e)\backslash e)\backslash e) = T(x, y\backslash e)$ by Theorem 721. Hence we are done by Theorem 209. □

Theorem 1402. $T(x, T(y/z, y\backslash z)) = T(x, y/z)$.

Proof. We have $T(x, ((y/z)\backslash e)\backslash e) = T(x, y/z)$ by Theorem 721. Hence we are done by Theorem 1391. □

Theorem 1403. $((y\backslash e)\backslash e) \cdot T(x, y) = x \cdot T(y, y\backslash e)$.

Proof. We have $T(y, y\backslash e) \cdot T(x, T(y, y\backslash e)) = x \cdot T(y, y\backslash e)$ by Proposition 46. Then $((y\backslash e)\backslash e) \cdot T(x, T(y, y\backslash e)) = x \cdot T(y, y\backslash e)$ by Theorem 464. Hence we are done by Theorem 722. □

Theorem 1404. $T(((e/y) \cdot x) \cdot y, y\backslash e) = R(x, e/y, y)$.

Proof. We have $(((e/y) \cdot x) \cdot y)/e = ((e/y) \cdot x) \cdot y$ by Proposition 27. Then

$$(((e/y) \cdot x) \cdot y)/((e/y) \cdot y) = ((e/y) \cdot x) \cdot y \tag{812}$$

by Axiom 6. We have $R(x, e/y, y) = T((((e/y) \cdot x) \cdot y)/((e/y) \cdot y), e/y)$ by Theorem 110. Then $R(x, e/y, y) = T(((e/y) \cdot x) \cdot y, e/y)$ by (812). Hence we are done by Theorem 1397. □

Theorem 1405. $x \cdot K(x, x\backslash e) = T(x, T(x, y)\backslash e)$.

Proof. We have $T(x, y) \cdot ((x \cdot (T(x, y)\backslash e))/x) = T(x \cdot ((x \cdot (x\backslash e))/x), y)$ by Theorem 345. Then $T(x, y) \cdot ((x \cdot (T(x, y)\backslash e))/x) = T(x \cdot (e/x), y)$ by Axiom 4. Then $T(x, y) \cdot ((T(x, y)\backslash x)/x) = T(x \cdot (e/x), y)$ by Theorem 175. Then

$$T(x, y) \cdot ((T(x, y)\backslash x)/x) = T(K(x, x\backslash e), y) \tag{813}$$

by Theorem 250. We have $x \cdot (T(x, T(x, y)\backslash e)/x) = T(x, T(x, y)\backslash e)$ by Theorem 757. Then $x \cdot (T(x, y) \cdot ((T(x, y)\backslash x)/x)) = T(x, T(x, y)\backslash e)$ by Theorem 1351. Then $x \cdot T(K(x, x\backslash e), y) = T(x, T(x, y)\backslash e)$ by (813). Hence we are done by Theorem 723. □

Theorem 1406. $(K(y\backslash e, y) \cdot x) \cdot ((y\backslash e)\backslash e) = x \cdot y$.

Proof. We have $(x \cdot K(y\backslash e, y)) \cdot ((y\backslash e)\backslash e) = x \cdot y$ by Theorem 908. Hence we are done by Theorem 923. □

Theorem 1407. $K(y\backslash e, y) \cdot (x \cdot y) = x \cdot (e/(e/y))$.

Proof. We have $(x \cdot y) \cdot K(y\backslash e, y) = x \cdot (e/(e/y))$ by Theorem 1365. Hence we are done by Theorem 923. □

Theorem 1408. $x \cdot ((y\backslash e)\backslash e) = K(y, y\backslash e) \cdot (x \cdot y)$.

Proof.

$$\begin{array}{lll} & (x \cdot ((y\backslash e)\backslash e)) \cdot K(y\backslash e, y) & \\ = & x \cdot y & \text{by Theorem 655} \\ = & (K(y, y\backslash e) \cdot (x \cdot y)) \cdot K(y\backslash e, y) & \text{by Theorem 924.} \end{array}$$

Then $(x \cdot ((y\backslash e)\backslash e)) \cdot K(y\backslash e, y) = (K(y, y\backslash e) \cdot (x \cdot y)) \cdot K(y\backslash e, y)$. Hence we are done by Proposition 10. □

Theorem 1409. $((y\backslash e)\backslash e) \cdot x = (y \cdot x) \cdot K(y, y\backslash e)$.

Proof.

$$\begin{array}{lll} & K(y\backslash e, y) \cdot (((y\backslash e)\backslash e) \cdot x) & \\ = & y \cdot x & \text{by Theorem 1376} \\ = & K(y\backslash e, y) \cdot ((y \cdot x) \cdot K(y, y\backslash e)) & \text{by Theorem 1182.} \end{array}$$

Then $K(y\backslash e, y) \cdot (((y\backslash e)\backslash e) \cdot x) = K(y\backslash e, y) \cdot ((y \cdot x) \cdot K(y, y\backslash e))$. Hence we are done by Proposition 9. □

Theorem 1410. $T(x\backslash T(x, y), y\backslash e) = x\backslash T(x, y)$.

Proof. We have $T(e/y, K(y\backslash(y/(x\backslash e)), y)) = e/y$ by Theorem 367. Then

$$T(e/y, x\backslash T(x, y)) = e/y \tag{814}$$

by Theorem 198. We have $(x\backslash T(x, y)) \cdot T(e/y, x\backslash T(x, y)) = (e/y) \cdot (x\backslash T(x, y))$ by Proposition 46. Then

$$(x\backslash T(x, y)) \cdot (e/y) = (e/y) \cdot (x\backslash T(x, y)) \tag{815}$$

by (814). We have $(e/y) \cdot T(x\backslash T(x, y), y\backslash e) = (x\backslash T(x, y)) \cdot (e/y)$ by Theorem 1398. Hence we are done by (815) and Proposition 7. □

Theorem 1411. $T(y, T(y, x)\backslash e) = (y\backslash e)\backslash e$.

Proof. We have $y \cdot (y\backslash((y\backslash e)\backslash e)) = (y\backslash e)\backslash e$ by Axiom 4. Then $y \cdot K(y, e/y) = (y\backslash e)\backslash e$ by Theorem 559. Then $y \cdot K(y, y\backslash e) = (y\backslash e)\backslash e$ by Theorem 780. Hence we are done by Theorem 1405. □

Theorem 1412. $T(y, T(y, x)\backslash e) = T(y, y\backslash e)$.

Proof. We have $y \cdot K(y, y\backslash e) = T(y, y\backslash e)$ by Theorem 562. Hence we are done by Theorem 1405. □

Theorem 1413. $(K(y, y\backslash e) \cdot x) \cdot y = x \cdot T(y, y\backslash e)$.

Proof. We have $(K(y, y\backslash e) \cdot x) \cdot y = K(y, y\backslash e) \cdot (x \cdot y)$ by Theorem 1225. Hence we are done by Theorem 925. □

Theorem 1414. $K(x\backslash e, ((x\backslash e)\backslash e) \cdot y) = K(x\backslash e, x \cdot y)$.

Proof. We have $T(x\backslash e,(((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot K(x\backslash e,(x\backslash e)\backslash e)) = T(x\backslash e,((x\cdot y)\cdot(e/x))/(x\backslash e))$ by Theorem 626. Then

$$T(x\backslash e,(((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot K(x\backslash e,x)) = T(x\backslash e,((x\cdot y)\cdot(e/x))/(x\backslash e)) \tag{816}$$

by Theorem 779. We have $(((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot(K(x\backslash e,x)\cdot T(e/x,(((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot K(x\backslash e,x))) = L(e/x,K(x\backslash e,x),((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot((((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot K(x\backslash e,x))$ by Theorem 1328. Then $(((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot((x\backslash e)\cdot(x\cdot T(x\backslash e,(((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot K(x\backslash e,x)))) = L(e/x,K(x\backslash e,x),((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot((((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot K(x\backslash e,x))$ by Theorem 567. Then $(((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot((x\backslash e)\cdot(x\cdot T(x\backslash e,((x\cdot y)\cdot(e/x))/(x\backslash e)))) = L(e/x,K(x\backslash e,x),((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot((((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot K(x\backslash e,x))$ by (816). Then $(e/x)\cdot((((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot K(x\backslash e,x)) = (((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot((x\backslash e)\cdot(x\cdot T(x\backslash e,((x\cdot y)\cdot(e/x))/(x\backslash e))))$ by Theorem 644. Then

$$(((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot((x\backslash e)\cdot(x\cdot T(x\backslash e,((x\cdot y)\cdot(e/x))/(x\backslash e)))) = T((x\backslash e)\cdot(((x\cdot y)\cdot(e/x))/(x\backslash e)),K(x\backslash e,x)) \tag{817}$$

by Theorem 1385. We have $((x\cdot y)\cdot(e/x))\backslash((((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot((x\backslash e)\cdot(x\cdot T(x\backslash e,((x\cdot y)\cdot(e/x))/(x\backslash e))))) = L(x\cdot T(x\backslash e,((x\cdot y)\cdot(e/x))/(x\backslash e)),x\backslash e,((x\cdot y)\cdot(e/x))/(x\backslash e))$ by Theorem 4. Then $((x\cdot y)\cdot(e/x))\backslash((((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot((x\backslash e)\cdot(x\cdot T(x\backslash e,((x\cdot y)\cdot(e/x))/(x\backslash e))))) = K(e/x,((x\cdot y)\cdot(e/x))/(x\backslash e))$ by Theorem 654. Then

$$((x\cdot y)\cdot(e/x))\backslash T((x\backslash e)\cdot(((x\cdot y)\cdot(e/x))/(x\backslash e)),K(x\backslash e,x)) = K(e/x,((x\cdot y)\cdot(e/x))/(x\backslash e)) \tag{818}$$

by (817). We have $T((x\backslash e)\cdot(((x\cdot y)\cdot(e/x))/(x\backslash e)),K(x\backslash e,x)) = (e/x)\cdot((((x\cdot y)\cdot(e/x))/(x\backslash e))\cdot K(x\backslash e,x))$ by Theorem 1385. Then $T((x\backslash e)\cdot(((x\cdot y)\cdot(e/x))/(x\backslash e)),K(x\backslash e,x)) = (e/x)\cdot(((x\cdot y)\cdot(e/x))/(e/x))$ by Theorem 1362. Then

$$((x\cdot y)\cdot(e/x))\backslash((e/x)\cdot(((x\cdot y)\cdot(e/x))/(e/x))) = K(e/x,((x\cdot y)\cdot(e/x))/(x\backslash e)) \tag{819}$$

by (818). We have $(x\backslash K(e/x,((x\cdot y)\cdot(e/x))/(e/x)))\cdot((x\backslash e)\backslash e) = K(x\backslash e,((x\cdot y)\cdot(e/x))/(e/x))$ by Theorem 1387. Then $(x\backslash(((x\cdot y)\cdot(e/x))\backslash((e/x)\cdot(((x\cdot y)\cdot(e/x))/(e/x)))))\cdot((x\backslash e)\backslash e) = K(x\backslash e,((x\cdot y)\cdot(e/x))/(e/x))$ by Theorem 3. Then

$$(x\backslash K(e/x,((x\cdot y)\cdot(e/x))/(x\backslash e)))\cdot((x\backslash e)\backslash e) = K(x\backslash e,((x\cdot y)\cdot(e/x))/(e/x)) \tag{820}$$

by (819). We have $(x\backslash K(e/x,((x\cdot y)\cdot(e/x))/(x\backslash e)))\cdot((x\backslash e)\backslash e) = K(x\backslash e,((x\cdot y)\cdot(e/x))/(x\backslash e))$ by Theorem 1387. Then $K(x\backslash e,((x\cdot y)\cdot(e/x))/(e/x)) = K(x\backslash e,((x\cdot y)\cdot(e/x))/(x\backslash e))$ by (820). Then $K(x\backslash e,x\cdot y) = K(x\backslash e,((x\cdot y)\cdot(e/x))/(x\backslash e))$ by Axiom 5. Then

$$K(x\backslash e,((x\cdot y)/(x\backslash e))\cdot(e/x)) = K(x\backslash e,x\cdot y) \tag{821}$$

by Theorem 1348. We have $(((x\cdot y)/(x\backslash e))\cdot(x\backslash e))\cdot K(x,x\backslash e) = ((x\cdot y)/(x\backslash e))\cdot(e/x)$ by Theorem 1364. Then $(x\cdot y)\cdot K(x,x\backslash e) = ((x\cdot y)/(x\backslash e))\cdot(e/x)$ by Axiom 6. Then $K(x\backslash e,(x\cdot y)\cdot K(x,x\backslash e)) = K(x\backslash e,x\cdot y)$ by (821). Hence we are done by Theorem 1409. ☐

Theorem 1415. $K(T(e/x,y)\backslash e,z) = K(T(x\backslash e,y)\backslash e,z)$.

Proof. We have $K(e/T(x\backslash e,y),z) = K(T(x\backslash e,y)\backslash e,z)$ by Theorem 732. Hence we are done by Theorem 917. ☐

Theorem 1416. $K(y,y\backslash e) = K(T(y,x),y\backslash e)$.

Proof. We have $K(T(y,x),T(y,x)\backslash e) = K(T(y,x),y\backslash e)$ by Theorem 711. Hence we are done by Theorem 744. ☐

Theorem 1417. $(e/T(x,y))\cdot K(x\backslash e,x) = T(x,y)\backslash e$.

Proof.

$$
\begin{array}{rcl}
& ((e/T(x,y)) \cdot K(x\backslash e, x)) \cdot K(x, x\backslash e) & \\
= & e/T(x,y) & \text{by Theorem 426} \\
= & (T(x,y)\backslash e) \cdot K(x, x\backslash e) & \text{by Theorem 746.}
\end{array}
$$

Then $((e/T(x,y))\cdot K(x\backslash e,x))\cdot K(x,x\backslash e) = (T(x,y)\backslash e)\cdot K(x,x\backslash e)$. Hence we are done by Proposition 10. □

Theorem 1418. $(y\backslash e)\backslash(T(y,x)\backslash e) = T(y,x)\backslash y$.

Proof. We have $(y\backslash e)\backslash(T(y,x)\backslash e) = (y\backslash T(y,x))\backslash e$ by Theorem 638. Hence we are done by Theorem 749. □

Theorem 1419. $T(y, x \cdot T(x\backslash e, y)) = y$.

Proof. We have $T(y, K(y\backslash(y/x), y)) = y$ by Theorem 755. Hence we are done by Theorem 196. □

Theorem 1420. $R(x, K(y,z), z) = ((x \cdot K(y,z)) \cdot z)/(z \cdot K(y,z))$.

Proof. We have $R(x, K(y,z), z) = ((x \cdot K(y,z)) \cdot z)/(K(y,z) \cdot z)$ by Definition 5. Hence we are done by Theorem 933. □

Theorem 1421. $K((y/x)\backslash e, y/x) = K(y\backslash x, x\backslash y)$.

Proof. We have $K(y\backslash x, (((x \cdot y)/x) \cdot (y\backslash x))\backslash((x \cdot y)/x)) = K(y\backslash x, (y\backslash x)\backslash e)$ by Theorem 392. Then $K(y\backslash x, T(x,y)\backslash((x \cdot y)/x)) = K(y\backslash x, (y\backslash x)\backslash e)$ by Theorem 179. Then

$$K(y\backslash x, T(y/x, T(x,y))) = K(y\backslash x, (y\backslash x)\backslash e) \tag{822}$$

by Theorem 1083. We have $K(y\backslash x, (y\backslash x)\backslash e) = K(y\backslash x, x\backslash y)$ by Theorem 612. Then

$$K(y\backslash x, T(y/x, T(x,y))) = K(y\backslash x, x\backslash y) \tag{823}$$

by (822). We have $K(((x\cdot y)/x)\backslash(((x\cdot y)/x)/T(y/x,T(x,y))), T(y/x,T(x,y))) = K(T(y/x,T(x,y))\backslash e, T(y/x,T(x,y)))$ by Theorem 1371. Then $K(((x\cdot y)/x)\backslash T(x,y), T(y/x,T(x,y))) = K(T(y/x,T(x,y))\backslash e, T(y/x,T(x,y)))$ by Theorem 1334. Then $K(T(y/x,T(x,y))\backslash e, T(y/x,T(x,y))) = K(y\backslash x, T(y/x,T(x,y)))$ by Theorem 1340. Then $K(T(y/x,T(x,y))\backslash e, T(y/x,T(x,y))) = K(y\backslash x, x\backslash y)$ by (823). Hence we are done by Theorem 929. □

Theorem 1422. $y \cdot (x \cdot T(x\backslash e, y)) = x \cdot ((y\backslash(x\backslash y)) \cdot y)$.

Proof. We have $T(y, x \cdot T(x\backslash e, y)) = y$ by Theorem 1419. Then

$$T(x \cdot T(x\backslash e, y), y) = x \cdot T(x\backslash e, y) \tag{824}$$

by Proposition 21. We have $y \cdot T(x \cdot T(x\backslash e, y), y) = (x \cdot T(x\backslash e, y)) \cdot y$ by Proposition 46. Then $y \cdot T(x \cdot T(x\backslash e, y), y) = x \cdot ((y\backslash(x\backslash y)) \cdot y)$ by Theorem 236. Hence we are done by (824). □

Theorem 1423. $K(y\backslash x, x\backslash y) = K(x/y, (x/y)\backslash e)$.

Proof. We have $K(x/y, (x/y)\backslash e) = K(x/y, y/x)$ by Theorem 402. Then $K(x/y, (x/y)\backslash e) = K((y/x)\backslash e, y/x)$ by Theorem 429. Hence we are done by Theorem 1421. □

Theorem 1424. $T(x\backslash T(x,y), x) = x\backslash T(x,y)$.

Proof. We have $T(T(x,y)/x, x) = x\backslash T(x,y)$ by Proposition 47. Hence we are done by Theorem 758. □

Theorem 1425. $K(y\backslash(y/(x\backslash e)), y) \cdot x = T(x,y)$.

Proof. We have $(x\backslash T(x,y)) \cdot x = T(x,y)$ by Theorem 759. Hence we are done by Theorem 198. ☐

Theorem 1426. $x\backslash T(x,y) = K(y\backslash(y/T(x\backslash e,x)),y)$.

Proof. We have $T(x\backslash T(x,y),x) = K(y\backslash(y/T(x\backslash e,x)),y)$ by Theorem 575. Hence we are done by Theorem 1424. ☐

Theorem 1427. $K(z\backslash y,y\backslash z) \cdot (x \cdot (y/z)) = x \cdot (((y/z)\backslash e)\backslash e)$.

Proof. We have $K(y/z,(y/z)\backslash e) \cdot (x \cdot (y/z)) = x \cdot (((y/z)\backslash e)\backslash e)$ by Theorem 1408. Hence we are done by Theorem 1423. ☐

Theorem 1428. $T(y\backslash e,T(y,x)\backslash y) = y\backslash e$.

Proof. We have $T(y\backslash e,K(y\backslash(y/((x\backslash y)/T(y,x))),y)) = y\backslash e$ by Theorem 607. Hence we are done by Theorem 932. ☐

Theorem 1429. $y\backslash(((y \cdot x)/y)\backslash x) = T(y,R(x,x\backslash e,y))\backslash e$.

Proof. We have $y\backslash(T(y,R(x,x\backslash e,y))\backslash y) = T(y,R(x,x\backslash e,y))\backslash e$ by Theorem 1339. Hence we are done by Theorem 769. ☐

Theorem 1430. $x\backslash(y\backslash T(y,x)) = (x \cdot (T(y,x)\backslash y))\backslash e$.

Proof. We have $x\backslash(T(x,R(T(y,x),T(y,x)\backslash e,x))\backslash x) = T(x,R(T(y,x),T(y,x)\backslash e,x))\backslash e$ by Theorem 1339. Then $x\backslash(((T(y,x)\backslash y) \cdot x)\backslash x) = T(x,R(T(y,x),T(y,x)\backslash e,x))\backslash e$ by Theorem 591. Then $((T(y,x)\backslash y) \cdot x)\backslash e = x\backslash(((T(y,x)\backslash y) \cdot x)\backslash x)$ by Theorem 591. Then $x\backslash((x \cdot (T(y,x)\backslash y))\backslash x) = ((T(y,x)\backslash y) \cdot x)\backslash e$ by Theorem 740. Then $x\backslash((x \cdot (T(y,x)\backslash y))\backslash x) = (x \cdot (T(y,x)\backslash y))\backslash e$ by Theorem 740. Hence we are done by Theorem 771. ☐

Theorem 1431. $(x \cdot K(x,y))\backslash x = K(x,y)\backslash e$.

Proof. We have $T(x,R(y,x,(y \cdot x)\backslash x))\backslash x = K(x,y)\backslash e$ by Theorem 949. Hence we are done by Theorem 871. ☐

Theorem 1432. $(x\backslash T(x,y)) \cdot (y\backslash e) = y\backslash(x\backslash T(x,y))$.

Proof. We have $(T(y,R(T(x,y),T(x,y)\backslash e,y))\backslash y) \cdot T(y\backslash e,T(y,R(T(x,y),T(x,y)\backslash e,y))\backslash y) = (y\backslash e) \cdot (T(y,R(T(x,y),T(x,y)\backslash e,y))\backslash y)$ by Proposition 46. Then

$$(T(y,R(T(x,y),T(x,y)\backslash e,y))\backslash y) \cdot (y\backslash e) = (y\backslash e) \cdot (T(y,R(T(x,y),T(x,y)\backslash e,y))\backslash y) \tag{825}$$

by Theorem 1428. We have $(y\backslash e) \cdot ((y\backslash e)\backslash(T(y,R(T(x,y),T(x,y)\backslash e,y))\backslash e)) = T(y,R(T(x,y),T(x,y)\backslash e,y))\backslash e$ by Axiom 4. Then $(y\backslash e) \cdot (T(y,R(T(x,y),T(x,y)\backslash e,y))\backslash y) = T(y,R(T(x,y),T(x,y)\backslash e,y))\backslash e$ by Theorem 1418. Then $(T(y,R(T(x,y),T(x,y)\backslash e,y))\backslash y) \cdot (y\backslash e) = T(y,R(T(x,y),T(x,y)\backslash e,y))\backslash e$ by (825). Then $(T(y,R(T(x,y),T(x,y)\backslash e,y))\backslash y)\backslash$ $y\backslash e$ by Proposition 2. Then

$$(x\backslash T(x,y))\backslash(T(y,R(T(x,y),T(x,y)\backslash e,y))\backslash e) = y\backslash e \tag{826}$$

by Theorem 770. We have $y\backslash(T(y,R(T(x,y),T(x,y)\backslash e,y))\backslash y) = T(y,R(T(x,y),T(x,y)\backslash e,y))\backslash e$ by Theorem 1339. Then $y\backslash(x\backslash T(x,y)) = T(y,R(T(x,y),T(x,y)\backslash e,y))\backslash e$ by Theorem 770. Then

$$(x\backslash T(x,y))\backslash(y\backslash(x\backslash T(x,y))) = y\backslash e \tag{827}$$

by (826). We have $(x\backslash T(x,y)) \cdot ((x\backslash T(x,y))\backslash(y\backslash(x\backslash T(x,y)))) = y\backslash(x\backslash T(x,y))$ by Axiom 4. Hence we are done by (827). ☐

Theorem 1433. $T(y,y\backslash K(x,y)) = T(y,y\backslash e)$.

Proof. We have $T(y,T(y,R(T(((y{\cdot}x)\backslash y)\backslash e,y),T(((y{\cdot}x)\backslash y)\backslash e,y)\backslash e,y))\backslash e) = T(y,y\backslash e)$ by Theorem 1412. Then $T(y,y\backslash(((y\cdot T(((y\cdot x)\backslash y)\backslash e,y))/y)\backslash T(((y\cdot x)\backslash y)\backslash e,y))) = T(y,y\backslash e)$ by Theorem 1429. Then $T(y,y\backslash((((y{\cdot}x)\backslash y)\backslash e)\backslash T(((y{\cdot}x)\backslash y)\backslash e,y))) = T(y,y\backslash e)$ by Proposition 48. Then $T(y,y\backslash((((y{\cdot}x)\backslash y)\backslash e)\backslash(((y{\cdot}x)\backslash y)\backslash K(x,y)))) = T(y,y\backslash e)$ by Theorem 538. Hence we are done by Theorem 735. ☐

Theorem 1434. $K(x,y)/T(y,y\backslash e) = y\backslash K(x,y)$.

Proof. We have $K(x,y)/T(y,y\backslash K(x,y)) = y\backslash K(x,y)$ by Theorem 15. Hence we are done by Theorem 1433. ☐

Theorem 1435. $(y\backslash K(x,y))\cdot K(y,y\backslash e) = K(x,y)/y$.

Proof. We have $(K(x,y)/T(y,y\backslash e))\cdot K(y,y\backslash e) = K(x,y)/y$ by Theorem 1361. Hence we are done by Theorem 1434. ☐

Theorem 1436. $(y\backslash K(x,y))\cdot y = K(x,y)\cdot K(y\backslash e,y)$.

Proof. We have $(K(x,y)/T(y,y\backslash e))\cdot y = K(x,y)\cdot K(y\backslash e,y)$ by Theorem 1148. Hence we are done by Theorem 1434. ☐

Theorem 1437. $y\cdot(K(x,y)/y) = K(x,y)\cdot K(y,y\backslash e)$.

Proof. We have $y\cdot((y\backslash K(x,y))\cdot K(y,y\backslash e)) = K(x,y)\cdot K(y,y\backslash e)$ by Theorem 622. Hence we are done by Theorem 1435. ☐

Theorem 1438. $K(y,y\backslash e)\cdot K(x,y) = y\cdot(K(x,y)/y)$.

Proof. We have $K(y,y\backslash e)\cdot K(x,y) = K(x,y)\cdot K(y,y\backslash e)$ by Theorem 1356. Hence we are done by Theorem 1437. ☐

Theorem 1439. $(y\backslash e)\cdot(y\cdot(K(x,y)/y)) = (e/y)\cdot K(x,y)$.

Proof. We have $(y\backslash e)\cdot(K(x,y)\cdot K(y,y\backslash e)) = (e/y)\cdot K(x,y)$ by Theorem 1185. Hence we are done by Theorem 1437. ☐

Theorem 1440. $R(T(x,y\cdot K(z\backslash e,z)),y,K(z\backslash e,z)) = L(T(x,y),y,K(z\backslash e,z))$.

Proof. We have $(y{\cdot}K(z\backslash e,z))\backslash((x{\cdot}y){\cdot}K(z\backslash e,z)) = R(T(x,y{\cdot}K(z\backslash e,z)),y,K(z\backslash e,z))$ by Proposition 81. Then

$$(K(z\backslash e,z)\cdot y)\backslash((x\cdot y)\cdot K(z\backslash e,z)) = R(T(x,y\cdot K(z\backslash e,z)),y,K(z\backslash e,z)) \tag{828}$$

by Theorem 923. We have $(K(z\backslash e,z){\cdot}y)\backslash(K(z\backslash e,z){\cdot}(x{\cdot}y)) = L(T(x,y),y,K(z\backslash e,z))$ by Proposition 62. Then $(K(z\backslash e,z)\cdot y)\backslash((x\cdot y)\cdot K(z\backslash e,z)) = L(T(x,y),y,K(z\backslash e,z))$ by Theorem 923. Hence we are done by (828). ☐

Theorem 1441. $y\cdot(T(((y\cdot x)\backslash y)\backslash e,y)\backslash(((y\cdot x)\backslash y)\backslash e)) = K(x,y)\backslash y$.

Proof. We have $y/K(x,y) = K(x,y)\backslash y$ by Theorem 766. Hence we are done by Theorem 776. ☐

Theorem 1442. $K(x,y)/y = e/(K(x,y)\backslash y)$.

