HF 100 Benchmark

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Contents

Chapter 1

100 Primary Theorems

1.1 Definitions and Axioms

All the theorems have the same set of definitions and axioms. We will take primitive names for the first five ordinals: 0, 1, 2, 3 and 4. Note that 0 will also be our notation for the empty set. All the axioms below obviously hold in every (reasonable) set theoretic model. The definition of atleast2 is nonstandard, but can be proven to mean that the set in question has at least two elements.

Definition 1.1. *We define* $\subseteq$ *to be a binary relation such that* $X \subseteq Y$ *holds iff (if and only if)* $\forall x.x \in X \Rightarrow x \in Y$.

Definition 1.2. *We define* disj *to be a binary relation such that* disj X Y *holds iff* $X \cap Y = 0$.

Definition 1.3. *We define* atleast2 *to be a unary predicate such that* atleast2 X *holds iff* $\exists Y \in X.X \not\subseteq \wp\, Y$.

Definition 1.4. *We define* atleast3 *to be a unary predicate such that* atleast3 X *holds iff* $\exists Y \subseteq X.X \not\subseteq Y \wedge$ atleast2 Y.

Definition 1.5. *We define* atleast4 *to be a unary predicate such that* atleast4 X *holds iff* $\exists Y \subseteq X.X \not\subseteq Y \wedge$ atleast3 Y.

Definition 1.6. *We define* atleast5 *to be a unary predicate such that* atleast5 X *holds iff* $\exists Y \subseteq X.X \not\subseteq Y \wedge$ atleast4 Y.

Definition 1.7. *We define* atleast6 *to be a unary predicate such that* atleast6 X *holds iff* $\exists Y \subseteq X.X \not\subseteq Y \wedge$ atleast5 Y.

Definition 1.8. *We define* atleast7 *to be a unary predicate such that* atleast7 X *holds iff* $\exists Y \subseteq X.X \not\subseteq Y \wedge$ atleast6 Y.

Definition 1.9. *We define* exactly2 *to be a unary predicate such that* exactly2 X *holds iff* atleast2 $X \wedge \neg$atleast3 X.

Definition 1.10. *We define* exactly3 *to be a unary predicate such that* exactly3 X *holds iff* atleast3 $X \wedge \neg$atleast4 X.

Definition 1.11. *We define* exactly4 *to be a unary predicate such that* exactly4 X *holds iff* atleast4 $X \wedge \neg$atleast5 X.

Definition 1.12. *We define* exactly5 *to be a unary predicate such that* exactly5 X *holds iff* atleast5 $X \wedge \neg$atleast6 X.

Definition 1.13. *We define* exactly6 *to be a unary predicate such that* exactly6 X *holds iff* atleast6 $X \wedge \neg$atleast7 X.

Axiom 1.1. $\forall XY.X \subseteq Y \Rightarrow Y \subseteq X \Rightarrow X = Y$.

Axiom 1.2. $\forall x.x \notin x$.

Axiom 1.3. $\forall xy.x \in y \Rightarrow y \notin x$.

Axiom 1.4. $\forall x.x \notin 0$.

Axiom 1.5. $\forall i.i \in 1 \Leftrightarrow i = 0$.

Axiom 1.6. $\forall i.i \in 2 \Leftrightarrow i = 0 \vee i = 1$.

Axiom 1.7. $\forall i.i \in 3 \Leftrightarrow i = 0 \vee i = 1 \vee i = 2$.

Axiom 1.8. $\forall i.i \in 4 \Leftrightarrow i = 0 \vee i = 1 \vee i = 2 \vee i = 3$.

Axiom 1.9. $\forall XY.Y \in \wp\ X \Leftrightarrow Y \subseteq X$.

Axiom 1.10. $\forall xy.y \in \{x\} \Leftrightarrow y = x$.

Axiom 1.11. $\forall XYz.z \in X \cup Y \Leftrightarrow z \in X \vee z \in Y$.

Axiom 1.12. $\forall XYz.z \in X \cap Y \Leftrightarrow z \in X \wedge z \in Y$.

Axiom 1.13. $\forall XYz.z \in X \setminus Y \Leftrightarrow z \in X \wedge z \notin Y$.

1.2 The 100 Theorems

We now list the 100 theorems we are interested in using as a benchmark for first-order automated theorem provers. Each of the theorems states that a certain hereditarily finite set (logically, a ground term that would be interpreted as a hereditarily finite in a model of set theory) has a specific cardinality $n \in \{2, 3, 4, 5, 6\}$. It should not be too difficult for the reader to verify these (at least morally) by determining the elements of the set in question. A first order theorem prover, of course, would need to prove the statement starting from only the definitions and axioms above.

Theorem 1.1. exactly2 2.

Theorem 1.2. exactly2 $(\{1\} \cup \{\{1\}\})$.

Theorem 1.3. exactly2 $(3 \setminus \{1\})$.

Theorem 1.4. exactly2 $(((\wp\ 2) \cup \{\{2\}\}) \cap (4 \setminus \{1\}))$.

Theorem 1.5. exactly2 $((\{\{2\}\} \cup 4) \cap ((\wp\ 2) \setminus \{1\}))$.

Theorem 1.6. exactly2 $(((\wp\ 2) \cup \{(\wp\ 2)\}) \cap (4 \setminus \{1\}))$.

Theorem 1.7. exactly2 $((4 \cup \{(\wp\ 2)\}) \cap (\{2\} \cup (\wp\ \{2\})))$.

Theorem 1.8. exactly2 $((\wp\ 2) \setminus (\wp\ \{1\}))$.

Theorem 1.9. exactly2 $((\wp\ 2) \setminus 2)$.

Theorem 1.10. exactly2 $(1 \cup \{3\})$.

Theorem 1.11. exactly2 $(4 \setminus 2)$.

Theorem 1.12. exactly2 $((\{\{1\}\} \cup 4) \cap (\{2\} \cup (\wp\ \{2\})))$.

Theorem 1.13. exactly2 $(1 \cup \{(\wp\ 2)\})$.

Theorem 1.14. exactly2 $(\{((\wp\ 2) \setminus (\wp\ \{2\}))\} \cup \{(\wp\ 2)\})$.

Theorem 1.15. exactly2 $((4 \cup \{(\wp\ 2)\}) \cap ((\wp\ 2) \setminus (\wp\ \{2\})))$.

Theorem 1.16. exactly3 $((\wp\ 2) \setminus \{2\})$.

Theorem 1.17. exactly3 3.

Theorem 1.18. exactly2 $(3 \setminus \{2\})$.

Theorem 1.19. exactly3 $((\wp\ 2) \setminus \{1\})$.

Theorem 1.20. exactly2 $(((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}) \setminus \{0\})$.

Theorem 1.21. exactly3 $(\{2\} \cup (\wp\ \{2\}))$.

Theorem 1.22. exactly3 $(((\wp\ 2) \cup \{\{2\}\}) \cap ((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\}))$.

Theorem 1.23. exactly3 $(4 \setminus \{2\})$.

Theorem 1.24. exactly2 $((4 \setminus \{2\}) \setminus \{1\})$.

Theorem 1.25. exactly2 $((4 \setminus \{2\}) \setminus \{3\})$.

Theorem 1.26. exactly3 $(4 \setminus \{1\})$.

Theorem 1.27. exactly2 $((4 \setminus \{1\}) \setminus \{0\})$.

Theorem 1.28. exactly2 $((4 \setminus \{1\}) \setminus \{2\})$.

Theorem 1.29. exactly2 $((4 \setminus \{1\}) \setminus \{3\})$.

Theorem 1.30. exactly3 $(2 \cup \{(\wp\ 2)\})$.

Theorem 1.31. exactly3 $(\{\{\{1\}\}\} \cup (1 \cup \{(\wp\ 2)\}))$.

Theorem 1.32. exactly3 $(((\wp\ 2) \cup \{\{2\}\}) \cap (4 \cup \{((\wp\ 2) \setminus 2)\}))$.

Theorem 1.33. exactly3 $(((\wp\ 2) \cup \{\{2\}\}) \cap (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})))$.

Theorem 1.34. *The intersection of*

$$((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\})$$

and

$$(\{\{1\}\} \cup (\{\{2\}\} \cup 4))$$

has exactly 3 elements.

Theorem 1.35. *The intersection of*

$$((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\})$$

and

$$(\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}))$$

has exactly 3 elements.

Theorem 1.36. *The intersection of*

$$((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\})$$

and

$$(((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}))$$

has exactly 3 elements.

Theorem 1.37. exactly3 $((3 \cup \{(3 \setminus \{1\})\}) \cap (\wp\ (3 \setminus \{1\})))$.

Theorem 1.38. exactly3 $((\{\{1\}\} \cup (\{\{2\}\} \cup 4)) \cap (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})))$.

Theorem 1.39. exactly4 $(\wp\ 2)$.

Theorem 1.40. exactly3 $((\wp\ 2) \setminus \{0\})$.

Theorem 1.41. exactly3 $((\wp\ 2) \setminus \{\{1\}\})$.

Theorem 1.42. exactly4 $(3 \cup \{(\wp\ \{1\})\})$.

Theorem 1.43. exactly4 $(3 \cup \{\{2\}\})$.

Theorem 1.44. exactly3 $((3 \cup \{\{2\}\}) \setminus \{2\})$.

Theorem 1.45. exactly4 4.

Theorem 1.46. exactly3 $(4 \setminus \{0\})$.

Theorem 1.47. exactly3 $(4 \setminus \{3\})$.

Theorem 1.48. exactly4 $((\{2\} \cup \{\{2\}\}) \cup (1 \cup \{3\}))$.

Theorem 1.49. exactly4 $(2 \cup (\{\{1\}\} \cup \{3\}))$.

Theorem 1.50. exactly4 $(2 \cup (\{3\} \cup \{(\wp\ 2)\}))$.

Theorem 1.51. exactly4 $((\wp\ \{1\}) \cup (4 \setminus 2))$.

Theorem 1.52. exactly4 $(((\wp\ 2) \setminus 2) \cup (\wp\ \{2\}))$.

Theorem 1.53. exactly4 $(3 \cup \{((\wp\ 2) \setminus 2)\})$.

Theorem 1.54. exactly4 $(((\wp\ 2) \setminus \{2\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\})$.

Theorem 1.55. exactly4 $(((\wp\ 2) \setminus \{1\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\})$.

Theorem 1.56. exactly4 $(3 \cup \{(\wp\ 2)\})$.

Theorem 1.57. exactly4 $(((\wp\ 2) \cup \{\{2\}\}) \cap (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})))$.

Theorem 1.58. exactly4 $(((\wp\ 2) \cup \{\{2\}\}) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})))$.

Theorem 1.59. exactly4 $((\{\{1\}\} \cup (\{\{2\}\} \cup 4)) \cap (4 \cup \{((\wp\ 2) \setminus 2)\}))$.

Theorem 1.60. exactly4 $((4 \cup \{((\wp\ 2) \setminus 2)\}) \cap (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})))$.

Theorem 1.61. exactly3 $((3 \cup \{(\wp\ 2)\}) \setminus \{0\})$.

Theorem 1.62. exactly3 $((3 \cup \{(\wp\ 2)\}) \setminus \{1\})$.

Theorem 1.63. exactly3 $((3 \cup \{(\wp\ 2)\}) \setminus \{2\})$.

Theorem 1.64. exactly3 $((\{\{\{1\}\}\} \cup (2 \cup \{(\wp\ 2)\})) \setminus \{0\})$.

Theorem 1.65. exactly4 $((((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{0\})$.

Theorem 1.66. exactly5 $((\wp\ 2) \cup \{\{2\}\})$.

Theorem 1.67. exactly5 $(\{\{1\}\} \cup 4)$.

Theorem 1.68. exactly5 $(\{\{\{1\}\}\} \cup 4)$.

Theorem 1.69. exactly5 $(\{(\wp\ \{1\})\} \cup 4)$.

Theorem 1.70. exactly5 $(\{\{2\}\} \cup 4)$.

Theorem 1.71. exactly4 $((\{\{2\}\} \cup 4) \setminus \{1\})$.

Theorem 1.72. exactly4 $((\{\{2\}\} \cup 4) \setminus \{2\})$.

Theorem 1.73. exactly4 $((\{\{2\}\} \cup 4) \setminus \{3\})$.

Theorem 1.74. exactly5 $(4 \cup \{((\wp\ 2) \setminus 2)\})$.

Theorem 1.75. exactly5 $((\wp\ 2) \cup \{(\wp\ 2)\})$.

Theorem 1.76. exactly5 $(4 \cup \{(\wp\ 2)\})$.

Theorem 1.77. exactly4 $((4 \cup \{(\wp\ 2)\}) \setminus \{0\})$.

Theorem 1.78. exactly4 $((4 \cup \{(\wp\ 2)\}) \setminus \{1\})$.

Theorem 1.79. exactly4 $((4 \cup \{(\wp\ 2)\}) \setminus \{2\})$.

Theorem 1.80. exactly4 $((4 \cup \{(\wp\ 2)\}) \setminus \{3\})$.

Theorem 1.81. exactly6 $(\{\{1\}\} \cup (\{\{2\}\} \cup 4))$.

Theorem 1.82. exactly6 $(\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Theorem 1.83. exactly6 $(\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Theorem 1.84. exactly3 $(((\wp\ 2) \cup \{\{2\}\}) \setminus (4 \setminus \{1\}))$.

Theorem 1.85. exactly3 $((\{\{1\}\} \cup 4) \setminus (\{2\} \cup (\wp\ \{2\})))$.

Theorem 1.86. exactly3 $((\{\{2\}\} \cup 4) \setminus ((\wp\ 2) \setminus \{1\}))$.

Theorem 1.87. exactly3 $(((\wp\ 2) \cup \{(\wp\ 2)\}) \setminus (4 \setminus \{1\}))$.

Theorem 1.88. exactly3 $((4 \cup \{(\wp\ 2)\}) \setminus ((\wp\ 2) \setminus (\wp\ \{2\})))$.

