Some Unproven Theorems(?) In Megalodon

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1 Surreal Operations and Relations

Notation. We use $<$ as an infix operator corresponding to applying term $SNoLt$. We use $\leq$ as an infix operator corresponding to applying term $SNoLe$. We use $+$ as a right associative infix operator corresponding to applying term add_SNo. We use $--$ as a prefix operator corresponding to applying term $minus\_SNo$. We use $*$ as a right associative infix operator corresponding to applying term mul_SNo. We use $\div$ as an infix operator corresponding to applying term div_SNo. We use superscripts for the operation corresponding to applying term exp_SNo_nat.

2 Integers and Diadic Rationals as Rings

Definition 2.1. *We define* subring_with_id *to be*

$$\begin{array}{c}\lambda RR'.Ring\_with\_id\ R \wedge Ring\_with\_id\ R' \wedge \\ unpack\_b\_b\_e\_e\_o\ R\ (\lambda R\ plus\ mult\ zero\ one. \\ unpack\_b\_b\_e\_e\_o\ R'\ (\lambda R'\ plus'\ mult'\ zero'\ one'. \\ R \subseteq R' \wedge zero = zero' \wedge one = one' \wedge \\ (\forall xy \in R.plus\ x\ y = plus'\ x\ y) \wedge \\ (\forall xy \in R.mult\ x\ y = mult'\ x\ y)))\end{array}$$

of type $\iota \to \iota \to o$.

Let Z be int.

Theorem 2.1. $Z \subseteq UnivOf\ Empty$.

Proof. *Proof unfinished.* □

Theorem 2.2. $\forall n \in Z. SNo\ n$.

Proof. *Proof unfinished.* □

Definition 2.2. *We define* intRing *to be* $pack\_b\_b\_e\_e\ Z\ add\_SNo\ mul\_SNo\ 0\ 1$ *of type* ι.

Let ZR be intRing.

Theorem 2.3. $CRing\_with\_id$ intRing.

Proof. *Proof unfinished.* □

Definition 2.3. *We define* diadicrational *to be* $SNoS\_\ omega$ *of type* ι.

Let DQ be diadicrational.

Theorem 2.4. $\forall x \in UnivOf\ Empty. SNo\ x \to x \in DQ$.

Proof. *Proof unfinished.* □

Theorem 2.5. $DQ \subseteq UnivOf\ Empty$.

Proof. *Proof unfinished.* □

Theorem 2.6. $\forall x \in DQ. SNo\ x$.

Proof. *Proof unfinished.* □

Theorem 2.7. $Z \subseteq DQ$.

Proof. *Proof unfinished.* □

Theorem 2.8. $\forall n \in int. \forall m \in omega. \forall x. SNo\ x \to x * 2^m = n \to x \in DQ$.

Proof. *Proof unfinished.* □

Theorem 2.9. $\forall x \in DQ. \exists n \in int. \exists m \in omega. x * 2^m = n$.

Proof. *Proof unfinished.* □

Definition 2.4. *We define* diadicrationalRing *to be*

$$pack\_b\_b\_e\_e\ \text{diadicrational}\ add\_SNo\ mul\_SNo\ 0\ 1$$

of type ι.

Let DQR be diadicrationalRing.

Theorem 2.10. $CRing\_with\_id\ DQ$.

Proof. *Proof unfinished.* □

Theorem 2.11. subring_with_id $ZR\ DQR$.

Proof. *Proof unfinished.* □

Theorem 2.12. $\neg Field\ ZR$.

Proof. *Proof unfinished.* □

Theorem 2.13. $\neg Field\ DQR$.

Proof. *Proof unfinished.* □

3 Ring Homomorphisms and Ideals

Definition 3.1. *We define* Ring_with_id_Hom *to be*

$$\begin{gathered} \lambda RR'g.Ring\_with\_id\ R \wedge Ring\_with\_id\ R' \wedge \\ unpack\_b\_b\_e\_e\_o\ R\ (\lambda R\ plus\ mult\ zero\ one. \\ unpack\_b\_b\_e\_e\_o\ R'\ (\lambda R'\ plus'\ mult'\ zero'\ one'. \\ g \in {R'}^{R} \wedge g\ zero = zero' \wedge g\ one = one' \wedge \\ (\forall xy \in R.g\ (plus\ x\ y) = plus'\ (g\ x)\ (g\ y)) \wedge \\ (\forall xy \in R.g\ (mult\ x\ y) = mult'\ (g\ x)\ (g\ y)))) \end{gathered}$$

of type $\iota \to \iota \to \iota \to o$.

Definition 3.2. *We define* Ring_with_id_Ker *to be*

$$\lambda RR'g.unpack\_b\_b\_e\_e\_i\ R'\ (\lambda R'\ plus'\ mult'\ zero'\ one'.\{x \in R | g\ x = zero'\})$$

of type $\iota \to \iota \to \iota \to \iota$.