Proof. We have $R((R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)\cdot((T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))/T(T(y,R(T(((y{\cdot}x)\backslash y)\backslash e,y),T(((y{\cdot}x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y{\cdot}x)\backslash y)\backslash e,y),T(((y{\cdot}x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))/(e/T(T(y,R(T(((y{\cdot}x)\backslash y)\backslash e,y),T(((y{\cdot}x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y{\cdot}x)\backslash y)\backslash e,y),T(((y{\cdot}x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))))/R(T(((y{\cdot}x)\backslash y)\backslash e,y),T(((y{\cdot}x)\backslash y)\backslash e,y)\backslash e,y),e/T(T(y,R(T(((y{\cdot}x)\backslash y)\backslash e,y),T(((y{\cdot}x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y{\cdot}x)\backslash y)\backslash e,y),T(((y{\cdot}x)\backslash y)\backslash e,y)\backslash e,y))\backslash y),(e/T(T(y,R(T(((y{\cdot}x)\backslash y)\backslash e,y),T(((y{\cdot}x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y{\cdot}x)\backslash y)\backslash e,y),T(((y{\cdot}x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\backslash e)/((e/T(T(y,R(T(((y{\cdot}x)\backslash y)\backslash e,y),T(((y{\cdot}x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y{\cdot}x)\backslash y)\backslash e,y),T(((y{\cdot}x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\backslash e) = ((R(T(((y{\cdot}x)\backslash y)\backslash e,y),T(((y{\cdot}x)\backslash y)\backslash e,y)\backslash e,y){\cdot}((T(y,R(T(((y{\cdot}x)\backslash y)\backslash e,y),T(((y{\cdot}x)\backslash y)\backslash e,y)\backslash e,y))/T(T(y,R(T(((y{\cdot}x)\backslash y)\backslash e,y),T(((y{\cdot}x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y{\cdot}x)\backslash y)\backslash e,y),T(((y{\cdot}x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))/(e/T(T(y,R(T(((y{\cdot}x)\backslash y)\backslash e,y),T(((y{\cdot}$

$x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y)))))/R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot$ $x)\backslash y)\backslash e,y)\backslash e,y))\cdot(e/T(T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot$ $x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))$ by Theorem 73. Then $R((R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)\cdot((T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))/T(T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))/(e/T(T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y)))))/R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y),e/T(T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y),(e/T(T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\backslash e)/((e/T(T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\backslash e) = (e/T(T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\cdot((R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)\cdot((e/T(T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\backslash(T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))/T(T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y)))))/R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))$ by Theorem 125. Then $((R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)\cdot$ $((T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))/T(T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\cdot((e/T(T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\backslash e)))/R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))/((e/T(T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\backslash e) = (e/T(T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\cdot((R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)\cdot((e/T(T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\backslash(T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))/T(T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y)))))/R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))$ by Theorem 1332. Then $((R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)\cdot$ $((T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))/T(T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\cdot((e/T(T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\backslash e)))/R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))/((e/T(T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\backslash e) = (e/T(T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\cdot((R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)\cdot T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)))/R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot$ $x)\backslash y)\backslash e,y)\backslash e,y))$ by Theorem 1345. Then $((R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)\cdot((T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))/T(T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\cdot T(T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y)))/R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))/((e/T(T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\backslash e) = (e/T(T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\cdot((R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)\cdot T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)))/R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot$ $x)\backslash y)\backslash e,y)\backslash e,y))$ by Proposition 25. Then $((R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)\cdot T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)))/R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))/((e/T(T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\backslash e) = (e/T(T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\cdot((R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)\cdot T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)))/R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot$ $x)\backslash y)\backslash e,y)\backslash e,y))$ by Axiom 6. Then $((R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)\cdot T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)))/R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))/T(T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y) = (e/T(T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\cdot((R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)\cdot T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)))/R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot$ $x)\backslash y)\backslash e,y)\backslash e,y))$ by Proposition 25. Then $(e/T(T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\cdot y = ((R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)\cdot T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)))/R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))/T(T(y,R(T(((y\cdot$ $x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y)$ by Proposition 48. Then $y/T(T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y) =$ $(e/T(T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y))\backslash y))\cdot$ y by Proposition 48. Then $(y/T(T(y,R(T(((y\cdot x)\backslash y)\backslash e,y),T(((y\cdot x)\backslash y)\backslash e,y)\backslash e,y)),T(y,R(T(((y\cdot$

$x)\backslash y)\backslash e, y), T(((y\cdot x)\backslash y)\backslash e, y)\backslash e, y))\backslash y))/y = e/T(T(y, R(T(((y\cdot x)\backslash y)\backslash e, y), T(((y\cdot x)\backslash y)\backslash e, y)\backslash e, y)), T(y, R(T(((y\cdot x)\backslash y)\backslash e, y), T(((y\cdot x)\backslash y)\backslash e, y)\backslash e, y))\backslash y)$ by Proposition 1. Then $(T(y, R(T(((y\cdot x)\backslash y)\backslash e, y), T(((y\cdot x)\backslash y)\backslash e, y)\backslash e, y))\backslash y)/y = e/T(T(y, R(T(((y\cdot x)\backslash y)\backslash e, y), T(((y\cdot x)\backslash y)\backslash e, y)\backslash e, y)), T(y, R(T(((y\cdot x)\backslash y)\backslash e, y), T(((y\cdot x)\backslash y)\backslash e, y)\backslash e, y))\backslash y)$ by Theorem 15. Then $e/((T(y, R(T(((y\cdot x)\backslash y)\backslash e, y), T(((y\cdot x)\backslash y)\backslash e, y)\backslash e, y))\backslash y)\backslash y) = (T(y, R(T(((y\cdot x)\backslash y)\backslash e, y), T(((y\cdot x)\backslash y)\backslash e, y)\backslash e, y))\backslash y)/y$ by Proposition 49. Then $e/T(y, R(T(((y\cdot x)\backslash y)\backslash e, y), T(((y\cdot x)\backslash y)\backslash e, y)\backslash e, y)) = (T(y, R(T(((y\cdot x)\backslash y)\backslash e, y), T(((y\cdot x)\backslash y)\backslash e, y)\backslash e, y))\backslash y)/y$ by Theorem 930. Then $((((y\cdot x)\backslash y)\backslash e)\backslash T(((y\cdot x)\backslash y)\backslash e, y))/y = e/T(y, R(T(((y\cdot x)\backslash y)\backslash e, y), T(((y\cdot x)\backslash y)\backslash e, y)\backslash e, y))$ by Theorem 770. Then $e/(y\cdot(T(((y\cdot x)\backslash y)\backslash e, y)\backslash(((y\cdot x)\backslash y)\backslash e))) = ((((y\cdot x)\backslash y)\backslash e)\backslash T(((y\cdot x)\backslash y)\backslash e, y))/y$ by Theorem 742. Then $e/(y\cdot(T(((y\cdot x)\backslash y)\backslash e, y)\backslash(((y\cdot x)\backslash y)\backslash e))) = (((y\cdot x)\backslash y)\cdot T(((y\cdot x)\backslash y)\backslash e, y))/y$ by Theorem 729. Then $e/(K(x, y)\backslash y) = (((y\cdot x)\backslash y)\cdot T(((y\cdot x)\backslash y)\backslash e, y))/y$ by Theorem 1441. Hence we are done by Theorem 539. ☐

Theorem 1443. $(K(x, y)/y)\backslash e = K(x, y)\backslash y$.

Proof. We have $(e/(K(x, y)\backslash y))\backslash e = K(x, y)\backslash y$ by Proposition 25. Hence we are done by Theorem 1442. ☐

Theorem 1444. $y/(e/(K(x, y)/y)) = y\cdot(e/(e/(y\backslash K(x, y))))$.

Proof. We have $y/(e/(e/(K(x, y)\backslash y))) = y\cdot(e/(e/(y\backslash K(x, y))))$ by Theorem 647. Hence we are done by Theorem 1442. ☐

Theorem 1445. $K(K(y, y\backslash e)\cdot x, y) = K(x, y)$.

Proof. We have $K(K(y\backslash e, y)\cdot(K(y, y\backslash e)\cdot x), (y\backslash e)\backslash e) = (((y\backslash e)\backslash e)\cdot(K(y\backslash e, y)\cdot(K(y, y\backslash e)\cdot x)))\backslash((K(y\backslash e, y)\cdot(K(y, y\backslash e)\cdot x))\cdot((y\backslash e)\backslash e))$ by Definition 2. Then $K(K(y\backslash e, y)\cdot(K(y, y\backslash e)\cdot x), (y\backslash e)\backslash e) = (y\cdot(K(y, y\backslash e)\cdot x))\backslash((K(y\backslash e, y)\cdot(K(y, y\backslash e)\cdot x))\cdot((y\backslash e)\backslash e))$ by Theorem 1169. Then

$$(y\cdot(K(y, y\backslash e)\cdot x))\backslash((K(y, y\backslash e)\cdot x)\cdot y) = K(K(y\backslash e, y)\cdot(K(y, y\backslash e)\cdot x), (y\backslash e)\backslash e) \tag{829}$$

by Theorem 1406. We have $K(K(y, y\backslash e)\cdot x, y) = (y\cdot(K(y, y\backslash e)\cdot x))\backslash((K(y, y\backslash e)\cdot x)\cdot y)$ by Definition 2. Then $K(K(y, y\backslash e)\cdot x, y) = K(K(y\backslash e, y)\cdot(K(y, y\backslash e)\cdot x), (y\backslash e)\backslash e)$ by (829). Then $K(x, (y\backslash e)\backslash e) = K(K(y, y\backslash e)\cdot x, y)$ by Theorem 693. Hence we are done by Theorem 779. ☐

Theorem 1446. $K(y\backslash x, y) = K(((y\backslash e)\backslash e)\backslash x, y)$.

Proof. We have $K(K(y, y\backslash e)\cdot(((y\backslash e)\backslash e)\backslash x), y) = K(((y\backslash e)\backslash e)\backslash x, y)$ by Theorem 1445. Hence we are done by Theorem 680. ☐

Theorem 1447. $K((e/y)\cdot x, y) = K((y\backslash e)\cdot x, y)$.

Proof. We have $K(K(y, y\backslash e)\cdot((y\backslash e)\cdot x), y) = K((y\backslash e)\cdot x, y)$ by Theorem 1445. Hence we are done by Theorem 697. ☐

Theorem 1448. $y\backslash K(x, y) = (K(x, y)\backslash y)\backslash e$.

Proof. We have $(y\cdot(T(((y\cdot x)\backslash y)\backslash e, y)\backslash(((y\cdot x)\backslash y)\backslash e)))\backslash e = y\backslash((((y\cdot x)\backslash y)\backslash e)\backslash T(((y\cdot x)\backslash y)\backslash e, y))$ by Theorem 1430. Then $(K(x, y)\backslash y)\backslash e = y\backslash((((y\cdot x)\backslash y)\backslash e)\backslash T(((y\cdot x)\backslash y)\backslash e, y))$ by Theorem 1441. Then $y\backslash(((y\cdot x)\backslash y)\cdot T(((y\cdot x)\backslash y)\backslash e, y)) = (K(x, y)\backslash y)\backslash e$ by Theorem 729. Hence we are done by Theorem 539. ☐

Theorem 1449. $e/(y\backslash K(x, y)) = K(x, y)\backslash y$.

Proof. We have $e/((K(x, y)\backslash y)\backslash e) = K(x, y)\backslash y$ by Proposition 24. Hence we are done by Theorem 1448. ☐

Theorem 1450. $T(x, z\backslash K(y, z)) = T(x, K(y, z)/z)$.

Proof. We have $T(e/(K(y,z)\backslash z), K(y,z)\backslash z) = (K(y,z)\backslash z)\backslash e$ by Proposition 47. Then $T(K(y,z)/z, K(y,z)\backslash z) = (K(y,z)\backslash z)\backslash e$ by Theorem 1442. Then

$$T(K(y,z)/z, K(y,z)\backslash z) = z\backslash K(y,z) \tag{830}$$

by Theorem 1448. We have $T(x, T(K(y,z)/z, K(y,z)\backslash z)) = T(x, K(y,z)/z)$ by Theorem 1402. Hence we are done by (830). □

Theorem 1451. *$(K(y,z)/z) \cdot T(x, z\backslash K(y,z)) = x \cdot (K(y,z)/z)$.*

Proof. We have $(K(y,z)/z) \cdot T(x, K(y,z)/z) = x \cdot (K(y,z)/z)$ by Proposition 46. Hence we are done by Theorem 1450. □

Theorem 1452. *$K(K(x,y)\backslash y, y\backslash K(x,y)) = K(y, y\backslash e)$.*

Proof. We have

$$y \cdot (e/(e/(y\backslash K(x,y)))) = y/(e/(K(x,y)/y)) \tag{831}$$

by Theorem 1444. Then $y \cdot (e/(K(x,y)\backslash y)) = y/(e/(K(x,y)/y))$ by Theorem 1449. Then $y/(e/(K(x,y)/y)) = y \cdot (K(x,y)/y)$ by Theorem 1442. Then

$$y \cdot (e/(e/(y\backslash K(x,y)))) = y \cdot (K(x,y)/y) \tag{832}$$

by (831). We have $K(y, y\backslash e) \cdot K(x,y) = y \cdot (K(x,y)/y)$ by Theorem 1438. Then

$$K(y, y\backslash e) \cdot K(x,y) = y \cdot (e/(e/(y\backslash K(x,y)))) \tag{833}$$

by (832). We have $K(K(x,y)\backslash y, y\backslash K(x,y)) \cdot K(x,y) = y \cdot (e/(e/(y\backslash K(x,y))))$ by Theorem 731. Then $K(K(x,y)\backslash y, y\backslash K(x,y)) \cdot K(x,y) = K(y, y\backslash e) \cdot K(x,y)$ by (833). Hence we are done by Proposition 10. □

Theorem 1453. *$K(x, z\backslash K(y,z)) = K(x, K(y,z)/z)$.*

Proof. We have

$$(K(y,z)/z) \cdot T(z, z\backslash K(y,z)) = z \cdot (K(y,z)/z) \tag{834}$$

by Theorem 1451. Then $(K(y,z)/z) \cdot T(z, z\backslash K(y,z)) = z \cdot (e/(K(y,z)\backslash z))$ by Theorem 1442. Then $z \cdot (e/(K(y,z)\backslash z)) = z \cdot (K(y,z)/z)$ by (834). Then

$$z \cdot (e/(e/(z\backslash K(y,z)))) = z \cdot (K(y,z)/z) \tag{835}$$

by Theorem 1449. We have $K((z\backslash K(y,z))\backslash e, z\backslash K(y,z)) \cdot (z \cdot (z\backslash K(y,z))) = z \cdot (e/(e/(z\backslash K(y,z))))$ by Theorem 1407. Then $K(K(y,z)\backslash z, z\backslash K(y,z)) \cdot (z \cdot (z\backslash K(y,z))) = z \cdot (e/(e/(z\backslash K(y,z))))$ by Theorem 666. Then $K(z, z\backslash e) \cdot (z \cdot (z\backslash K(y,z))) = z \cdot (e/(e/(z\backslash K(y,z))))$ by Theorem 1452. Then

$$K(z, z\backslash e) \cdot (z \cdot (z\backslash K(y,z))) = z \cdot (K(y,z)/z) \tag{836}$$

by (835). We have $K(z, z\backslash e) \cdot (K(z\backslash e, z) \cdot (z \cdot (K(y,z)/z))) = z \cdot (K(y,z)/z)$ by Theorem 442. Then $K(z, z\backslash e) \cdot (K(z\backslash e, z) \cdot (z \cdot (K(y,z)/z))) = K(z, z\backslash e) \cdot (z \cdot (z\backslash K(y,z)))$ by (836). Then $K(z\backslash e, z) \cdot (z \cdot (K(y,z)/z)) = z \cdot (z\backslash K(y,z))$ by Proposition 9. Then $K(z\backslash e, z) \cdot (z \cdot (K(y,z)/z)) = z \cdot ((K(y,z)\backslash z)\backslash e)$ by Theorem 1448. Then

$$K(z\backslash e, z) \cdot (z \cdot (K(y,z)/z)) = z \cdot (((K(y,z)/z)\backslash e)\backslash e) \tag{837}$$

by Theorem 1443. We have $K(z\backslash K(y,z), K(y,z)\backslash z) \cdot (z \cdot (K(y,z)/z)) = z \cdot (((K(y,z)/z)\backslash e)\backslash e)$ by Theorem 1427. Then

$$K(z\backslash e, z) = K(z\backslash K(y,z), K(y,z)\backslash z) \tag{838}$$

by (837) and Proposition 8. We have $(K(y,z) \cdot K((K(y,z)\backslash z)\backslash e, K(y,z)\backslash z))/((K(y,z)\backslash z) \cdot K(y,z)) = R((K(y,z)\backslash z)\backslash e, K(y,z)\backslash z, K(y,z))$ by Theorem 1399. Then $(K(y,z) \cdot K((K(y,z)\backslash z)\backslash e, K(y,z)\backslash z))/z =$

$R((K(y,z)\backslash z)\backslash e, K(y,z)\backslash z, K(y,z))$ by Theorem 765. Then $(K(y,z) \cdot K(z\backslash K(y,z), K(y,z)\backslash z))/z = R((K(y,z)\backslash z)\backslash e, K(y,z)\backslash z, K(y,z))$ by Theorem 666. Then $(K(y,z)\cdot K(z\backslash e, z))/z = R((K(y,z)\backslash z)\backslash e, K(y,z)\backslash z, K(y,z))$ by (838). Then

$$R((K(y,z)\backslash z)\backslash e, K(y,z)\backslash z, K(y,z)) = ((z\backslash K(y,z)) \cdot z)/z \tag{839}$$

by Theorem 1436. We have $((z\backslash K(y,z)) \cdot z)/z = z\backslash K(y,z)$ by Axiom 5. Then

$$R((K(y,z)\backslash z)\backslash e, K(y,z)\backslash z, K(y,z)) = z\backslash K(y,z) \tag{840}$$

by (839). We have $K(x, T(K(y,z)/((K(y,z)\backslash z) \cdot K(y,z)), (K(y,z)/((K(y,z)\backslash z) \cdot K(y,z)))\backslash e)) = K(x, ((K(y,z)/((K(y,z)\backslash z) \cdot K(y,z)))\backslash e)\backslash e)$ by Theorem 1342. Then $K(x, T(K(y,z)/((K(y,z)\backslash z) \cdot K(y,z)), ((K(y,z)\backslash z)\cdot K(y,z))/K(y,z))) = K(x, ((K(y,z)/((K(y,z)\backslash z)\cdot K(y,z)))\backslash e)\backslash e)$ by Theorem 1034. Then $K(x, T(K(y,z)/((K(y,z)\backslash z)\cdot K(y,z)), K(y,z)\backslash z)) = K(x, ((K(y,z)/((K(y,z)\backslash z)\cdot K(y,z)))\backslash e)\backslash e)$ by Axiom 5. Then $K(x, R((K(y,z)\backslash z)\backslash e, K(y,z)\backslash z, K(y,z))) = K(x, ((K(y,z)/((K(y,z)\backslash z)\cdot K(y,z)))\backslash e)\backslash e)$ by Theorem 89. Then $K(x, R((K(y,z)\backslash z)\backslash e, K(y,z)\backslash z, K(y,z))) = K(x, K(y,z)/((K(y,z)\backslash z)\cdot K(y,z)))$ by Theorem 779. Then $K(x, R((K(y,z)\backslash z)\backslash e, K(y,z)\backslash z, K(y,z))) = K(x, K(y,z)/z)$ by Theorem 765. Hence we are done by (840). □

Theorem 1454. $(x \cdot (e/z))\backslash(y \cdot (e/z)) = (x \cdot (z\backslash e))\backslash(y \cdot (z\backslash e))$.

Proof. We have $L((x\cdot(e/z))\backslash(K(z, z\backslash e)\cdot(y\cdot(z\backslash e))), x\cdot(e/z), K(z\backslash e, z)) = (K(z\backslash e, z)\cdot(x\cdot(e/z)))\backslash(K(z\backslash e, z)\cdot(K(z, z\backslash e)\cdot(y\cdot(z\backslash e))))$ by Proposition 53. Then $L((x\cdot(e/z))\backslash(K(z, z\backslash e)\cdot(y\cdot(z\backslash e))), x\cdot(e/z), K(z\backslash e, z)) = (K(z\backslash e, z)\cdot(x\cdot(e/z)))\backslash(y\cdot(z\backslash e))$ by Theorem 693. Then $(x\cdot(e/z))\backslash(K(z, z\backslash e)\cdot(y\cdot(z\backslash e))) = (K(z\backslash e, z)\cdot(x \cdot (e/z)))\backslash(y \cdot (z\backslash e))$ by Theorem 967. Then $(K(z\backslash e, z) \cdot (x \cdot (e/z)))\backslash(y \cdot (z\backslash e)) = (x \cdot (e/z))\backslash((y \cdot (z\backslash e)) \cdot K(z, z\backslash e))$ by Theorem 1183. Then $(x \cdot (z\backslash e))\backslash(y \cdot (z\backslash e)) = (x \cdot (e/z))\backslash((y \cdot (z\backslash e)) \cdot K(z, z\backslash e))$ by Theorem 1400. Hence we are done by Theorem 1364. □

Theorem 1455. $(x \cdot y)\backslash((x \cdot y)/x) = L(x\backslash e, T(x,y), y)$.

Proof. We have $(x \cdot y)\backslash(y \cdot (T(x,y) \cdot (x\backslash e))) = L(x\backslash e, T(x,y), y)$ by Theorem 1321. Then $(x \cdot y)\backslash(y \cdot (x\backslash T(x,y))) = L(x\backslash e, T(x,y), y)$ by Theorem 146. Hence we are done by Theorem 977. □

Theorem 1456. $K(y\backslash(y/(x\backslash e)), y) = T(y, T(x,y))\backslash y$.

Proof. We have $K(y\backslash(y/(x\backslash e)), y) = x\backslash T(x,y)$ by Theorem 198. Hence we are done by Theorem 986. □

Theorem 1457. $x \cdot K(x, (x/y)/x) = T(x, y\backslash e)$.

Proof. We have $x\cdot K(x, (x/y)/x) = T(x, x/(x\cdot y))$ by Theorem 870. Hence we are done by Theorem 1003. □

Theorem 1458. $x\backslash T(y, x\backslash e) = y \cdot R(e/x, x, x\backslash y)$.

Proof. We have $x\backslash T(y, x\backslash e) = y \cdot ((x\backslash y)/y)$ by Theorem 1248. Hence we are done by Theorem 37. □

Theorem 1459. $((y \cdot x) \cdot (y\backslash e))\backslash R(x, y, y\backslash e) = T(y, R(x, y, y\backslash e))\backslash y$.

Proof. We have $((y \cdot R(x, y, y\backslash e))/y)\backslash R(x, y, y\backslash e) = T(y, R(x, y, y\backslash e))\backslash y$ by Theorem 984. Then $R((y \cdot x)/y, y, y\backslash e)\backslash R(x, y, y\backslash e) = T(y, R(x, y, y\backslash e))\backslash y$ by Theorem 123. Hence we are done by Proposition 71. □

Theorem 1460. $L(z, x, x\backslash T(x,y)) = z$.

Proof. We have

$$L(x \cdot z, x\backslash T(x,y), x) = x \cdot z \tag{841}$$

by Theorem 1269. We have $L(x \cdot z, x\backslash T(x,y), x) = x \cdot L(z, x, x\backslash T(x,y))$ by Theorem 950. Then $x \cdot L(z, x, x\backslash T(x,y)) = x \cdot z$ by (841). Hence we are done by Proposition 9. □

Theorem 1461. $T(T(z,x\backslash e)\backslash z,y)=K(z\backslash(z/T(x,y)),z)$.

Proof. We have $T(x\cdot T(x\backslash e,z),y)=K(z\backslash(z/T(x,y)),z)$ by Theorem 296. Hence we are done by Theorem 1028. ☐

Theorem 1462. $(T(z,y/x)\backslash z)\cdot y=x\cdot T(x\backslash y,z)$.

Proof. We have

$$(y/x)\cdot((T(z,y/x)\backslash z)\cdot y)=T(y/x,z)\cdot y \tag{842}$$

by Theorem 1314. We have $(y/x)\cdot(x\cdot T(x\backslash y,z))=T(y/x,z)\cdot y$ by Theorem 1094. Hence we are done by (842) and Proposition 7. ☐

Theorem 1463. $(T(x,y)\backslash x)\cdot z=x\cdot(T(x,y)\backslash z)$.

Proof. We have $(x\backslash T(x,y))\backslash z=x\cdot(T(x,y)\backslash z)$ by Theorem 1272. Hence we are done by Theorem 1281. ☐

Theorem 1464. $K(y\backslash x,y)=K(x/y,y)$.

Proof. We have $K(T(x/y,y),y)=K(x/y,y)$ by Theorem 1053. Hence we are done by Proposition 47. ☐

Theorem 1465. $K(y,y\backslash x)=K(y,x/y)$.

Proof. We have $K(T(y,y\backslash x),y\backslash x)=K(y,x/y)$ by Theorem 1250. Hence we are done by Theorem 1053. ☐

Theorem 1466. $K(y,x)\backslash e=K(x,y)$.

Proof. We have $K(T(y,x),x)\backslash e=K(x,y)$ by Theorem 1036. Hence we are done by Theorem 1053. ☐

Theorem 1467. $K(y,x)\backslash z=K(x,y)\cdot z$.

Proof. We have $K(T(y,x),x)\cdot(K(T(y,x),x)\backslash z)=z$ by Axiom 4. Then

$$K(y,(y\cdot x)/y)\cdot(K(T(y,x),x)\backslash z)=z \tag{843}$$

by Theorem 1016.

$$\begin{array}{rcl} & (K(x,y)\backslash e)\cdot(K(T(y,x),x)\backslash z) & \\ = & z & \text{by (843), Theorem 1035} \\ = & (K(x,y)\backslash e)\cdot(K(x,y)\cdot z) & \text{by Theorem 1051.} \end{array}$$

Then $(K(x,y)\backslash e)\cdot(K(T(y,x),x)\backslash z)=(K(x,y)\backslash e)\cdot(K(x,y)\cdot z)$. Then $K(T(y,x),x)\backslash z=K(x,y)\cdot z$ by Proposition 9. Hence we are done by Theorem 1053. ☐

Theorem 1468. $(y\cdot(x/y))\backslash x=K(y\backslash x,y)$.

Proof. We have $(y\cdot(x/y))\backslash x=K(x/y,y)$ by Theorem 448. Hence we are done by Theorem 1464. ☐

Theorem 1469. $(x\cdot(y/x))\cdot K(x\backslash y,x)=y$.

Proof. We have $(x\cdot(y/x))\cdot((x\cdot(y/x))\backslash y)=y$ by Axiom 4. Hence we are done by Theorem 1468. ☐

Theorem 1470. $T(x,y/(e/z))=T(x,y/(z\backslash e))$.

Proof. We have $R(T(x,(y/(z\backslash e))\cdot K(z\backslash e,z)),y/(z\backslash e),K(z\backslash e,z))=L(T(x,y/(z\backslash e)),y/(z\backslash e),K(z\backslash e,z))$ by Theorem 1440. Then $R(T(x,(y/(z\backslash e))\cdot K(z\backslash e,z)),y/(z\backslash e),K(z\backslash e,z))=T(x,y/(z\backslash e))$ by Theorem 967. Then $R(T(x,y/(e/z)),y/(z\backslash e),K(z\backslash e,z))=T(x,y/(z\backslash e))$ by Theorem 1362. Hence we are done by Theorem 1293. ☐

Theorem 1471. $z\backslash T(z\cdot(x/z),y) = T(x,y)/z$.

Proof. We have $(e/z)\cdot T((e/z)\backslash(x/z),y) = T((x/z)\cdot z,y)/z$ by Theorem 498. Then

$$(x/z)\cdot K(y\backslash(y/((x/z)\backslash(e/z))),y) = T((x/z)\cdot z,y)/z \tag{844}$$

by Theorem 284. We have

$$(e/z)\cdot T((e/z)\backslash(x/z),y) = (x/z)\cdot K(y\backslash(y/((x/z)\backslash(e/z))),y) \tag{845}$$

by Theorem 284.

$$\begin{array}{lll} & (T(y,(x/z)/(e/z))\backslash y)\cdot(x/z) & \\ = & (x/z)\cdot K(y\backslash(y/((x/z)\backslash(e/z))),y) & \text{by (845), Theorem 1462} \\ = & T(x,y)/z & \text{by (844), Axiom 6.} \end{array}$$

Then $(T(y,(x/z)/(e/z))\backslash y)\cdot(x/z) = T(x,y)/z$. Then $y\cdot(T(y,(x/z)/(e/z))\backslash(x/z)) = T(x,y)/z$ by Theorem 1463. Then

$$y\cdot(T(y,(x/z)/(z\backslash e))\backslash(x/z)) = T(x,y)/z \tag{846}$$

by Theorem 1470. We have $(T(y,(x/z)/(z\backslash e))\backslash y)\cdot(x/z) = (z\backslash e)\cdot T((z\backslash e)\backslash(x/z),y)$ by Theorem 1462. Then

$$y\cdot(T(y,(x/z)/(z\backslash e))\backslash(x/z)) = (z\backslash e)\cdot T((z\backslash e)\backslash(x/z),y) \tag{847}$$

by Theorem 1463. We have $(z\backslash e)\cdot T((z\backslash e)\backslash(x/z),y) = z\backslash T(z\cdot(x/z),y)$ by Theorem 140. Then $y\cdot(T(y,(x/z)/(z\backslash e))\backslash(x/z)) = z\backslash T(z\cdot(x/z),y)$ by (847). Hence we are done by (846). ☐

Theorem 1472. $z\backslash T(z\cdot x,y) = T(x\cdot z,y)/z$.