Theorem 1.89. exactly3 $((4 \cup \{(\wp\ 2)\}) \setminus (\{2\} \cup (\wp\ \{2\})))$.

Theorem 1.90. exactly4 $(((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\})) \cap (\{\{2\}\} \cup 4))$.

Theorem 1.91. exactly5 $((\{\{1\}\} \cup (\{\{2\}\} \cup 4)) \setminus \{0\})$.

Theorem 1.92. exactly5 $((\{\{1\}\} \cup (\{\{2\}\} \cup 4)) \setminus \{2\})$.

Theorem 1.93. exactly5 $((\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{1\})$.

Theorem 1.94. exactly5 $((\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{2\})$.

Theorem 1.95. exactly5 $((\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{3\})$.

Theorem 1.96. exactly5 $((\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{0\})$.

Theorem 1.97. exactly5 $((\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{1\})$.

Theorem 1.98. exactly5 $((\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{2\})$.

Theorem 1.99. exactly5 $((\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{3\})$.

Theorem 1.100. *The intersection of*

$$(((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}))$$

and

$$(\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$$

has exactly 5 elements.

1.3 Propositions, Lemmas and Theorems

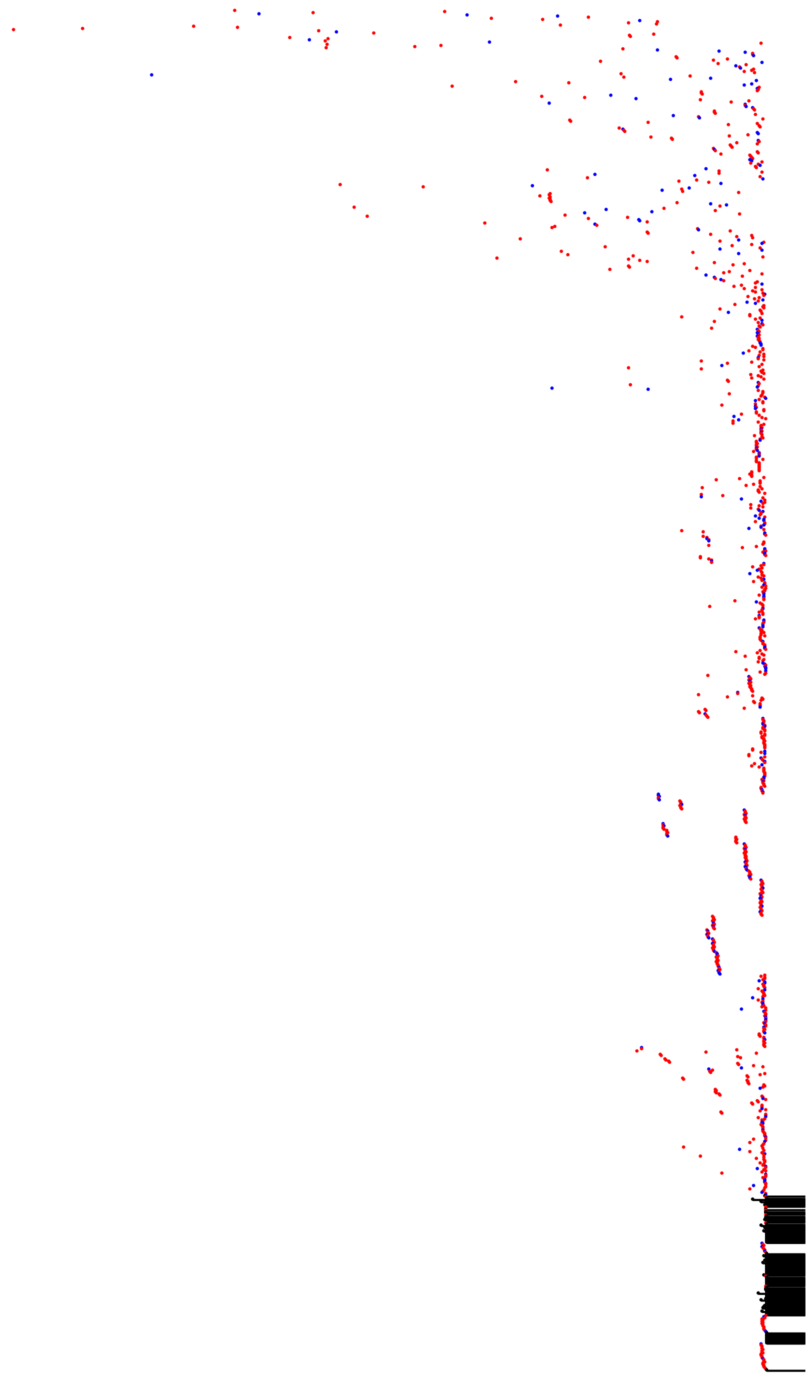

Figure 1.1: Dependency Graph

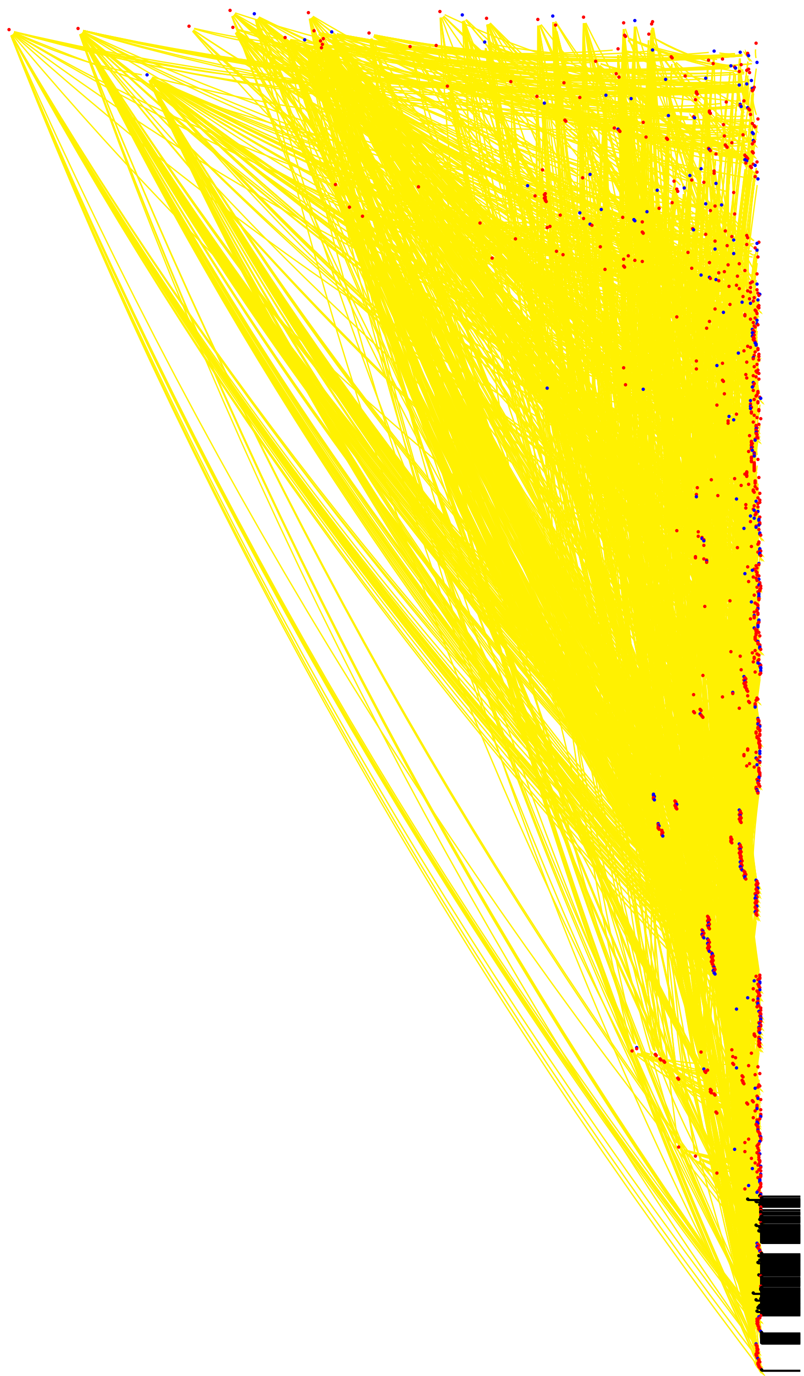

Figure 1.2: Dependency Graph

Figure 1.3: Dependency Graph with just lemmas and primary theorems

Figure 1.4: Dependency Graph for Just Theorem 1

Chapter 2

Presentation with Intermediate Lemmas

Lemma 2.1. $1 \in 2$.

Proof. Trivial. $\square$

Lemma 2.2. $\forall i \in 2.i = 0 \vee i = 1$.

Proof. Trivial. $\square$

Lemma 2.3. $0 \in 3$.

Proof. Trivial. $\square$

Lemma 2.4. $0 \in 4$.

Proof. Trivial. $\square$

Lemma 2.5. $\forall XYz.z \in Y \Rightarrow z \in X \cup Y$.

Proof. Trivial. $\square$

Lemma 2.6. $\forall XYz.z \in X \setminus Y \Rightarrow z \notin Y$.

Proof. Trivial. $\square$

Lemma 2.7. $\forall XYZ : \iota.X \subseteq Y \Rightarrow Y \subseteq Z \Rightarrow X \subseteq Z$.

Proof. Trivial. $\square$

Lemma 2.8. $\forall XY : \iota.Y \subseteq X \cup Y$.

Proof. Use Lemma 2.5. $\square$

Lemma 2.9. $\forall XYZ : \iota.X \subseteq Z \Rightarrow Y \subseteq Z \Rightarrow X \cup Y \subseteq Z$.

Proof. Trivial. □

Lemma 2.10. $\forall XYZ.Y \subseteq Z \Rightarrow X \cup Y \subseteq X \cup Z$.

Proof. Use Lemma 2.8, Lemma 2.7 and Lemma 2.9. □

Lemma 2.11. $\forall XYZ : \iota.X \cup (Y \cup Z) = (X \cup Y) \cup Z$.

Proof. Use Lemma 2.5. □

Lemma 2.12. $\forall XYZ : \iota.Z \subseteq Y \Rightarrow X \setminus Y \subseteq X \setminus Z$.

Proof. Use Lemma 2.6. □

Lemma 2.13. $\forall XYZ.(\forall x \in X.x \notin Y \Rightarrow x \in Z) \Rightarrow X \setminus Y \subseteq Z$.

Proof. Use Lemma 2.6. □

Lemma 2.14. $\forall XYx.\mathsf{disj}\ X\ Y \Rightarrow x \in X \Rightarrow x \notin Y$.

Proof. Trivial. □

Lemma 2.15. $\forall xy.y \neq x \Rightarrow y \notin \{x\}$.

Proof. Trivial. □

Lemma 2.16. $\forall XYz.z \notin X \Rightarrow z \notin X \setminus Y$.

Proof. Trivial. □

Lemma 2.17. $\forall XYz.z \in Y \Rightarrow z \notin X \setminus Y$.

Proof. Use Lemma 2.6. □

Lemma 2.18. $\forall XY.X \in Y \Rightarrow \mathsf{atleast2}\ (\wp\ Y)$.

Proof. Trivial. □

Lemma 2.19. $\forall xy.x \neq y \Rightarrow \mathsf{atleast2}\ (\{x\} \cup \{y\})$.

Proof. Use Lemma 2.5. □

Lemma 2.20. $\forall Xx.x \notin X \Rightarrow \mathsf{atleast2}\ X \Rightarrow \mathsf{atleast3}\ (X \cup \{x\})$.

Proof. Use Lemma 2.5. □

Lemma 2.21. $\forall X.\forall x \in X.\neg\mathsf{atleast5}\ X \Rightarrow \neg\mathsf{atleast4}\ (X \setminus \{x\})$.

Proof. Use Lemma 2.6. □

Lemma 2.22. $\forall XY.Y \subseteq X \Rightarrow X \nsubseteq Y \Rightarrow \exists x \in X.Y \subseteq X \setminus \{x\}$.