Theorem 3.1. Ring_with_id_Hom $ZR\ DQR\ (\lambda x \in int.x)$.

Proof. *Proof unfinished.* □

Theorem 3.2. Ring_with_id_Ker $ZR\ DQR\ (\lambda x \in int.x) = \{0\}$.

Proof. *Proof unfinished.* □

Definition 3.3. *We define* Ring_with_id_LeftIdeal *to be*

$$\lambda RI:\iota.Ring\_with\_id\ R\wedge \\ unpack\_b\_b\_e\_e\_o\ R\ (\lambda R\ plus\ mult\ zero\ one. \\ I\subseteq R\wedge\forall r\in R.\forall x\in I.mult\ r\ x\in I)$$

of type $\iota\to\iota\to o$.

Definition 3.4. *We define* Ring_with_id_RightIdeal *to be*

$$\lambda RI:\iota.Ring\_with\_id\ R\wedge \\ unpack\_b\_b\_e\_e\_o\ R\ (\lambda R\ plus\ mult\ zero\ one. \\ I\subseteq R\wedge\forall r\in R.\forall x\in I.mult\ x\ r\in I)$$

of type $\iota\to\iota\to o$.

Definition 3.5. *We define* Ring_with_id_Ideal *to be*

$\lambda RI:\iota.Ring\_with\_id\ R\wedge$ Ring_with_id_LeftIdeal $R\ I\wedge$ Ring_with_id_RightIdeal $R\ I$

of type $\iota\to\iota\to o$.

Theorem 3.3.

$\forall RR'g.$Ring_with_id_Hom $R\ R'\ g\to$ Ring_with_id_Ideal R (Ring_with_id_Ker $R\ R'\ g$).

Proof. *Proof unfinished.* □

Theorem 3.4.

$$\forall RI.\text{Ring\_with\_id\_Ideal}\ R\ I\to\exists R'.Ring\_with\_id\ R'\wedge \\ \exists g.\text{Ring\_with\_id\_Hom}\ R\ R'\ g\wedge\text{Ring\_with\_id\_Ker}\ R\ R'\ g=I \\ \wedge surj\ (R\ 0)\ (R'\ 0)\ (\lambda x.g\ x).$$

Proof. *Proof unfinished.* □

Theorem 3.5.

$$\forall RI.CRing\_with\_id\ R\to\text{Ring\_with\_id\_Ideal}\ R\ I\to\exists R'.CRing\_with\_id\ R'\wedge \\ \exists g.\text{Ring\_with\_id\_Hom}\ R\ R'\ g\wedge\text{Ring\_with\_id\_Ker}\ R\ R'\ g=I \\ \wedge surj\ (R\ 0)\ (R'\ 0)\ (\lambda x.g\ x).$$

Proof. *Proof unfinished.* □

Definition 3.6. *We define* Ring_with_id_PrimeIdeal *to be*

$$\lambda RP.\mathsf{Ring\_with\_id\_Ideal}\ R\ P \wedge P \neq R\ 0 \wedge$$
$$\forall AB.\mathsf{Ring\_with\_id\_Ideal}\ R\ A \rightarrow$$
$$\mathsf{Ring\_with\_id\_Ideal}\ R\ B \rightarrow A \cap B \subseteq P \rightarrow A \subseteq P \vee B \subseteq P$$

of type $\iota \rightarrow \iota \rightarrow o$.

Theorem 3.6.

$$\forall P.\mathsf{Ring\_with\_id\_Ideal}\ ZR\ P \rightarrow 1 \notin P \rightarrow$$
$$\forall p \in P.prime\_nat\ p \rightarrow \mathsf{Ring\_with\_id\_PrimeIdeal}\ ZR\ P.$$

Proof. *Proof unfinished.* □

Theorem 3.7. $\forall P.\mathsf{Ring\_with\_id\_PrimeIdeal}\ ZR\ P \rightarrow 1 \notin P \wedge \exists p \in P.prime\_nat\ p.$

Proof. *Proof unfinished.* □

Definition 3.7. *We define* Ring_with_id_univar_poly *to be*

$$\lambda R.unpack\_b\_b\_e\_e\_i\ R\ (\lambda R\ plus\ mult\ zero\ one.$$
$$\{p \in R^{omega} | \exists n \in omega.\forall m \in omega \setminus n.p\ m = zero\})$$

of type $\iota \rightarrow \iota$.

Definition 3.8. *We define* Ring_with_id_univar_poly_Ring *to be*

$$\lambda R.unpack\_b\_b\_e\_e\_i\ R\ (\lambda R' plusmultzeroone.$$
$$pack\_b\_b\_e\_e\ (\mathsf{Ring\_with\_id\_univar\_poly}\ R)$$
$$(\lambda pp'.\lambda i \in omega.plus\ (p\ i)\ (p'\ i))$$
$$(\lambda pp'.\lambda i \in omega.mult\ (p\ i)\ (p'\ i))$$
$$(\lambda i \in omega.zero)$$
$$(\lambda i \in omega.\mathsf{if}\ i = 0\ \mathsf{then}\ one\ \mathsf{else}\ zero))$$

of type $\iota \rightarrow \iota$.