Proof. We have $z\backslash T(z\cdot((x\cdot z)/z),y) = T(x\cdot z,y)/z$ by Theorem 1471. Hence we are done by Axiom 5. ☐

Theorem 1473. $(y\backslash T(x,z))\cdot y = T((y\backslash x)\cdot y,z)$.

Proof. We have $T((y\backslash x)\cdot y,z)/y = y\backslash T(y\cdot(y\backslash x),z)$ by Theorem 1472. Then

$$T((y\backslash x)\cdot y,z)/y = y\backslash T(x,z) \tag{848}$$

by Axiom 4. We have $(T((y\backslash x)\cdot y,z)/y)\cdot y = T((y\backslash x)\cdot y,z)$ by Axiom 6. Hence we are done by (848). ☐

Theorem 1474. $K(y\backslash((y\backslash e)\backslash x),y) = K(x,y)$.

Proof. We have $R(e/(e/y),e/y,(e/y)\backslash x) = ((e/y)\backslash x)/x$ by Theorem 37. Then

$$R(e/(e/y),y\backslash e,(e/y)\backslash x) = ((e/y)\backslash x)/x \tag{849}$$

by Theorem 1384. We have $((e/y)\backslash x)\backslash(x\cdot(((e/y)\backslash x)/x)) = K(x,((e/y)\backslash x)/x)$ by Theorem 3. Then

$$((e/y)\backslash x)\backslash(x\cdot R(e/(e/y),y\backslash e,(e/y)\backslash x)) = K(x,((e/y)\backslash x)/x) \tag{850}$$

by (849). We have $K(x,((((e/y)\backslash x)/x)\backslash e)\backslash e) = K(x,((e/y)\backslash x)/x)$ by Theorem 779. Then $K(x,((y\backslash e)\backslash x)/x) = K(x,((e/y)\backslash x)/x)$ by Theorem 1380. Then

$$((e/y)\backslash x)\backslash(x\cdot R(e/(e/y),y\backslash e,(e/y)\backslash x)) = K(x,((y\backslash e)\backslash x)/x) \tag{851}$$

by (850). We have $x\cdot R(e/(e/y),e/y,(e/y)\backslash x) = (e/y)\backslash T(x,(e/y)\backslash e)$ by Theorem 1458. Then $x\cdot R(e/(e/y),e/y,(e/y)\backslash x) = (e/y)\backslash T(x,y)$ by Proposition 25. Then $x\cdot R(e/(e/y),y\backslash e,(e/y)\backslash x) = (e/y)\backslash T(x,y)$ by Theorem 1384. Then

$$((e/y)\backslash x)\backslash((e/y)\backslash T(x,y)) = K(x,((y\backslash e)\backslash x)/x) \tag{852}$$

by (851). We have $K(((y\backslash e)\backslash e)\backslash((y\backslash e)\backslash x), y) = K(y\backslash((y\backslash e)\backslash x), y)$ by Theorem 1446. Then

$$K(y\backslash((e/y)\backslash x), y) = K(y\backslash((y\backslash e)\backslash x), y) \tag{853}$$

by Theorem 1374. We have $((e/y)\backslash x)\backslash(y \cdot T(y\backslash((e/y)\backslash x), y)) = K(y\backslash((e/y)\backslash x), y)$ by Theorem 46. Then $((e/y)\backslash x)\backslash((e/y)\backslash T(x, y)) = K(y\backslash((e/y)\backslash x), y)$ by Theorem 1335. Then $((e/y)\backslash x)\backslash((e/y)\backslash T(x, y)) = K(y\backslash((y\backslash e)\backslash x), y)$ by (853). Then

$$K(x, ((y\backslash e)\backslash x)/x) = K(y\backslash((y\backslash e)\backslash x), y) \tag{854}$$

by (852). We have

$$K(x, (y\backslash e)\backslash e) = T(T((y\backslash e)\backslash e, x)\backslash((y\backslash e)\backslash e), ((y\backslash e)\backslash e) \cdot x) \tag{855}$$

by Theorem 1273.

$$\begin{array}{rcll} & & x\backslash T(x, T(y\backslash e, ((y\backslash e)\backslash e) \cdot x)\backslash e) & \\ & = & T(x\backslash T(x, (y\backslash e)\backslash e), ((y\backslash e)\backslash e) \cdot x) & \text{by Theorem 1014} \\ & = & K(x, (y\backslash e)\backslash e) & \text{by (855), Theorem 1030.} \end{array}$$

Then $x\backslash T(x, T(y\backslash e, ((y\backslash e)\backslash e) \cdot x)\backslash e) = K(x, (y\backslash e)\backslash e)$. Then

$$x\backslash T(x, T(y\backslash e, (y\backslash e)\backslash x)\backslash e) = K(x, (y\backslash e)\backslash e) \tag{856}$$

by Theorem 1302. We have $x \cdot (x\backslash T(x, T(y\backslash e, (y\backslash e)\backslash x)\backslash e)) = T(x, T(y\backslash e, (y\backslash e)\backslash x)\backslash e)$ by Axiom 4. Then

$$x \cdot K(x, (y\backslash e)\backslash e) = T(x, T(y\backslash e, (y\backslash e)\backslash x)\backslash e) \tag{857}$$

by (856). We have $T(x, (((y\backslash e)\backslash x)\backslash x)\backslash e) = x \cdot K(x, ((y\backslash e)\backslash x)/x)$ by Theorem 1001. Then $T(x, T(y\backslash e, (y\backslash e)\backslash x)\backslash e) = x \cdot K(x, ((y\backslash e)\backslash x)/x)$ by Proposition 49. Then $x \cdot K(x, ((y\backslash e)\backslash x)/x) = x \cdot K(x, (y\backslash e)\backslash e)$ by (857). Then $K(x, ((y\backslash e)\backslash x)/x) = K(x, (y\backslash e)\backslash e)$ by Proposition 9. Then

$$K(y\backslash((y\backslash e)\backslash x), y) = K(x, (y\backslash e)\backslash e) \tag{858}$$

by (854). We have $K(x, (y\backslash e)\backslash e) = K(x, y)$ by Theorem 779. Hence we are done by (858). ☐

Theorem 1475. $K(y\backslash x, y) = K((y\backslash e) \cdot x, y)$.

Proof. We have $K(y\backslash((y\backslash e)\backslash((y\backslash e) \cdot x)), y) = K((y\backslash e) \cdot x, y)$ by Theorem 1474. Hence we are done by Axiom 3. ☐

Theorem 1476. $K((e/y) \cdot x, y) = K(y\backslash x, y)$.

Proof. We have $K((e/y) \cdot x, y) = K((y\backslash e) \cdot x, y)$ by Theorem 1447. Hence we are done by Theorem 1475. ☐

Theorem 1477. $K(x, (x\backslash e) \cdot y) = K(x, x\backslash y)$.

Proof. We have $K((e/x) \cdot ((x\backslash y) \cdot x), x) = K(x\backslash((x\backslash y) \cdot x), x)$ by Theorem 1476. Then $K((e/x) \cdot ((x\backslash y) \cdot x), x) = K(T(x\backslash y, x), x)$ by Definition 3. Then

$$K(T((e/x) \cdot y, x), x) = K(T(x\backslash y, x), x) \tag{859}$$

by Theorem 1333. We have $K(K(x, x\backslash e) \cdot T((x\backslash e) \cdot y, x), x) = K(T((x\backslash e) \cdot y, x), x)$ by Theorem 1445. Then $K(T(K(x, x\backslash e) \cdot ((x\backslash e) \cdot y), x), x) = K(T((x\backslash e) \cdot y, x), x)$ by Theorem 1135. Then $K(T((e/x) \cdot y, x), x) = K(T((x\backslash e) \cdot y, x), x)$ by Theorem 697. Then

$$K(T(x\backslash y, x), x) = K(T((x\backslash e) \cdot y, x), x) \tag{860}$$

by (859). We have $K(T((x\backslash e) \cdot y, x), x)\backslash e = K(x, (x\backslash e) \cdot y)$ by Theorem 1036. Then

$$K(T(x\backslash y, x), x)\backslash e = K(x, (x\backslash e) \cdot y) \tag{861}$$

by (860). We have $K(T(x\backslash y, x), x)\backslash e = K(x, x\backslash y)$ by Theorem 1036. Hence we are done by (861). ☐

Theorem 1478. $K(x\backslash e,(x\backslash e)\backslash y) = K(x\backslash e, x\cdot y)$.

Proof. We have $K(x\backslash e,((x\backslash e)\backslash e)\cdot y) = K(x\backslash e, x\cdot y)$ by Theorem 1414. Hence we are done by Theorem 1477. ☐

Theorem 1479. $T(x\cdot T(y,y\backslash e),T(y,y\backslash e)\backslash e) = T(y\backslash e,y)\backslash R(x,y,y\backslash e)$.

Proof. We have $R(x,(y\backslash e)\backslash e,((y\backslash e)\backslash e)\backslash e) = R(x,(y\backslash e)\backslash e,y\backslash e)$ by Theorem 1370. Then $R(x,(y\backslash e)\backslash e,((y\backslash e)\backslash e)\backslash e) = R(x,e/(y\backslash e),y\backslash e)$ by Theorem 1151. Then $R(x,(y\backslash e)\backslash e,T(y\backslash e,y)) = R(x,e/(y\backslash e),y\backslash e)$ by Theorem 209. Then $R(x,(y\backslash e)\backslash e,T(y\backslash e,y)) = R(x,y,y\backslash e)$ by Proposition 24. Then

$$R(x,T(y,y\backslash e),T(y\backslash e,y)) = R(x,y,y\backslash e) \tag{862}$$

by Proposition 49. We have $(T(y,y\backslash e)\backslash e)\backslash R(x,T(y,y\backslash e),T(y,y\backslash e)\backslash e) = T(x\cdot T(y,y\backslash e),T(y,y\backslash e)\backslash e)$ by Theorem 1326. Then $(T(y,y\backslash e)\backslash e)\backslash R(x,T(y,y\backslash e),T(y,y\backslash e)\backslash e) = T(((y\backslash e)\backslash e)\cdot T(x,y),T(y,y\backslash e)\backslash e)$ by Theorem 1403. Then $T(y\backslash e,y)\backslash R(x,T(y,y\backslash e),T(y,y\backslash e)\backslash e) = T(((y\backslash e)\backslash e)\cdot T(x,y),T(y,y\backslash e)\backslash e)$ by Theorem 1276. Then $T(y\backslash e,y)\backslash R(x,T(y,y\backslash e),T(y\backslash e,y)) = T(((y\backslash e)\backslash e)\cdot T(x,y),T(y,y\backslash e)\backslash e)$ by Theorem 1276. Then $T(y\backslash e,y)\backslash R(x,y,y\backslash e) = T(((y\backslash e)\backslash e)\cdot T(x,y),T(y,y\backslash e)\backslash e)$ by (862). Hence we are done by Theorem 1403. ☐

Theorem 1480. $K(x,y) = L(K(x,y),y\backslash e,y)$.

Proof. We have $((T(y,x)\backslash y)/(y\cdot(y\backslash T(y,y\backslash e))))\cdot(((T(y,x)\backslash y)/(y\cdot(y\backslash T(y,y\backslash e))))\backslash(((T(y,x)\backslash y)/(y\cdot(y\backslash T(y,y\backslash e))))\cdot T(y,y\backslash e))) = ((T(y,x)\backslash y)/(y\cdot(y\backslash T(y,y\backslash e))))\cdot T(y,y\backslash e)$ by Axiom 4. Then $((T(y,x)\backslash y)/(y\cdot(y\backslash T(y,y\backslash e))))\cdot(y\cdot(y\backslash(((T(y,x)\backslash y)/(y\cdot(y\backslash T(y,y\backslash e))))\backslash(((T(y,x)\backslash y)/(y\cdot(y\backslash T(y,y\backslash e))))\cdot T(y,y\backslash e))))) = ((T(y,x)\backslash y)/(y\cdot(y\backslash T(y,y\backslash e))))\cdot T(y,y\backslash e)$ by Axiom 4. Then

$$((T(y,x)\backslash y)/(y\cdot(y\backslash T(y,y\backslash e))))\cdot(y\cdot(y\backslash T(y,y\backslash e))) = ((T(y,x)\backslash y)/(y\cdot(y\backslash T(y,y\backslash e))))\cdot T(y,y\backslash e) \tag{863}$$

by Axiom 3. We have $((T(y,x)\backslash y)/(y\cdot(y\backslash T(y,y\backslash e))))\cdot(y\cdot(y\backslash T(y,y\backslash e))) = T(y,x)\backslash y$ by Axiom 6. Then $((T(y,x)\backslash y)/(y\cdot(y\backslash T(y,y\backslash e))))\cdot T(y,y\backslash e) = T(y,x)\backslash y$ by (863). Then

$$((T(y,x)\backslash y)/((y\backslash e)\backslash e))\cdot T(y,y\backslash e) = T(y,x)\backslash y \tag{864}$$

by Theorem 795. We have $(y\cdot(T(y,x)\backslash e))/T(y,T(y,x)\backslash e) = T(y,x)\backslash e$ by Theorem 5. Then $(T(y,x)\backslash y)/T(y,T(y,x)\backslash e) = T(y,x)\backslash e$ by Theorem 175. Then $(T(y,x)\backslash y)/((y\backslash e)\backslash e) = T(y,x)\backslash e$ by Theorem 1411. Then

$$(T(y,x)\backslash e)\cdot T(y,y\backslash e) = T(y,x)\backslash y \tag{865}$$

by (864). We have

$$T((T(y,x)\backslash e)\cdot T(y,y\backslash e),T(y,y\backslash e)\backslash e) = T(y\backslash e,y)\backslash R(T(y,x)\backslash e,y,y\backslash e) \tag{866}$$

by Theorem 1479. Then

$$T(T(y,x)\backslash y,T(y,y\backslash e)\backslash e) = T(y\backslash e,y)\backslash R(T(y,x)\backslash e,y,y\backslash e) \tag{867}$$

by (865). We have

$$T((T(y,x)\backslash e)\cdot T(y,y\backslash e),T(y\backslash e,y)) = T(y\backslash e,y)\backslash R(T(y,x)\backslash e,y,y\backslash e) \tag{868}$$

by (866) and Theorem 1276. We have $T((T(y,x)\backslash e)\cdot T(y,y\backslash e),T(y\backslash e,y)) = T((T(y,x)\backslash e)\cdot T(y,y\backslash e),y\backslash e)$ by Theorem 1401. Then

$$T(y\backslash e,y)\backslash R(T(y,x)\backslash e,y,y\backslash e) = T((T(y,x)\backslash e)\cdot T(y,y\backslash e),y\backslash e) \tag{869}$$

by (868). We have $T(((e/y)\cdot((y\backslash e)\backslash(T(y,x)\backslash e)))\cdot y,y\backslash e) = R((y\backslash e)\backslash(T(y,x)\backslash e),e/y,y)$ by Theorem 1404. Then $T((K(y,y\backslash e)\cdot(T(y,x)\backslash e))\cdot y,y\backslash e) = R((y\backslash e)\backslash(T(y,x)\backslash e),e/y,y)$ by Theorem 1375. Then $T((T(y,x)\backslash e)\cdot T(y,y\backslash e),y\backslash e) = R((y\backslash e)\backslash(T(y,x)\backslash e),e/y,y)$ by Theorem 1413. Then

$$T(y\backslash e,y)\backslash R(T(y,x)\backslash e,y,y\backslash e) = R((y\backslash e)\backslash(T(y,x)\backslash e),e/y,y) \tag{870}$$

by (869). We have $R((y\backslash e)\backslash(T(y,x)\backslash e), e/y, y) = R((y\backslash e)\backslash(T(y,x)\backslash e), y\backslash e, y)$ by Theorem 1151. Then $T(y\backslash e, y)\backslash R(T(y,x)\backslash e, y, y\backslash e) = R((y\backslash e)\backslash(T(y,x)\backslash e), y\backslash e, y)$ by (870). Then $T(T(y,x)\backslash y, T(y, y\backslash e)\backslash e) = R((y\backslash e)\backslash(T(y,x)\backslash e), y\backslash e, y)$ by (867). Then

$$R((y\backslash e)\backslash(T(y,x)\backslash e), y\backslash e, y) = T(T(y,x)\backslash y, T(y\backslash e, y)) \tag{871}$$

by Theorem 1276. We have $T(T(y,x)\backslash y, T(y\backslash e, y)) = T(T(y,x)\backslash y, y\backslash e)$ by Theorem 1401. Then

$$R((y\backslash e)\backslash(T(y,x)\backslash e), y\backslash e, y) = T(T(y,x)\backslash y, y\backslash e) \tag{872}$$

by (871). We have $T(y\backslash e, T(y,x)\backslash y) = y\backslash e$ by Theorem 1428. Then $T(T(y,x)\backslash y, y\backslash e) = T(y,x)\backslash y$ by Proposition 21. Then $R((y\backslash e)\backslash(T(y,x)\backslash e), y\backslash e, y) = T(y,x)\backslash y$ by (872). Then

$$R(T(y,x)\backslash y, y\backslash e, y) = T(y,x)\backslash y \tag{873}$$

by Theorem 1418. We have $y \cdot (T(y,x)\backslash e) = T(y,x)\backslash y$ by Theorem 175. Then

$$y \cdot ((e/T(y,x)) \cdot K(y\backslash e, y)) = T(y,x)\backslash y \tag{874}$$

by Theorem 1417. We have $(y \cdot (e/T(y,x))) \cdot K(y\backslash e, y) = y \cdot ((e/T(y,x)) \cdot K(y\backslash e, y))$ by Theorem 668. Then

$$(y \cdot (e/T(y,x))) \cdot K(y\backslash e, y) = T(y,x)\backslash y \tag{875}$$

by (874). We have $(e/y) \cdot ((y \cdot (e/T(y,x))) \cdot K(y\backslash e, y)) = (y\backslash e) \cdot (y \cdot (e/T(y,x)))$ by Theorem 1396. Then

$$(e/y) \cdot (T(y,x)\backslash y) = (y\backslash e) \cdot (y \cdot (e/T(y,x))) \tag{876}$$

by (875). We have $((y\backslash e) \cdot (y \cdot (e/T(y,x)))) \cdot y = L((e/T(y,x)) \cdot y, y\backslash e, y)$ by Theorem 893. Then

$$((e/y) \cdot (T(y,x)\backslash y)) \cdot y = L((e/T(y,x)) \cdot y, y\backslash e, y) \tag{877}$$

by (876). We have $(e/T(y,x)) \cdot y = y/T(y,x)$ by Theorem 550. Then $(e/T(y,x)) \cdot y = T(y,x)\backslash y$ by Theorem 753. Then

$$L(T(y,x)\backslash y, y\backslash e, y) = ((e/y) \cdot (T(y,x)\backslash y)) \cdot y \tag{878}$$

by (877). We have $(K(y\backslash(y/((x\backslash y)/T(y,x))), y) \cdot (e/y)) \cdot y = R(K(y\backslash(y/((x\backslash y)/T(y,x))), y), y\backslash e, y)$ by Theorem 911. Then $((e/y) \cdot K(y\backslash(y/((x\backslash y)/T(y,x))), y)) \cdot y = R(K(y\backslash(y/((x\backslash y)/T(y,x))), y), y\backslash e, y)$ by Theorem 1354. Then $((e/y)\cdot(T(y,x)\backslash y))\cdot y = R(K(y\backslash(y/((x\backslash y)/T(y,x))), y), y\backslash e, y)$ by Theorem 932. Then $R(K(y\backslash(y/((x\backslash y)/T(y,x))), y), y\backslash e, y) = L(T(y,x)\backslash y, y\backslash e, y)$ by (878). Then $R(T(y,x)\backslash y, y\backslash e, y) = L(T(y,x)\backslash y, y\backslash e, y)$ by Theorem 932. Then

$$T(y,x)\backslash y = L(T(y,x)\backslash y, y\backslash e, y) \tag{879}$$

by (873). We have $L(L(T(y,x)\backslash y, T(y,x), x), y\backslash e, y) = L(L(T(y,x)\backslash y, y\backslash e, y), T(y,x), x)$ by Axiom 11. Then $L((y \cdot x)\backslash(x \cdot y), y\backslash e, y) = L(L(T(y,x)\backslash y, y\backslash e, y), T(y,x), x)$ by Theorem 798. Then

$$L(T(y,x)\backslash y, T(y,x), x) = L((y \cdot x)\backslash(x \cdot y), y\backslash e, y) \tag{880}$$

by (879). We have $L(T(y,x)\backslash y, T(y,x), x) = (y \cdot x)\backslash(x \cdot y)$ by Theorem 798. Then

$$L((y \cdot x)\backslash(x \cdot y), y\backslash e, y) = (y \cdot x)\backslash(x \cdot y) \tag{881}$$

by (880). We have $K(x,y) = (y \cdot x)\backslash(x \cdot y)$ by Definition 2. Then $K(x,y) = L((y \cdot x)\backslash(x \cdot y), y\backslash e, y)$ by (881). Hence we are done by Definition 2. □

Theorem 1481. *$K(x,y)/y = (e/y) \cdot K(x,y)$.*

Proof. We have $(y\backslash e)\cdot(y\cdot(K(x,y)/y)) = L(K(x,y), y\backslash e, y)/y$ by Theorem 1355. Then $(e/y)\cdot K(x,y) = L(K(x,y), y\backslash e, y)/y$ by Theorem 1439. Hence we are done by Theorem 1480. □

Theorem 1482. $T(y,x)\backslash y = K(x,y)$.

Proof. We have $T(y,x) \cdot K(x,y) = y$ by Theorem 1307. Hence we are done by Proposition 2. $\square$

Theorem 1483. $L(K(x,z),T(z,x),y) = (y \cdot T(z,x))\backslash(y \cdot z)$.

Proof. We have $L(K(x,z),T(z,x),y) = (y \cdot T(z,x))\backslash(y \cdot (T(z,x) \cdot K(x,z)))$ by Definition 4. Hence we are done by Theorem 1307. $\square$

Theorem 1484. $(y \cdot x)/(K(z,y) \cdot x) = T(y,z)$.

Proof. We have $(y{\cdot}x)/(K(z,y){\cdot}x) = K(z,y)\backslash y$ by Theorem 1050. Hence we are done by Theorem 1319. $\square$

Theorem 1485. $K(x,x\backslash(x/y)) = K(x,y\backslash e)$.

Proof. We have $x \cdot K(x,(x/y)/x) = T(x,y\backslash e)$ by Theorem 1457. Then

$$x \cdot K(x,x\backslash(x/y)) = T(x,y\backslash e) \tag{882}$$

by Theorem 1465. We have $T(x,y\backslash e) = x \cdot K(x,y\backslash e)$ by Theorem 1309. Then $x \cdot K(x,x\backslash(x/y)) = x \cdot K(x,y\backslash e)$ by (882). Hence we are done by Proposition 9. $\square$

Theorem 1486. $x \cdot T(x\backslash e,y) = K(x\backslash e,y)$.

Proof. We have

$$T(x\backslash e,y) = (x\backslash e) \cdot K(x\backslash e,y) \tag{883}$$

by Theorem 1309. We have $(x\backslash e) \cdot (x \cdot T(x\backslash e,y)) = T(x\backslash e,y)$ by Theorem 728. Then $(x\backslash e) \cdot (x \cdot T(x\backslash e,y)) = (x\backslash e) \cdot K(x\backslash e,y)$ by (883). Hence we are done by Proposition 9. $\square$

Theorem 1487. $L(K(y,z),y,x) = (x \cdot y)\backslash(x \cdot T(y,z))$.

Proof. We have $L(K(y,z),y,x) = (x \cdot y)\backslash(x \cdot (y \cdot K(y,z)))$ by Definition 4. Hence we are done by Theorem 1309. $\square$

Theorem 1488. $y\backslash((y/z) \cdot T(z,x)) = L(K(z,x),z,y/z)$.

Proof. We have $y\backslash((y/z) \cdot (z \cdot K(z,x))) = L(K(z,x),z,y/z)$ by Theorem 4. Hence we are done by Theorem 1309. $\square$

Theorem 1489. $(z \cdot K(x,y))\backslash(z \cdot T(x,y)) = L(x,K(x,y),z)$.

Proof. We have $(z{\cdot}K(x,y))\backslash(z{\cdot}(x{\cdot}K(x,y))) = L(T(x,K(x,y)),K(x,y),z)$ by Proposition 62. Then $(z{\cdot}K(x,y))\backslash(z{\cdot}(x{\cdot}K(x,y))) = L(x,K(x,y),z)$ by Theorem 736. Hence we are done by Theorem 1309. $\square$

Theorem 1490. $y \cdot (x \cdot T(K(x,y),z)) = (y \cdot x) \cdot T(K(x,y),z)$.

Proof. We have $y \cdot (x \cdot T(x\backslash T(x,y),z)) = (y \cdot x) \cdot T(K(x,y),z)$ by Theorem 1113. Hence we are done by Theorem 1308. $\square$

Theorem 1491. $K(y\backslash(y/x),y) = K(x\backslash e,y)$.

Proof. We have $K(y\backslash(y/x),y) = (x\backslash e)\backslash T(x\backslash e,y)$ by Theorem 730. Hence we are done by Theorem 1308. $\square$

Theorem 1492. $K(y,x) \cdot z = T(y,x) \cdot (y\backslash z)$.

Proof. We have $(y\backslash T(y,x)){\cdot}z = T(y,x){\cdot}(y\backslash z)$ by Theorem 1270. Hence we are done by Theorem 1308. $\square$

Theorem 1493. $K(K(x,y),z) = K(y,x) \cdot T(K(x,y),z)$.

Proof. We have $K(x,y)\backslash T(K(x,y),z) = K(y,x) \cdot T(K(x,y),z)$ by Theorem 1467. Hence we are done by Theorem 1308. ☐

Theorem 1494. $y \cdot K(x\backslash e, y) = x \cdot ((y\backslash(x\backslash y)) \cdot y)$.

Proof. We have $x \cdot ((y\backslash(x\backslash y)) \cdot y) = y \cdot (x \cdot T(x\backslash e, y))$ by Theorem 1422. Then $x \cdot ((y\backslash(x\backslash y)) \cdot y) = y \cdot (T(y, x\backslash e)\backslash y)$ by Theorem 1028. Hence we are done by Theorem 1482. ☐

Theorem 1495. $T(K(x, y\backslash e), z) = K(x, T(y,z)\backslash e)$.

Proof. We have $x\backslash T(x, T(y,z)\backslash e) = T(x\backslash T(x, y\backslash e), z)$ by Theorem 1014. Then $K(x, T(y,z)\backslash e) = T(x\backslash T(x, y\backslash e), z)$ by Theorem 1308. Hence we are done by Theorem 1308. ☐

Theorem 1496. $K(x, T(y\backslash e, z)\backslash e) = T(K(x,y), z)$.

Proof. We have $K(x, T(y\backslash e, z)\backslash e) = T(K(x, (y\backslash e)\backslash e), z)$ by Theorem 1495. Hence we are done by Theorem 779. ☐

Theorem 1497. $T(K(x\backslash e, z), y) = K(T(x,y)\backslash e, z)$.

Proof. We have $T(T(z, x\backslash e)\backslash z, y) = K(z\backslash(z/T(x,y)), z)$ by Theorem 1461. Then

$$T(K(x\backslash e, z), y) = K(z\backslash(z/T(x,y)), z) \tag{884}$$

by Theorem 1482. We have $K(z\backslash(z/T(x,y)), z) = K(T(x,y)\backslash e, z)$ by Theorem 1491. Hence we are done by (884). ☐

Theorem 1498. $K(T(x\backslash e, z)\backslash e, y) = T(K(x,y), z)$.

Proof. We have $K(T(x\backslash e, z)\backslash e, y) = T(K((x\backslash e)\backslash e, y), z)$ by Theorem 1497. Hence we are done by Theorem 733. ☐

Theorem 1499. $K(x \cdot K(z, x\backslash e), y) = T(K(x,y), z)$.