Proof. Trivial. □

Lemma 2.23. $\forall XY. X \subseteq Y \Rightarrow \mathsf{atleast4}\ X \Rightarrow \mathsf{atleast4}\ Y.$

Proof. Use Lemma 2.7. ☐

Lemma 2.24. $\forall z. \neg \mathsf{atleast2}\ \{z\}.$

Proof. Trivial. ☐

Lemma 2.25. $\forall XY. \mathsf{disj}\ X\ Y \Rightarrow \mathsf{atleast2}\ Y \Rightarrow \forall y \in Y. X \cup Y \not\subseteq X \cup \{y\}.$

Proof. Use Lemma 2.5, Lemma 2.14, Lemma 2.7 and Lemma 2.24. ☐

Lemma 2.26. $\forall XY. \mathsf{disj}\ X\ Y \Rightarrow \mathsf{atleast2}\ X \Rightarrow \mathsf{atleast2}\ Y \Rightarrow \mathsf{atleast4}\ (X \cup Y).$

Proof. Use Lemma 2.14, Lemma 2.20, Lemma 2.25 and Lemma 2.10. ☐

Lemma 2.27. $\forall XY. \mathsf{disj}\ X\ Y \Rightarrow \mathsf{atleast2}\ X \Rightarrow \mathsf{atleast3}\ Y \Rightarrow \mathsf{atleast5}\ (X \cup Y).$

Proof. Use Lemma 2.14, Lemma 2.26, Lemma 2.5 and Lemma 2.10. ☐

Lemma 2.28. $\forall Xz. X \in \wp\ \{z\} \Rightarrow X = 0 \vee X = \{z\}.$

Proof. Trivial. ☐

Lemma 2.29. $\forall z. \neg \mathsf{atleast3}\ (\wp\ \{z\}).$

Proof. Use Lemma 2.28. ☐

Lemma 2.30. $\forall XYZ. Z \subseteq X \cup Y \Rightarrow \forall w \in X. w \notin Z \Rightarrow \mathsf{atleast3}\ Z \Rightarrow \mathsf{atleast3}\ ((X \setminus \{w\}) \cup Y).$

Proof. Use Lemma 2.5, Lemma 2.15 and Lemma 2.7. ☐

Lemma 2.31. $\forall X. \forall x \in X. \mathsf{atleast4}\ X \Rightarrow \mathsf{atleast3}\ (X \setminus \{x\}).$

Proof. Use Lemma 2.5, Lemma 2.24, Lemma 2.30, Lemma 2.6 and Lemma 2.7. ☐

Lemma 2.32. $\forall X. \forall x \in X. \mathsf{exactly4}\ X \Rightarrow \mathsf{exactly3}\ (X \setminus \{x\}).$

Proof. Use Lemma 2.21 and Lemma 2.31. ☐

Lemma 2.33. $\forall X. \forall x \in X. \mathsf{atleast5}\ X \Rightarrow \mathsf{atleast4}\ (X \setminus \{x\}).$

Proof. Use Lemma 2.5, Lemma 2.24, Lemma 2.15, Lemma 2.23, Lemma 2.6, Lemma 2.30 and Lemma 2.7. ☐

Lemma 2.34. $\forall XYZ. Z \subseteq X \cup Y \Rightarrow \forall w \in X. w \notin Z \Rightarrow \mathsf{atleast5}\ Z \Rightarrow \mathsf{atleast5}\ X \vee \mathsf{atleast2}\ Y.$

Proof. Use Lemma 2.5, Lemma 2.15, Lemma 2.7, Lemma 2.6, Lemma 2.23 and Lemma 2.30. ☐

Lemma 2.35. $\forall X. \forall x \in X. \mathsf{atleast3}\ (X \setminus \{x\}) \Rightarrow \mathsf{atleast4}\ X.$

Proof. Use Lemma 2.6. ☐

Lemma 2.36. $\forall X. \forall x \in X.\mathsf{atleast4}\ (X \setminus \{x\}) \Rightarrow \mathsf{atleast5}\ X$.

Proof. Use Lemma 2.6. ☐

Lemma 2.37. $\forall XY. \forall z \in X \cup Y.\mathsf{atleast4}\ ((X \cup Y) \setminus \{z\}) \Rightarrow \mathsf{atleast3}\ X \vee \mathsf{atleast3}\ Y$.

Proof. Use Lemma 2.36, Lemma 2.5, Lemma 2.15, Lemma 2.23, Lemma 2.6, Lemma 2.30 and Lemma 2.7. ☐

Lemma 2.38. $0 \notin 0$.

Proof. Trivial. ☐

Lemma 2.39. $2 \notin 2$.

Proof. Trivial. ☐

Lemma 2.40. $3 \notin 2$.

Proof. Trivial. ☐

Lemma 2.41. $2 \not\subseteq 1$.

Proof. Use Lemma 2.1. ☐

Lemma 2.42. $4 \not\subseteq 3$.

Proof. Trivial. ☐

Lemma 2.43. $0 \neq 2$.

Proof. Trivial. ☐

Lemma 2.44. $0 \neq 3$.

Proof. Use Lemma 2.3. ☐

Lemma 2.45. $2 \neq 3$.

Proof. Trivial. ☐

Lemma 2.46. $\forall X \subseteq \{1\}.X = 0 \vee X = \{1\}$.

Proof. Trivial. ☐

Lemma 2.47. $\forall X \subseteq \{\{1\}\}.X = 0 \vee X = \{\{1\}\}$.

Proof. Trivial. ☐

Lemma 2.48. $0 \in (\wp\ \{1\})$.

Proof. Trivial. □

Lemma 2.49. $0 \in (\wp\ 2)$.

Proof. Trivial. □

Lemma 2.50. $0 \notin \{2\}$.

Proof. Use Lemma 2.43 and Lemma 2.15. □

Lemma 2.51. $1 \not\subseteq \{1\}$.

Proof. Use Lemma 2.15. □

Lemma 2.52. $1 \notin (\wp\ \{1\})$.

Proof. Use Lemma 2.51. □

Lemma 2.53. $\{1\} \subseteq 2$.

Proof. Use Lemma 2.1. □

Lemma 2.54. $\{1\} \in ((\wp\ 2) \setminus 2)$.

Proof. Use Lemma 2.15, Lemma 2.2 and Lemma 2.53. □

Lemma 2.55. $1 \in ((\wp\ 2) \setminus \{2\})$.

Proof. Use Lemma 2.1 and Lemma 2.15. □

Lemma 2.56. $0 \neq (3 \setminus \{1\})$.

Proof. Use Lemma 2.15, Lemma 2.3 and Lemma 2.38. □

Lemma 2.57. $\{1\} \in (\{1\} \cup \{\{1\}\})$.

Proof. Use Lemma 2.5. □

Lemma 2.58. $0 \notin (\{1\} \cup \{\{1\}\})$.

Proof. Use Lemma 2.15. □

Lemma 2.59. $\{1\} \in ((\wp\ 2) \setminus \{2\})$.

Proof. Use Lemma 2.15 and Lemma 2.53. □

Lemma 2.60. $3 \notin \{1\}$.

Proof. Use Lemma 2.1 and Lemma 2.15. □

Lemma 2.61. $\{1\} \not\subseteq \{2\}$.

Proof. Use Lemma 2.1 and Lemma 2.15. □

Lemma 2.62. $\{1\} \in ((\wp\ 2) \setminus (\wp\ \{2\}))$.

Proof. Use Lemma 2.61 and Lemma 2.53. □

Lemma 2.63. $\{1\} \subseteq ((\wp\ 2) \setminus \{2\})$.

Proof. Use Lemma 2.55. □

Lemma 2.64. $(3 \setminus \{1\}) \notin 2$.

Proof. Use Lemma 2.1 and Lemma 2.15. □

Lemma 2.65. $((\wp\ 2) \setminus 2) \nsubseteq 2$.

Proof. Use Lemma 2.39. □

Lemma 2.66. $2 \notin \{\{1\}\}$.

Proof. Use Lemma 2.15. □

Lemma 2.67. $\{1\} \neq (\{1\} \cup \{\{1\}\})$.

Proof. Use Lemma 2.57. □

Lemma 2.68. $\{1\} \neq (\wp\ 2)$.

Proof. Use Lemma 2.1 and Lemma 2.15. □

Lemma 2.69. $\{\{1\}\} \subseteq ((\wp\ 2) \setminus \{2\})$.

Proof. Use Lemma 2.59. □

Lemma 2.70. $\{1\} \notin ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Lemma 2.17. □

Lemma 2.71. $\forall X.\{1\} \in X \Rightarrow 0 \in X \Rightarrow (\wp\ \{1\}) \subseteq X$.

Proof. Use Lemma 2.46. □

Lemma 2.72. $\forall X \subseteq (\wp\ \{1\}).0 \notin X \Rightarrow X \subseteq \{\{1\}\}$.

Proof. Use Lemma 2.46. □

Lemma 2.73. $\forall X \subseteq (\wp\ \{1\}).\{1\} \in X \Rightarrow 0 \notin X \Rightarrow X = \{\{1\}\}$.

Proof. Use Lemma 2.72. □

Lemma 2.74. $\forall X \subseteq (\wp\ \{1\}).\{1\} \notin X \Rightarrow X \subseteq 1$.

Proof. Use Lemma 2.46. □

Lemma 2.75. $\forall X \subseteq (\wp\ \{1\}).\{1\} \notin X \Rightarrow X = 0 \lor X = 1 \lor X = \{\{1\}\} \lor X = (\wp\ \{1\})$.

Proof. Use Lemma 2.46 and Lemma 2.74. □

Lemma 2.76. $\forall X \subseteq (\wp\ \{1\}).X = 0 \vee X = 1 \vee X = \{\{1\}\} \vee X = (\wp\ \{1\})$.

Proof. Use Lemma 2.75, Lemma 2.73 and Lemma 2.71. □

Lemma 2.77. $(\wp\ 2) \nsubseteq (\wp\ \{1\})$.

Proof. Use Lemma 2.15. □

Lemma 2.78. $3 \notin (\wp\ 2)$.

Proof. Trivial. □

Lemma 2.79. $(\wp\ 2) \notin (\wp\ 2)$.

Proof. Trivial. □

Lemma 2.80. $2 \neq \{2\}$.

Proof. Use Lemma 2.1 and Lemma 2.15. □

Lemma 2.81. $(\wp\ 2) \notin 3$.

Proof. Use Lemma 2.39, Lemma 2.1, Lemma 2.49 and Lemma 2.38. □

Lemma 2.82. $(3 \setminus \{1\}) \subseteq (\wp\ 2)$.

Proof. Use Lemma 2.49 and Lemma 2.6. □

Lemma 2.83. $\forall x \in (\wp\ 2).x = 0 \vee x = 1 \vee x = \{1\} \vee x = 2$.

Proof. Use Lemma 2.2. □

Lemma 2.84. $\forall x \in ((\wp\ 2) \setminus (\wp\ \{1\})).x = 1 \vee x = 2$.

Proof. Use Lemma 2.6, Lemma 2.48 and Lemma 2.83. □

Lemma 2.85. $\mathsf{disj}\ ((\wp\ 2) \setminus 2)\ (\wp\ \{2\})$.

Proof. Use Lemma 2.39, Lemma 2.16, Lemma 2.17 and Lemma 2.28. □

Lemma 2.86. $3 \neq ((\wp\ 2) \setminus 2)$.

Proof. Use Lemma 2.1 and Lemma 2.17. □

Lemma 2.87. $((\wp\ 2) \setminus 2) \subseteq ((\wp\ 2) \setminus (\wp\ \{2\}))$.

Proof. Use Lemma 2.1, Lemma 2.15, Lemma 2.62, Lemma 2.6 and Lemma 2.83. □

Lemma 2.88. $3 \notin ((\wp\ 2) \setminus \{1\})$.

Proof. Use Lemma 2.78 and Lemma 2.16. □

Lemma 2.89. $3 \neq (\wp\ 2)$.

Proof. Use Lemma 2.53 and Lemma 2.15. □

Lemma 2.90. $\{\{1\}\} \in \{\{\{1\}\}\}$.

Proof. Trivial. □

Lemma 2.91. $1 \in (\wp\ (\wp\ \{1\}))$.

Proof. Use Lemma 2.48. □

Lemma 2.92. $\{\{1\}\} \in (\wp\ (\wp\ \{1\}))$.

Proof. Trivial. □

Lemma 2.93. $\{\{1\}\} \in (\{\{1\}\} \cup \{\{\{1\}\}\})$.

Proof. Use Lemma 2.90 and Lemma 2.5. □

Lemma 2.94. $\{\{1\}\} \in (\wp\ (\{1\} \cup \{\{1\}\}))$.

Proof. Use Lemma 2.8. □

Lemma 2.95. $\{\{1\}\} \in ((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\})$.

Proof. Use Lemma 2.15 and Lemma 2.94. □

Lemma 2.96. $1 \in ((\{1\} \cup \{\{1\}\}) \cup (\wp\ \{\{1\}\}))$.

Proof. Trivial. □

Lemma 2.97. $\{\{1\}\} \in ((\{1\} \cup \{\{1\}\}) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Lemma 2.5. □

Lemma 2.98. $2 \in (4 \setminus \{1\})$.

Proof. Use Lemma 2.1 and Lemma 2.15. □

Lemma 2.99. $\{2\} \notin 4$.

Proof. Use Lemma 2.1, Lemma 2.15, Lemma 2.80 and Lemma 2.50. □

Lemma 2.100. $3 \notin ((\wp\ 2) \cup \{\{2\}\})$.

Proof. Use Lemma 2.1, Lemma 2.15 and Lemma 2.78. □

Lemma 2.101. $3 \notin (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Lemma 2.1, Lemma 2.15 and Lemma 2.45. □

Lemma 2.102. $(\wp\ 2) \notin (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Lemma 2.53, Lemma 2.15 and Lemma 2.39. □

Lemma 2.103. $\{2\} \in (3 \cup \{\{2\}\})$.

Proof. Use Lemma 2.5. □

Lemma 2.104. $((\wp\ 2) \setminus 2) \in \{((\wp\ 2) \setminus 2)\}$.

Proof. Trivial. □

Lemma 2.105. $(3 \setminus \{1\}) \in \{(3 \setminus \{1\})\}$.

Proof. Trivial. □

Lemma 2.106. $0 \in (\wp\ (3 \setminus \{1\}))$.

Proof. Trivial. □

Lemma 2.107. $1 \in (\wp\ (3 \setminus \{1\}))$.

Proof. Use Lemma 2.15 and Lemma 2.3. □

Lemma 2.108. $1 \notin \{3\}$.

Proof. Use Lemma 2.1 and Lemma 2.15. □

Lemma 2.109. $2 \notin \{3\}$.

Proof. Use Lemma 2.45 and Lemma 2.15. □

Lemma 2.110. $2 \notin (1 \cup \{3\})$.

Proof. Use Lemma 2.109 and Lemma 2.1. □

Lemma 2.111. $\{2\} \notin (1 \cup \{3\})$.

Proof. Use Lemma 2.1 and Lemma 2.15. □

Lemma 2.112. $3 \in (\{1\} \cup \{3\})$.

Proof. Use Lemma 2.5. □

Lemma 2.113. $1 \in (4 \setminus \{2\})$.

Proof. Use Lemma 2.1 and Lemma 2.15. □

Lemma 2.114. $0 \notin (\{\{1\}\} \cup \{3\})$.

Proof. Use Lemma 2.44 and Lemma 2.15. □

Lemma 2.115. $1 \notin (\{\{1\}\} \cup \{3\})$.

Proof. Use Lemma 2.108 and Lemma 2.15. □

Lemma 2.116. $0 \notin (4 \setminus 2)$.

Proof. Use Lemma 2.17. □

Lemma 2.117. $\{1\} \notin (4 \setminus 2)$.