Theorem 3.8.

$\forall R.Ring\_with\_id\ R \rightarrow Ring\_with\_id\ (\mathsf{Ring\_with\_id\_univar\_poly\_Ring}\ R).$

Proof. *Proof unfinished.* □

Theorem 3.9.

$\forall R.CRing\_with\_id\ R \rightarrow CRing\_with\_id\ (\mathsf{Ring\_with\_id\_univar\_poly\_Ring}\ R).$

Proof. *Proof unfinished.* □

4 Topology on Diadic Rationals

Theorem 4.1. *diadic_open Empty.*

Proof. We will prove

$$\begin{gathered} Empty \subseteq SNoS\_ \ omega \wedge \\ \forall x \in Empty. \exists n \in omega. \forall y \in SNoS\_ \ omega. \\ x + -eps\_ \ n < y \to y < x + eps\_ \ n \to y \in Empty \end{gathered}$$

Apply *andI* to the current goal. Apply *Subq_Empty* to the current goal. Let x be given. Assume Hx: $x \in Empty$. We will prove *False*. Exact *EmptyE x Hx*. □

Theorem 4.2. *diadic_open DQ.*

Proof. *Proof unfinished.* □

Theorem 4.3. $\forall XY \subseteq DQ. diadic\_open\ X \to diadic\_open\ Y \to diadic\_open\ (X \cap Y).$

Proof. *Proof unfinished.* □

Theorem 4.4. $\forall C. (\forall X \in C. diadic\_open\ X) \to diadic\_open\ (Union\ C).$

Proof. *Proof unfinished.* □

Theorem 4.5.

$$\begin{gathered} \forall xy \in DQ. x \neq y \to \exists UV. \\ diadic\_open\ U \wedge diadic\_open\ V \wedge x \in U \wedge y \in V \wedge U \cap V = 0. \end{gathered}$$

Proof. *Proof unfinished.* □

Definition 4.1. *We define* diadic_rationals_Cauchy_seq *to be*

$$\begin{gathered} \lambda x. x \in DQ^{omega} \wedge \forall n \in omega. \exists M \in omega. \\ \forall ij \in omega \setminus M. -eps\_ \ n < x\ i + -x\ j \wedge x\ i + -x\ j < eps\_ \ n \end{gathered}$$

of type $\iota \to o$.

Definition 4.2. *We define* diadic_rationals_seq_lim *to be*

$$\begin{gathered} \lambda xy. x \in DQ^{omega} \wedge \forall n \in omega. \exists M \in omega. \\ \forall i \in omega \setminus M. -eps\_ \ n < x\ i + -y \wedge x\ i + -y < eps\_ \ n \end{gathered}$$

of type $\iota \to \iota \to o$.

Theorem 4.6. $\exists x.\text{diadic\_rationals\_Cauchy\_seq } x \land \forall y \in DQ.\neg\text{diadic\_rationals\_seq\_lim } x\ y.$

Proof. Proof unfinished. □

Theorem 4.7. $\forall x.SNo\ x \to x \neq 0 \to \exists r.SNo\ r \to x * r = 1.$

Proof. Proof unfinished. □

Theorem 4.8. $\forall xy.SNo\ x \to SNo\ y \to y \neq 0 \to \exists q.SNo\ q \to y * q = x.$

Proof. Proof unfinished. □

5 Reals

Theorem 5.1. $\forall xy \in real.x + y \in real.$

Proof. Proof unfinished. □

Theorem 5.2. $\forall xy \in real.x * y \in real.$

Proof. Proof unfinished. □

Theorem 5.3. $\forall xy \in real.x : / : y \in real.$

Proof. Proof unfinished. □

Theorem 5.4. $explicit\_Field\ real\ 0\ 1\ add\_SNo\ mul\_SNo.$

Proof. Proof unfinished. □

Theorem 5.5. $explicit\_OrderedField\ real\ 0\ 1\ add\_SNo\ mul\_SNo\ SNoLe.$

Proof. Proof unfinished. □

Theorem 5.6. $explicit\_Reals\ real\ 0\ 1\ add\_SNo\ mul\_SNo\ SNoLe.$

Proof. Proof unfinished. □

Theorem 5.7.

$$\forall U.TransSet\ U \to ZF\_closed\ U \to omega \in U \to$$
$$explicit\_Field\ \{x \in U | SNo\ x\}\ 0\ 1\ add\_SNo\ mul\_SNo.$$

Proof. Proof unfinished. □

Theorem 5.8.

$$\forall U.TransSet\ U \to ZF\_closed\ U \to omega \in U \to$$
$$explicit\_Field\ \{x \in U | SNo\ x\}\ 0\ 1\ add\_SNo\ mul\_SNo.$$

Proof. Proof unfinished. □