Proof. We have $K(x\backslash e, x) \cdot ((z\backslash T(z, x\backslash e)) \cdot ((x\backslash e)\backslash e)) = x \cdot T(z\backslash T(z, x\backslash e), (x\backslash e)\backslash e)$ by Theorem 1378. Then

$$K(x\backslash e, x) \cdot ((x\backslash e)\backslash(z\backslash T(z, x\backslash e))) = x \cdot T(z\backslash T(z, x\backslash e), (x\backslash e)\backslash e) \tag{885}$$

by Theorem 1432. We have $K(x\backslash e, x) \cdot ((x\backslash e)\backslash(z\backslash T(z, x\backslash e))) = (e/x)\backslash(z\backslash T(z, x\backslash e))$ by Theorem 436. Then $x \cdot T(z\backslash T(z, x\backslash e), (x\backslash e)\backslash e) = (e/x)\backslash(z\backslash T(z, x\backslash e))$ by (885). Then

$$x \cdot (z\backslash T(z, x\backslash e)) = (e/x)\backslash(z\backslash T(z, x\backslash e)) \tag{886}$$

by Theorem 1410. We have $(e/x)\backslash(T(e/x, T(z, e/x))\backslash(e/x)) = T(e/x, T(z, e/x))\backslash e$ by Theorem 1339. Then

$$(e/x)\backslash(z\backslash T(z, e/x)) = T(e/x, T(z, e/x))\backslash e \tag{887}$$

by Theorem 986. We have $K(T(e/x, T(z, e/x))\backslash e, y) = K(T(x\backslash e, T(z, e/x))\backslash e, y)$ by Theorem 1415. Then $K((e/x)\backslash(z\backslash T(z, e/x)), y) = K(T(x\backslash e, T(z, e/x))\backslash e, y)$ by (887). Then $K(T(x\backslash e, T(z, e/x))\backslash e, y) = K((e/x)\backslash(z\backslash T(z, x\backslash e)), y)$ by Theorem 1397. Then $K(T(x\backslash e, T(z, e/x))\backslash e, y) = K(x \cdot (z\backslash T(z, x\backslash e)), y)$ by (886). Then $K(T(x\backslash e, T(z, x\backslash e))\backslash e, y) = K(x \cdot (z\backslash T(z, x\backslash e)), y)$ by Theorem 1397. Then $K(T(x\backslash e, z)\backslash e, y) = K(x \cdot (z\backslash T(z, x\backslash e)), y)$ by Theorem 1032. Then $K(x \cdot (z\backslash T(z, x\backslash e)), y) = T(K(x,y), z)$ by Theorem 1498. Hence we are done by Theorem 1308. ☐

Theorem 1500. $K(x, (y\backslash z)\backslash e) = L(K(x, z\backslash y), z\backslash y, z)$.

Proof. We have

$$T(x, R((y\backslash z)\backslash e, ((y\backslash z)\backslash e)\backslash e, x)) = (x\backslash T(x, (y\backslash z)\backslash e)) \cdot x \tag{888}$$

by Theorem 989. We have $T(x, R((y\backslash z)\backslash e, ((y\backslash z)\backslash e)\backslash e, x)) = (y\backslash (z\cdot((x\cdot(z\backslash y))/x)))\cdot x$ by Theorem 953. Then $(y\backslash (z \cdot ((x \cdot (z\backslash y))/x))) \cdot x = (x\backslash T(x, (y\backslash z)\backslash e)) \cdot x$ by (888). Then

$$y\backslash (z \cdot ((x \cdot (z\backslash y))/x)) = x\backslash T(x, (y\backslash z)\backslash e) \tag{889}$$

by Proposition 10. We have $y\backslash (z \cdot ((z\backslash y) \cdot (x\backslash T(x, z\backslash y)))) = L(x\backslash T(x, z\backslash y), z\backslash y, z)$ by Theorem 449. Then $y\backslash (z \cdot ((x \cdot (z\backslash y))/x)) = L(x\backslash T(x, z\backslash y), z\backslash y, z)$ by Theorem 977. Then

$$L(x\backslash T(x, z\backslash y), z\backslash y, z) = x\backslash T(x, (y\backslash z)\backslash e) \tag{890}$$

by (889).

$$\begin{array}{rll} & K(x, (y\backslash z)\backslash e) & \\ = & x\backslash T(x, (y\backslash z)\backslash e) & \text{by Theorem 1308} \\ = & L(K(x, z\backslash y), z\backslash y, z) & \text{by (890), Theorem 1308.} \end{array}$$

Hence we are done. □

Theorem 1501. $((z\backslash x) \cdot z) \cdot K(z, y) = (z\backslash (x \cdot K(z, y))) \cdot z.$

Proof. We have $(z\backslash (x \cdot (((y \cdot z)/y)\backslash z))) \cdot z = (((y \cdot z)/y) \cdot T(((y \cdot z)/y)\backslash x, z)) \cdot (((y \cdot z)/y)\backslash z)$ by Theorem 1096. Then $(z\backslash (x \cdot (T(y, z)\backslash y))) \cdot z = (((y \cdot z)/y) \cdot T(((y \cdot z)/y)\backslash x, z)) \cdot (((y \cdot z)/y)\backslash z)$ by Theorem 984. Then $((y \cdot (z \cdot T(z\backslash T(x, y), z)))/y) \cdot (((y \cdot z)/y)\backslash z) = (z\backslash (x \cdot (T(y, z)\backslash y))) \cdot z$ by Theorem 1360. Then $((y \cdot ((z\backslash T(x, y)) \cdot z))/y) \cdot (((y \cdot z)/y)\backslash z) = (z\backslash (x \cdot (T(y, z)\backslash y))) \cdot z$ by Proposition 46. Then $((y \cdot ((z\backslash T(x, y)) \cdot z))/y) \cdot (T(y, z)\backslash y) = (z\backslash (x \cdot (T(y, z)\backslash y))) \cdot z$ by Theorem 984. Then $((y \cdot T((z\backslash x) \cdot z, y))/y) \cdot (T(y, z)\backslash y) = (z\backslash (x \cdot (T(y, z)\backslash y))) \cdot z$ by Theorem 1473. Then $((z\backslash x) \cdot z) \cdot (T(y, z)\backslash y) = (z\backslash (x\cdot(T(y, z)\backslash y)))\cdot z$ by Proposition 48. Then $(z\backslash (x\cdot K(z, y)))\cdot z = ((z\backslash x)\cdot z)\cdot(T(y, z)\backslash y)$ by Theorem 1482. Hence we are done by Theorem 1482. □

Theorem 1502. $x \cdot (z \cdot K(y\backslash e, z)) = (x \cdot K(y\backslash e, z)) \cdot z.$

Proof. We have $(x \cdot y) \cdot (z \cdot T(z\backslash L(y\backslash z, y, x), z)) = (x \cdot (y \cdot T(y\backslash e, z))) \cdot z$ by Theorem 884. Then $(x\cdot y)\cdot((z\backslash L(y\backslash z, y, x))\cdot z) = (x\cdot(y\cdot T(y\backslash e, z)))\cdot z$ by Proposition 46. Then $(x\cdot y)\cdot((z\backslash L(y\backslash z, y, x))\cdot z) = (x \cdot K(y\backslash e, z)) \cdot z$ by Theorem 1486. Then

$$(x \cdot y) \cdot L((z\backslash (y\backslash z)) \cdot z, y, x) = (x \cdot K(y\backslash e, z)) \cdot z \tag{891}$$

by Theorem 1299. We have $(x \cdot y) \cdot L((z\backslash (y\backslash z)) \cdot z, y, x) = x \cdot (y \cdot ((z\backslash (y\backslash z)) \cdot z))$ by Proposition 52. Then $(x \cdot K(y\backslash e, z)) \cdot z = x \cdot (y \cdot ((z\backslash (y\backslash z)) \cdot z))$ by (891). Hence we are done by Theorem 1494. □

Theorem 1503. $R(z, K(x\backslash e, y), y) = z.$

Proof. We have

$$(z \cdot K(x\backslash e, y)) \cdot y = z \cdot (y \cdot K(x\backslash e, y)) \tag{892}$$

by Theorem 1502. We have $K(x\backslash e, y) \cdot y = y \cdot K(x\backslash e, y)$ by Theorem 933. Then $(z \cdot K(x\backslash e, y)) \cdot y = z \cdot (K(x\backslash e, y) \cdot y)$ by (892). Hence we are done by Theorem 64. □

Theorem 1504. $R(z, K(x, y), y) = z.$

Proof. We have $R(z, K((e/x)\backslash e, y), y) = z$ by Theorem 1503. Hence we are done by Proposition 25. □

Theorem 1505. $K(y, x) = y\backslash T(y, R(x, y, y\backslash e)).$

Proof. We have $K((((y\cdot x)/((y\cdot x)\cdot(e/y)))\backslash e)\backslash e,((y\cdot x)/((y\cdot x)\cdot(y\backslash e)))\backslash e) = K((y\cdot x)/((y\cdot x)\cdot(e/y)),((y\cdot x)/((y\cdot x)\cdot(y\backslash e)))\backslash e)$ by Theorem 733. Then

$$K((y\cdot x)/((y\cdot x)\cdot(y\backslash e)),((y\cdot x)/((y\cdot x)\cdot(y\backslash e)))\backslash e) = K((y\cdot x)/((y\cdot x)\cdot(e/y)),((y\cdot x)/((y\cdot x)\cdot(y\backslash e)))\backslash e) \tag{893}$$

by Theorem 1394. We have $K((y\cdot x)/((y\cdot x)\cdot(y\backslash e)),((y\cdot x)/((y\cdot x)\cdot(y\backslash e)))\backslash e) = K(((y\cdot x)\cdot(y\backslash e))\backslash(y\cdot x),(y\cdot x)\backslash((y\cdot x)\cdot(y\backslash e)))$ by Theorem 1423. Then

$$K((y\cdot x)/((y\cdot x)\cdot(e/y)),((y\cdot x)/((y\cdot x)\cdot(y\backslash e)))\backslash e) = K(((y\cdot x)\cdot(y\backslash e))\backslash(y\cdot x),(y\cdot x)\backslash((y\cdot x)\cdot(y\backslash e))) \tag{894}$$

by (893). We have $K(((y\cdot x)\cdot(y\backslash e))\backslash(y\cdot x),(y\cdot x)\backslash((y\cdot x)\cdot(y\backslash e))) = K((y\backslash e)\backslash e,y\backslash e)$ by Theorem 664. Then

$$K((y\cdot x)/((y\cdot x)\cdot(e/y)),((y\cdot x)/((y\cdot x)\cdot(y\backslash e)))\backslash e) = K((y\backslash e)\backslash e,y\backslash e) \tag{895}$$

by (894). We have $K((y\backslash e)\backslash e,y\backslash e) = K(y,y\backslash e)$ by Theorem 733. Then

$$K((y\cdot x)/((y\cdot x)\cdot(e/y)),((y\cdot x)/((y\cdot x)\cdot(y\backslash e)))\backslash e) = K(y,y\backslash e) \tag{896}$$

by (895). We have $K((y\cdot x)/((y\cdot x)\cdot(e/y)),((((y\cdot x)/((y\cdot x)\cdot(e/y)))\backslash e)\backslash e)\backslash e) = K((y\cdot x)/((y\cdot x)\cdot(e/y)),((y\cdot x)/((y\cdot x)\cdot(e/y)))\backslash e)$ by Theorem 779. Then

$$K((y\cdot x)/((y\cdot x)\cdot(e/y)),((y\cdot x)/((y\cdot x)\cdot(y\backslash e)))\backslash e) = K((y\cdot x)/((y\cdot x)\cdot(e/y)),((y\cdot x)/((y\cdot x)\cdot(e/y)))\backslash e) \tag{897}$$

by Theorem 1394. We have $(((y\cdot x)\cdot(e/y))\cdot((y\cdot x)/((y\cdot x)\cdot(e/y))))\cdot K((y\cdot x)/((y\cdot x)\cdot(e/y)),((y\cdot x)/((y\cdot x)\cdot(e/y)))\backslash e) = ((y\cdot x)\cdot(e/y))\cdot((((y\cdot x)/((y\cdot x)\cdot(e/y)))\backslash e)\backslash e)$ by Theorem 407. Then $(((y\cdot x)\cdot(e/y))\cdot((y\cdot x)/((y\cdot x)\cdot(e/y))))\cdot K((y\cdot x)/((y\cdot x)\cdot(e/y)),((y\cdot x)/((y\cdot x)\cdot(y\backslash e)))\backslash e) = ((y\cdot x)\cdot(e/y))\cdot((((y\cdot x)/((y\cdot x)\cdot(e/y)))\backslash e)\backslash e)$ by (897). Then $(((y\cdot x)\cdot(e/y))\cdot((y\cdot x)/((y\cdot x)\cdot(e/y))))\cdot K(y,y\backslash e) = ((y\cdot x)\cdot(e/y))\cdot((((y\cdot x)/((y\cdot x)\cdot(e/y)))\backslash e)\backslash e)$ by (896). Then

$$(((y\cdot x)\cdot(e/y))\cdot((y\cdot x)/((y\cdot x)\cdot(e/y))))\cdot K(y,y\backslash e) = ((y\cdot x)\cdot(e/y))\cdot((y\cdot x)/((y\cdot x)\cdot(y\backslash e))) \tag{898}$$

by Theorem 1394. We have $(K(y\backslash e,y)\cdot((y\cdot x)\cdot(e/y)))\cdot((y\cdot x)/((y\cdot x)\cdot(y\backslash e))) = K(y\backslash e,y)\cdot(((y\cdot x)\cdot(e/y))\cdot((y\cdot x)/((y\cdot x)\cdot(y\backslash e))))$ by Theorem 968. Then

$$((y\cdot x)\cdot(y\backslash e))\cdot((y\cdot x)/((y\cdot x)\cdot(y\backslash e))) = K(y\backslash e,y)\cdot(((y\cdot x)\cdot(e/y))\cdot((y\cdot x)/((y\cdot x)\cdot(y\backslash e)))) \tag{899}$$

by Theorem 1400. We have $K(y\backslash e,y)\cdot((((y\cdot x)\cdot(y\backslash e))\cdot((y\cdot x)/((y\cdot x)\cdot(y\backslash e))))\cdot K(y,y\backslash e)) = ((y\cdot x)\cdot(y\backslash e))\cdot((y\cdot x)/((y\cdot x)\cdot(y\backslash e)))$ by Theorem 1182. Then $K(y\backslash e,y)\cdot((((y\cdot x)\cdot(y\backslash e))\cdot((y\cdot x)/((y\cdot x)\cdot(y\backslash e))))\cdot K(y,y\backslash e)) = K(y\backslash e,y)\cdot(((y\cdot x)\cdot(e/y))\cdot((y\cdot x)/((y\cdot x)\cdot(y\backslash e))))$ by (899). Then $(((y\cdot x)\cdot(y\backslash e))\cdot((y\cdot x)/((y\cdot x)\cdot(y\backslash e))))\cdot K(y,y\backslash e) = ((y\cdot x)\cdot(e/y))\cdot((y\cdot x)/((y\cdot x)\cdot(y\backslash e)))$ by Proposition 9. Then

$$((y\cdot x)\cdot(e/y))\cdot((y\cdot x)/((y\cdot x)\cdot(e/y))) = ((y\cdot x)\cdot(y\backslash e))\cdot((y\cdot x)/((y\cdot x)\cdot(y\backslash e))) \tag{900}$$

by (898) and Proposition 8. We have $(((y\cdot x)\cdot(e/y))\cdot((y\cdot x)/((y\cdot x)\cdot(e/y))))\cdot K(((y\cdot x)\cdot(e/y))\backslash(y\cdot x),(y\cdot x)\cdot(e/y)) = y\cdot x$ by Theorem 1469. Then $(((y\cdot x)\cdot(e/y))\cdot((y\cdot x)/((y\cdot x)\cdot(e/y))))\cdot K(L(y,y\backslash e,y\cdot x),(y\cdot x)\cdot(e/y)) = y\cdot x$ by Theorem 649. Then

$$(((y\cdot x)\cdot(y\backslash e))\cdot((y\cdot x)/((y\cdot x)\cdot(y\backslash e))))\cdot K(L(y,y\backslash e,y\cdot x),(y\cdot x)\cdot(e/y)) = y\cdot x \tag{901}$$

by (900). We have

$$(((y\cdot x)\cdot(y\backslash e))\cdot((((y\cdot x)\cdot(y\backslash e))/(y\backslash e))/((y\cdot x)\cdot(y\backslash e))))\cdot(y\backslash e) = T((y\cdot x)\cdot(y\backslash e),(y\backslash e)\backslash e) \tag{902}$$

by Theorem 1005. We have $(y\cdot T(x,(y\backslash e)\backslash e))\cdot(y\backslash e) = T((y\cdot x)\cdot(y\backslash e),(y\backslash e)\backslash e)$ by Theorem 1337. Then $((y\cdot x)\cdot(y\backslash e))\cdot((((y\cdot x)\cdot(y\backslash e))/(y\backslash e))/((y\cdot x)\cdot(y\backslash e))) = y\cdot T(x,(y\backslash e)\backslash e)$ by (902) and Proposition 8. Then

$$((y\cdot x)\cdot(y\backslash e))\cdot((y\cdot x)/((y\cdot x)\cdot(y\backslash e))) = y\cdot T(x,(y\backslash e)\backslash e) \tag{903}$$

by Axiom 5. We have $y \cdot T(x,(y\backslash e)\backslash e) = x \cdot y$ by Theorem 1395. Then $((y \cdot x) \cdot (y\backslash e)) \cdot ((y \cdot x)/((y \cdot x) \cdot (y\backslash e))) = x \cdot y$ by (903). Then

$$(x \cdot y) \cdot K(L(y, y\backslash e, y \cdot x), (y \cdot x) \cdot (e/y)) = y \cdot x \tag{904}$$

by (901). We have $T(y,(y \cdot x) \cdot (e/y)) = K(((y \cdot x) \cdot (e/y))\backslash(((y \cdot x) \cdot (e/y))/(e/y)), (y \cdot x) \cdot (e/y)) \cdot y$ by Theorem 201. Then $T(y,(y \cdot x) \cdot (e/y)) = K(((y \cdot x) \cdot (e/y))\backslash(y \cdot x), (y \cdot x) \cdot (e/y)) \cdot y$ by Axiom 5. Then

$$K(L(y, y\backslash e, y \cdot x), (y \cdot x) \cdot (e/y)) \cdot y = T(y,(y \cdot x) \cdot (e/y)) \tag{905}$$

by Theorem 649. We have $T(y,(y \cdot x) \cdot (e/y)) = T(y,(y \cdot x) \cdot (y\backslash e))$ by Theorem 1366. Then $T(y,(y \cdot x) \cdot (e/y)) = (y\backslash T(y,(y \cdot x) \cdot (y\backslash e))) \cdot y$ by Theorem 759. Then $y\backslash T(y,(y \cdot x) \cdot (y\backslash e)) = K(L(y, y\backslash e, y \cdot x), (y \cdot x) \cdot (e/y))$ by (905) and Proposition 8. Then

$$(x \cdot y) \cdot (y\backslash T(y,(y \cdot x) \cdot (y\backslash e))) = y \cdot x \tag{906}$$

by (904). We have $T(y, T((y \cdot x) \cdot (y\backslash e), y)) = T(y,(y \cdot x) \cdot (y\backslash e))$ by Theorem 1032. Then $T(y, R(x, y, y\backslash e)) = T(y,(y \cdot x) \cdot (y\backslash e))$ by Theorem 1331. Then $(x \cdot y) \cdot (y\backslash T(y, R(x, y, y\backslash e))) = y \cdot x$ by (906). Hence we are done by Theorem 460. □

Theorem 1506. $T(K(y,x),z) = T(K(x,y),z)\backslash e$.

Proof. We have $T((x\backslash e) \cdot ((((x \cdot y) \cdot (x\backslash e)) \cdot ((x\backslash e)\backslash e))/((x \cdot y) \cdot (x\backslash e))), z) = K((x \cdot y) \cdot (x\backslash e), (((x \cdot y) \cdot (x\backslash e))/T(x\backslash e, z))/((x \cdot y) \cdot (x\backslash e)))$ by Theorem 344. Then $T(x\backslash((((x \cdot y) \cdot (x\backslash e)) \cdot x)/((x \cdot y) \cdot (x\backslash e))), z) = K((x \cdot y) \cdot (x\backslash e), (((x \cdot y) \cdot (x\backslash e))/T(x\backslash e, z))/((x \cdot y) \cdot (x\backslash e)))$ by Theorem 546. Then

$$T(((x \cdot y) \cdot (x\backslash e))\backslash T((x \cdot y) \cdot (x\backslash e), x), z) = K((x \cdot y) \cdot (x\backslash e), (((x \cdot y) \cdot (x\backslash e))/T(x\backslash e, z))/((x \cdot y) \cdot (x\backslash e))) \tag{907}$$

by Theorem 979. We have $(((x \cdot y) \cdot (x\backslash e))\backslash T((x \cdot y) \cdot (x\backslash e), T(x\backslash e, z)\backslash e))\backslash e = T((x \cdot y) \cdot (x\backslash e), T(x\backslash e, z)\backslash e)\backslash((x \cdot y) \cdot (x\backslash e))$ by Theorem 749. Then $K((x \cdot y) \cdot (x\backslash e), (((x \cdot y) \cdot (x\backslash e))/T(x\backslash e, z))/((x \cdot y) \cdot (x\backslash e)))\backslash e = T((x \cdot y) \cdot (x\backslash e), T(x\backslash e, z)\backslash e)\backslash((x \cdot y) \cdot (x\backslash e))$ by Theorem 1004. Then

$$T(((x \cdot y) \cdot (x\backslash e))\backslash T((x \cdot y) \cdot (x\backslash e), x), z)\backslash e = T((x \cdot y) \cdot (x\backslash e), T(x\backslash e, z)\backslash e)\backslash((x \cdot y) \cdot (x\backslash e)) \tag{908}$$

by (907). We have $K(((x \cdot y) \cdot (x\backslash e))\backslash(((x \cdot y) \cdot (x\backslash e))/T(x\backslash e, z)), (x \cdot y) \cdot (x\backslash e)) = T((x \cdot y) \cdot (x\backslash e), T(x\backslash e, z)\backslash e)\backslash((x \cdot y) \cdot (x\backslash e))$ by Theorem 1029. Then $T(T((x \cdot y) \cdot (x\backslash e), (x\backslash e)\backslash e)\backslash((x \cdot y) \cdot (x\backslash e)), z) = T((x \cdot y) \cdot (x\backslash e), T(x\backslash e, z)\backslash e)\backslash((x \cdot y) \cdot (x\backslash e))$ by Theorem 1461. Then $T(T((x \cdot y) \cdot (x\backslash e), (x\backslash e)\backslash e)\backslash((x \cdot y) \cdot (x\backslash e)), z) = T(((x \cdot y) \cdot (x\backslash e))\backslash T((x \cdot y) \cdot (x\backslash e), x), z)\backslash e$ by (908). Then

$$T(T((x \cdot y) \cdot (x\backslash e), x)\backslash((x \cdot y) \cdot (x\backslash e)), z) = T(((x \cdot y) \cdot (x\backslash e))\backslash T((x \cdot y) \cdot (x\backslash e), x), z)\backslash e \tag{909}$$

by Theorem 721. We have $e/(T(((x \cdot y) \cdot (x\backslash e))\backslash T((x \cdot y) \cdot (x\backslash e), x), z)\backslash e) = T(((x \cdot y) \cdot (x\backslash e))\backslash T((x \cdot y) \cdot (x\backslash e), x), z)$ by Proposition 24. Then

$$e/T(T((x \cdot y) \cdot (x\backslash e), x)\backslash((x \cdot y) \cdot (x\backslash e)), z) = T(((x \cdot y) \cdot (x\backslash e))\backslash T((x \cdot y) \cdot (x\backslash e), x), z) \tag{910}$$

by (909). We have $((T((x \cdot y) \cdot (x\backslash e), x)\backslash((x \cdot y) \cdot (x\backslash e)))\backslash e)\backslash K(((x \cdot y) \cdot (x\backslash e))\backslash T((x \cdot y) \cdot (x\backslash e), x), T((x \cdot y) \cdot (x\backslash e), x)\backslash((x \cdot y) \cdot (x\backslash e))) = T((x \cdot y) \cdot (x\backslash e), x)\backslash((x \cdot y) \cdot (x\backslash e))$ by Theorem 1150. Then

$$((T((x \cdot y) \cdot (x\backslash e), x)\backslash((x \cdot y) \cdot (x\backslash e)))\backslash e)\backslash e = T((x \cdot y) \cdot (x\backslash e), x)\backslash((x \cdot y) \cdot (x\backslash e)) \tag{911}$$

by Theorem 741. We have $e/T(((T((x \cdot y) \cdot (x\backslash e), x)\backslash((x \cdot y) \cdot (x\backslash e)))\backslash e)\backslash e, z) = T(T((x \cdot y) \cdot (x\backslash e), x)\backslash((x \cdot y) \cdot (x\backslash e)), z)\backslash e$ by Theorem 918. Then $e/T(T((x \cdot y) \cdot (x\backslash e), x)\backslash((x \cdot y) \cdot (x\backslash e)), z) = T(T((x \cdot y) \cdot (x\backslash e), x)\backslash((x \cdot y) \cdot (x\backslash e)), z)\backslash e$ by (911). Then $T(((x \cdot y) \cdot (x\backslash e))\backslash T((x \cdot y) \cdot (x\backslash e), x), z) = T(T((x \cdot y) \cdot (x\backslash e), x)\backslash((x \cdot y) \cdot (x\backslash e)), z)\backslash e$ by (910). Then

$$T(R(y, x, x\backslash e)\backslash((x \cdot y) \cdot (x\backslash e)), z)\backslash e = T(((x \cdot y) \cdot (x\backslash e))\backslash T((x \cdot y) \cdot (x\backslash e), x), z) \tag{912}$$

by Theorem 1331. We have $(((x \cdot y) \cdot (e/x))\backslash T((x \cdot y) \cdot (e/x), x))\backslash e = T((x \cdot y) \cdot (e/x), x)\backslash((x \cdot y) \cdot (e/x))$ by Theorem 749. Then $(((x \cdot y) \cdot (e/x))\backslash((y \cdot x) \cdot (e/x)))\backslash e = T((x \cdot y) \cdot (e/x), x)\backslash((x \cdot y) \cdot (e/x))$ by Theorem 611. Then $((y \cdot x) \cdot (e/x))\backslash((x \cdot y) \cdot (e/x)) = (((x \cdot y) \cdot (e/x))\backslash((y \cdot x) \cdot (e/x)))\backslash e$ by Theorem 611. Then $(((x \cdot y) \cdot (x\backslash e))\backslash((y \cdot x) \cdot (x\backslash e)))\backslash e = ((y \cdot x) \cdot (e/x))\backslash((x \cdot y) \cdot (e/x))$ by Theorem 1454. Then

$$(((x \cdot y) \cdot (x\backslash e))\backslash R(y, x, x\backslash e))\backslash e = ((y \cdot x) \cdot (e/x))\backslash((x \cdot y) \cdot (e/x)) \tag{913}$$

by Proposition 68. We have $((y \cdot x) \cdot (e/x))\backslash((x \cdot y) \cdot (e/x)) = ((y \cdot x) \cdot (x\backslash e))\backslash((x \cdot y) \cdot (x\backslash e))$ by Theorem 1454. Then $(((x \cdot y) \cdot (x\backslash e))\backslash R(y, x, x\backslash e))\backslash e = ((y \cdot x) \cdot (x\backslash e))\backslash((x \cdot y) \cdot (x\backslash e))$ by (913). Then $(((x \cdot y) \cdot (x\backslash e))\backslash R(y, x, x\backslash e))\backslash e = R(y, x, x\backslash e)\backslash((x \cdot y) \cdot (x\backslash e))$ by Proposition 68. Then

$$(T(x, R(y, x, x\backslash e))\backslash x)\backslash e = R(y, x, x\backslash e)\backslash((x \cdot y) \cdot (x\backslash e)) \tag{914}$$

by Theorem 1459. We have $(T(x, R(y, x, x\backslash e))\backslash x)\backslash e = x\backslash T(x, R(y, x, x\backslash e))$ by Theorem 1197. Then $R(y, x, x\backslash e)\backslash((x \cdot y) \cdot (x\backslash e)) = x\backslash T(x, R(y, x, x\backslash e))$ by (914). Then $R(y, x, x\backslash e)\backslash((x \cdot y) \cdot (x\backslash e)) = K(x, y)$ by Theorem 1505. Then $T(((x \cdot y) \cdot (x\backslash e))\backslash T((x \cdot y) \cdot (x\backslash e), x), z) = T(K(x, y), z)\backslash e$ by (912). Then

$$T(((x \cdot y) \cdot (x\backslash e))\backslash R(y, x, x\backslash e), z) = T(K(x, y), z)\backslash e \tag{915}$$

by Theorem 1331. We have $(x \cdot K(x, y))\backslash x = K(x, y)\backslash e$ by Theorem 1431. Then $(x \cdot K(x, y))\backslash x = K(T(y, x), x)$ by Theorem 1037. Then

$$(x \cdot K(x, y))\backslash x = K(y, x) \tag{916}$$

by Theorem 1053. We have $x \cdot (x\backslash T(x, R(y, x, x\backslash e))) = T(x, R(y, x, x\backslash e))$ by Axiom 4. Then $x \cdot K(x, y) = T(x, R(y, x, x\backslash e))$ by Theorem 1505. Then

$$T(x, R(y, x, x\backslash e))\backslash x = K(y, x) \tag{917}$$

by (916). We have $((x \cdot y) \cdot (x\backslash e))\backslash R(y, x, x\backslash e) = T(x, R(y, x, x\backslash e))\backslash x$ by Theorem 1459. Then $((x \cdot y) \cdot (x\backslash e))\backslash R(y, x, x\backslash e) = K(y, x)$ by (917). Hence we are done by (915). □

Theorem 1507. $K(x, y \cdot K(z, y\backslash e)) = T(K(x, y), z)$.