Proof. Use Lemma 2.1, Lemma 2.15 and Lemma 2.16. □

Lemma 2.118. $2 \in (4 \setminus (\wp\ \{2\}))$.

Proof. Use Lemma 2.1 and Lemma 2.15. □

Lemma 2.119. $\{\{1\}\} \in (\{\{\{1\}\}\} \cup 4)$.

Proof. Use Lemma 2.90. □

Lemma 2.120. $1 \in (\{\{2\}\} \cup 4)$.

Proof. Use Lemma 2.5. □

Lemma 2.121. $\{2\} \in (\{\{2\}\} \cup 4)$.

Proof. Trivial. □

Lemma 2.122. $((\wp\ 2) \setminus 2) \notin (\{\{2\}\} \cup 4)$.

Proof. Use Lemma 2.86, Lemma 2.1, Lemma 2.17, Lemma 2.54 and Lemma 2.15. □

Lemma 2.123. $2 \in (\{\{2\}\} \cup 4)$.

Proof. Use Lemma 2.5. □

Lemma 2.124. $((\wp\ 2) \setminus 2) \in (4 \cup \{((\wp\ 2) \setminus 2)\})$.

Proof. Use Lemma 2.104 and Lemma 2.5. □

Lemma 2.125. $((\wp\ 2) \setminus (\wp\ \{2\})) \in (((\wp\ 2) \setminus \{2\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\})$.

Proof. Use Lemma 2.5. □

Lemma 2.126. $((\wp\ 2) \setminus (\wp\ \{2\})) \in (((\wp\ 2) \setminus \{1\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\})$.

Proof. Use Lemma 2.5. □

Lemma 2.127. $\{2\} \notin \{(\wp\ 2)\}$.

Proof. Use Lemma 2.53 and Lemma 2.15. □

Lemma 2.128. $(\wp\ 2) \in (1 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 2.5. □

Lemma 2.129. $0 \in (3 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 2.3. □

Lemma 2.130. $0 \in (\{\{\{1\}\}\} \cup (1 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.5. □

Lemma 2.131. $\{\{1\}\} \in (\{\{\{1\}\}\} \cup (1 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.90. □

Lemma 2.132. $\{\{1\}\} \in (\{\{\{1\}\}\} \cup (2 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.90. □

Lemma 2.133. $(\wp\ 2) \in (\{\{\{1\}\}\} \cup (2 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.5. □

Lemma 2.134. $\{2\} \in (\{\{2\}\} \cup \{(\wp\ 2)\})$.

Proof. Trivial. □

Lemma 2.135. $0 \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.5. □

Lemma 2.136. $0 \notin (\{3\} \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 2.49, Lemma 2.38, Lemma 2.15 and Lemma 2.44. □

Lemma 2.137. $1 \notin (\{3\} \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 2.1, Lemma 2.15 and Lemma 2.108. □

Lemma 2.138. $2 \in (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.5. □

Lemma 2.139. $\{2\} \notin (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.127, Lemma 2.99, Lemma 2.1 and Lemma 2.15. □

Lemma 2.140. $0 \in (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.4 and Lemma 2.5. □

Lemma 2.141. $1 \in (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.5. □

Lemma 2.142. $2 \in (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.5. □

Lemma 2.143. $\forall X.\{1\} \in X \Rightarrow 1 \in X \Rightarrow (\{1\} \cup \{\{1\}\}) \subseteq X$.

Proof. Trivial. □

Lemma 2.144. $\forall X \subseteq (\{1\} \cup \{\{1\}\}).\{1\} \in X \Rightarrow 1 \notin X \Rightarrow X = \{\{1\}\}$.

Proof. Trivial. □

Lemma 2.145. $\forall x \in ((\wp\ 2) \setminus \{2\}).x = 0 \vee x = 1 \vee x = \{1\}$.

Proof. Use Lemma 2.6 and Lemma 2.83. □

Lemma 2.146. $\forall x \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup \{\{\{1\}\}\}).x = 1 \vee x = \{1\} \vee x = 2 \vee x = \{\{1\}\}$.

Proof. Use Lemma 2.6 and Lemma 2.83. □

Lemma 2.147. $\forall x \in 4.x \neq 1 \Rightarrow x = 0 \vee x = 2 \vee x = 3$.

Proof. Trivial. □

Lemma 2.148. $2 \subseteq ((\wp\ (\wp\ \{1\})) \setminus \{(\wp\ \{1\})\})$.

Proof. Use Lemma 2.15, Lemma 2.91, Lemma 2.48, Lemma 2.38 and Lemma 2.2. □

Lemma 2.149. $(4 \setminus \{2\}) \not\subseteq 2$.

Proof. Use Lemma 2.45, Lemma 2.15 and Lemma 2.40. □

Lemma 2.150. $(2 \cup \{(\wp\ 2)\}) \not\subseteq 2$.

Proof. Use Lemma 2.5. □

Lemma 2.151. $4 \subseteq \{\{1\}\} \cup (\{\{2\}\} \cup 4)$.

Proof. Use Lemma 2.8 and Lemma 2.7. □

Lemma 2.152. $(4 \cup \{((\wp\ 2) \setminus 2)\}) \not\subseteq 4$.

Proof. Use Lemma 2.124, Lemma 2.86, Lemma 2.1, Lemma 2.17 and Lemma 2.54. □

Lemma 2.153. $4 \subseteq \{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 2.8 and Lemma 2.7. □

Lemma 2.154. $(\wp\ \{\{1\}\}) \subseteq (\{\{\{1\}\}\} \cup (1 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.131, Lemma 2.130 and Lemma 2.47. □

Lemma 2.155. $(4 \setminus \{1\}) \not\subseteq (3 \setminus \{1\})$.

Proof. Use Lemma 2.60 and Lemma 2.16. □

Lemma 2.156. $(\wp\ 2) \subseteq ((\wp\ 2) \cup \{(\wp\ 2)\})$.

Proof. Trivial. □

Lemma 2.157. $((\wp\ 2) \cup \{(\wp\ 2)\}) \nsubseteq (\wp\ 2)$.

Proof. Use Lemma 2.5 and Lemma 2.79. □

Lemma 2.158. $((\wp\ 2) \setminus (\wp\ \{1\})) \subseteq (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Lemma 2.1, Lemma 2.15, Lemma 2.50, Lemma 2.84 and Lemma 2.7. □

Lemma 2.159. $(4 \cup \{(\wp\ 2)\}) \subseteq (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.8. □

Lemma 2.160. $(\{\{2\}\} \cup 4) \subseteq (\{\{1\}\} \cup (\{\{2\}\} \cup 4))$.

Proof. Use Lemma 2.8. □

Lemma 2.161. $2 \setminus \{0\} \subseteq \{1\}$.

Proof. Use Lemma 2.6 and Lemma 2.2. □

Lemma 2.162. $\{1\} \subseteq 2 \setminus \{0\}$.

Proof. Use Lemma 2.15 and Lemma 2.1. □

Lemma 2.163. $2 \setminus \{0\} = \{1\}$.

Proof. Use Lemma 2.162 and Lemma 2.161. □

Lemma 2.164. $2 \setminus \{1\} = 1$.

Proof. Use Lemma 2.15, Lemma 2.6 and Lemma 2.2. □

Lemma 2.165. $3 \setminus \{0\} = ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Lemma 2.43, Lemma 2.15, Lemma 2.84, Lemma 2.6 and Lemma 2.52. □

Lemma 2.166. $\forall x \in ((\wp\ 2) \setminus \{2\}).x \neq 0 \Rightarrow x \in (\{1\} \cup \{\{1\}\})$.

Proof. Use Lemma 2.57 and Lemma 2.145. □

Lemma 2.167. $((\wp\ 2) \setminus \{2\}) \setminus \{1\} \subseteq (\wp\ \{1\})$.

Proof. Use Lemma 2.6, Lemma 2.48 and Lemma 2.145. □

Lemma 2.168. $(3 \setminus \{1\}) \setminus \{0\} \subseteq \{2\}$.

Proof. Use Lemma 2.6. □

Lemma 2.169. $(3 \setminus \{1\}) \setminus \{0\} = \{2\}$.

Proof. Use Lemma 2.43, Lemma 2.15, Lemma 2.1 and Lemma 2.168. □

Lemma 2.170. $((\wp\ 2) \setminus (\wp\ \{1\})) \setminus \{2\} = \{1\}$.

Proof. Use Lemma 2.1, Lemma 2.15, Lemma 2.52, Lemma 2.6 and Lemma 2.84. □

Lemma 2.171. $\forall x \in ((\wp\ 2) \setminus \{1\}).x \neq 0 \Rightarrow x \in ((\wp\ 2) \setminus 2)$.

Proof. Use Lemma 2.39, Lemma 2.54, Lemma 2.6 and Lemma 2.83. □

Lemma 2.172. $((\wp\ 2) \setminus \{1\}) \setminus \{\{1\}\} \subseteq (3 \setminus \{1\})$.

Proof. Use Lemma 2.6, Lemma 2.1, Lemma 2.15, Lemma 2.3 and Lemma 2.83. □

Lemma 2.173. $\forall x \in ((\wp\ 2) \setminus (\wp\ \{2\})).x \neq \{1\} \Rightarrow x \in ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Lemma 2.15, Lemma 2.52, Lemma 2.6 and Lemma 2.83. □

Lemma 2.174. $\forall x \in ((\wp\ 2) \setminus (\wp\ \{2\})).x \neq 2 \Rightarrow x \in (\{1\} \cup \{\{1\}\})$.

Proof. Use Lemma 2.57, Lemma 2.6 and Lemma 2.83. □

Lemma 2.175. $((\wp\ 2) \setminus (\wp\ \{2\})) \setminus \{2\} \subseteq (\{1\} \cup \{\{1\}\})$.

Proof. Use Lemma 2.6 and Lemma 2.174. □

Lemma 2.176. $((\wp\ 2) \setminus (\wp\ \{2\})) \setminus \{2\} = (\{1\} \cup \{\{1\}\})$.

Proof. Use Lemma 2.15, Lemma 2.62, Lemma 2.1, Lemma 2.50 and Lemma 2.175. □

Lemma 2.177. $(\wp\ 2) \setminus \{\{1\}\} \subseteq 3$.

Proof. Use Lemma 2.6, Lemma 2.3 and Lemma 2.83. □

Lemma 2.178. $3 \subseteq (\wp\ 2) \setminus \{\{1\}\}$.

Proof. Use Lemma 2.15 and Lemma 2.49. □

Lemma 2.179. $\forall x \in ((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}).x \neq 0 \Rightarrow x \in (\{\{1\}\} \cup \{\{\{1\}\}\})$.

Proof. Use Lemma 2.93, Lemma 2.6, Lemma 2.144 and Lemma 2.143. □

Lemma 2.180. $((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}) \setminus \{\{1\}\} = (\wp\ \{\{1\}\})$.

Proof. Use Lemma 2.15, Lemma 2.95, Lemma 2.47, Lemma 2.6, Lemma 2.144 and Lemma 2.143. □

Lemma 2.181. $(((\wp\ 2) \setminus (\wp\ \{2\})) \cup \{\{\{1\}\}\}) \subseteq (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{0\}$.

Proof. Use Lemma 2.5, Lemma 2.43, Lemma 2.1, Lemma 2.15, Lemma 2.62, Lemma 2.50 and Lemma 2.146. □

Lemma 2.182. $\forall x \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})).x \neq 1 \Rightarrow x \in (((\wp\ 2) \setminus 2) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Lemma 2.5, Lemma 2.39, Lemma 2.54, Lemma 2.47, Lemma 2.6 and Lemma 2.83. □

Lemma 2.183. $(((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{1\} \subseteq (((\wp\ 2) \setminus 2) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Lemma 2.6 and Lemma 2.182. □

Lemma 2.184. $(((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{\{1\}\} = (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Lemma 2.15, Lemma 2.5, Lemma 2.1, Lemma 2.50, Lemma 2.47, Lemma 2.84, Lemma 2.6, Lemma 2.52 and Lemma 2.83. □

Lemma 2.185. $\forall x \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})).x \neq \{\{1\}\} \Rightarrow x \in (\wp\ 2)$.

Proof. Use Lemma 2.53, Lemma 2.49, Lemma 2.47, Lemma 2.6 and Lemma 2.83. □

Lemma 2.186. $(1 \cup \{3\}) \subseteq (4 \setminus \{2\}) \setminus \{1\}$.

Proof. Use Lemma 2.1, Lemma 2.45, Lemma 2.15, Lemma 2.50 and Lemma 2.4. □

Lemma 2.187. $(4 \setminus \{2\}) \setminus \{3\} \subseteq 2$.

Proof. Use Lemma 2.6 and Lemma 2.1. □

Lemma 2.188. $(4 \setminus 2) \setminus \{2\} \subseteq \{3\}$.

Proof. Use Lemma 2.6 and Lemma 2.1. □

Lemma 2.189. $\forall x \in (4 \setminus 2).x \neq 3 \Rightarrow x \in \{2\}$.

Proof. Use Lemma 2.6 and Lemma 2.1. □

Lemma 2.190. $(4 \setminus 2) \setminus \{3\} = \{2\}$.

Proof. Use Lemma 2.45, Lemma 2.15, Lemma 2.39, Lemma 2.6 and Lemma 2.189. □

Lemma 2.191. $\forall x \in (4 \setminus \{1\}).x \neq 0 \Rightarrow x \in (4 \setminus 2)$.

Proof. Use Lemma 2.40, Lemma 2.39, Lemma 2.6 and Lemma 2.147. □

Lemma 2.192. $\forall x \in (4 \setminus \{1\}).x \neq 2 \Rightarrow x \in (1 \cup \{3\})$.

Proof. Use Lemma 2.5, Lemma 2.6 and Lemma 2.147. □

Lemma 2.193. $(4 \setminus \{1\}) \setminus \{3\} = (3 \setminus \{1\})$.

Proof. Use Lemma 2.45, Lemma 2.98, Lemma 2.44, Lemma 2.15, Lemma 2.4, Lemma 2.6, Lemma 2.1, Lemma 2.3 and Lemma 2.147. □

Lemma 2.194. $(4 \setminus (\wp\ \{2\})) \setminus \{1\} \subseteq (4 \setminus 2)$.

Proof. Use Lemma 2.6, Lemma 2.40 and Lemma 2.39. □

Lemma 2.195. $(4 \setminus 2) \subseteq (4 \setminus (\wp \{2\})) \setminus \{1\}$.

Proof. Use Lemma 2.1, Lemma 2.15, Lemma 2.118 and Lemma 2.6. □

Lemma 2.196. $4 \setminus \{0\} \subseteq (4 \setminus (\wp \{2\}))$.

Proof. Use Lemma 2.6, Lemma 2.1, Lemma 2.15, Lemma 2.118 and Lemma 2.50. □

Lemma 2.197. $\forall x \in 4.x \neq 3 \Rightarrow x \in 3$.

Proof. Use Lemma 2.3. □

Lemma 2.198. $3 \subseteq 4 \setminus \{3\}$.

Proof. Use Lemma 2.45, Lemma 2.1, Lemma 2.44, Lemma 2.15 and Lemma 2.4. □

Lemma 2.199. $4 \setminus \{3\} = 3$.

Proof. Use Lemma 2.198, Lemma 2.6 and Lemma 2.197. □

Lemma 2.200. $\{2\} \cup \wp \{2\} \subseteq (\{2\} \cup (\wp \{2\})) \cup \{(3 \setminus \{1\})\}$.

Proof. Trivial. □

Lemma 2.201. $2 \subseteq 3 \cup \{(3 \setminus \{1\})\}$.

Proof. Use Lemma 2.3, Lemma 2.2 and Lemma 2.7. □

Lemma 2.202. $2 \subseteq \wp (3 \setminus \{1\})$.

Proof. Use Lemma 2.107, Lemma 2.106 and Lemma 2.2. □

Lemma 2.203. $\{1\} \cup \wp \{2\} \subseteq \{\{2\}\} \cup 4$.

Proof. Use Lemma 2.4, Lemma 2.5, Lemma 2.28, Lemma 2.120 and Lemma 2.9. □

Lemma 2.204. $(3 \setminus \{1\}) \subseteq 4$.

Proof. Use Lemma 2.4 and Lemma 2.6. □

Lemma 2.205. $(3 \setminus \{1\}) \subseteq (\{\{1\}\} \cup 4)$.

Proof. Use Lemma 2.8, Lemma 2.204 and Lemma 2.7. □

Lemma 2.206. $((\wp 2) \setminus (\wp \{1\})) \subseteq (4 \cup \{(\wp 2)\})$.

Proof. Use Lemma 2.84 and Lemma 2.7. □

Lemma 2.207. $\forall u \in 3.u \in \wp (3 \setminus \{1\}) \Rightarrow u \in 2$.

Proof. Use Lemma 2.1 and Lemma 2.17. □

Lemma 2.208. $\forall x \in 4.x \in 3 \vee x = 3$.

Proof. Use Lemma 2.3. ☐

Lemma 2.209. $\forall x \in (\wp\ 2) \cup \{\{2\}\}.x \in (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \Rightarrow x \notin \{\wp\ 2\}$.

Proof. Use Lemma 2.53, Lemma 2.15 and Lemma 2.79. ☐

Lemma 2.210. $((\wp\ 2) \cup \{\{2\}\}) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})) = (3 \cup \{\{2\}\})$.

Proof. Use Lemma 2.153, Lemma 2.4, Lemma 2.7, Lemma 2.9, Lemma 2.8, Lemma 2.49, Lemma 2.5, Lemma 2.68, Lemma 2.15, Lemma 2.1, Lemma 2.3 and Lemma 2.83. ☐

Lemma 2.211. $((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\}) \cap (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) = (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Lemma 2.11, Lemma 2.200, Lemma 2.53, Lemma 2.15, Lemma 2.16, Lemma 2.3, Lemma 2.50, Lemma 2.1 and Lemma 2.17. ☐

Lemma 2.212. $((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\}) \cap (((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) = (\{2\} \cup (\wp$

Proof. Use Lemma 2.8, Lemma 2.7, Lemma 2.1, Lemma 2.15, Lemma 2.9, Lemma 2.200, Lemma 2.53, Lemma 2.16, Lemma 2.3, Lemma 2.50 and Lemma 2.39. ☐

Lemma 2.213. $\forall x \in \{\{1\}\} \cup (\{\{2\}\} \cup 4).x \in \{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}) \Rightarrow x \in \{2\} \cup (\wp\ \{2\})$.

Proof. Use Lemma 2.11, Lemma 2.89, Lemma 2.39, Lemma 2.1, Lemma 2.49, Lemma 2.38, Lemma 2.53, Lemma 2.15 and Lemma 2.68. ☐

Lemma 2.214. $(3 \setminus \{1\}) \subseteq ((\wp\ 2) \cup \{\{2\}\}) \cap (4 \setminus \{1\})$.

Proof. Use Lemma 2.98, Lemma 2.15, Lemma 2.4, Lemma 2.6, Lemma 2.82 and Lemma 2.7. ☐

Lemma 2.215. $(\{\{1\}\} \cup 4) \cap (\{2\} \cup (\wp\ \{2\})) = (3 \setminus \{1\})$.

Proof. Use Lemma 2.5, Lemma 2.6, Lemma 2.205, Lemma 2.50, Lemma 2.1, Lemma 2.15, Lemma 2.101, Lemma 2.208 and Lemma 2.61. ☐

Lemma 2.216. $((\wp\ 2) \cup \{(\wp\ 2)\}) \cap (4 \setminus \{1\}) = (3 \setminus \{1\})$.

Proof. Use Lemma 2.98, Lemma 2.15, Lemma 2.4, Lemma 2.6, Lemma 2.82, Lemma 2.7, Lemma 2.89, Lemma 2.78 and Lemma 2.208. ☐

Lemma 2.217. $\neg\mathsf{atleast2}\ 1$.

Proof. Trivial. ☐

Lemma 2.218. $\neg\mathsf{atleast3}\ (\wp\ \{2\})$.

Proof. Use Lemma 2.29. ☐

Lemma 2.219. $\neg\mathsf{atleast3}\ (1 \cup \{3\})$.

Proof. Use Lemma 2.24 and Lemma 2.7. □

Lemma 2.220. $\neg\mathsf{atleast4}\ (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Lemma 2.218, Lemma 2.24, Lemma 2.30, Lemma 2.6, Lemma 2.7 and Lemma 2.5. □

Lemma 2.221. $\neg\mathsf{atleast4}\ (2 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.2, Lemma 2.30 and Lemma 2.6. □

Lemma 2.222. $\neg\mathsf{atleast5}\ (((\wp\ 2) \setminus (\wp\ \{2\})) \cup \{\{\{1\}\}\})$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.176, Lemma 2.24, Lemma 2.15, Lemma 2.1, Lemma 2.50, Lemma 2.84, Lemma 2.6, Lemma 2.173, Lemma 2.170, Lemma 2.62, Lemma 2.83, Lemma 2.39, Lemma 2.54, Lemma 2.5, Lemma 2.23 and Lemma 2.30. □

Lemma 2.223. $\neg\mathsf{atleast5}\ (((\wp\ 2) \setminus \{1\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\})$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.15, Lemma 2.53, Lemma 2.43, Lemma 2.49, Lemma 2.46, Lemma 2.6, Lemma 2.48, Lemma 2.83, Lemma 2.29, Lemma 2.1, Lemma 2.172, Lemma 2.3, Lemma 2.217, Lemma 2.169, Lemma 2.24, Lemma 2.171, Lemma 2.54, Lemma 2.39, Lemma 2.5, Lemma 2.23 and Lemma 2.30. □

Lemma 2.224. $\forall x \in \{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}).\neg\mathsf{atleast4}\ ((\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \setminus \{x\})$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.24, Lemma 2.218, Lemma 2.35, Lemma 2.30, Lemma 2.6, Lemma 2.36, Lemma 2.5, Lemma 2.15 and Lemma 2.23. □

Lemma 2.225. $\neg\mathsf{atleast5}\ (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.22, Lemma 2.23 and Lemma 2.224. □

Lemma 2.226. $\forall x.\mathsf{atleast5}\ ((((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{x\}) \Rightarrow x \neq 2$.

Proof. Use Lemma 2.1, Lemma 2.15, Lemma 2.5, Lemma 2.62, Lemma 2.50, Lemma 2.43, Lemma 2.47, Lemma 2.6, Lemma 2.97, Lemma 2.57, Lemma 2.96, Lemma 2.83, Lemma 2.29, Lemma 2.24, Lemma 2.7, Lemma 2.23 and Lemma 2.30. □

Lemma 2.227. $\neg\mathsf{atleast6}\ (\{\{\{1\}\}\} \cup 4)$.

Proof. Use Lemma 2.22, Lemma 2.23, Lemma 2.199, Lemma 2.7, Lemma 2.1, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.2, Lemma 2.6, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.113, Lemma 2.44, Lemma 2.50, Lemma 2.4, Lemma 2.187, Lemma 2.186, Lemma 2.5, Lemma 2.219, Lemma 2.45, Lemma 2.112, Lemma 2.193, Lemma 2.60, Lemma 2.192, Lemma 2.98, Lemma 2.191, Lemma 2.190, Lemma 2.40, Lemma 2.188, Lemma 2.196, Lemma 2.118, Lemma 2.52, Lemma 2.195, Lemma 2.194 and Lemma 2.34. □

Lemma 2.228. $\neg\mathsf{atleast6}\ ((\wp\ 2) \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 2.22, Lemma 2.23, Lemma 2.7, Lemma 2.15, Lemma 2.55, Lemma 2.50, Lemma 2.49, Lemma 2.2, Lemma 2.6, Lemma 2.1, Lemma 2.145, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.59, Lemma 2.46, Lemma 2.167, Lemma 2.29, Lemma 2.166, Lemma 2.178, Lemma 2.177, Lemma 2.43, Lemma 2.3, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.53, Lemma 2.48, Lemma 2.83, Lemma 2.172, Lemma 2.171, Lemma 2.54, Lemma 2.39, Lemma 2.62, Lemma 2.176, Lemma 2.173 and Lemma 2.34. □

Lemma 2.229. $\mathsf{atleast2}\ (3 \setminus \{1\})$.

Proof. Use Lemma 2.1, Lemma 2.15 and Lemma 2.3. □

Lemma 2.230. $\mathsf{atleast2}\ (4 \setminus 2)$.

Proof. Use Lemma 2.40 and Lemma 2.39. □

Lemma 2.231. $\mathsf{atleast3}\ (2 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 2.1 and Lemma 2.150. □

Lemma 2.232. $\mathsf{atleast4}\ (\wp\ 2)$.

Proof. Use Lemma 2.15, Lemma 2.39, Lemma 2.54, Lemma 2.50, Lemma 2.1, Lemma 2.17, Lemma 2.87 and Lemma 2.49. □

Lemma 2.233. $\mathsf{atleast4}\ (3 \cup \{((\wp\ 2) \setminus 2)\})$.

Proof. Use Lemma 2.1, Lemma 2.3, Lemma 2.2, Lemma 2.104, Lemma 2.5, Lemma 2.17 and Lemma 2.54. □

Lemma 2.234. $\mathsf{atleast5}\ (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Lemma 2.1, Lemma 2.3, Lemma 2.2, Lemma 2.5, Lemma 2.15, Lemma 2.52, Lemma 2.62, Lemma 2.70, Lemma 2.8, Lemma 2.158 and Lemma 2.9. □

Lemma 2.235. $\mathsf{atleast5}\ ((\wp\ 2) \cup \{\{2\}\})$.

Proof. Use Lemma 2.232, Lemma 2.5 and Lemma 2.39. □

Lemma 2.236. $\mathsf{atleast5}\ (\{\{1\}\} \cup 4)$.

Proof. Use Lemma 2.1, Lemma 2.3, Lemma 2.2, Lemma 2.42, Lemma 2.4, Lemma 2.15 and Lemma 2.8. □

Lemma 2.237. $\mathsf{atleast5}\ (\{\{2\}\} \cup 4)$.

Proof. Use Lemma 2.1, Lemma 2.3, Lemma 2.2, Lemma 2.42, Lemma 2.4, Lemma 2.121, Lemma 2.99 and Lemma 2.8. □

Lemma 2.238. atleast6 $(\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.1, Lemma 2.3, Lemma 2.2, Lemma 2.42, Lemma 2.4, Lemma 2.5, Lemma 2.89, Lemma 2.39, Lemma 2.49, Lemma 2.38, Lemma 2.127, Lemma 2.99 and Lemma 2.8. □

Lemma 2.239. atleast5 $((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})))$.

Proof. Use Lemma 2.1, Lemma 2.3, Lemma 2.2, Lemma 2.103, Lemma 2.80, Lemma 2.50, Lemma 2.5, Lemma 2.53, Lemma 2.15, Lemma 2.81, Lemma 2.153, Lemma 2.4, Lemma 2.7, Lemma 2.9 and Lemma 2.135. □

Theorem 2.1. exactly2 2.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.2 and Lemma 2.1. □

Theorem 2.2. exactly2 $(\{1\} \cup \{\{1\}\})$.

Proof. Use Lemma 2.15, Lemma 2.24, Lemma 2.7 and Lemma 2.19. □

Theorem 2.3. exactly2 $(3 \setminus \{1\})$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.6, Lemma 2.217, Lemma 2.169, Lemma 2.24 and Lemma 2.229. □

Theorem 2.4. exactly2 $(((\wp\ 2) \cup \{\{2\}\}) \cap (4 \setminus \{1\}))$.