Proof. We have $K(y \cdot K(z, y\backslash e), x)\backslash e = K(x, y \cdot K(z, y\backslash e))$ by Theorem 1466. Then $T(K(y, x), z)\backslash e = K(x, y \cdot K(z, y\backslash e))$ by Theorem 1499. Hence we are done by Theorem 1506. □

Theorem 1508. $K(z, K(y, x)) = K(y, x) \cdot T(K(x, y), z)$.

Proof. We have

$$K(y, x) \cdot (T(K(y, x), z)\backslash e) = T(K(y, x), z)\backslash K(y, x) \tag{918}$$

by Theorem 175.

$$\begin{aligned} & K(z, K(y, x)) \\ = \quad & T(K(y, x), z)\backslash K(y, x) && \text{by Theorem 1482} \\ = \quad & K(y, x) \cdot T(K(x, y), z) && \text{by (918), Theorem 1506.} \end{aligned}$$

Hence we are done. □

Theorem 1509. $K(z, K(y, x)) = K(K(x, y), z)$.

Proof. We have $K(y, x) \cdot T(K(x, y), z) = K(K(x, y), z)$ by Theorem 1493. Hence we are done by Theorem 1508. □

Theorem 1510. $K(x, x\backslash(x/L(y\backslash e, z, w))) = L(x\backslash T(x, y), z, w)$.

Proof. We have $((w \cdot (z \cdot (y\backslash e)))/L(y\backslash e, z, w))\backslash((w \cdot (z \cdot (y\backslash e))) \cdot ((x \cdot ((w \cdot (z \cdot (y\backslash e)))\backslash((w \cdot (z \cdot (y\backslash e)))/L(y\backslash e, z, w))))/x)) = L(y\backslash e, z, w) \cdot ((x \cdot (L(y\backslash e, z, w)\backslash e))/x)$ by Theorem 330. Then $(w \cdot z)\backslash((w \cdot (z \cdot (y\backslash e))) \cdot ((x \cdot ((w \cdot (z \cdot (y\backslash e)))\backslash((w \cdot (z \cdot (y\backslash e)))/L(y\backslash e, z, w))))/x)) = L(y\backslash e, z, w) \cdot ((x \cdot (L(y\backslash e, z, w)\backslash e))/x)$ by Theorem 453. Then $(w \cdot z)\backslash((w \cdot (z \cdot (y\backslash e))) \cdot ((x \cdot ((w \cdot (z \cdot (y\backslash e)))\backslash(w \cdot z)))/x)) = L(y\backslash e, z, w) \cdot ((x \cdot (L(y\backslash e, z, w)\backslash e))/x)$ by Theorem 453. Then $(w \cdot z)\backslash(w \cdot ((z \cdot (y\backslash e)) \cdot ((x \cdot ((z \cdot (y\backslash e))\backslash z))/x))) = L(y\backslash e, z, w) \cdot ((x \cdot (L(y\backslash e, z, w)\backslash e))/x)$ by Theorem 1102. Then

$$(w \cdot z)\backslash(w \cdot (z \cdot ((y\backslash e) \cdot ((x \cdot ((y\backslash e)\backslash e))/x)))) = L(y\backslash e, z, w) \cdot ((x \cdot (L(y\backslash e, z, w)\backslash e))/x) \tag{919}$$

by Theorem 170. We have $L((y\backslash e) \cdot ((x \cdot ((y\backslash e)\backslash e))/x), z, w) = (w \cdot z)\backslash(w \cdot (z \cdot ((y\backslash e) \cdot ((x \cdot ((y\backslash e)\backslash e))/x))))$ by Definition 4. Then

$$L((y\backslash e) \cdot ((x \cdot ((y\backslash e)\backslash e))/x), z, w) = L(y\backslash e, z, w) \cdot ((x \cdot (L(y\backslash e, z, w)\backslash e))/x) \tag{920}$$

by (919). We have $L(y\backslash e, z, w) \cdot ((x \cdot (L(y\backslash e, z, w)\backslash e))/x) = x\backslash T(x, L(y\backslash e, z, w)\backslash e)$ by Theorem 980. Then

$$L((y\backslash e) \cdot ((x \cdot ((y\backslash e)\backslash e))/x), z, w) = x\backslash T(x, L(y\backslash e, z, w)\backslash e) \tag{921}$$

by (920).

$$\begin{aligned} & K(x, L(y\backslash e, z, w)\backslash e) \\ = \; & x\backslash T(x, L(y\backslash e, z, w)\backslash e) && \text{by Theorem 1308} \\ = \; & L(x\backslash T(x, (y\backslash e)\backslash e), z, w) && \text{by (921), Theorem 980.} \end{aligned}$$

Then

$$K(x, L(y\backslash e, z, w)\backslash e) = L(x\backslash T(x, (y\backslash e)\backslash e), z, w). \tag{922}$$

$$\begin{aligned} & K(x, x\backslash(x/L(y\backslash e, z, w))) \\ = \; & K(x, L(y\backslash e, z, w)\backslash e) && \text{by Theorem 1485} \\ = \; & L(x\backslash T(x, y), z, w) && \text{by (922), Theorem 721.} \end{aligned}$$

Hence we are done. □

Theorem 1511. $L(K(y, x), z, w)\backslash e = L(K(x, y), z, w)$.

Proof. We have $K(x\backslash(x/L(y\backslash e, z, w)), x)\backslash e = K(x, x\backslash(x/L(y\backslash e, z, w)))$ by Theorem 1466. Then $L(y\backslash T(y, x), z, w)\backslash e = K(x, x\backslash(x/L(y\backslash e, z, w)))$ by Theorem 1358. Then $L(x\backslash T(x, y), z, w) = L(y\backslash T(y, x), z, w)\backslash e$ by Theorem 1510. Then $L(K(x, y), z, w) = L(y\backslash T(y, x), z, w)\backslash e$ by Theorem 1308. Hence we are done by Theorem 1308. □

Theorem 1512. $(x/(z \cdot K(y, z))) \cdot z = (x \cdot z)/(z \cdot K(y, z))$.

Proof. We have $(x \cdot z)/z = x$ by Axiom 5. Then

$$(((x \cdot z)/(z \cdot (T(((z \cdot y)\backslash z)\backslash e, z)/(((z \cdot y)\backslash z)\backslash e)))) \cdot (z \cdot (T(((z \cdot y)\backslash z)\backslash e, z)/(((z \cdot y)\backslash z)\backslash e))))/z = x \tag{923}$$

by Axiom 6. We have $((((x \cdot z)/(z \cdot (T(((z \cdot y)\backslash z)\backslash e, z)/(((z \cdot y)\backslash z)\backslash e)))) \cdot (z \cdot (T(((z \cdot y)\backslash z)\backslash e, z)/(((z \cdot y)\backslash z)\backslash e))))/z) \cdot z = ((x \cdot z)/(z \cdot (T(((z \cdot y)\backslash z)\backslash e, z)/(((z \cdot y)\backslash z)\backslash e)))) \cdot (z \cdot (T(((z \cdot y)\backslash z)\backslash e, z)/(((z \cdot y)\backslash z)\backslash e)))$ by Axiom 6. Then $((((((x \cdot z)/(z \cdot (T(((z \cdot y)\backslash z)\backslash e, z)/(((z \cdot y)\backslash z)\backslash e)))) \cdot (z \cdot (T(((z \cdot y)\backslash z)\backslash e, z)/(((z \cdot y)\backslash z)\backslash e))))/z)/(z \cdot (T(((z \cdot y)\backslash z)\backslash e, z)/(((z \cdot y)\backslash z)\backslash e)))) \cdot (z \cdot (T(((z \cdot y)\backslash z)\backslash e, z)/(((z \cdot y)\backslash z)\backslash e)))) \cdot z = ((x \cdot z)/(z \cdot (T(((z \cdot y)\backslash z)\backslash e, z)/(((z \cdot y)\backslash z)\backslash e)))) \cdot (z \cdot (T(((z \cdot y)\backslash z)\backslash e, z)/(((z \cdot y)\backslash z)\backslash e)))$ by Axiom 6. Then

$$((x/(z \cdot (T(((z \cdot y)\backslash z)\backslash e, z)/(((z \cdot y)\backslash z)\backslash e)))) \cdot (z \cdot (T(((z \cdot y)\backslash z)\backslash e, z)/(((z \cdot y)\backslash z)\backslash e)))) \cdot z = ((x \cdot z)/(z \cdot (T(((z \cdot y)\backslash z)\backslash e, z)/(((z \cdot y)\backslash z)\backslash e)))) \tag{924}$$

by (923). We have $((x/(z\cdot(T(((z\cdot y)\backslash z)\backslash e,z)/(((z\cdot y)\backslash z)\backslash e))))\cdot((L(((z\cdot y)\backslash z)\backslash e,e/(((z\cdot y)\backslash z)\backslash e),z)\cdot z)/L(((z\cdot y)\backslash z)\backslash e,e/(((z\cdot y)\backslash z)\backslash e),z)))\cdot z = ((x/(z\cdot(T(((z\cdot y)\backslash z)\backslash e,z)/(((z\cdot y)\backslash z)\backslash e))))\cdot z)\cdot((L(((z\cdot y)\backslash z)\backslash e,e/(((z\cdot y)\backslash z)\backslash e),z)\cdot z)/L(((z\cdot y)\backslash z)\backslash e,e/(((z\cdot y)\backslash z)\backslash e),z))$ by Theorem 860. Then $((x/(z\cdot(T(((z\cdot y)\backslash z)\backslash e,z)/(((z\cdot y)\backslash z)\backslash e))))\cdot(z\cdot(T(((z\cdot y)\backslash z)\backslash e,z)/(((z\cdot y)\backslash z)\backslash e))))\cdot z = ((x/(z\cdot(T(((z\cdot y)\backslash z)\backslash e,z)/(((z\cdot y)\backslash z)\backslash e))))\cdot z)\cdot((L(((z\cdot y)\backslash z)\backslash e,e/(((z\cdot y)\backslash z)\backslash e),z)\cdot z)/L(((z\cdot y)\backslash z)\backslash e,e/(((z\cdot y)\backslash z)\backslash e),z))$ by Theorem 883. Then $((x/(z\cdot(T(((z\cdot y)\backslash z)\backslash e,z)/(((z\cdot y)\backslash z)\backslash e))))\cdot z)\cdot(z\cdot(T(((z\cdot y)\backslash z)\backslash e,z)/(((z\cdot y)\backslash z)\backslash e))) = ((x/(z\cdot(T(((z\cdot y)\backslash z)\backslash e,z)/(((z\cdot y)\backslash z)\backslash e))))\cdot(z\cdot(T(((z\cdot y)\backslash z)\backslash e,z)/(((z\cdot y)\backslash z)\backslash e))))\cdot z$ by Theorem 883. Then $((x\cdot z)/(z\cdot(T(((z\cdot y)\backslash z)\backslash e,z)/(((z\cdot y)\backslash z)\backslash e))))\cdot(z\cdot(T(((z\cdot y)\backslash z)\backslash e,z)/(((z\cdot y)\backslash z)\backslash e))) = ((x/(z\cdot(T(((z\cdot y)\backslash z)\backslash e,z)/(((z\cdot y)\backslash z)\backslash e))))\cdot z)\cdot(z\cdot(T(((z\cdot y)\backslash z)\backslash e,z)/(((z\cdot y)\backslash z)\backslash e)))$ by (924). Then $(x\cdot z)/(z\cdot(T(((z\cdot y)\backslash z)\backslash e,z)/(((z\cdot y)\backslash z)\backslash e))) = (x/(z\cdot(T(((z\cdot y)\backslash z)\backslash e,z)/(((z\cdot y)\backslash z)\backslash e))))\cdot z$ by Proposition 10. Then $(x\cdot z)/(z\cdot((((z\cdot y)\backslash z)\backslash e)\backslash T(((z\cdot y)\backslash z)\backslash e,z))) = (x/(z\cdot(T(((z\cdot y)\backslash z)\backslash e,z)/(((z\cdot y)\backslash z)\backslash e))))\cdot z$ by Theorem 758. Then $(x\cdot z)/(z\cdot((((z\cdot y)\backslash z)\backslash e)\backslash T(((z\cdot y)\backslash z)\backslash e,z))) = (x/(z\cdot((((z\cdot y)\backslash z)\backslash e)\backslash T(((z\cdot y)\backslash z)\backslash e,z))))\cdot z$ by Theorem 758. Then $(x\cdot z)/(z\cdot((((z\cdot y)\backslash z)\backslash e)\backslash(((z\cdot y)\backslash z)\backslash K(y,z)))) = (x/(z\cdot((((z\cdot y)\backslash z)\backslash e)\backslash T(((z\cdot y)\backslash z)\backslash e,z))))\cdot z$ by Theorem 538. Then $(x\cdot z)/(z\cdot K(y,z)) = (x/(z\cdot((((z\cdot y)\backslash z)\backslash e)\backslash T(((z\cdot y)\backslash z)\backslash e,z))))\cdot z$ by Theorem 735. Then $(x/(z\cdot(((z\cdot y)\backslash z)\cdot T(((z\cdot y)\backslash z)\backslash e,z))))\cdot z = (x\cdot z)/(z\cdot K(y,z))$ by Theorem 729. Hence we are done by Theorem 539. ☐

Theorem 1513. $((x\cdot K(y,z))/(z\cdot K(y,z)))\cdot z = R(x,K(y,z),z)$.

Proof. We have $((x\cdot K(y,z))\cdot z)/(z\cdot K(y,z)) = R(x,K(y,z),z)$ by Theorem 1420. Hence we are done by Theorem 1512. ☐

Theorem 1514. $x\cdot(z\cdot K(y,z)) = (x\cdot z)\cdot K(y,z)$.

Proof. We have $(((x\cdot z)\cdot K(y,z))/(z\cdot K(y,z)))\cdot z = R(x\cdot z,K(y,z),z)$ by Theorem 1513. Then

$$R(x,z,K(y,z))\cdot z = R(x\cdot z,K(y,z),z) \tag{925}$$

by Definition 5. We have $R(x\cdot z,K(y,z),z) = x\cdot z$ by Theorem 1504. Then $R(x,z,K(y,z))\cdot z = x\cdot z$ by (925). Then

$$R(x,z,K(y,z)) = x \tag{926}$$

by Proposition 10. We have $R(x,z,K(y,z))\cdot(z\cdot K(y,z)) = (x\cdot z)\cdot K(y,z)$ by Proposition 54. Hence we are done by (926). ☐

Theorem 1515. $L(K(y,z),z,x) = K(y,z)$.

Proof. We have $(x\cdot z)\cdot K(y,z) = x\cdot(z\cdot K(y,z))$ by Theorem 1514. Hence we are done by Theorem 54. ☐

Theorem 1516. $a(x,z,K(y,z)) = e$.

Proof. We have $L(K(y,z),z,x) = K(y,z)$ by Theorem 1515. Hence we are done by Proposition 20. ☐

Theorem 1517. $K(x,z\backslash y) = K(x,(y\backslash z)\backslash e)$.

Proof. We have $L(K(x,z\backslash y),z\backslash y,z) = K(x,(y\backslash z)\backslash e)$ by Theorem 1500. Hence we are done by Theorem 1515. ☐

Theorem 1518. $L(K(y,z),y,x) = K(y,z)$.

Proof. We have $L(K(z,y),y,x)\backslash e = L(K(y,z),y,x)$ by Theorem 1511. Then

$$K(z,y)\backslash e = L(K(y,z),y,x) \tag{927}$$

by Theorem 1515. We have $K(z,y)\backslash e = K(y,z)$ by Theorem 1466. Hence we are done by (927). ☐

Theorem 1519. $K(y,z) = x\backslash((x/y)\cdot T(y,z))$.

Proof. We have $L(K(y,z),y,x/y) = x\backslash((x/y)\cdot T(y,z))$ by Theorem 1488. Hence we are done by Theorem 1518. ☐

Theorem 1520. $K(x,z) = (y\cdot T(z,x))\backslash(y\cdot z)$.

Proof. We have $a(y,T(z,x),K(T(x,T(z,x)),T(z,x))) = e$ by Theorem 1516. Then $a(y,T(z,x),K(x,z)) = e$ by Theorem 1313. Then

$$L(K(x,z),T(z,x),y) = K(x,z) \tag{928}$$

by Proposition 19. We have $L(K(x,z),T(z,x),y) = (y\cdot T(z,x))\backslash(y\cdot z)$ by Theorem 1483. Hence we are done by (928). ☐

Theorem 1521. $(x\cdot y)\cdot K(y,z) = x\cdot T(y,z)$.

Proof. We have $(x\cdot y)\backslash(x\cdot T(y,z)) = L(K(y,z),y,x)$ by Theorem 1487. Then

$$(x\cdot y)\backslash(x\cdot T(y,z)) = K(y,z) \tag{929}$$

by Theorem 1518. We have $(x\cdot y)\cdot((x\cdot y)\backslash(x\cdot T(y,z))) = x\cdot T(y,z)$ by Axiom 4. Hence we are done by (929). ☐

Theorem 1522. $x\cdot K(y,z) = (x/y)\cdot T(y,z)$.

Proof. We have $x\cdot(x\backslash((x/y)\cdot T(y,z))) = (x/y)\cdot T(y,z)$ by Axiom 4. Hence we are done by Theorem 1519. ☐

Theorem 1523. $(y\cdot T(z,x))\cdot K(x,z) = y\cdot z$.

Proof. We have $(y\cdot T(z,x))\cdot((y\cdot T(z,x))\backslash(y\cdot z)) = y\cdot z$ by Axiom 4. Hence we are done by Theorem 1520. ☐

Theorem 1524. $(x\cdot K(y,z))\cdot y = x\cdot T(y,z)$.

Proof. We have $((x/y)\cdot T(y,z))\cdot y = x\cdot T(y,z)$ by Theorem 1344. Hence we are done by Theorem 1522. ☐

Theorem 1525. $x\cdot K(y,z) = (x/T(z,y))\cdot z$.

Proof. We have $((x/T(z,y))\cdot T(z,y))\cdot K(y,z) = (x/T(z,y))\cdot z$ by Theorem 1523. Hence we are done by Axiom 6. ☐

Theorem 1526. $(x\cdot K(z,y))\backslash x = K(y,z)$.

Proof. We have $((x/z)\cdot T(z,y))\backslash((x/z)\cdot z) = K(y,z)$ by Theorem 1520. Then $((x/z)\cdot T(z,y))\backslash x = K(y,z)$ by Axiom 6. Hence we are done by Theorem 1522. ☐

Theorem 1527. $(z\cdot K(y,x))\cdot K(x,y) = z$.

Proof. We have $(z\cdot K(y,x))\cdot((z\cdot K(y,x))\backslash z) = z$ by Axiom 4. Hence we are done by Theorem 1526. ☐

Theorem 1528. $x = L(x,K(x,y),z)$.

Proof. We have $((z/x)\cdot T(x,y))\backslash(z\cdot T(x,y)) = x$ by Theorem 1346. Then

$$(z\cdot K(x,y))\backslash(z\cdot T(x,y)) = x \tag{930}$$

by Theorem 1522. We have $(z\cdot K(x,y))\backslash(z\cdot T(x,y)) = L(x,K(x,y),z)$ by Theorem 1489. Hence we are done by (930). ☐

Theorem 1529. $(y\backslash x)\cdot K(y,z) = y\backslash(x\cdot K(y,z))$.

Proof. We have $((y\backslash x)\cdot y)\cdot K(y,z) = (y\backslash x)\cdot T(y,z)$ by Theorem 1521. Then

$$((y\backslash x)\cdot y)\cdot K(y,z) = ((y\backslash x)\cdot K(y,z))\cdot y \tag{931}$$

by Theorem 1524. We have $(y\backslash(x\cdot K(y,z)))\cdot y = ((y\backslash x)\cdot y)\cdot K(y,z)$ by Theorem 1501. Hence we are done by (931) and Proposition 8. □

Theorem 1530. $y\cdot(x\cdot K(y,z)) = (y\cdot x)\cdot K(y,z)$.

Proof. We have $y\cdot(y\backslash((y\cdot x)\cdot K(y,z))) = (y\cdot x)\cdot K(y,z)$ by Axiom 4. Then $y\cdot((y\backslash(y\cdot x))\cdot K(y,z)) = (y\cdot x)\cdot K(y,z)$ by Theorem 1529. Hence we are done by Axiom 3. □

Theorem 1531. $T(x\cdot y, K(y,z)) = y\cdot T(x, T(y,z))$.

Proof. We have $K(z\backslash(z/(y\backslash e)),z)\backslash(L(x,y,K(z\backslash(z/(y\backslash e)),z))\cdot(K(z\backslash(z/(y\backslash e)),z)\cdot y)) = y\cdot T(x,K(z\backslash(z/(y\backslash e)),z)\cdot y)$ by Theorem 805. Then $K(z\backslash(z/(y\backslash e)),z)\backslash(L(x,y,K(z\backslash(z/(y\backslash e)),z))\cdot T(y,z)) = y\cdot T(x,K(z\backslash(z/(y\backslash e)),z)\cdot y)$ by Theorem 1425. Then $K(z\backslash(z/(y\backslash e)),z)\backslash(L(x,y,K(z\backslash(z/(y\backslash e)),z))\cdot T(y,z)) = y\cdot T(x,T(y,z))$ by Theorem 1425. Then $(T(z,T(y,z))\backslash z)\backslash(L(x,y,K(z\backslash(z/(y\backslash e)),z))\cdot T(y,z)) = y\cdot T(x,T(y,z))$ by Theorem 1456. Then

$$(T(z,T(y,z))\backslash z)\backslash(L(x,y,T(z,T(y,z))\backslash z)\cdot T(y,z)) = y\cdot T(x,T(y,z)) \tag{932}$$

by Theorem 1456. We have $L(x,y,y\backslash T(y,z)) = x$ by Theorem 1460. Then $L(x,y,T(z,T(y,z))\backslash z) = x$ by Theorem 986. Then

$$(T(z,T(y,z))\backslash z)\backslash(x\cdot T(y,z)) = y\cdot T(x,T(y,z)) \tag{933}$$

by (932). We have $(T(y,z)\cdot(z\backslash T(z,T(y,z))))\cdot(T(z,T(y,z))\backslash z) = T(y,z)$ by Theorem 1211. Then

$$y\cdot(T(z,T(y,z))\backslash z) = T(y,z) \tag{934}$$

by Theorem 978. We have $T(y,z) = y\cdot K(y,z)$ by Theorem 1309. Then $y\cdot(T(z,T(y,z))\backslash z) = y\cdot K(y,z)$ by (934). Then $T(z,T(y,z))\backslash z = K(y,z)$ by Proposition 9. Then

$$K(y,z)\backslash(x\cdot T(y,z)) = y\cdot T(x,T(y,z)) \tag{935}$$

by (933). We have $T(x\cdot y,K(y,z)) = K(y,z)\backslash((x\cdot y)\cdot K(y,z))$ by Definition 3. Then $T(x\cdot y,K(y,z)) = K(y,z)\backslash(x\cdot T(y,z))$ by Theorem 1521. Hence we are done by (935). □

Theorem 1532. $T(y,x) = T(y, x\cdot K(y,z))$.

Proof. We have $(T(y,z)\cdot(T(T(y,z),y\backslash e)\backslash T(y,z)))\backslash(T(y,z)\cdot T(y,z)) = T(T(y,z),y\backslash e)$ by Theorem 570. Then $(T(y,z)\cdot(y\cdot T(y\backslash e,T(y,z))))\backslash(T(y,z)\cdot T(y,z)) = T(T(y,z),y\backslash e)$ by Theorem 1028. Then $(y\cdot(T(y,z)\cdot T(y\backslash e,T(y,z))))\backslash(T(y,z)\cdot T(y,z)) = T(T(y,z),y\backslash e)$ by Theorem 172. Then $(y\cdot((y\backslash e)\cdot T(y,z)))\backslash(T(y,z)\cdot T(y,z)) = T(T(y,z),y\backslash e)$ by Proposition 46. Then $L(T(y,z),y\backslash e,y)\backslash(T(y,z)\cdot T(y,z)) = T(T(y,z),y\backslash e)$ by Proposition 56. Then

$$T(e/(e/y),z)\backslash(T(y,z)\cdot T(y,z)) = T(T(y,z),y\backslash e) \tag{936}$$

by Theorem 264. We have $T(e/(e/y),z)\backslash(T(T(e/(e/y),z),e/y)\cdot T(y,z)) = T(T(e/(e/y),z),e/y)\cdot(T(e/(e/y),z)\backslash T(y,z))$ by Theorem 173. Then $T(e/(e/y),z)\backslash(T((e/y)\backslash e,z)\cdot T(y,z)) = T(T(e/(e/y),z),e/y)\cdot(T(e/(e/y),z)\backslash T(y,z))$ by Proposition 51. Then $T((e/y)\backslash e,z)\cdot(T(e/(e/y),z)\backslash T(y,z)) = T(e/(e/y),z)\backslash(T((e/y)\backslash e,z)\cdot T(y,z))$ by Proposition 51. Then $T(e/(e/y),z)\backslash(T((e/y)\backslash e,z)\cdot T(y,z)) = T((e/y)\backslash e,z)\cdot K(T(y,z),e/y)$ by Theorem 401. Then $T(e/(e/y),z)\backslash(T(y,z)\cdot T(y,z)) = T((e/y)\backslash e,z)\cdot K(T(y,z),e/y)$ by Proposition 25. Then $T((e/y)\backslash e,z)\cdot K(T(y,z),e/y) = T(T(y,z),y\backslash e)$ by (936). Then $T(y,z)\cdot K(T(y,z),e/y) = T(T(y,z),y\backslash e)$ by Proposition 25. Then $T(y,z)\cdot K(T(y,z),y\backslash e) = T(T(y,z),y\backslash e)$ by Theorem 780. Then

$$T(y,z)\cdot K(y,y\backslash e) = T(T(y,z),y\backslash e) \tag{937}$$

by Theorem 1416. We have $T(T(y,z),y\backslash e)=T((y\backslash e)\backslash e,z)$ by Theorem 18. Then

$$T(y,z)\cdot K(y,y\backslash e)=T((y\backslash e)\backslash e,z) \tag{938}$$

by (937). We have $T(y,z)\cdot K(y,y\backslash e)=y\cdot(T(y,z)\cdot(e/y))$ by Theorem 1349. Then

$$T((y\backslash e)\backslash e,z)=y\cdot(T(y,z)\cdot(e/y)) \tag{939}$$

by (938). We have $R((y\cdot x)/T(y,y\backslash e),K(T(y,z),T(y,z)\backslash e),T(y,z))=(y\cdot x)/T(y,y\backslash e)$ by Theorem 409. Then

$$R((y\cdot x)/T(y,y\backslash e),K(y,y\backslash e),T(y,z))=(y\cdot x)/T(y,y\backslash e) \tag{940}$$

by Theorem 744. We have $R((y\cdot x)/T(y,y\backslash e),K(y,y\backslash e),T(y,z))\cdot(K(y,y\backslash e)\cdot T(y,z))=(((y\cdot x)/T(y,y\backslash e))\cdot K(y,y\backslash e))\cdot T(y,z)$ by Proposition 54. Then $R((y\cdot x)/T(y,y\backslash e),K(y,y\backslash e),T(y,z))\cdot(y\cdot(T(y,z)\cdot(e/y)))=(((y\cdot x)/T(y,y\backslash e))\cdot K(y,y\backslash e))\cdot T(y,z)$ by Theorem 275. Then $((y\cdot x)/T(y,y\backslash e))\cdot(y\cdot(T(y,z)\cdot(e/y)))=(((y\cdot x)/T(y,y\backslash e))\cdot K(y,y\backslash e))\cdot T(y,z)$ by (940). Then $((y\cdot x)/y)\cdot T(y,z)=((y\cdot x)/T(y,y\backslash e))\cdot(y\cdot(T(y,z)\cdot(e/y)))$ by Theorem 1361. Then $((y\cdot x)/T(y,y\backslash e))\cdot T((y\backslash e)\backslash e,z)=((y\cdot x)/y)\cdot T(y,z)$ by (939). Then

$$((y\cdot x)/((y\backslash e)\backslash e))\cdot T((y\backslash e)\backslash e,z)=((y\cdot x)/y)\cdot T(y,z) \tag{941}$$

by Proposition 49. We have $K(z\backslash(z/((y\cdot x)\backslash((y\cdot x)/((y\backslash e)\backslash e)))),z)=(y\cdot x)\backslash(((y\cdot x)/((y\backslash e)\backslash e))\cdot T((y\backslash e)\backslash e,z))$ by Theorem 852. Then $K(z\backslash(z/T((y\cdot x)\backslash((y\cdot x)/y),y)),z)=(y\cdot x)\backslash(((y\cdot x)/((y\backslash e)\backslash e))\cdot T((y\backslash e)\backslash e,z))$ by Theorem 421. Then $K(z\backslash(z/T((y\cdot x)\backslash((y\cdot x)/y),y)),z)=(y\cdot x)\backslash(((y\cdot x)/y)\cdot T(y,z))$ by (941). Then $K(z\backslash(z/T(L(y\backslash e,T(y,x),x),y)),z)=(y\cdot x)\backslash(((y\cdot x)/y)\cdot T(y,z))$ by Theorem 1455. Then

$$K(z\backslash(z/L(T(y\backslash e,y),T(y,x),x)),z)=(y\cdot x)\backslash(((y\cdot x)/y)\cdot T(y,z)) \tag{942}$$

by Axiom 8. We have $L(K(z\backslash(z/T(y\backslash e,y)),z),T(y,x),x)=K(z\backslash(z/L(T(y\backslash e,y),T(y,x),x)),z)$ by Theorem 1359. Then $L(y\backslash T(y,z),T(y,x),x)=K(z\backslash(z/L(T(y\backslash e,y),T(y,x),x)),z)$ by Theorem 1426. Then

$$(y\cdot x)\backslash(((y\cdot x)/y)\cdot T(y,z))=L(y\backslash T(y,z),T(y,x),x) \tag{943}$$

by (942). We have $(y\cdot x)\cdot((y\cdot x)\backslash(((y\cdot x)/y)\cdot T(y,z)))=((y\cdot x)/y)\cdot T(y,z)$ by Axiom 4. Then

$$(y\cdot x)\cdot L(y\backslash T(y,z),T(y,x),x)=((y\cdot x)/y)\cdot T(y,z) \tag{944}$$

by (943). We have $(y\cdot x)\cdot L(y\backslash T(y,z),T(y,x),x)=x\cdot(T(y,x)\cdot(y\backslash T(y,z)))$ by Theorem 58. Then $((y\cdot x)/y)\cdot T(y,z)=x\cdot(T(y,x)\cdot(y\backslash T(y,z)))$ by (944). Then $x\cdot(T(y,z)\cdot(y\backslash T(y,x)))=((y\cdot x)/y)\cdot T(y,z)$ by Theorem 833. Then $x\cdot(K(y,z)\cdot T(y,x))=((y\cdot x)/y)\cdot T(y,z)$ by Theorem 1492. Then

$$(y\cdot x)\cdot K(y,z)=x\cdot(K(y,z)\cdot T(y,x)) \tag{945}$$

by Theorem 1522. We have $(y\cdot x)\cdot K(y,z)=y\cdot(x\cdot K(y,z))$ by Theorem 1530. Then

$$x\cdot(K(y,z)\cdot T(y,x))=y\cdot(x\cdot K(y,z)) \tag{946}$$

by (945). We have $x\backslash(L(y,K(y,z),x)\cdot(x\cdot K(y,z)))=K(y,z)\cdot T(y,x\cdot K(y,z))$ by Theorem 805. Then $x\backslash(y\cdot(x\cdot K(y,z)))=K(y,z)\cdot T(y,x\cdot K(y,z))$ by Theorem 1528. Then

$$x\backslash(x\cdot(K(y,z)\cdot T(y,x)))=K(y,z)\cdot T(y,x\cdot K(y,z)) \tag{947}$$

by (946). We have $x\backslash(x\cdot(K(y,z)\cdot T(y,x)))=K(y,z)\cdot T(y,x)$ by Axiom 3. Then $K(y,z)\cdot T(y,x\cdot K(y,z))=K(y,z)\cdot T(y,x)$ by (947). Hence we are done by Proposition 9. ☐

Theorem 1533. $T(z,x)=T(z,x\cdot K(y,z))$.

Proof. We have $T(z,(x\cdot K(y,z))\cdot K(z,y))=T(z,x\cdot K(y,z))$ by Theorem 1532. Hence we are done by Theorem 1527. ☐

Theorem 1534. $T(y, K(x, y) \cdot z) = T(y, z)$.

Proof. We have $(x\backslash z) \cdot T((x\backslash z)\backslash z, y) = T(x, y) \cdot (x\backslash z)$ by Theorem 48. Then

$$z \cdot K(y\backslash(y/(z\backslash(x\backslash z))), y) = T(x, y) \cdot (x\backslash z) \tag{948}$$

by Theorem 284. We have $T(y, z \cdot K(y\backslash(y/(z\backslash(x\backslash z))), y)) = T(y, z)$ by Theorem 1533. Then $T(y, T(x, y) \cdot (x\backslash z)) = T(y, z)$ by (948). Hence we are done by Theorem 1492. □

Theorem 1535. $T(T(z, w), x) = T(T(z, x), K(y, z) \cdot w)$.

Proof. We have $T(T(z, K(y, z) \cdot w), x) = T(T(z, x), K(y, z) \cdot w)$ by Axiom 7. Hence we are done by Theorem 1534. □

Theorem 1536. $(y \cdot z)/T(T(y, z), x) = K(x, y) \cdot z$.

Proof. We have $(y \cdot z)/((K(x, y) \cdot z)\backslash(y \cdot z)) = K(x, y) \cdot z$ by Proposition 24. Then $(y \cdot z)/T((y \cdot z)/(K(x, y) \cdot z), K(x, y) \cdot z) = K(x, y) \cdot z$ by Proposition 47. Then $(y \cdot z)/T(T(y, x), K(x, y) \cdot z) = K(x, y) \cdot z$ by Theorem 1484. Hence we are done by Theorem 1535. □

Theorem 1537. $K(x, y) \cdot z = z \cdot K(x, T(y, z))$.

Proof. We have $(z \cdot T(y, z))/T(T(y, z), x) = (z/T(T(y, z), x)) \cdot T(y, z)$ by Theorem 239. Then $(y \cdot z)/T(T(y, z), x) = (z/T(T(y, z), x)) \cdot T(y, z)$ by Proposition 46. Then $z \cdot K(x, T(y, z)) = (y \cdot z)/T(T(y, z), x)$ by Theorem 1525. Hence we are done by Theorem 1536. □

Theorem 1538. $T(K(x, y), z) = K(x, T(y, z))$.

Proof. We have $z \cdot K(x, T(y, z)) = K(x, y) \cdot z$ by Theorem 1537. Hence we are done by Theorem 11. □

Theorem 1539. $K(z, K(y, x)) = K(x, K(y, z))$.

Proof. We have $K(y\backslash e, x/(y\backslash e)) = (y\backslash e)\backslash T(y\backslash e, R(x/(y\backslash e), y\backslash e, (y\backslash e)\backslash e))$ by Theorem 1505. Then

$$K(y\backslash e, x/(y\backslash e)) = (y\backslash e)\backslash T(y\backslash e, x \cdot ((y\backslash e)\backslash e)) \tag{949}$$

by Proposition 71. We have $K(y\backslash e, x/(y\backslash e)) = K(y\backslash e, (y\backslash e)\backslash x)$ by Theorem 1465. Then

$$(y\backslash e)\backslash T(y\backslash e, x \cdot ((y\backslash e)\backslash e)) = K(y\backslash e, (y\backslash e)\backslash x) \tag{950}$$

by (949). We have $T(y\backslash e, x \cdot (e/(y\backslash e))) = T(y\backslash e, x \cdot ((y\backslash e)\backslash e))$ by Theorem 1366. Then

$$T(y\backslash e, x \cdot y) = T(y\backslash e, x \cdot ((y\backslash e)\backslash e)) \tag{951}$$

by Proposition 24. We have $(y\backslash e)\backslash T(y\backslash e, x \cdot ((y\backslash e)\backslash e)) = y \cdot T(y\backslash e, x \cdot ((y\backslash e)\backslash e))$ by Theorem 729. Then $(y\backslash e)\backslash T(y\backslash e, x \cdot ((y\backslash e)\backslash e)) = y \cdot T(y\backslash e, x \cdot y)$ by (951). Then

$$K(y\backslash e, (y\backslash e)\backslash x) = y \cdot T(y\backslash e, x \cdot y) \tag{952}$$

by (950). We have $K(y\backslash e, (y\backslash e)\backslash x) = K(y\backslash e, y \cdot x)$ by Theorem 1478. Then $y \cdot T(y\backslash e, x \cdot y) = K(y\backslash e, y \cdot x)$ by (952). Then

$$y\backslash K(y\backslash e, y \cdot x) = T(y\backslash e, x \cdot y) \tag{953}$$

by Proposition 2. We have $(y\backslash e)\backslash(T(y\backslash e, (((y \cdot x) \cdot (y\backslash e))\backslash(y\backslash e))\backslash e)\backslash(y\backslash e)) = T(y\backslash e, (((y \cdot x) \cdot (y\backslash e))\backslash(y\backslash e))\backslash e)\backslash e$ by Theorem 1339. Then $(y\backslash e)\backslash(((y\backslash e) \cdot K(y\backslash e, y \cdot x))\backslash(y\backslash e)) = T(y\backslash e, (((y \cdot x) \cdot (y\backslash e))\backslash(y\backslash e))\backslash e)\backslash e$ by Theorem 1312. Then $T(y\backslash e, (((y \cdot x) \cdot (y\backslash e))\backslash(y\backslash e))\backslash e)\backslash e = (y\backslash e)\backslash(K(y\backslash e, y \cdot x)\backslash e)$ by Theorem 1431. Then

$$((y\backslash e) \cdot K(y\backslash e, y \cdot x))\backslash e = (y\backslash e)\backslash(K(y\backslash e, y \cdot x)\backslash e) \tag{954}$$

by Theorem 1312. We have $(y\backslash e)\cdot K(y\backslash e,y\cdot x)=y\backslash K(e/y,y\cdot x)$ by Theorem 1386. Then $(y\backslash e)\cdot K(y\backslash e,y\cdot x)=y\backslash K(y\backslash e,y\cdot x)$ by Theorem 732. Then $(y\backslash K(y\backslash e,y\cdot x))\backslash e=(y\backslash e)\backslash(K(y\backslash e,y\cdot x)\backslash e)$ by (954). Then $(y\backslash e)\backslash K(y\cdot x,y\backslash e)=(y\backslash K(y\backslash e,y\cdot x))\backslash e$ by Theorem 1466. Then

$$T(y\backslash e,x\cdot y)\backslash e=(y\backslash e)\backslash K(y\cdot x,y\backslash e) \tag{955}$$

by (953). We have $(e/(y\backslash e))\cdot K(y\cdot x,y\backslash e)=K(y\cdot x,y\backslash e)/(y\backslash e)$ by Theorem 1481. Then

$$y\cdot K(y\cdot x,y\backslash e)=K(y\cdot x,y\backslash e)/(y\backslash e) \tag{956}$$

by Proposition 24. We have $K(z,K(y\cdot x,y\backslash e)/(y\backslash e))=K(z,(y\backslash e)\backslash K(y\cdot x,y\backslash e))$ by Theorem 1453. Then $K(z,y\cdot K(y\cdot x,y\backslash e))=K(z,(y\backslash e)\backslash K(y\cdot x,y\backslash e))$ by (956). Then

$$K(z,T(y\backslash e,x\cdot y)\backslash e)=K(z,y\cdot K(y\cdot x,y\backslash e)) \tag{957}$$

by (955). We have $K(z,T(y\backslash e,x\cdot y)\backslash e)=T(K(z,y),x\cdot y)$ by Theorem 1496. Then $K(z,y\cdot K(y\cdot x,y\backslash e))=T(K(z,y),x\cdot y)$ by (957). Then

$$T(K(z,y),y\cdot x)=T(K(z,y),x\cdot y) \tag{958}$$

by Theorem 1507. We have $T(K(z,y),y\cdot x)\cdot K(y,z)=K(y,z)\cdot T(K(z,y),y\cdot x)$ by Theorem 1357. Then $T(K(z,y),y\cdot x)\cdot K(y,z)=K(K(z,y),y\cdot x)$ by Theorem 1493. Then

$$T(K(z,y),x\cdot y)\cdot K(y,z)=K(K(z,y),y\cdot x) \tag{959}$$

by (958). We have $T(K(z,y),x\cdot y)\cdot K(y,z)=K(y,z)\cdot T(K(z,y),x\cdot y)$ by Theorem 1357. Then $T(K(z,y),x\cdot y)\cdot K(y,z)=K(x\cdot y,K(y,z))$ by Theorem 1508. Then

$$K(K(z,y),y\cdot x)=K(x\cdot y,K(y,z)) \tag{960}$$

by (959). We have $K(K(z,y),y\cdot x)=K(y\cdot x,K(y,z))$ by Theorem 1509. Then

$$K(x\cdot y,K(y,z))=K(y\cdot x,K(y,z)) \tag{961}$$

by (960). We have $T((y\cdot x)/x,z)\cdot(((y\cdot x)/x)\backslash(y\cdot x))=(T(z,(y\cdot x)/x)\backslash z)\cdot(y\cdot x)$ by Theorem 1275. Then

$$T((y\cdot x)/x,z)\cdot(((y\cdot x)/x)\backslash(y\cdot x))=z\cdot(T(z,(y\cdot x)/x)\backslash(y\cdot x)) \tag{962}$$

by Theorem 1463. We have $K(z\backslash(z/((y\cdot x)\backslash(((y\cdot x)/x)\backslash(y\cdot x)))),z)=(y\cdot x)\backslash((((y\cdot x)/x)\backslash(y\cdot x))\cdot T((((y\cdot x)/x)\backslash(y\cdot x))\backslash(y\cdot x),z))$ by Theorem 285. Then $K(z\backslash(z/((y\cdot x)\backslash(((y\cdot x)/x)\backslash(y\cdot x)))),z)=(y\cdot x)\backslash(T((y\cdot x)/x,z)\cdot(((y\cdot x)/x)\backslash(y\cdot x)))$ by Theorem 48. Then

$$K(z\backslash(z/((y\cdot x)\backslash(((y\cdot x)/x)\backslash(y\cdot x)))),z)=(y\cdot x)\backslash(z\cdot(T(z,(y\cdot x)/x)\backslash(y\cdot x))) \tag{963}$$

by (962). We have $T(T(z,(y\cdot x)/x)\backslash z,y\cdot x)=(y\cdot x)\backslash((T(z,(y\cdot x)/x)\backslash z)\cdot(y\cdot x))$ by Definition 3. Then $T(T(z,(y\cdot x)/x)\backslash z,y\cdot x)=(y\cdot x)\backslash(z\cdot(T(z,(y\cdot x)/x)\backslash(y\cdot x)))$ by Theorem 1463. Then

$$K(z\backslash(z/((y\cdot x)\backslash(((y\cdot x)/x)\backslash(y\cdot x)))),z)=T(T(z,(y\cdot x)/x)\backslash z,y\cdot x) \tag{964}$$

by (963). We have $K(z\backslash(z/((y\cdot x)\backslash(((y\cdot x)/x)\backslash(y\cdot x)))),z)=T(z,((y\cdot x)\backslash(((y\cdot x)/x)\backslash(y\cdot x)))\backslash e)\backslash z$ by Theorem 1029. Then $T(T(z,(y\cdot x)/x)\backslash z,y\cdot x)=T(z,((y\cdot x)\backslash(((y\cdot x)/x)\backslash(y\cdot x)))\backslash e)\backslash z$ by (964). Then $K(((y\cdot x)\backslash(((y\cdot x)/x)\backslash(y\cdot x)))\backslash e,z)=T(T(z,(y\cdot x)/x)\backslash z,y\cdot x)$ by Theorem 1482. Then $T(K((y\cdot x)/x,z),y\cdot x)=K(((y\cdot x)\backslash(((y\cdot x)/x)\backslash(y\cdot x)))\backslash e,z)$ by Theorem 1482. Then

$$K(((y\cdot x)\backslash x)\backslash e,z)=T(K((y\cdot x)/x,z),y\cdot x) \tag{965}$$

by Proposition 25. We have $K(((y\cdot x)\backslash x)\backslash e,z)\backslash e=K(z,((y\cdot x)\backslash x)\backslash e)$ by Theorem 1466. Then $T(K((y\cdot x)/x,z),y\cdot x)\backslash e=K(z,((y\cdot x)\backslash x)\backslash e)$ by (965). Then $T(K(z,(y\cdot x)/x),y\cdot x)=K(z,((y\cdot x)\backslash x)\backslash e)$ by Theorem 1506. Then

$$T(K(z,y),y\cdot x)=K(z,((y\cdot x)\backslash x)\backslash e) \tag{966}$$

by Axiom 5. We have $K(y,z) \cdot T(K(z,y), y \cdot x) = K(y \cdot x, K(y,z))$ by Theorem 1508. Then $K(y,z) \cdot K(z, ((y \cdot x)\backslash x)\backslash e) = K(y \cdot x, K(y,z))$ by (966). Then $K(y,z) \cdot K(z, x\backslash(y \cdot x)) = K(y \cdot x, K(y,z))$ by Theorem 1517. Then

$$K(y,z) \cdot K(z, T(y,x)) = K(y \cdot x, K(y,z)) \tag{967}$$

by Definition 3. We have $K(y,z) \cdot T(K(z,y), x) = K(x, K(y,z))$ by Theorem 1508. Then $K(y,z) \cdot K(z, T(y,x)) = K(x, K(y,z))$ by Theorem 1538. Then $K(x, K(y,z)) = K(y \cdot x, K(y,z))$ by (967). Then

$$K(x \cdot y, K(y,z)) = K(x, K(y,z)) \tag{968}$$

by (961). We have $(x \cdot y)\backslash((y \cdot x) \cdot T(K(x,y), z)) = K(y,x) \cdot T(K(y,x)\backslash e, z)$ by Theorem 1127. Then $(x \cdot y)\backslash((y \cdot x) \cdot T(K(x,y), z)) = K(y,x) \cdot T(K(x,y), z)$ by Theorem 1466. Then $(x \cdot y)\backslash(y \cdot (x \cdot T(K(x,y), z))) = K(y,x) \cdot T(K(x,y), z)$ by Theorem 1490. Then

$$(x \cdot y)\backslash(y \cdot (x \cdot T(K(x,y), z))) = K(z, K(y,x)) \tag{969}$$

by Theorem 1508. We have $T(x, T(y,z)) = x \cdot K(x, T(y,z))$ by Theorem 1309. Then $T(x, T(y,z)) = x \cdot T(K(x,y), z)$ by Theorem 1538. Then $(x \cdot y)\backslash(y \cdot T(x, T(y,z))) = K(z, K(y,x))$ by (969). Then

$$(x \cdot y)\backslash T(x \cdot y, K(y,z)) = K(z, K(y,x)) \tag{970}$$

by Theorem 1531. We have $K(x \cdot y, K(y,z)) = (x \cdot y)\backslash T(x \cdot y, K(y,z))$ by Theorem 1308. Then $K(x \cdot y, K(y,z)) = K(z, K(y,x))$ by (970). Hence we are done by (968). □

The following was first conjectured in [9].

Theorem 1540. $K(K(x,y), z) = K(x, K(y,z))$.

Proof. We have $K(K(x,y), z) = K(z, K(y,x))$ by Theorem 1509. Hence we are done by Theorem 1539. □

The following was first conjectured in [9].

Theorem 1541. $K(y, K(x,z)) = K(x, K(y,z))$.

Proof. We have $K(K(z,y), x) \cdot K(x, K(z,y)) = e$ by Theorem 1054. Then

$$K(x, K(y,z)) \cdot K(x, K(z,y)) = e \tag{971}$$

by Theorem 1509.

$$\begin{aligned} & K(y, K(x,z)) \cdot K(K(x,z), y) \\ = & \quad e && \text{by Theorem 1054} \\ = & \quad K(x, K(y,z)) \cdot K(K(x,z), y) && \text{by (971), Theorem 1540.} \end{aligned}$$

Then $K(y, K(x,z)) \cdot K(K(x,z), y) = K(x, K(y,z)) \cdot K(K(x,z), y)$. Hence we are done by Proposition 10. □

Theorem 1542. $K(y, K(z,x)) = K(x, K(y,z))$.

Proof. We have $K(z, K(y,x)) = K(x, K(y,z))$ by Theorem 1539. Hence we are done by Theorem 1541. □

The following was first conjectured in [9].

Theorem 1543. $K(x, K(z,y)) = K(x, K(y,z))$.

Proof. We have $K(z, K(y,x)) = K(x, K(y,z))$ by Theorem 1539. Hence we are done by Theorem 1542. □

Theorem 1544. $R(x, T(y,z)\backslash y, y\backslash T(y,z)) = (x \cdot (T(y,z)\backslash y)) \cdot (y\backslash T(y,z))$.

Proof. We have $(x \cdot (T(y,z)\backslash y)) \cdot ((T(y,z)\backslash y)\backslash e) = R(x, T(y,z)\backslash y, (T(y,z)\backslash y)\backslash e)$ by Proposition 68. Then $(x \cdot (T(y,z)\backslash y)) \cdot (y\backslash T(y,z)) = R(x, T(y,z)\backslash y, (T(y,z)\backslash y)\backslash e)$ by Theorem 1197. Hence we are done by Theorem 1197. □

Theorem 1545. $x\backslash T(y,x) = T(x,y)\backslash y$.

Proof. We have $x \cdot (T(x,y)\backslash y) = T(y, T(x,y))$ by Theorem 177. Then $x\backslash T(y, T(x,y)) = T(x,y)\backslash y$ by Proposition 2. Hence we are done by Theorem 1032. □

Theorem 1546. $y/(x\backslash T(y,x)) = T(x,y)$.

Proof. We have $y/(T(x,y)\backslash y) = T(x,y)$ by Proposition 24. Hence we are done by Theorem 1545. □

Theorem 1547. $x/(K(z,y) \cdot x) = K(y,z)$.

Proof. We have $x/(K(y,z)\backslash x) = K(y,z)$ by Proposition 24. Hence we are done by Theorem 1467. □

Theorem 1548. $((x \cdot y)/x) \cdot K(y,x) = y$.

Proof. We have $(y \cdot (T(x,y)\backslash x)) \cdot ((y \cdot (T(x,y)\backslash x))\backslash y) = y$ by Axiom 4. Then $(y \cdot (T(x,y)\backslash x)) \cdot (x\backslash T(x,y)) = y$ by Theorem 771. Then

$$R(y, T(x,y)\backslash x, x\backslash T(x,y)) = y \tag{972}$$

by Theorem 1544. We have $R((x \cdot y)/x, T(x,y)\backslash x, x\backslash T(x,y)) = (x \cdot R(y, T(x,y)\backslash x, x\backslash T(x,y)))/x$ by Theorem 123. Then

$$R((x \cdot y)/x, T(x,y)\backslash x, x\backslash T(x,y)) = (x \cdot y)/x \tag{973}$$

by (972). We have $(((x \cdot y)/x) \cdot (T(x,y)\backslash x)) \cdot (x\backslash T(x,y)) = R((x \cdot y)/x, T(x,y)\backslash x, x\backslash T(x,y))$ by Theorem 1544. Then

$$(((x \cdot y)/x) \cdot (T(x,y)\backslash x)) \cdot (x\backslash T(x,y)) = (x \cdot y)/x \tag{974}$$

by (973). We have $y \cdot (x\backslash T(x,y)) = (x \cdot y)/x$ by Theorem 977. Then $y = ((x \cdot y)/x) \cdot (T(x,y)\backslash x)$ by (974) and Proposition 8. Hence we are done by Theorem 1482. □

Theorem 1549. $T(x, K(z,y)) = T(x, K(y,z))$.

Proof. We have

$$T(x, K(y,z)) = x \cdot K(x, K(y,z)) \tag{975}$$

by Theorem 1309.

$$\begin{aligned} & T(x, K(z,y)) \\ = \; & x \cdot K(x, K(z,y)) && \text{by Theorem 1309} \\ = \; & T(x, K(y,z)) && \text{by (975), Theorem 1543.} \end{aligned}$$

Hence we are done. □

Theorem 1550. $(z \cdot K(x,y))/z = T(K(x,y), z)$.

Proof. We have $K(K(x,y), z) = K(x, K(y,z))$ by Theorem 1540. Then

$$K(K(x,y), z) = K(z, K(x,y)) \tag{976}$$

by Theorem 1542. We have $T(K(x,y), z) \cdot K(z, K(x,y)) = K(x,y)$ by Theorem 1307. Then

$$T(K(x,y), z) \cdot K(K(x,y), z) = K(x,y) \tag{977}$$

by (976). We have $((z \cdot K(x,y))/z) \cdot K(K(x,y), z) = K(x,y)$ by Theorem 1548. Then $((z \cdot K(x,y))/z) \cdot K(K(x,y), z) = T(K(x,y), z) \cdot K(K(x,y), z)$ by (977). Hence we are done by Proposition 10. □

Theorem 1551. $T(K(y,z),x) \cdot x = x \cdot K(y,z)$.

Proof. We have $((x \cdot K(y,z))/x) \cdot x = x \cdot K(y,z)$ by Axiom 6. Hence we are done by Theorem 1550. □

Theorem 1552. $K(y,T(z,x)) \cdot x = x \cdot K(y,z)$.

Proof. We have $T(K(y,z),x) \cdot x = x \cdot K(y,z)$ by Theorem 1551. Hence we are done by Theorem 1538. □

Theorem 1553. $K(y,T(T(z,x),x)) = K(y,z)$.

Proof. We have

$$K(y,T(T(z,x),x)) \cdot x = x \cdot K(y,T(z,x)) \tag{978}$$

by Theorem 1552. We have $K(y,z) \cdot x = x \cdot K(y,T(z,x))$ by Theorem 1537. Hence we are done by (978) and Proposition 8. □

Theorem 1554. $z/(z \cdot K(y,x)) = T(K(x,y),z)$.

Proof. We have $K(y,x) \cdot T(z,K(y,x)) = z \cdot K(y,x)$ by Proposition 46. Then

$$K(x,y)\backslash T(z,K(y,x)) = z \cdot K(y,x) \tag{979}$$

by Theorem 1467. We have $z/(K(x,y)\backslash T(z,K(x,y))) = T(K(x,y),z)$ by Theorem 1546. Then $z/(K(x,y)\backslash T(z,K(y,x))) = T(K(x,y),z)$ by Theorem 1549. Hence we are done by (979). □

Theorem 1555. $K(x,T(z,y)) = K(T(x,y),z)$.

Proof. We have $y/(K(z,T(x,y)) \cdot y) = K(T(x,y),z)$ by Theorem 1547. Then $y/(y \cdot K(z,x)) = K(T(x,y),z)$ by Theorem 1552. Then

$$T(K(x,z),y) = K(T(x,y),z) \tag{980}$$

by Theorem 1554. We have $K(x,T(z,y)) = T(K(x,z),y)$ by Theorem 1538. Hence we are done by (980). □

The following was first conjectured in [9].

Theorem 1556. $K(T(y,x),T(z,x)) = K(y,z)$.

Proof. We have $K(y,T(T(z,x),x)) = K(y,z)$ by Theorem 1553. Hence we are done by Theorem 1555. □

Theorem 1557. $e/x = L(y,x,e/x)/(x \cdot y)$.