Proof. Use Theorem 2.3, Lemma 2.214, Lemma 2.100, Lemma 2.6 and Lemma 2.208. □

Theorem 2.5. exactly2 $((\{\{2\}\} \cup 4) \cap ((\wp\ 2) \setminus \{1\}))$.

Proof. Use Theorem 2.3, Lemma 2.1, Lemma 2.15, Lemma 2.49, Lemma 2.6, Lemma 2.8, Lemma 2.204, Lemma 2.7, Lemma 2.17, Lemma 2.88, Lemma 2.208, Lemma 2.39 and Lemma 2.16. □

Theorem 2.6. exactly2 $(((\wp\ 2) \cup \{(\wp\ 2)\}) \cap (4 \setminus \{1\}))$.

Proof. Use Theorem 2.3 and Lemma 2.216. □

Theorem 2.7. exactly2 $((4 \cup \{(\wp\ 2)\}) \cap (\{2\} \cup (\wp\ \{2\})))$.

Proof. Use Theorem 2.3, Lemma 2.5, Lemma 2.6, Lemma 2.4, Lemma 2.7, Lemma 2.50, Lemma 2.1, Lemma 2.15, Lemma 2.102, Lemma 2.101 and Lemma 2.208. □

Theorem 2.8. exactly2 $((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.170, Lemma 2.24, Lemma 2.1, Lemma 2.15, Lemma 2.6, Lemma 2.84, Lemma 2.41 and Lemma 2.52. □

Theorem 2.9. $\mathsf{exactly2}\ ((\wp\ 2) \setminus 2)$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.15, Lemma 2.54, Lemma 2.6, Lemma 2.1, Lemma 2.83, Lemma 2.24 and Lemma 2.39. □

Theorem 2.10. $\mathsf{exactly2}\ (1 \cup \{3\})$.

Proof. Use Lemma 2.44, Lemma 2.24, Lemma 2.7 and Lemma 2.19. □

Theorem 2.11. $\mathsf{exactly2}\ (4 \setminus 2)$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.6, Lemma 2.190, Lemma 2.24, Lemma 2.45, Lemma 2.15, Lemma 2.40, Lemma 2.188, Lemma 2.1 and Lemma 2.230. □

Theorem 2.12. $\mathsf{exactly2}\ ((\{\{1\}\} \cup 4) \cap (\{2\} \cup (\wp\ \{2\})))$.

Proof. Use Theorem 2.3 and Lemma 2.215. □

Theorem 2.13. $\mathsf{exactly2}\ (1 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 2.49, Lemma 2.38, Lemma 2.24, Lemma 2.7 and Lemma 2.19. □

Theorem 2.14. $\mathsf{exactly2}\ (\{((\wp\ 2) \setminus (\wp\ \{2\}))\} \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 2.49, Lemma 2.17, Lemma 2.24, Lemma 2.7 and Lemma 2.19. □

Theorem 2.15. $\mathsf{exactly2}\ ((4 \cup \{(\wp\ 2)\}) \cap ((\wp\ 2) \setminus (\wp\ \{2\})))$.

Proof. Use Theorem 2.8, Lemma 2.1, Lemma 2.15, Lemma 2.50, Lemma 2.84, Lemma 2.206, Lemma 2.68, Lemma 2.17 and Lemma 2.28. □

Theorem 2.16. $\mathsf{exactly3}\ ((\wp\ 2) \setminus \{2\})$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.15, Lemma 2.55, Lemma 2.50, Lemma 2.49, Lemma 2.2, Lemma 2.6, Lemma 2.1, Lemma 2.145, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.59, Lemma 2.46, Lemma 2.167, Lemma 2.29, Lemma 2.166, Lemma 2.19, Lemma 2.58, Lemma 2.69, Lemma 2.63 and Lemma 2.9. □

Theorem 2.17. $\mathsf{exactly3}\ 3$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.1, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.2, Lemma 2.6, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.169, Lemma 2.165, Lemma 2.170 and Lemma 2.84. □

Theorem 2.18. $\mathsf{exactly2}\ (3 \setminus \{2\})$.

Proof. Use Theorem 2.17, Lemma 2.6, Lemma 2.5, Lemma 2.24 and Lemma 2.7. □

Theorem 2.19. $\mathsf{exactly3}\ ((\wp\ 2) \setminus \{1\})$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.15, Lemma 2.53, Lemma 2.43, Lemma 2.49, Lemma 2.46, Lemma 2.6, Lemma 2.48, Lemma 2.83, Lemma 2.29, Lemma 2.1, Lemma 2.172, Lemma 2.3, Lemma 2.217, Lemma 2.169, Lemma 2.24, Lemma 2.171, Lemma 2.54, Lemma 2.39, Lemma 2.17 and Lemma 2.12. □

Theorem 2.20. exactly2 $(((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}) \setminus \{0\})$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.15, Lemma 2.67, Lemma 2.46, Lemma 2.6, Lemma 2.48, Lemma 2.144, Lemma 2.143, Lemma 2.29, Lemma 2.180, Lemma 2.95, Lemma 2.179, Lemma 2.24, Lemma 2.18, Lemma 2.47 and Lemma 2.5. □

Theorem 2.21. exactly3 $(\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Lemma 2.220, Lemma 2.229, Lemma 2.5, Lemma 2.80, Lemma 2.50, Lemma 2.16 and Lemma 2.6. □

Theorem 2.22. exactly3 $(((\wp\ 2) \cup \{\{2\}\}) \cap ((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\}))$.

Proof. Use Theorem 2.21, Lemma 2.200, Lemma 2.5, Lemma 2.28, Lemma 2.15, Lemma 2.3, Lemma 2.50, Lemma 2.1 and Lemma 2.39. □

Theorem 2.23. exactly3 $(4 \setminus \{2\})$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.6, Lemma 2.1, Lemma 2.113, Lemma 2.44, Lemma 2.15, Lemma 2.50, Lemma 2.4, Lemma 2.2, Lemma 2.187, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.186, Lemma 2.5, Lemma 2.219, Lemma 2.45, Lemma 2.112 and Lemma 2.149. □

Theorem 2.24. exactly2 $((4 \setminus \{2\}) \setminus \{1\})$.

Proof. Use Theorem 2.23, Lemma 2.113, Lemma 2.6, Lemma 2.5, Lemma 2.24 and Lemma 2.7. □

Theorem 2.25. exactly2 $((4 \setminus \{2\}) \setminus \{3\})$.

Proof. Use Theorem 2.23, Lemma 2.45, Lemma 2.15, Lemma 2.6, Lemma 2.5, Lemma 2.24 and Lemma 2.7. □

Theorem 2.26. exactly3 $(4 \setminus \{1\})$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.6, Lemma 2.193, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.217, Lemma 2.169, Lemma 2.24, Lemma 2.45, Lemma 2.60, Lemma 2.4, Lemma 2.192, Lemma 2.219, Lemma 2.44, Lemma 2.98, Lemma 2.1, Lemma 2.191, Lemma 2.190, Lemma 2.40, Lemma 2.188, Lemma 2.229 and Lemma 2.155. □

Theorem 2.27. exactly2 $((4 \setminus \{1\}) \setminus \{0\})$.

Proof. Use Theorem 2.26, Lemma 2.15, Lemma 2.4, Lemma 2.6, Lemma 2.5, Lemma 2.24 and Lemma 2.7. □

Theorem 2.28. exactly2 $((4 \setminus \{1\}) \setminus \{2\})$.

Proof. Use Theorem 2.26, Lemma 2.98, Lemma 2.6, Lemma 2.5, Lemma 2.24 and Lemma 2.7. □

Theorem 2.29. exactly2 $((4 \setminus \{1\}) \setminus \{3\})$.

Proof. Use Theorem 2.26, Lemma 2.60, Lemma 2.6, Lemma 2.5, Lemma 2.24 and Lemma 2.7. □

Theorem 2.30. exactly3 $(2 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 2.221 and Lemma 2.231. □

Theorem 2.31. exactly3 $(\{\{\{1\}\}\} \cup (1 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.24, Lemma 2.7, Lemma 2.30, Lemma 2.6, Lemma 2.5, Lemma 2.18, Lemma 2.128, Lemma 2.66 and Lemma 2.154. □

Theorem 2.32. exactly3 $(((\wp\ 2) \cup \{\{2\}\}) \cap (4 \cup \{((\wp\ 2) \setminus 2)\}))$.

Proof. Use Theorem 2.17, Lemma 2.4, Lemma 2.7, Lemma 2.49, Lemma 2.54, Lemma 2.15, Lemma 2.65, Lemma 2.100 and Lemma 2.208. □

Theorem 2.33. exactly3 $(((\wp\ 2) \cup \{\{2\}\}) \cap (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})))$.

Proof. Use Theorem 2.21, Lemma 2.11, Lemma 2.5, Lemma 2.28 and Lemma 2.209. □

Theorem 2.34. exactly3 $(((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\}) \cap (\{\{1\}\} \cup (\{\{2\}\} \cup 4)))$.

Proof. Use Theorem 2.21, Lemma 2.8, Lemma 2.4, Lemma 2.5, Lemma 2.28, Lemma 2.9, Lemma 2.7, Lemma 2.200, Lemma 2.17, Lemma 2.1, Lemma 2.15, Lemma 2.56, Lemma 2.3 and Lemma 2.50. □

Theorem 2.35. exactly3 $(((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\}) \cap (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})))$.

Proof. Use Theorem 2.21 and Lemma 2.211. □

Theorem 2.36. exactly3 $(((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\}) \cap (((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})))$.

Proof. Use Theorem 2.21 and Lemma 2.212. □

Lemma 2.240. $\forall u \in 3.u \in \wp\ (3 \setminus \{1\}) \Rightarrow u \notin \{0\} \Rightarrow u \in \{1\} \cup \{3 \setminus \{1\}\}$.

Proof. Use Lemma 2.1 and Lemma 2.17. □

Lemma 2.241. $\forall u \in 3.u \in \wp\ (3 \setminus \{1\}) \Rightarrow u \notin \{1\} \Rightarrow u \in \{0\} \cup \{3 \setminus \{1\}\}$.

Proof. Use Lemma 2.1 and Lemma 2.17. □

Lemma 2.242. $\neg\mathsf{atleast3}\ (((3 \cup \{(3 \setminus \{1\})\}) \cap (\wp\ (3 \setminus \{1\}))) \setminus \{3 \setminus \{1\}\})$.

Proof. Use Lemma 2.207, Lemma 2.13, Lemma 2.7, Lemma 2.22, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24 and Lemma 2.2. □

Theorem 2.37. $\mathsf{exactly3}\ ((3 \cup \{(3 \setminus \{1\})\}) \cap (\wp\ (3 \setminus \{1\})))$.

Proof. Use Lemma 2.242, Lemma 2.1, Lemma 2.17, Lemma 2.5, Lemma 2.241, Lemma 2.13, Lemma 2.7, Lemma 2.24, Lemma 2.240, Lemma 2.22, Lemma 2.105, Lemma 2.64, Lemma 2.202 and Lemma 2.201. □

Theorem 2.38. $\mathsf{exactly3}\ ((\{\{1\}\} \cup (\{\{2\}\} \cup 4)) \cap (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})))$.

Proof. Use Theorem 2.21, Lemma 2.11, Lemma 2.8, Lemma 2.4, Lemma 2.5, Lemma 2.28, Lemma 2.9, Lemma 2.7 and Lemma 2.213. □

Theorem 2.39. $\mathsf{exactly4}\ (\wp\ 2)$.

Proof. Use Lemma 2.22, Lemma 2.23, Lemma 2.7, Lemma 2.15, Lemma 2.55, Lemma 2.50, Lemma 2.49, Lemma 2.2, Lemma 2.6, Lemma 2.1, Lemma 2.145, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.59, Lemma 2.46, Lemma 2.167, Lemma 2.29, Lemma 2.166, Lemma 2.178, Lemma 2.177, Lemma 2.43, Lemma 2.3, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.53, Lemma 2.48, Lemma 2.83, Lemma 2.172, Lemma 2.171, Lemma 2.54, Lemma 2.39, Lemma 2.62, Lemma 2.176, Lemma 2.173 and Lemma 2.232. □

Theorem 2.40. $\mathsf{exactly3}\ ((\wp\ 2) \setminus \{0\})$.

Proof. Use Theorem 2.39, Lemma 2.49 and Lemma 2.32. □

Theorem 2.41. $\mathsf{exactly3}\ ((\wp\ 2) \setminus \{\{1\}\})$.

Proof. Use Theorem 2.39, Lemma 2.53 and Lemma 2.32. □

Theorem 2.42. $\mathsf{exactly4}\ (3 \cup \{(\wp\ \{1\})\})$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.1, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.2, Lemma 2.6, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.5, Lemma 2.23, Lemma 2.30, Lemma 2.52, Lemma 2.48 and Lemma 2.38. □

Theorem 2.43. $\mathsf{exactly4}\ (3 \cup \{\{2\}\})$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.1, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.2, Lemma 2.6, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.5, Lemma 2.23, Lemma 2.30, Lemma 2.103, Lemma 2.80 and Lemma 2.50. □

Theorem 2.44. $\mathsf{exactly3}\ ((3 \cup \{\{2\}\}) \setminus \{2\})$.

Proof. Use Theorem 2.43 and Lemma 2.32. ∎

Theorem 2.45. $\mathsf{exactly4}\ 4$.

Proof. Use Lemma 2.22, Lemma 2.23, Lemma 2.199, Lemma 2.7, Lemma 2.1, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.2, Lemma 2.6, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.113, Lemma 2.44, Lemma 2.50, Lemma 2.4, Lemma 2.187, Lemma 2.186, Lemma 2.5, Lemma 2.219, Lemma 2.45, Lemma 2.112, Lemma 2.193, Lemma 2.60, Lemma 2.192, Lemma 2.98, Lemma 2.191, Lemma 2.190, Lemma 2.40, Lemma 2.188, Lemma 2.196, Lemma 2.118, Lemma 2.52, Lemma 2.195, Lemma 2.194 and Lemma 2.42. ∎

Theorem 2.46. $\mathsf{exactly3}\ (4 \setminus \{0\})$.

Proof. Use Theorem 2.45, Lemma 2.4 and Lemma 2.32. ∎

Theorem 2.47. $\mathsf{exactly3}\ (4 \setminus \{3\})$.

Proof. Use Theorem 2.45 and Lemma 2.32. ∎

Theorem 2.48. $\mathsf{exactly4}\ ((\{2\} \cup \{\{2\}\}) \cup (1 \cup \{3\}))$.

Proof. Use Theorem 2.10, Lemma 2.80, Lemma 2.24, Lemma 2.7, Lemma 2.19, Lemma 2.111, Lemma 2.110, Lemma 2.5, Lemma 2.15, Lemma 2.23, Lemma 2.6, Lemma 2.30 and Lemma 2.26. ∎

Theorem 2.49. $\mathsf{exactly4}\ (2 \cup (\{\{1\}\} \cup \{3\}))$.

Proof. Use Lemma 2.1, Lemma 2.15, Lemma 2.24, Lemma 2.7, Lemma 2.19, Theorem 2.1, Lemma 2.115, Lemma 2.114, Lemma 2.2, Lemma 2.5, Lemma 2.23, Lemma 2.6, Lemma 2.30 and Lemma 2.26. ∎

Theorem 2.50. $\mathsf{exactly4}\ (2 \cup (\{3\} \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.89, Lemma 2.24, Lemma 2.7, Lemma 2.19, Theorem 2.1, Lemma 2.137, Lemma 2.136, Lemma 2.2, Lemma 2.5, Lemma 2.15, Lemma 2.23, Lemma 2.6, Lemma 2.30 and Lemma 2.26. ∎

Theorem 2.51. $\mathsf{exactly4}\ ((\wp\ \{1\}) \cup (4 \setminus 2))$.

Proof. Use Theorem 2.11, Lemma 2.29, Lemma 2.18, Lemma 2.117, Lemma 2.116, Lemma 2.46, Lemma 2.5, Lemma 2.15, Lemma 2.23, Lemma 2.6, Lemma 2.30, Lemma 2.7 and Lemma 2.26. ∎

Theorem 2.52. $\mathsf{exactly4}\ (((\wp\ 2) \setminus 2) \cup (\wp\ \{2\}))$.

Proof. Use Lemma 2.29, Lemma 2.18, Theorem 2.9, Lemma 2.85, Lemma 2.5, Lemma 2.15, Lemma 2.23, Lemma 2.6, Lemma 2.30, Lemma 2.7 and Lemma 2.26. ∎

Theorem 2.53. exactly4 $(3 \cup \{((\wp\ 2) \setminus 2)\})$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.1, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.2, Lemma 2.6, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.5, Lemma 2.23, Lemma 2.30 and Lemma 2.233. □

Theorem 2.54. exactly4 $(((\wp\ 2) \setminus \{2\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\})$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.15, Lemma 2.55, Lemma 2.50, Lemma 2.49, Lemma 2.2, Lemma 2.6, Lemma 2.1, Lemma 2.145, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.59, Lemma 2.46, Lemma 2.167, Lemma 2.29, Lemma 2.166, Lemma 2.5, Lemma 2.23, Lemma 2.30, Lemma 2.19, Lemma 2.58, Lemma 2.69, Lemma 2.63, Lemma 2.9, Lemma 2.125, Lemma 2.39 and Lemma 2.16. □

Theorem 2.55. exactly4 $(((\wp\ 2) \setminus \{1\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\})$.

Proof. Use Lemma 2.223, Lemma 2.15, Lemma 2.39, Lemma 2.54, Lemma 2.49, Lemma 2.17, Lemma 2.53, Lemma 2.12, Lemma 2.126, Lemma 2.1 and Lemma 2.16. □

Theorem 2.56. exactly4 $(3 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.1, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.2, Lemma 2.6, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.5, Lemma 2.23, Lemma 2.30 and Lemma 2.81. □

Theorem 2.57. exactly4 $(((\wp\ 2) \cup \{\{2\}\}) \cap (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})))$.

Proof. Use Theorem 2.39, Lemma 2.138, Lemma 2.5, Lemma 2.4, Lemma 2.83 and Lemma 2.139. □

Theorem 2.58. exactly4 $(((\wp\ 2) \cup \{\{2\}\}) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})))$.

Proof. Use Theorem 2.43 and Lemma 2.210. □

Theorem 2.59. exactly4 $((\{\{1\}\} \cup (\{\{2\}\} \cup 4)) \cap (4 \cup \{((\wp\ 2) \setminus 2)\}))$.

Proof. Use Theorem 2.45, Lemma 2.151, Lemma 2.122, Lemma 2.1, Lemma 2.17 and Lemma 2.15. □

Theorem 2.60. exactly4 $((4 \cup \{((\wp\ 2) \setminus 2)\}) \cap (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})))$.

Proof. Use Theorem 2.45, Lemma 2.8, Lemma 2.7, Lemma 2.1, Lemma 2.17, Lemma 2.15, Lemma 2.86 and Lemma 2.54. □

Theorem 2.61. exactly3 $((3 \cup \{(\wp\ 2)\}) \setminus \{0\})$.

Proof. Use Theorem 2.56, Lemma 2.129 and Lemma 2.32. □

Theorem 2.62. exactly3 $((3 \cup \{(\wp\ 2)\}) \setminus \{1\})$.

Proof. Use Theorem 2.56 and Lemma 2.32. □

Theorem 2.63. exactly3 $((3 \cup \{(\wp\ 2)\}) \setminus \{2\})$.

Proof. Use Theorem 2.56 and Lemma 2.32. □

Theorem 2.64. exactly3 $((\{\{\{1\}\}\} \cup (2 \cup \{(\wp\ 2)\})) \setminus \{0\})$.

Proof. Use Lemma 2.221, Lemma 2.24, Lemma 2.5, Lemma 2.15, Lemma 2.23, Lemma 2.6, Lemma 2.30, Lemma 2.7, Lemma 2.1, Lemma 2.48, Lemma 2.92, Lemma 2.2, Lemma 2.148, Lemma 2.133, Lemma 2.77, Lemma 2.16, Lemma 2.132, Lemma 2.76 and Lemma 2.32. □

Theorem 2.65. exactly4 $((((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{0\})$.

Proof. Use Lemma 2.22, Lemma 2.7, Lemma 2.1, Lemma 2.15, Lemma 2.62, Lemma 2.50, Lemma 2.5, Lemma 2.83, Lemma 2.6, Lemma 2.185, Lemma 2.23, Lemma 2.55, Lemma 2.49, Lemma 2.2, Lemma 2.145, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.59, Lemma 2.46, Lemma 2.167, Lemma 2.29, Lemma 2.166, Lemma 2.178, Lemma 2.177, Lemma 2.43, Lemma 2.3, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.53, Lemma 2.48, Lemma 2.172, Lemma 2.171, Lemma 2.54, Lemma 2.39, Lemma 2.176, Lemma 2.173, Lemma 2.226, Lemma 2.184, Lemma 2.30, Lemma 2.47, Lemma 2.183, Lemma 2.181, Lemma 2.90, Lemma 2.222, Lemma 2.234 and Lemma 2.33. □

Theorem 2.66. exactly5 $((\wp\ 2) \cup \{\{2\}\})$.

Proof. Use Lemma 2.22, Lemma 2.23, Lemma 2.7, Lemma 2.15, Lemma 2.55, Lemma 2.50, Lemma 2.49, Lemma 2.2, Lemma 2.6, Lemma 2.1, Lemma 2.145, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.59, Lemma 2.46, Lemma 2.167, Lemma 2.29, Lemma 2.166, Lemma 2.178, Lemma 2.177, Lemma 2.43, Lemma 2.3, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.53, Lemma 2.48, Lemma 2.83, Lemma 2.172, Lemma 2.171, Lemma 2.54, Lemma 2.39, Lemma 2.62, Lemma 2.176, Lemma 2.173, Lemma 2.34 and Lemma 2.235. □

Theorem 2.67. exactly5 $(\{\{1\}\} \cup 4)$.

Proof. Use Lemma 2.22, Lemma 2.23, Lemma 2.199, Lemma 2.7, Lemma 2.1, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.2, Lemma 2.6, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.113, Lemma 2.44, Lemma 2.50, Lemma 2.4, Lemma 2.187, Lemma 2.186, Lemma 2.5, Lemma 2.219, Lemma 2.45, Lemma 2.112, Lemma 2.193, Lemma 2.60, Lemma 2.192, Lemma 2.98, Lemma 2.191, Lemma 2.190, Lemma 2.40, Lemma 2.188, Lemma 2.196, Lemma 2.118, Lemma 2.52, Lemma 2.195, Lemma 2.194, Lemma 2.34 and Lemma 2.236. □

Theorem 2.68. $\mathsf{exactly5}\ (\{\{\{1\}\}\} \cup 4)$.

Proof. Use Lemma 2.227, Lemma 2.1, Lemma 2.3, Lemma 2.2, Lemma 2.42, Lemma 2.4, Lemma 2.119, Lemma 2.15 and Lemma 2.8. □

Theorem 2.69. $\mathsf{exactly5}\ (\{(\wp\ \{1\})\} \cup 4)$.

Proof. Use Lemma 2.22, Lemma 2.23, Lemma 2.199, Lemma 2.7, Lemma 2.1, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.2, Lemma 2.6, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.113, Lemma 2.44, Lemma 2.50, Lemma 2.4, Lemma 2.187, Lemma 2.186, Lemma 2.5, Lemma 2.219, Lemma 2.45, Lemma 2.112, Lemma 2.193, Lemma 2.60, Lemma 2.192, Lemma 2.98, Lemma 2.191, Lemma 2.190, Lemma 2.40, Lemma 2.188, Lemma 2.196, Lemma 2.118, Lemma 2.52, Lemma 2.195, Lemma 2.194, Lemma 2.34, Lemma 2.42, Lemma 2.48, Lemma 2.38 and Lemma 2.8. □

Theorem 2.70. $\mathsf{exactly5}\ (\{\{2\}\} \cup 4)$.

Proof. Use Lemma 2.22, Lemma 2.23, Lemma 2.199, Lemma 2.7, Lemma 2.1, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.2, Lemma 2.6, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.113, Lemma 2.44, Lemma 2.50, Lemma 2.4, Lemma 2.187, Lemma 2.186, Lemma 2.5, Lemma 2.219, Lemma 2.45, Lemma 2.112, Lemma 2.193, Lemma 2.60, Lemma 2.192, Lemma 2.98, Lemma 2.191, Lemma 2.190, Lemma 2.40, Lemma 2.188, Lemma 2.196, Lemma 2.118, Lemma 2.52, Lemma 2.195, Lemma 2.194, Lemma 2.34 and Lemma 2.237. □

Theorem 2.71. $\mathsf{exactly4}\ ((\{\{2\}\} \cup 4) \setminus \{1\})$.

Proof. Use Theorem 2.70, Lemma 2.120, Lemma 2.6 and Lemma 2.33. □

Theorem 2.72. $\mathsf{exactly4}\ ((\{\{2\}\} \cup 4) \setminus \{2\})$.

Proof. Use Theorem 2.70, Lemma 2.123, Lemma 2.6 and Lemma 2.33. □

Theorem 2.73. $\mathsf{exactly4}\ ((\{\{2\}\} \cup 4) \setminus \{3\})$.

Proof. Use Theorem 2.70, Lemma 2.5, Lemma 2.6 and Lemma 2.33. □

Theorem 2.74. $\mathsf{exactly5}\ (4 \cup \{((\wp\ 2) \setminus 2)\})$.

Proof. Use Lemma 2.22, Lemma 2.23, Lemma 2.199, Lemma 2.7, Lemma 2.1, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.2, Lemma 2.6, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.113, Lemma 2.44, Lemma 2.50, Lemma 2.4, Lemma 2.187, Lemma 2.186, Lemma 2.5, Lemma 2.219, Lemma 2.45, Lemma 2.112, Lemma 2.193, Lemma 2.60, Lemma 2.192, Lemma 2.98, Lemma 2.191, Lemma 2.190, Lemma 2.40, Lemma 2.188, Lemma 2.196, Lemma 2.118, Lemma 2.52, Lemma 2.195, Lemma 2.194, Lemma 2.34, Lemma 2.42 and Lemma 2.152. □

Theorem 2.75. exactly5 $((\wp\ 2) \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 2.228, Lemma 2.232, Lemma 2.157 and Lemma 2.156. □

Theorem 2.76. exactly5 $(4 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 2.22, Lemma 2.23, Lemma 2.199, Lemma 2.7, Lemma 2.1, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.2, Lemma 2.6, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.113, Lemma 2.44, Lemma 2.50, Lemma 2.4, Lemma 2.187, Lemma 2.186, Lemma 2.5, Lemma 2.219, Lemma 2.45, Lemma 2.112, Lemma 2.193, Lemma 2.60, Lemma 2.192, Lemma 2.98, Lemma 2.191, Lemma 2.190, Lemma 2.40, Lemma 2.188, Lemma 2.196, Lemma 2.118, Lemma 2.52, Lemma 2.195, Lemma 2.194, Lemma 2.34, Lemma 2.42, Lemma 2.89, Lemma 2.39, Lemma 2.49 and Lemma 2.38. □

Theorem 2.77. exactly4 $((4 \cup \{(\wp\ 2)\}) \setminus \{0\})$.

Proof. Use Theorem 2.76, Lemma 2.4, Lemma 2.6 and Lemma 2.33. □

Theorem 2.78. exactly4 $((4 \cup \{(\wp\ 2)\}) \setminus \{1\})$.

Proof. Use Theorem 2.76, Lemma 2.6 and Lemma 2.33. □

Theorem 2.79. exactly4 $((4 \cup \{(\wp\ 2)\}) \setminus \{2\})$.