Proof. We have $e \cdot L(y,x,e/x) = L(y,x,e/x)$ by Axiom 1. Then

$$L(y,x,e/x) \cdot (L(y,x,e/x)\backslash(e \cdot L(y,x,e/x))) = L(y,x,e/x) \tag{981}$$

by Axiom 4. We have $L(y,x,e/x) \cdot (L(y,x,e/x)\backslash(e \cdot L(y,x,e/x))) = e \cdot L(y,x,e/x)$ by Axiom 4. Then $((L(y,x,e/x) \cdot (L(y,x,e/x)\backslash(e \cdot L(y,x,e/x))))/(x \cdot y)) \cdot (x \cdot y) = e \cdot L(y,x,e/x)$ by Axiom 6. Then $(L(y,x,e/x)/(x \cdot y)) \cdot (x \cdot y) = e \cdot L(y,x,e/x)$ by (981). Then

$$(L(y,x,e/x)/(x \cdot y)) \cdot (x \cdot y) = ((e/x) \cdot x) \cdot L(y,x,e/x) \tag{982}$$

by Axiom 6. We have $(e/x) \cdot (x \cdot y) = ((e/x) \cdot x) \cdot L(y,x,e/x)$ by Proposition 52. Hence we are done by (982) and Proposition 8. □

Theorem 1558. $y \cdot ((x\backslash y)\backslash e) = R(x,x\backslash y,(x\backslash y)\backslash e)$.

Proof. We have $(x \cdot (x\backslash y)) \cdot ((x\backslash y)\backslash e) = R(x,x\backslash y,(x\backslash y)\backslash e)$ by Proposition 68. Hence we are done by Axiom 4. □

Theorem 1559. $T(L(y,z,y),x) = T(T((z\cdot y)/z,x), y\cdot z)$.

Proof. We have $T(T((z\cdot y)/z, y\cdot z),x) = T(T((z\cdot y)/z,x), y\cdot z)$ by Axiom 7. Hence we are done by Theorem 92. □

Theorem 1560. $L(w\backslash(u\cdot x),y,z) = L(L((u\backslash w)\backslash x,y,z), u\backslash w, u)$.

Proof. We have $L(L((u\backslash w)\backslash x, u\backslash w, u),y,z) = L(L((u\backslash w)\backslash x,y,z), u\backslash w, u)$ by Axiom 11. Hence we are done by Theorem 60. □

Theorem 1561. $K(x\backslash e,x)\cdot((x\backslash e)\backslash e) = x$.

Proof. We have $K(x\backslash e,x)\cdot(K(x\backslash e,x)\backslash x) = x$ by Axiom 4. Then $K(x\backslash e,x)\cdot T(x,x\backslash e) = x$ by Theorem 1319. Hence we are done by Proposition 49. □

Theorem 1562. $((x\cdot(y\backslash z))/z)\cdot y = y\cdot R(y\backslash x,y,y\backslash z)$.

Proof. We have $R(x/y,y,y\backslash z)\cdot y = y\cdot R(y\backslash x,y,y\backslash z)$ by Proposition 83. Hence we are done by Theorem 66. □

Theorem 1563. $L(T(y,x),z,y) = T(T((z\cdot y)/z,x), y\cdot z)$.

Proof. We have $L(T(y,x),z,y) = T(L(y,z,y),x)$ by Axiom 8. Hence we are done by Theorem 1559. □

Theorem 1564. $(e/y)\backslash(T(x,z)/y) = T((e/y)\backslash(x/y),z)$.

Proof. We have $(e/y)\cdot T((e/y)\backslash(x/y),z) = T((x/y)\cdot y,z)/y$ by Theorem 498. Then $(e/y)\backslash(T((x/y)\cdot y,z)/y) = T((e/y)\backslash(x/y),z)$ by Proposition 2. Hence we are done by Axiom 6. □

Theorem 1565. $L(z,x,y)\backslash(z\backslash L(z,x,y)) = z\backslash e$.

Proof. We have $L(z,x,y)\cdot(z\backslash e) = z\backslash L(z,x,y)$ by Theorem 837. Hence we are done by Proposition 2. □

Theorem 1566. $K(y\backslash(y/x),y)/x = T(e/x,y)$.

Proof. We have $(x\cdot T(x\backslash e,y))/x = T(e/x,y)$ by Theorem 49. Hence we are done by Theorem 196. □

Theorem 1567. $y\cdot L(y\backslash(x\cdot z),w,u) = x\cdot((x\backslash y)\cdot L((x\backslash y)\backslash z,w,u))$.

Proof. We have $y\cdot L(L((x\backslash y)\backslash z,w,u), x\backslash y, x) = x\cdot((x\backslash y)\cdot L((x\backslash y)\backslash z,w,u))$ by Theorem 52. Hence we are done by Theorem 1560. □

Theorem 1568. $(e/T(y,z))\cdot T(y,x) = T(y,x)/T(y,z)$.

Proof. We have $(e/T(y,z))\backslash(T(y,x)/T(y,z)) = T((e/T(y,z))\backslash(y/T(y,z)),x)$ by Theorem 1564. Then

$$(e/T(y,z))\backslash(T(y,x)/T(y,z)) = T(y,x) \tag{983}$$

by Theorem 1345. We have $(e/T(y,z))\cdot((e/T(y,z))\backslash(T(y,x)/T(y,z))) = T(y,x)/T(y,z)$ by Axiom 4. Hence we are done by (983). □

Theorem 1569. $T(e/T(y,z),T(y,x)) = T(y,x)\backslash(T(y,x)/T(y,z))$.

Proof. We have $T(e/T(y,z),T(y,x)) = T(y,x)\backslash((e/T(y,z))\cdot T(y,x))$ by Definition 3. Hence we are done by Theorem 1568. □

Theorem 1570. $L(e/(e/z),x,y) = L(L(z,x,y), z\backslash e, z)$.

Proof. We have $L(L(z,z\backslash e,z),x,y) = L(L(z,x,y),z\backslash e,z)$ by Axiom 11. Hence we are done by Theorem 263. □

Theorem 1571. $L(w,x,y)\cdot(z\cdot w)=w\cdot(L(w,x,y)\cdot T(z,w))$.

Proof. We have $L(w,x,y)\cdot(w\cdot T(z,w))=w\cdot(L(w,x,y)\cdot T(z,w))$ by Theorem 276. Hence we are done by Proposition 46. ☐

Theorem 1572. $w\cdot(L(w,y,z)\backslash((w\backslash L(w,y,z))\cdot x))=L(x,w\backslash L(w,y,z),w)$.

Proof. We have $L(w,y,z)\backslash(w\cdot((w\backslash L(w,y,z))\cdot x))=L(x,w\backslash L(w,y,z),w)$ by Theorem 449. Hence we are done by Theorem 277. ☐

Theorem 1573. $x\backslash K(x,L(x,y,z)/x)=T(e/x,L(x,y,z))$.

Proof. We have $x\cdot T(e/x,L(x,y,z))=K(x,L(x,y,z)/x)$ by Theorem 282. Hence we are done by Proposition 2. ☐

Theorem 1574. $(x\cdot(x\cdot L(x\backslash e,y,z)))/x=x\cdot L(x\backslash e,y,z)$.

Proof. We have $(x\cdot L(x\backslash e,y,z))\cdot x=x\cdot(x\cdot L(x\backslash e,y,z))$ by Theorem 303. Hence we are done by Proposition 1. ☐

Theorem 1575. $x\backslash(y\cdot L(y\backslash(x\cdot z),w,u))=(x\backslash y)\cdot L((x\backslash y)\backslash z,w,u)$.

Proof. We have $x\cdot((x\backslash y)\cdot L((x\backslash y)\backslash z,w,u))=y\cdot L(y\backslash(x\cdot z),w,u)$ by Theorem 1567. Hence we are done by Proposition 2. ☐

Theorem 1576. $x\backslash(y\cdot L(y\backslash x,z,w))=(x\backslash y)\cdot L((x\backslash y)\backslash e,z,w)$.

Proof. We have $x\backslash(y\cdot L(y\backslash(x\cdot e),z,w))=(x\backslash y)\cdot L((x\backslash y)\backslash e,z,w)$ by Theorem 1575. Hence we are done by Axiom 2. ☐

Theorem 1577. $(x\backslash L(x,y,z))\cdot((x\backslash e)\backslash e)=L((x\backslash e)\backslash e,y,z)$.

Proof. We have $(L(x,y,z)\cdot(x\backslash e))\cdot((x\backslash e)\backslash e)=R(L(x,y,z),x\backslash e,(x\backslash e)\backslash e)$ by Proposition 68. Then

$$(x\backslash L(x,y,z))\cdot((x\backslash e)\backslash e)=R(L(x,y,z),x\backslash e,(x\backslash e)\backslash e)\tag{984}$$

by Theorem 837. We have $R(L(x,y,z),x\backslash e,(x\backslash e)\backslash e)=L(R(x,x\backslash e,(x\backslash e)\backslash e),y,z)$ by Axiom 10. Then $(x\backslash L(x,y,z))\cdot((x\backslash e)\backslash e)=L(R(x,x\backslash e,(x\backslash e)\backslash e),y,z)$ by (984). Then $L(e\cdot((x\backslash e)\backslash e),y,z)=(x\backslash L(x,y,z))\cdot((x\backslash e)\backslash e)$ by Theorem 1558. Hence we are done by Axiom 1. ☐

Theorem 1578. $L(x\cdot L(x\backslash e,w,u),y,z)=L(x,y,z)\cdot L(L(x,y,z)\backslash e,w,u)$.

Proof. We have $((z\cdot y)\backslash(z\cdot(y\cdot x)))\cdot L(((z\cdot y)\backslash(z\cdot(y\cdot x)))\backslash e,w,u)=(z\cdot y)\backslash((z\cdot(y\cdot x))\cdot L((z\cdot(y\cdot x))\backslash(z\cdot y),w,u))$ by Theorem 1576. Then $L(x,y,z)\cdot L(((z\cdot y)\backslash(z\cdot(y\cdot x)))\backslash e,w,u)=(z\cdot y)\backslash((z\cdot(y\cdot x))\cdot L((z\cdot(y\cdot x))\backslash(z\cdot y),w,u))$ by Definition 4. Then

$$(z\cdot y)\backslash((z\cdot(y\cdot x))\cdot L((z\cdot(y\cdot x))\backslash(z\cdot y),w,u))=L(x,y,z)\cdot L(L(x,y,z)\backslash e,w,u)\tag{985}$$

by Definition 4. We have $(z\cdot(y\cdot x))\cdot L(L((y\cdot x)\backslash y,y\cdot x,z),w,u)=z\cdot((y\cdot x)\cdot L((y\cdot x)\backslash y,w,u))$ by Theorem 57. Then $(z\cdot(y\cdot x))\cdot L((z\cdot(y\cdot x))\backslash(z\cdot y),w,u)=z\cdot((y\cdot x)\cdot L((y\cdot x)\backslash y,w,u))$ by Proposition 53. Then $(z\cdot y)\backslash(z\cdot((y\cdot x)\cdot L((y\cdot x)\backslash y,w,u)))=L(x,y,z)\cdot L(L(x,y,z)\backslash e,w,u)$ by (985). Then

$$(z\cdot y)\backslash(z\cdot(y\cdot(x\cdot L(x\backslash e,w,u))))=L(x,y,z)\cdot L(L(x,y,z)\backslash e,w,u)\tag{986}$$

by Theorem 135. We have $L(x\cdot L(x\backslash e,w,u),y,z)=(z\cdot y)\backslash(z\cdot(y\cdot(x\cdot L(x\backslash e,w,u))))$ by Definition 4. Hence we are done by (986). ☐

Theorem 1579. $L(y,x,y)=L(y,x,e/(e/y))$.

Proof. We have $T(y, K(y, y\backslash e) \cdot ((e/(e/y)) \cdot x)) = T(y, (e/(e/y)) \cdot x)$ by Theorem 698. Then

$$T(y, y \cdot x) = T(y, (e/(e/y)) \cdot x) \tag{987}$$

by Theorem 685. We have $(x \cdot T(y, (e/(e/y)) \cdot x))/x = T((x \cdot y)/x, (e/(e/y)) \cdot x)$ by Theorem 50. Then

$$(x \cdot T(y, y \cdot x))/x = T((x \cdot y)/x, (e/(e/y)) \cdot x) \tag{988}$$

by (987). We have $(x \cdot T(y, y \cdot x))/x = L(y, x, y)$ by Theorem 80. Then

$$T((x \cdot y)/x, (e/(e/y)) \cdot x) = L(y, x, y) \tag{989}$$

by (988). We have $(x \cdot T(e/(e/y), y\backslash e))/x = T((x \cdot (e/(e/y)))/x, y\backslash e)$ by Theorem 50. Then

$$(x \cdot y)/x = T((x \cdot (e/(e/y)))/x, y\backslash e) \tag{990}$$

by Theorem 242. We have $T(T((x \cdot (e/(e/y)))/x, y\backslash e), (e/(e/y)) \cdot x) = L(T(e/(e/y), y\backslash e), x, e/(e/y))$ by Theorem 1563. Then $T(T((x \cdot (e/(e/y)))/x, y\backslash e), (e/(e/y)) \cdot x) = L(y, x, e/(e/y))$ by Theorem 242. Then $T((x \cdot y)/x, (e/(e/y)) \cdot x) = L(y, x, e/(e/y))$ by (990). Hence we are done by (989). □

Theorem 1580. *$(L(x, y, x)\backslash e)\backslash e = T((((y \cdot x)/y)\backslash e)\backslash e, x \cdot y)$.*

Proof. We have $(T((y \cdot x)/y, x \cdot y)\backslash e)\backslash e = T((((y \cdot x)/y)\backslash e)\backslash e, x \cdot y)$ by Theorem 919. Hence we are done by Theorem 92. □

Theorem 1581. *$L((y\backslash e)\backslash e, x, y) = (L(y, x, y)\backslash e)\backslash e$.*

Proof. We have $T(T((x \cdot y)/x, ((x \cdot y)/x)\backslash e), y \cdot x) = L(T(y, ((x \cdot y)/x)\backslash e), x, y)$ by Theorem 1563. Then $T((((x \cdot y)/x)\backslash e)\backslash e, y \cdot x) = L(T(y, ((x \cdot y)/x)\backslash e), x, y)$ by Proposition 49. Then $(L(y, x, y)\backslash e)\backslash e = L(T(y, ((x \cdot y)/x)\backslash e), x, y)$ by Theorem 1580. Hence we are done by Theorem 1392. □

Theorem 1582. *$K(y\backslash (y/T(y, x)), y) = K(y\backslash e, y)$.*

Proof. We have $K(y\backslash (y/T(y, x)), y) = T(K(y\backslash e, y), x)$ by Theorem 577. Hence we are done by Theorem 716. □

Theorem 1583. *$L(y, T(y, x)\backslash e, T(y, x)) = e/(e/y)$.*

Proof. We have $y \cdot (T(y, x) \cdot T(T(y, x)\backslash e, y)) = T(y, x) \cdot ((T(y, x)\backslash e) \cdot y)$ by Theorem 520. Then

$$y \cdot K(y\backslash (y/T(y, x)), y) = T(y, x) \cdot ((T(y, x)\backslash e) \cdot y) \tag{991}$$

by Theorem 196. We have $T(y, x) \cdot ((T(y, x)\backslash e) \cdot y) = L(y, T(y, x)\backslash e, T(y, x))$ by Proposition 56. Then

$$y \cdot K(y\backslash (y/T(y, x)), y) = L(y, T(y, x)\backslash e, T(y, x)) \tag{992}$$

by (991). We have $y \cdot K(y\backslash e, y) = e/(e/y)$ by Theorem 262. Then $y \cdot K(y\backslash (y/T(y, x)), y) = e/(e/y)$ by Theorem 1582. Hence we are done by (992). □

Theorem 1584. *$K(y\backslash x, (x/y)\backslash e) = K(x/y, (x/y)\backslash e)$.*

Proof. We have $K(T(x/y, y), (x/y)\backslash e) = K(x/y, (x/y)\backslash e)$ by Theorem 1416. Hence we are done by Proposition 47. □

Theorem 1585. *$T(e/T(x, y), x) = T(x, y)\backslash e$.*

Proof. We have $T(e/T(x, y), x) = x\backslash (x/T(x, y))$ by Theorem 551. Hence we are done by Theorem 752. □

Theorem 1586. *$K(x\backslash e, x)/T(x, y) = T(x, y)\backslash e$.*

Proof. We have $K(x\backslash(x/T(x,y)),x)/T(x,y) = T(e/T(x,y),x)$ by Theorem 1566. Then $K(x\backslash e,x)/T(x,y) = T(e/T(x,y),x)$ by Theorem 1582. Hence we are done by Theorem 1585. □

Theorem 1587. $T(e/T(y,z),T(y,x)) = T(y,z)\backslash e$.

Proof. We have $T(y\cdot T(y\backslash e,T(y,x)),z) = K(T(y,x)\backslash(T(y,x)/T(y,z)),T(y,x))$ by Theorem 296. Then $T(y\cdot T(y\backslash e,y),z) = K(T(y,x)\backslash(T(y,x)/T(y,z)),T(y,x))$ by Theorem 1201. Then $K(T(e/T(y,z),T(y,x)),T(y,x)) = T(y\cdot T(y\backslash e,y),z)$ by Theorem 1569. Then $K(T(e/T(y,z),T(y,x)),T(y,x)) = T((y\backslash e)\cdot y,z)$ by Proposition 46. Then

$$K(T(e/T(y,z),T(y,x)),T(y,x)) = T(K(y\backslash e,y),z) \quad (993)$$

by Proposition 76. We have $T(K(y\backslash e,y),z) = K(y\backslash e,y)$ by Theorem 716. Then

$$K(T(e/T(y,z),T(y,x)),T(y,x)) = K(y\backslash e,y) \quad (994)$$

by (993). We have $K(T(y,x)\backslash(T(y,x)/T(y,z)),T(y,x))/T(y,z) = T(e/T(y,z),T(y,x))$ by Theorem 1566. Then $K(T(e/T(y,z),T(y,x)),T(y,x))/T(y,z) = T(e/T(y,z),T(y,x))$ by Theorem 1569. Then

$$K(y\backslash e,y)/T(y,z) = T(e/T(y,z),T(y,x)) \quad (995)$$

by (994). We have $K(y\backslash e,y)/T(y,z) = T(y,z)\backslash e$ by Theorem 1586. Hence we are done by (995). □

Theorem 1588. $T(e/T(x/y,z),y\backslash x) = T(x/y,z)\backslash e$.

Proof. We have $T(e/T(x/y,z),T(x/y,y)) = T(x/y,z)\backslash e$ by Theorem 1587. Hence we are done by Proposition 47. □

Theorem 1589. $K(y,((x\cdot y)/x)\backslash e) = K(y,y\backslash e)$.

Proof. We have $K((x\cdot y)\backslash x,x\backslash(x\cdot y)) = K(y\backslash e,y)$ by Theorem 664. Then

$$K(((x\cdot y)/x)\backslash e,(x\cdot y)/x) = K(y\backslash e,y) \quad (996)$$

by Theorem 1421. We have $K(((x\cdot y)/x)\backslash e,(x\cdot y)/x)\cdot((((x\cdot y)/x)\backslash e)\backslash e) = (x\cdot y)/x$ by Theorem 1561. Then

$$K(y\backslash e,y)\cdot((((x\cdot y)/x)\backslash e)\backslash e) = (x\cdot y)/x \quad (997)$$

by (996). We have $K(y\backslash e,y)\cdot(K(y,y\backslash e)\cdot((x\cdot y)/x)) = (x\cdot y)/x$ by Theorem 693. Then $K(y\backslash e,y)\cdot(K(y,y\backslash e)\cdot((x\cdot y)/x)) = K(y\backslash e,y)\cdot((((x\cdot y)/x)\backslash e)\backslash e)$ by (997). Then

$$K(y,y\backslash e)\cdot((x\cdot y)/x) = (((x\cdot y)/x)\backslash e)\backslash e \quad (998)$$

by Proposition 9. We have $K((x\cdot y)/x,((x\cdot y)/x)\backslash e)\cdot((x\cdot y)/x) = (((x\cdot y)/x)\backslash e)\backslash e$ by Theorem 552. Then $K(x\backslash(x\cdot y),((x\cdot y)/x)\backslash e)\cdot((x\cdot y)/x) = (((x\cdot y)/x)\backslash e)\backslash e$ by Theorem 1584. Then $K(y,((x\cdot y)/x)\backslash e)\cdot((x\cdot y)/x) = (((x\cdot y)/x)\backslash e)\backslash e$ by Axiom 3. Hence we are done by (998) and Proposition 8. □

Theorem 1590. $R(e/y,y,K(x,y)) = e/y$.

Proof. We have $(e/y)\cdot(y\cdot R(y\backslash(((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y))),y,y\backslash(e/((y\cdot x)\backslash y)))) = L(R(y\backslash(((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y))),y,y\backslash(e/((y\cdot x)\backslash y))),y,e/y)$ by Proposition 67. Then $(e/y)\cdot(y\cdot R(y\backslash(((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y))),y,y\backslash(e/((y\cdot x)\backslash y)))) = R((e/y)\cdot(((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y))),y,y\backslash(e/((y\cdot x)\backslash y)))$ by Theorem 81. Then $(e/y)\cdot((((((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y)))\cdot(y\backslash(e/((y\cdot x)\backslash y))))/(e/((y\cdot x)\backslash y)))\cdot y) = R((e/y)\cdot(((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y))),y,y\backslash(e/((y\cdot x)\backslash y)))$ by Theorem 1562. Then $(e/y)\cdot((T(e/((y\cdot x)\backslash y),y)/(e/((y\cdot x)\backslash y)))\cdot y) = R((e/y)\cdot(((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y))),y,y\backslash(e/((y\cdot x)\backslash y)))$ by Theorem 179. Then $R((e/y)\cdot(((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y))),y,y\backslash(e/((y\cdot x)\backslash y))) = (e/y)\cdot(((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y))\cdot y)$ by Theorem 758. Then

$$R((e/y)\cdot(((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y))),y,y\backslash(e/((y\cdot x)\backslash y))) = (e/y)\cdot(y\cdot((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y))) \quad (999)$$

by Theorem 937. We have $(e/y)\cdot(y\cdot((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y)))=L((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y),y,e/y)$ by Proposition 67. Then

$$R((e/y)\cdot(((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y))),y,y\backslash(e/((y\cdot x)\backslash y)))=L((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y),y,e/y)\quad(1000)$$

by (999). We have $(e/y)\cdot(((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y)))=(((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y)))/y$ by Theorem 858. Then $(e/y)\cdot(((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y)))=y\backslash(((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y)))$ by Theorem 940. Then

$$R(y\backslash(((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y))),y,y\backslash(e/((y\cdot x)\backslash y)))=L((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y),y,e/y)\quad(1001)$$

by (1000). We have $T(((((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y)))\cdot(y\backslash(e/((y\cdot x)\backslash y))))/(y\cdot(y\backslash(e/((y\cdot x)\backslash y)))),y)=R(y\backslash(((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y))),y,y\backslash(e/((y\cdot x)\backslash y)))$ by Proposition 64. Then $T(T(e/((y\cdot x)\backslash y),y)/(y\cdot(y\backslash(e/((y\cdot x)\backslash y)))),y)=R(y\backslash(((e/((y\cdot x)\backslash y))\cdot y)/(e/((y\cdot x)\backslash y))),y,y\backslash(e/((y\cdot x)\backslash y)))$ by Theorem 179. Then $L((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y),y,e/y)=T(T(e/((y\cdot x)\backslash y),y)/(y\cdot(y\backslash(e/((y\cdot x)\backslash y)))),y)$ by (1001). Then $T(T(e/((y\cdot x)\backslash y),y)/(e/((y\cdot x)\backslash y)),y)=L((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y),y,e/y)$ by Axiom 4. Then

$$L((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y),y,e/y)=T((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y),y)\quad(1002)$$

by Theorem 758. We have $T(y,(e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y))=y$ by Theorem 936. Then $T((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y),y)=(e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y)$ by Proposition 21. Then

$$L((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y),y,e/y)=(e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y)\quad(1003)$$

by (1002). We have $L((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y),y,e/y)/(y\cdot((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y)))=e/y$ by Theorem 1557. Then

$$((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y))/(y\cdot((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y)))=e/y\quad(1004)$$

by (1003). We have $R(e/y,y,(e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y))=((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y))/(y\cdot((e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y)))$ by Proposition 79. Then $R(e/y,y,(e/((y\cdot x)\backslash y))\backslash T(e/((y\cdot x)\backslash y),y))=e/y$ by (1004). Then $R(e/y,y,((y\cdot x)\backslash y)\cdot T(((y\cdot x)\backslash y)\backslash e,y))=e/y$ by Theorem 102. Hence we are done by Theorem 539. ☐

Theorem 1591. $L((y\backslash e)\backslash e,x,y)=y\cdot(L(y,x,y)\cdot(e/y))$.

Proof. We have $(T((x\cdot y)/x,y\cdot x)\cdot(T(T((x\cdot y)/x,y\cdot x),((x\cdot y)/x)\backslash e)\backslash T((x\cdot y)/x,y\cdot x)))\backslash(T((x\cdot y)/x,y\cdot x)\cdot T((x\cdot y)/x,y\cdot x))=T(T((x\cdot y)/x,y\cdot x),((x\cdot y)/x)\backslash e)$ by Theorem 570. Then $(T((x\cdot y)/x,y\cdot x)\cdot(((x\cdot y)/x)\cdot T(((x\cdot y)/x)\backslash e,T((x\cdot y)/x,y\cdot x))))\backslash(T((x\cdot y)/x,y\cdot x)\cdot T((x\cdot y)/x,y\cdot x))=T(T((x\cdot y)/x,y\cdot x),((x\cdot y)/x)\backslash e)$ by Theorem 1028. Then $(((x\cdot y)/x)\cdot(T((x\cdot y)/x,y\cdot x)\cdot T(((x\cdot y)/x)\backslash e,T((x\cdot y)/x,y\cdot x))))\backslash(T((x\cdot y)/x,y\cdot x)\cdot T((x\cdot y)/x,y\cdot x))=T(T((x\cdot y)/x,y\cdot x),((x\cdot y)/x)\backslash e)$ by Theorem 172. Then $(((x\cdot y)/x)\cdot((((x\cdot y)/x)\backslash e)\cdot T((x\cdot y)/x,y\cdot x)))\backslash(T((x\cdot y)/x,y\cdot x)\cdot T((x\cdot y)/x,y\cdot x))=T(T((x\cdot y)/x,y\cdot x),((x\cdot y)/x)\backslash e)$ by Proposition 46. Then $L(T((x\cdot y)/x,y\cdot x),((x\cdot y)/x)\backslash e,(x\cdot y)/x)\backslash(T((x\cdot y)/x,y\cdot x)\cdot T((x\cdot y)/x,y\cdot x))=T(T((x\cdot y)/x,y\cdot x),((x\cdot y)/x)\backslash e)$ by Proposition 56. Then

$$T(e/(e/((x\cdot y)/x)),y\cdot x)\backslash(T((x\cdot y)/x,y\cdot x)\cdot T((x\cdot y)/x,y\cdot x))=T(T((x\cdot y)/x,y\cdot x),((x\cdot y)/x)\backslash e)\quad(1005)$$

by Theorem 264. We have $T(e/(e/((x\cdot y)/x)),y\cdot x)\backslash(T(T(e/(e/((x\cdot y)/x)),y\cdot x),e/((x\cdot y)/x))\cdot T((x\cdot y)/x,y\cdot x))=T(T(e/(e/((x\cdot y)/x)),y\cdot x),e/((x\cdot y)/x))\cdot(T(e/(e/((x\cdot y)/x)),y\cdot x)\backslash T((x\cdot y)/x,y\cdot x))$ by Theorem 173. Then $T(e/(e/((x\cdot y)/x)),y\cdot x)\backslash(T((e/((x\cdot y)/x))\backslash e,y\cdot x)\cdot T((x\cdot y)/x,y\cdot x))=T(T(e/(e/((x\cdot y)/x)),y\cdot x),e/((x\cdot y)/x))\cdot(T(e/(e/((x\cdot y)/x)),y\cdot x)\backslash T((x\cdot y)/x,y\cdot x))$ by Proposition 51. Then $T((e/((x\cdot y)/x))\backslash e,y\cdot x)\cdot(T(e/(e/((x\cdot y)/x)),y\cdot x)\backslash T((x\cdot y)/x,y\cdot x))=T(e/(e/((x\cdot y)/x)),y\cdot x)\backslash(T((e/((x\cdot y)/x))\backslash e,y\cdot x)\cdot T((x\cdot y)/x,y\cdot x))$ by Proposition 51. Then $T(e/(e/((x\cdot y)/x)),y\cdot x)\backslash(T((e/((x\cdot y)/x))\backslash e,y\cdot x)\cdot T((x\cdot y)/x,y\cdot x))=T((e/((x\cdot y)/x))\backslash e,y\cdot x)\cdot K(T((x\cdot y)/x,y\cdot x),e/((x\cdot y)/x))$ by Theorem 401. Then $T(e/(e/((x\cdot y)/x)),y\cdot x)\backslash(T((x\cdot y)/x,y\cdot x)\cdot T((x\cdot y)/x,y\cdot x))=$

$T((e/((x\cdot y)/x))\backslash e, y\cdot x)\cdot K(T((x\cdot y)/x, y\cdot x), e/((x\cdot y)/x))$ by Proposition 25. Then $T((e/((x\cdot y)/x))\backslash e, y\cdot x)\cdot K(T((x\cdot y)/x, y\cdot x), e/((x\cdot y)/x)) = T(T((x\cdot y)/x, y\cdot x), ((x\cdot y)/x)\backslash e)$ by (1005). Then $T((x\cdot y)/x, y\cdot x)\cdot K(T((x\cdot y)/x, y\cdot x), e/((x\cdot y)/x)) = T(T((x\cdot y)/x, y\cdot x), ((x\cdot y)/x)\backslash e)$ by Proposition 25. Then $T((x\cdot y)/x, y\cdot x)\cdot K(T((x\cdot y)/x, y\cdot x), ((x\cdot y)/x)\backslash e) = T(T((x\cdot y)/x, y\cdot x), ((x\cdot y)/x)\backslash e)$ by Theorem 780. Then

$$T((x\cdot y)/x, y\cdot x)\cdot K((x\cdot y)/x, ((x\cdot y)/x)\backslash e) = T(T((x\cdot y)/x, y\cdot x), ((x\cdot y)/x)\backslash e) \tag{1006}$$

by Theorem 1416. We have $T(T((x\cdot y)/x, y\cdot x), ((x\cdot y)/x)\backslash e) = T((((x\cdot y)/x)\backslash e)\backslash e, y\cdot x)$ by Theorem 18. Then

$$T((x\cdot y)/x, y\cdot x)\cdot K((x\cdot y)/x, ((x\cdot y)/x)\backslash e) = T((((x\cdot y)/x)\backslash e)\backslash e, y\cdot x) \tag{1007}$$

by (1006). We have $K((x\cdot y)/x, ((x\cdot y)/x)\backslash e)\cdot T((x\cdot y)/x, y\cdot x) = T((x\cdot y)/x, y\cdot x)\cdot K((x\cdot y)/x, ((x\cdot y)/x)\backslash e)$ by Theorem 1311. Then $K((x\cdot y)/x, ((x\cdot y)/x)\backslash e)\cdot T((x\cdot y)/x, y\cdot x) = T((((x\cdot y)/x)\backslash e)\backslash e, y\cdot x)$ by (1007). Then $K((x\cdot y)/x, ((x\cdot y)/x)\backslash e)\cdot L(y,x,y) = T((((x\cdot y)/x)\backslash e)\backslash e, y\cdot x)$ by Theorem 92. Then $K(x\backslash(x\cdot y), ((x\cdot y)/x)\backslash e)\cdot L(y,x,y) = T((((x\cdot y)/x)\backslash e)\backslash e, y\cdot x)$ by Theorem 1584. Then $K(y, ((x\cdot y)/x)\backslash e)\cdot L(y,x,y) = T((((x\cdot y)/x)\backslash e)\backslash e, y\cdot x)$ by Axiom 3. Then

$$K(y, y\backslash e)\cdot L(y,x,y) = T((((x\cdot y)/x)\backslash e)\backslash e, y\cdot x) \tag{1008}$$

by Theorem 1589. We have $T((((x\cdot y)/x)\backslash e)\backslash e, y\cdot x) = (L(y,x,y)\backslash e)\backslash e$ by Theorem 1580. Then

$$K(y, y\backslash e)\cdot L(y,x,y) = (L(y,x,y)\backslash e)\backslash e \tag{1009}$$

by (1008). We have $K(y, y\backslash e)\cdot L(y,x,y) = y\cdot(L(y,x,y)\cdot(e/y))$ by Theorem 281. Then

$$(L(y,x,y)\backslash e)\backslash e = y\cdot(L(y,x,y)\cdot(e/y)) \tag{1010}$$

by (1009). We have $L((y\backslash e)\backslash e, x, y) = (L(y,x,y)\backslash e)\backslash e$ by Theorem 1581. Hence we are done by (1010). ☐

Theorem 1592. $x\backslash K(L(x,y,z), x\backslash e) = (x\backslash L(x,y,z))/L(x,y,z)$.

Proof. We have $T((K(x, L(x,y,z)/x)\cdot K(L(x,y,z)/x, x))/(x\cdot K(L(x,y,z)/x, x)), x) = R(x\backslash K(x, L(x,y,z)/x), x, K(L(x,y,z)/x, x$ by Proposition 64. Then $T(e/(x\cdot K(L(x,y,z)/x, x)), x) = R(x\backslash K(x, L(x,y,z)/x), x, K(L(x,y,z)/x, x))$ by Theorem 1054. Then

$$R(T(e/x, L(x,y,z)), x, K(L(x,y,z)/x, x)) = T(e/(x\cdot K(L(x,y,z)/x, x)), x) \tag{1011}$$

by Theorem 1573. We have $R(T(e/x, L(x,y,z)), x, K(L(x,y,z)/x, x)) = T(R(e/x, x, K(L(x,y,z)/x, x)), L(x,y,z))$ by Axiom 9. Then $R(T(e/x, L(x,y,z)), x, K(L(x,y,z)/x, x)) = T(e/x, L(x,y,z))$ by Theorem 1590. Then $T(e/(x\cdot K(L(x,y,z)/x, x)), x) = T(e/x, L(x,y,z))$ by (1011). Then

$$T(e/(x\cdot K(x\backslash L(x,y,z), x)), x) = T(e/x, L(x,y,z)) \tag{1012}$$

by Theorem 1464. We have $x\cdot(L(x,y,z)\backslash((x\backslash L(x,y,z))\cdot x)) = L(x, x\backslash L(x,y,z), x)$ by Theorem 1572. Then $x\cdot K(x\backslash L(x,y,z), x) = L(x, x\backslash L(x,y,z), x)$ by Theorem 2. Then

$$T(e/L(x, x\backslash L(x,y,z), x), x) = T(e/x, L(x,y,z)) \tag{1013}$$

by (1012). We have $T(e/T(((x\backslash L(x,y,z))\cdot x)/(x\backslash L(x,y,z)), x\cdot(x\backslash L(x,y,z))), (x\backslash L(x,y,z))\backslash((x\backslash L(x,y,z))\cdot x)) = T(((x\backslash L(x,y,z))\cdot x)/(x\backslash L(x,y,z)), x\cdot(x\backslash L(x,y,z)))\backslash e$ by Theorem 1588. Then $T(e/L(x, x\backslash L(x,y,z), x), (x\backslash L(x,y,z))\backslash((x\backslash$ $x)) = T(((x\backslash L(x,y,z))\cdot x)/(x\backslash L(x,y,z)), x\cdot(x\backslash L(x,y,z)))\backslash e$ by Theorem 92. Then $T(e/L(x, x\backslash L(x,y,z), x), x) = T(((x\backslash L(x,y,z))\cdot x)/(x\backslash L(x,y,z)), x\cdot(x\backslash L(x,y,z)))\backslash e$ by Axiom 3. Then $T(e/L(x, x\backslash L(x,y,z), x), x) = L(x, x\backslash L(x,y,z), x)\backslash e$ by Theorem 92. Then

$$T(e/x, L(x,y,z)) = L(x, x\backslash L(x,y,z), x)\backslash e \tag{1014}$$

by (1013). We have $L((x\backslash e)\backslash e, x\backslash L(x,y,z), x) = (L(x, x\backslash L(x,y,z), x)\backslash e)\backslash e$ by Theorem 1581. Then

$$L((x\backslash e)\backslash e, x\backslash L(x,y,z), x) = T(e/x, L(x,y,z))\backslash e \tag{1015}$$

by (1014). We have

$$x \cdot (L(x, x\backslash L(x,y,z), x) \cdot (e/x)) = L((x\backslash e)\backslash e, x\backslash L(x,y,z), x) \tag{1016}$$

by Theorem 1591. Then $x\backslash L((x\backslash e)\backslash e, x\backslash L(x,y,z), x) = L(x, x\backslash L(x,y,z), x) \cdot (e/x)$ by Proposition 2. Then

$$x\backslash(T(e/x, L(x,y,z))\backslash e) = L(x, x\backslash L(x,y,z), x) \cdot (e/x) \tag{1017}$$

by (1015). We have $x \cdot (L(x,y,z)\backslash((x\backslash L(x,y,z)) \cdot ((x\backslash e)\backslash e))) = L((x\backslash e)\backslash e, x\backslash L(x,y,z), x)$ by Theorem 1572. Then $L(x, x\backslash L(x,y,z), x) \cdot (e/x) = L(x,y,z)\backslash((x\backslash L(x,y,z)) \cdot ((x\backslash e)\backslash e))$ by (1016) and Proposition 7. Then $L(x,y,z)\backslash L((x\backslash e)\backslash e, y, z) = L(x, x\backslash L(x,y,z), x) \cdot (e/x)$ by Theorem 1577. Then

$$L(x,y,z)\backslash L((x\backslash e)\backslash e, y, z) = x\backslash(T(e/x, L(x,y,z))\backslash e) \tag{1018}$$

by (1017). We have $((e/x)\backslash T(e/x, L(x,y,z)))\backslash e = T(e/x, L(x,y,z))\backslash(e/x)$ by Theorem 749. Then $(x \cdot T(x\backslash e, L(x,y,z)))\backslash e = T(e/x, L(x,y,z))\backslash(e/x)$ by Theorem 102. Then $K(x\backslash e, L(x,y,z))\backslash e = T(e/x, L(x,y,z))\backslash(e/x)$ by Theorem 1486. Then

$$T(e/x, L(x,y,z))\backslash(e/x) = K(L(x,y,z), x\backslash e) \tag{1019}$$

by Theorem 1466. We have $((e/x)\backslash e)\backslash(T(e/x, L(x,y,z))\backslash e) = T(e/x, L(x,y,z))\backslash(e/x)$ by Theorem 1418. Then $x\backslash(T(e/x, L(x,y,z))\backslash e) = T(e/x, L(x,y,z))\backslash(e/x)$ by Proposition 25. Then $K(L(x,y,z), x\backslash e) = x\backslash(T(e/x, L(x,y,z))\backslash e)$ by (1019). Then

$$L(x,y,z)\backslash L((x\backslash e)\backslash e, y, z) = K(L(x,y,z), x\backslash e) \tag{1020}$$

by (1018). We have $L(x,y,z)\backslash T(L(x,y,z), R(x\backslash e, (x\backslash e)\backslash e, L(x,y,z))) = (x\backslash e)\backslash((L(x,y,z) \cdot (x\backslash e))/L(x,y,z))$ by Theorem 760. Then $L(x,y,z)\backslash T(L(x,y,z), R(x\backslash e, (x\backslash e)\backslash e, L(x,y,z))) = (x\backslash e)\backslash((x\backslash L(x,y,z))/L(x,y,z))$ by Theorem 837. Then $L(x,y,z)\backslash T(L(x,y,z), R(e/x, x, L(x,y,z))) = (x\backslash e)\backslash((x\backslash L(x,y,z))/L(x,y,z))$ by Theorem 727. Then $L(x,y,z)\backslash L(T(x, R(e/x, x, L(x,y,z))), y, z) = (x\backslash e)\backslash((x\backslash L(x,y,z))/L(x,y,z))$ by Axiom 8. Then

$$L(x,y,z)\backslash L(T(x, x\backslash e), y, z) = (x\backslash e)\backslash((x\backslash L(x,y,z))/L(x,y,z)) \tag{1021}$$

by Theorem 361. We have $(x\backslash e)\backslash((x\backslash L(x,y,z))/L(x,y,z)) = x \cdot ((x\backslash L(x,y,z))/L(x,y,z))$ by Theorem 709. Then $L(x,y,z)\backslash L(T(x, x\backslash e), y, z) = x \cdot ((x\backslash L(x,y,z))/L(x,y,z))$ by (1021). Then $L(x,y,z)\backslash L((x\backslash e)\backslash e, y, z) = x \cdot ((x\backslash L(x,y,z))/L(x,y,z))$ by Proposition 49. Then $K(L(x,y,z), x\backslash e) = x \cdot ((x\backslash L(x,y,z))/L(x,y,z))$ by (1020). Hence we are done by Proposition 2. ☐

Theorem 1593. $L(K(x, y\backslash e), z, w) = K(x, L(y,z,w)\backslash e)$.

Proof. We have $((w \cdot (z \cdot y))/L(y,z,w))\backslash((w \cdot (z \cdot y)) \cdot ((x \cdot ((w \cdot (z \cdot y))\backslash((w \cdot (z \cdot y))/L(y,z,w))))/x)) = L(y,z,w) \cdot ((x \cdot (L(y,z,w)\backslash e))/x)$ by Theorem 330. Then $(w \cdot z)\backslash((w \cdot (z \cdot y)) \cdot ((x \cdot ((w \cdot (z \cdot y))\backslash((w \cdot (z \cdot y))/L(y,z,w))))/x)) = L(y,z,w) \cdot ((x \cdot (L(y,z,w)\backslash e))/x)$ by Theorem 453. Then $(w \cdot z)\backslash((w \cdot (z \cdot y)) \cdot ((x \cdot ((w \cdot (z \cdot y))\backslash(w \cdot z)))/x)) = L(y,z,w) \cdot ((x \cdot (L(y,z,w)\backslash e))/x)$ by Theorem 453. Then $(w \cdot z)\backslash(w \cdot ((z \cdot y) \cdot ((x \cdot ((z \cdot y)\backslash z))/x))) = L(y,z,w) \cdot ((x \cdot (L(y,z,w)\backslash e))/x)$ by Theorem 1102. Then

$$(w \cdot z)\backslash(w \cdot (z \cdot (y \cdot ((x \cdot (y\backslash e))/x)))) = L(y,z,w) \cdot ((x \cdot (L(y,z,w)\backslash e))/x) \tag{1022}$$

by Theorem 170. We have $L(y \cdot ((x \cdot (y\backslash e))/x), z, w) = (w \cdot z)\backslash(w \cdot (z \cdot (y \cdot ((x \cdot (y\backslash e))/x))))$ by Definition 4. Then

$$L(y \cdot ((x \cdot (y\backslash e))/x), z, w) = L(y,z,w) \cdot ((x \cdot (L(y,z,w)\backslash e))/x) \tag{1023}$$

by (1022). We have $L(y,z,w) \cdot ((x \cdot (L(y,z,w)\backslash e))/x) = x\backslash T(x, L(y,z,w)\backslash e)$ by Theorem 980. Then

$$L(y \cdot ((x \cdot (y\backslash e))/x), z, w) = x\backslash T(x, L(y,z,w)\backslash e) \tag{1024}$$

by (1023).

$$\begin{aligned} & K(x, L(y,z,w)\backslash e) \\ = \quad & x\backslash T(x, L(y,z,w)\backslash e) \quad \text{by Theorem 1308} \\ = \quad & L(x\backslash T(x, y\backslash e), z, w) \quad \text{by (1024), Theorem 980.} \end{aligned}$$

Then $K(x, L(y,z,w)\backslash e) = L(x\backslash T(x, y\backslash e), z, w)$. Hence we are done by Theorem 1308. □

Theorem 1594. *$(L(x,y,z)/x)\backslash(e/x) = e/L(x,y,z)$.*

Proof. We have $(e/x) \cdot ((x\backslash e) \cdot L((x\backslash e)\backslash((e/x)\backslash(x\backslash e)), y, z)) = ((e/x) \cdot L((e/x)\backslash e, y, z)) \cdot (x\backslash e)$ by Theorem 325. Then $(e/x) \cdot ((x\backslash e) \cdot L((x\backslash e)\backslash K(x\backslash e, x), y, z)) = ((e/x) \cdot L((e/x)\backslash e, y, z)) \cdot (x\backslash e)$ by Proposition 90. Then

$$(e/x) \cdot ((x\backslash e) \cdot L(x,y,z)) = ((e/x) \cdot L((e/x)\backslash e, y, z)) \cdot (x\backslash e) \tag{1025}$$

by Theorem 21. We have $(e/x) \cdot ((x\backslash e) \cdot L(x,y,z)) = (x\backslash e) \cdot ((e/x) \cdot L(x,y,z))$ by Theorem 535. Then $((e/x) \cdot L((e/x)\backslash e, y, z)) \cdot (x\backslash e) = (x\backslash e) \cdot ((e/x) \cdot L(x,y,z))$ by (1025). Then

$$T((e/x) \cdot L((e/x)\backslash e, y, z), x\backslash e) = (e/x) \cdot L(x,y,z) \tag{1026}$$

by Theorem 11. We have $(e/x) \cdot L(x,y,z) = L(x,y,z)/x$ by Theorem 166. Then $T((e/x) \cdot L((e/x)\backslash e, y, z), x\backslash e) = L(x,y,z)/x$ by (1026). Then $T((e/x) \cdot L(x,y,z), x\backslash e) = L(x,y,z)/x$ by Proposition 25. Then

$$T(L(x,y,z)/x, x\backslash e) = L(x,y,z)/x \tag{1027}$$

by Theorem 166. We have $(x\backslash e) \cdot (T(((L(x,y,z)/x) \cdot (e/(x\backslash e))) \cdot (x\backslash e), x\backslash e)/(x\backslash e)) = T((x\backslash e) \cdot ((L(x,y,z)/x) \cdot (e/(x\backslash e))), x\backslash e)$ by Theorem 584. Then $(x\backslash e) \cdot ((e/(x\backslash e)) \cdot T((e/(x\backslash e))\backslash((L(x,y,z)/x) \cdot (e/(x\backslash e))), x\backslash e)) = T((x\backslash e) \cdot ((L(x,y,z)/x) \cdot (e/(x\backslash e))), x\backslash e)$ by Theorem 498. Then $(x\backslash e) \cdot (T(L(x,y,z)/x, x\backslash e) \cdot (e/(x\backslash e))) = T((x\backslash e) \cdot ((L(x,y,z)/x) \cdot (e/(x\backslash e))), x\backslash e)$ by Theorem 466. Then $T((x\backslash e) \cdot ((L(x,y,z)/x) \cdot (e/(x\backslash e))), x\backslash e) = (x\backslash e) \cdot ((L(x,y,z)/x) \cdot (e/(x\backslash e)))$ by (1027). Then $T((x\backslash e) \cdot ((L(x,y,z)/x) \cdot (e/(x\backslash e))), x\backslash e) = (x\backslash e) \cdot ((L(x,y,z)/x) \cdot x)$ by Proposition 24. Then $T((x\backslash e) \cdot ((L(x,y,z)/x) \cdot (e/(x\backslash e))), x\backslash e) = (x\backslash e) \cdot L(x,y,z)$ by Axiom 6. Then $T((x\backslash e) \cdot ((L(x,y,z)/x) \cdot x), x\backslash e) = (x\backslash e) \cdot L(x,y,z)$ by Proposition 24. Then $T((x\backslash e) \cdot L(x,y,z), x\backslash e) = (x\backslash e) \cdot L(x,y,z)$ by Axiom 6. Then

$$T(x\backslash e, (x\backslash e) \cdot L(x,y,z)) = x\backslash e \tag{1028}$$

by Proposition 21. We have $L(x,y,z) \cdot T(x\backslash e, (x\backslash e) \cdot L(x,y,z)) = L(x\backslash e, L(x,y,z), x\backslash e) \cdot L(x,y,z)$ by Theorem 79. Then

$$L(x,y,z) \cdot (x\backslash e) = L(x\backslash e, L(x,y,z), x\backslash e) \cdot L(x,y,z) \tag{1029}$$

by (1028). We have $L(x,y,z) \cdot (x\backslash e) = x\backslash L(x,y,z)$ by Theorem 837. Then $L(x\backslash e, L(x,y,z), x\backslash e) \cdot L(x,y,z) = x\backslash L(x,y,z)$ by (1029). Then $(x\backslash L(x,y,z))/L(x,y,z) = L(x\backslash e, L(x,y,z), x\backslash e)$ by Proposition 1. Then

$$L(x\backslash e, L(x,y,z), x\backslash e) = x\backslash K(L(x,y,z), x\backslash e) \tag{1030}$$

by Theorem 1592. We have $L(L(x,y,z)\backslash e, L(x,y,z), e/x) = ((e/x) \cdot L(x,y,z))\backslash((e/x) \cdot e)$ by Proposition 53. Then

$$L(L(x,y,z)\backslash e, L(x,y,z), e/x) = (L(x,y,z)/x)\backslash((e/x) \cdot e) \tag{1031}$$

by Theorem 166. We have $L(x,y,z) \cdot L(L(x,y,z)\backslash e, L(x,y,z), e/x) = L(x \cdot L(x\backslash e, L(x,y,z), e/x), y, z)$ by Theorem 1578. Then $L(x,y,z) \cdot ((L(x,y,z)/x)\backslash((e/x) \cdot e)) = L(x \cdot L(x\backslash e, L(x,y,z), e/x), y, z)$ by (1031). Then $L(x,y,z) \cdot ((L(x,y,z)/x)\backslash(e/x)) = L(x \cdot L(x\backslash e, L(x,y,z), e/x), y, z)$ by Axiom 2. Then $L(x \cdot L(x\backslash e, L(x,y,z), x\backslash e), y, z) = L(x,y,z) \cdot ((L(x,y,z)/x)\backslash(e/x))$ by Theorem 1224. Then

$$L(x \cdot (x\backslash K(L(x,y,z), x\backslash e)), y, z) = L(x,y,z) \cdot ((L(x,y,z)/x)\backslash(e/x)) \tag{1032}$$

by (1030). We have $L(K(L(x,y,z),x\backslash e),y,z) = K(L(x,y,z),L(x,y,z)\backslash e)$ by Theorem 1593. Then $L(x\cdot(x\backslash K(L(x,y,z),x\backslash e)),y,z) = K(L(x,y,z),L(x,y,z)\backslash e)$ by Axiom 4. Then

$$K(L(x,y,z),L(x,y,z)\backslash e) = L(x,y,z)\cdot((L(x,y,z)/x)\backslash(e/x)) \tag{1033}$$

by (1032). We have $L(x,y,z)\cdot(e/L(x,y,z)) = K(L(x,y,z),L(x,y,z)\backslash e)$ by Theorem 250. Hence we are done by (1033) and Proposition 7. □

Theorem 1595. *$L(e/x,y,z)\backslash e = e/L(x\backslash e,y,z)$.*

Proof. We have $L(x\backslash T(x,x\cdot L(x\backslash e,y,z)),y,z) = K((x\cdot L(x\backslash e,y,z))\backslash((x\cdot L(x\backslash e,y,z))/L(x\backslash e,y,z)),x\cdot L(x\backslash e,y,z))$ by Theorem 1358. Then $L(x\backslash T(x,x\cdot L(x\backslash e,y,z)),y,z) = K((x\cdot L(x\backslash e,y,z))\backslash x,x\cdot L(x\backslash e,y,z))$ by Axiom 5. Then $L(x\backslash x,y,z) = K((x\cdot L(x\backslash e,y,z))\backslash x,x\cdot L(x\backslash e,y,z))$ by Theorem 304. Then

$$K((x\cdot L(x\backslash e,y,z))\backslash x,x\cdot L(x\backslash e,y,z)) = L(e,y,z) \tag{1034}$$

by Proposition 28. We have $L(e,y,z) = (z\cdot y)\backslash(z\cdot(y\cdot e))$ by Definition 4. Then

$$L(e,y,z) = (z\cdot y)\backslash(z\cdot y) \tag{1035}$$

by Axiom 2. We have $(z\cdot y)\backslash(z\cdot y) = e$ by Proposition 28. Then $L(e,y,z) = e$ by (1035). Then

$$K((x\cdot L(x\backslash e,y,z))\backslash x,x\cdot L(x\backslash e,y,z)) = e \tag{1036}$$

by (1034). We have $T(x,x\cdot L(x\backslash e,y,z))\cdot K((x\cdot L(x\backslash e,y,z))\backslash x,(x\cdot(x\cdot L(x\backslash e,y,z)))/x) = ((x\cdot L(x\backslash e,y,z))\backslash x)\cdot((x\cdot(x\cdot L(x\backslash e,y,z)))/x)$ by Theorem 1092. Then $x\cdot K((x\cdot L(x\backslash e,y,z))\backslash x,(x\cdot(x\cdot L(x\backslash e,y,z)))/x) = ((x\cdot L(x\backslash e,y,z))\backslash x)\cdot((x\cdot(x\cdot L(x\backslash e,y,z)))/x)$ by Theorem 304. Then $x\cdot K((x\cdot L(x\backslash e,y,z))\backslash x,x\cdot L(x\backslash e,y,z)) = ((x\cdot L(x\backslash e,y,z))\backslash x)\cdot((x\cdot(x\cdot L(x\backslash e,y,z)))/x)$ by Theorem 1574. Then

$$((x\cdot L(x\backslash e,y,z))\backslash x)\cdot((x\cdot(x\cdot L(x\backslash e,y,z)))/x) = x\cdot e \tag{1037}$$

by (1036). We have $(((x\cdot L(x\backslash e,y,z))\backslash x)\cdot((x\cdot(x\cdot L(x\backslash e,y,z)))/x))\cdot e = ((x\cdot L(x\backslash e,y,z))\backslash x)\cdot((x\cdot(x\cdot L(x\backslash e,y,z)))/x)$ by Axiom 2. Then $x\cdot e = (((x\cdot L(x\backslash e,y,z))\backslash x)\cdot((x\cdot(x\cdot L(x\backslash e,y,z)))/x))\cdot e$ by (1037). Then $x = ((x\cdot L(x\backslash e,y,z))\backslash x)\cdot((x\cdot(x\cdot L(x\backslash e,y,z)))/x)$ by Proposition 10. Then

$$((x\cdot L(x\backslash e,y,z))\backslash x)\cdot(x\cdot L(x\backslash e,y,z)) = x \tag{1038}$$

by Theorem 1574. We have $((e/x)\backslash e)/((e/x)\backslash L(e/x,y,z)) = L(e/x,y,z)\backslash e$ by Theorem 906. Then $x/((e/x)\backslash L(e/x,y,z)) = L(e/x,y,z)\backslash e$ by Proposition 25. Then

$$x/(x\cdot L(x\backslash e,y,z)) = L(e/x,y,z)\backslash e \tag{1039}$$

by Theorem 159. We have $(x/(x\cdot L(x\backslash e,y,z)))\cdot(x\cdot L(x\backslash e,y,z)) = x$ by Axiom 6. Then $(L(e/x,y,z)\backslash e)\cdot(x\cdot L(x\backslash e,y,z)) = x$ by (1039). Then $(L(e/x,y,z)\backslash e)\cdot(x\cdot L(x\backslash e,y,z)) = ((x\cdot L(x\backslash e,y,z))\backslash x)\cdot(x\cdot L(x\backslash e,y,z))$ by (1038). Then

$$L(e/x,y,z)\backslash e = (x\cdot L(x\backslash e,y,z))\backslash x \tag{1040}$$

by Proposition 10. We have $(L(x\backslash e,y,z)/(x\backslash e))\backslash(e/(x\backslash e)) = e/L(x\backslash e,y,z)$ by Theorem 1594. Then $(L(x\backslash e,y,z)/(x\backslash e))\backslash x = e/L(x\backslash e,y,z)$ by Proposition 24. Then $(x\cdot L(x\backslash e,y,z))\backslash x = e/L(x\backslash e,y,z)$ by Theorem 161. Hence we are done by (1040). □

Theorem 1596. *$e/L(x,y,z) = L(e/(e/x),y,z)\backslash e$.*

Proof. We have $e/L((e/x)\backslash e,y,z) = L(e/(e/x),y,z)\backslash e$ by Theorem 1595. Hence we are done by Proposition 25. □

The following was first conjectured in [9].

Theorem 1597. $e/(e/L(x,y,z)) = L(e/(e/x),y,z)$.

Proof. We have $e/(L(e/(e/x),y,z)\backslash e) = L(e/(e/x),y,z)$ by Proposition 24. Hence we are done by Theorem 1596. $\square$

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