Proof. Use Theorem 2.76, Lemma 2.6 and Lemma 2.33. □

Theorem 2.80. exactly4 $((4 \cup \{(\wp\ 2)\}) \setminus \{3\})$.

Proof. Use Theorem 2.76, Lemma 2.6 and Lemma 2.33. □

Theorem 2.81. exactly6 $(\{\{1\}\} \cup (\{\{2\}\} \cup 4))$.

Proof. Use Lemma 2.22, Lemma 2.23, Lemma 2.199, Lemma 2.7, Lemma 2.1, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.2, Lemma 2.6, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.113, Lemma 2.44, Lemma 2.50, Lemma 2.4, Lemma 2.187, Lemma 2.186, Lemma 2.5, Lemma 2.219, Lemma 2.45, Lemma 2.112, Lemma 2.193, Lemma 2.60, Lemma 2.192, Lemma 2.98, Lemma 2.191, Lemma 2.190, Lemma 2.40, Lemma 2.188, Lemma 2.196, Lemma 2.118, Lemma 2.52, Lemma 2.195, Lemma 2.194, Lemma 2.34, Lemma 2.237 and Lemma 2.160. □

Theorem 2.82. exactly6 $(\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.22, Lemma 2.23, Lemma 2.199, Lemma 2.7, Lemma 2.1, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.2, Lemma 2.6, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.113, Lemma 2.44, Lemma 2.50, Lemma 2.4, Lemma 2.187, Lemma 2.186, Lemma 2.5, Lemma 2.219, Lemma 2.45, Lemma 2.112, Lemma 2.193, Lemma 2.60, Lemma 2.192, Lemma 2.98, Lemma 2.191, Lemma 2.190, Lemma 2.40, Lemma 2.188, Lemma 2.196, Lemma 2.118, Lemma 2.52, Lemma 2.195, Lemma 2.194, Lemma 2.34, Lemma 2.42, Lemma 2.89, Lemma 2.39, Lemma 2.49, Lemma 2.38, Lemma 2.68 and Lemma 2.159. □

Theorem 2.83. exactly6 $(\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 2.22, Lemma 2.23, Lemma 2.199, Lemma 2.7, Lemma 2.1, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.2, Lemma 2.6, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.113, Lemma 2.44, Lemma 2.50, Lemma 2.4, Lemma 2.187, Lemma 2.186, Lemma 2.5, Lemma 2.219, Lemma 2.45, Lemma 2.112, Lemma 2.193, Lemma 2.60, Lemma 2.192, Lemma 2.98, Lemma 2.191, Lemma 2.190, Lemma 2.40, Lemma 2.188, Lemma 2.196, Lemma 2.118, Lemma 2.52, Lemma 2.195, Lemma 2.194, Lemma 2.34 and Lemma 2.238. □

Theorem 2.84. exactly3 $(((\wp\ 2) \cup \{\{2\}\}) \setminus (4 \setminus \{1\}))$.

Proof. Use Theorem 2.4, Theorem 2.66, Lemma 2.6, Lemma 2.14, Lemma 2.27, Lemma 2.5, Lemma 2.10, Lemma 2.15, Lemma 2.23, Lemma 2.30 and Lemma 2.7. □

Theorem 2.85. exactly3 $((\{\{1\}\} \cup 4) \setminus (\{2\} \cup (\wp\ \{2\})))$.

Proof. Use Theorem 2.12, Theorem 2.67, Lemma 2.6, Lemma 2.14, Lemma 2.27, Lemma 2.5, Lemma 2.10, Lemma 2.15, Lemma 2.23, Lemma 2.30 and Lemma 2.7. □

Theorem 2.86. exactly3 $((\{\{2\}\} \cup 4) \setminus ((\wp\ 2) \setminus \{1\}))$.

Proof. Use Theorem 2.5, Theorem 2.70, Lemma 2.6, Lemma 2.14, Lemma 2.27, Lemma 2.5, Lemma 2.10, Lemma 2.15, Lemma 2.23, Lemma 2.30 and Lemma 2.7. □

Theorem 2.87. exactly3 $(((\wp\ 2) \cup \{(\wp\ 2)\}) \setminus (4 \setminus \{1\}))$.

Proof. Use Theorem 2.6, Theorem 2.75, Lemma 2.6, Lemma 2.14, Lemma 2.27, Lemma 2.5, Lemma 2.10, Lemma 2.15, Lemma 2.23, Lemma 2.30 and Lemma 2.7. □

Theorem 2.88. exactly3 $((4 \cup \{(\wp\ 2)\}) \setminus ((\wp\ 2) \setminus (\wp\ \{2\})))$.

Proof. Use Theorem 2.15, Theorem 2.76, Lemma 2.6, Lemma 2.14, Lemma 2.27, Lemma 2.5, Lemma 2.10, Lemma 2.15, Lemma 2.23, Lemma 2.30 and Lemma 2.7. □

Theorem 2.89. exactly3 $((4 \cup \{(\wp\ 2)\}) \setminus (\{2\} \cup (\wp\ \{2\})))$.

Proof. Use Theorem 2.7, Theorem 2.76, Lemma 2.6, Lemma 2.14, Lemma 2.27, Lemma 2.5, Lemma 2.10, Lemma 2.15, Lemma 2.23, Lemma 2.30 and Lemma 2.7. □

Lemma 2.243. $(((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\})) \cap (\{\{2\}\} \cup 4)) \setminus \{\{2\}\} \subseteq 4 \setminus \{2\}$.

Proof. Use Lemma 2.6, Lemma 2.109, Lemma 2.1, Lemma 2.15 and Lemma 2.66. □

Lemma 2.244. $\neg \mathsf{atleast5}\ (((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\})) \cap (\{\{2\}\} \cup 4))$.

Proof. Use Lemma 2.109, Lemma 2.1, Lemma 2.15, Lemma 2.66, Lemma 2.5, Lemma 2.13, Lemma 2.23, Lemma 2.218, Lemma 2.24, Lemma 2.30, Lemma 2.6, Lemma 2.7, Lemma 2.22, Lemma 2.35, Lemma 2.88, Lemma 2.17, Lemma 2.120, Lemma 2.39, Lemma 2.16, Lemma 2.121, Theorem 2.86, Lemma 2.243, Lemma 2.113, Lemma 2.44, Lemma 2.50, Lemma 2.4, Lemma 2.2, Lemma 2.187, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.186, Lemma 2.219, Lemma 2.45 and Lemma 2.112. □

Theorem 2.90. $\mathsf{exactly4}\ (((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\})) \cap (\{\{2\}\} \cup 4))$.

Proof. Use Lemma 2.244, Lemma 2.1, Lemma 2.5, Lemma 2.2, Lemma 2.15, Lemma 2.60, Lemma 2.203, Lemma 2.11, Lemma 2.8 and Lemma 2.7. □

Theorem 2.91. $\mathsf{exactly5}\ ((\{\{1\}\} \cup (\{\{2\}\} \cup 4)) \setminus \{0\})$.

Proof. Use Theorem 2.81, Lemma 2.4, Lemma 2.5, Lemma 2.6, Lemma 2.24, Lemma 2.34 and Lemma 2.7. □

Theorem 2.92. $\mathsf{exactly5}\ ((\{\{1\}\} \cup (\{\{2\}\} \cup 4)) \setminus \{2\})$.

Proof. Use Theorem 2.81, Lemma 2.123, Lemma 2.5, Lemma 2.6, Lemma 2.24, Lemma 2.34 and Lemma 2.7. □

Theorem 2.93. $\mathsf{exactly5}\ ((\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{1\})$.

Proof. Use Theorem 2.82, Lemma 2.5, Lemma 2.6, Lemma 2.24, Lemma 2.34 and Lemma 2.7. □

Theorem 2.94. $\mathsf{exactly5}\ ((\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{2\})$.

Proof. Use Theorem 2.82, Lemma 2.138, Lemma 2.6, Lemma 2.5, Lemma 2.24, Lemma 2.34 and Lemma 2.7. □

Theorem 2.95. $\mathsf{exactly5}\ ((\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{3\})$.

Proof. Use Theorem 2.82, Lemma 2.5, Lemma 2.6, Lemma 2.24, Lemma 2.34 and Lemma 2.7. □

Theorem 2.96. $\mathsf{exactly5}\ ((\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{0\})$.

Proof. Use Theorem 2.83, Lemma 2.140, Lemma 2.6, Lemma 2.5, Lemma 2.24, Lemma 2.34 and Lemma 2.7. □

Theorem 2.97. $\mathsf{exactly5}\ ((\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{1\})$.

Proof. Use Theorem 2.83, Lemma 2.141, Lemma 2.6, Lemma 2.5, Lemma 2.24, Lemma 2.34 and Lemma 2.7. □

Theorem 2.98. $\mathsf{exactly5}\ ((\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{2\})$.

Proof. Use Theorem 2.83, Lemma 2.142, Lemma 2.6, Lemma 2.5, Lemma 2.24, Lemma 2.34 and Lemma 2.7. □

Theorem 2.99. $\mathsf{exactly5}\ ((\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{3\})$.

Proof. Use Theorem 2.83, Lemma 2.5, Lemma 2.6, Lemma 2.24, Lemma 2.34 and Lemma 2.7. □

Lemma 2.245. $\forall u \in (\wp\ 2) \setminus (\wp\ \{2\}).u \in \{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}) \Rightarrow u \in ((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\{\{2\}\} \cup \{(\wp$

Proof. Use Lemma 2.15, Lemma 2.68, Lemma 2.1, Lemma 2.52, Lemma 2.6 and Lemma 2.83. □

Lemma 2.246. $(((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))) \setminus \{\wp\ 2\}) \subseteq 3 \cup \{\{$

Proof. Use Lemma 2.103, Lemma 2.3, Lemma 2.28, Lemma 2.68, Lemma 2.15, Lemma 2.1, Lemma 2.6, Lemma 2.83 and Lemma 2.13. □

Theorem 2.100. $\mathsf{exactly5}\ ((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})))$.

Proof. Use Lemma 2.246, Lemma 2.7, Lemma 2.22, Lemma 2.1, Lemma 2.43, Lemma 2.15, Lemma 2.3, Lemma 2.2, Lemma 2.6, Lemma 2.164, Lemma 2.217, Lemma 2.163, Lemma 2.24, Lemma 2.169, Lemma 2.165, Lemma 2.170, Lemma 2.84, Lemma 2.5, Lemma 2.23, Lemma 2.30, Lemma 2.129, Lemma 2.28, Lemma 2.68, Lemma 2.83, Lemma 2.13, Lemma 2.134, Lemma 2.245, Lemma 2.37, Lemma 2.8, Lemma 2.218, Lemma 2.35, Lemma 2.36, Lemma 2.225 and Lemma 2.239. □

Chapter 3

Fine Level of Granularity

Proposition 3.1. $0 \in 1$.

Proof. Use Axiom 1.5. ☐

Proposition 3.2. $\forall i \in 1.i = 0$.

Proof. Use Axiom 1.5. ☐

Proposition 3.3. $0 \in 2$.

Proof. Use Axiom 1.6. ☐

Lemma 3.1. $1 \in 2$.

Proof. Use Axiom 1.6. ☐

Lemma 3.2. $\forall i \in 2.i = 0 \vee i = 1$.

Proof. Use Axiom 1.6. ☐

Lemma 3.3. $0 \in 3$.

Proof. Use Axiom 1.7. ☐

Proposition 3.4. $1 \in 3$.

Proof. Use Axiom 1.7. ☐

Proposition 3.5. $2 \in 3$.

Proof. Use Axiom 1.7. ☐

Proposition 3.6. $\forall i \in 3.i = 0 \vee i = 1 \vee i = 2$.

Proof. Use Axiom 1.7. ☐

Lemma 3.4. $0 \in 4$.

Proof. Use Axiom 1.8. □

Proposition 3.7. $1 \in 4$.

Proof. Use Axiom 1.8. □

Proposition 3.8. $2 \in 4$.

Proof. Use Axiom 1.8. □

Proposition 3.9. $3 \in 4$.

Proof. Use Axiom 1.8. □

Proposition 3.10. $\forall i \in 4.i = 0 \vee i = 1 \vee i = 2 \vee i = 3$.

Proof. Use Axiom 1.8. □

Proposition 3.11. $\forall XY : \iota.Y \subseteq X \Rightarrow Y \in (\wp\ X)$.

Proof. Use Axiom 1.9. □

Proposition 3.12. $\forall XY : \iota.Y \in \wp\ X \Rightarrow Y \subseteq X$.

Proof. Use Axiom 1.9. □

Proposition 3.13. $\forall x.x \in \{x\}$.

Proof. Use Axiom 1.10. □

Proposition 3.14. $\forall xy.y \in \{x\} \Rightarrow y = x$.

Proof. Use Axiom 1.10. □

Proposition 3.15. $\forall XYz.z \in X \Rightarrow z \in X \cup Y$.

Proof. Use Axiom 1.11. □

Lemma 3.5. $\forall XYz.z \in Y \Rightarrow z \in X \cup Y$.

Proof. Use Axiom 1.11. □

Proposition 3.16. $\forall XYz.z \in X \cup Y \Rightarrow z \in X \vee z \in Y$.

Proof. Use Axiom 1.11. □

Proposition 3.17. $\forall XYz.z \in X \Rightarrow z \in Y \Rightarrow z \in X \cap Y$.

Proof. Use Axiom 1.12. □

Proposition 3.18. $\forall XYz.z \in X \cap Y \Rightarrow z \in X$.

Proof. Use Axiom 1.12. □

Proposition 3.19. $\forall XYz.z \in X \cap Y \Rightarrow z \in Y$.

Proof. Use Axiom 1.12. ☐

Proposition 3.20. $\forall XYz.z \in X \Rightarrow z \notin Y \Rightarrow z \in X \setminus Y$.

Proof. Use Axiom 1.13. ☐

Proposition 3.21. $\forall XYz.z \in X \setminus Y \Rightarrow z \in X$.

Proof. Use Axiom 1.13. ☐

Lemma 3.6. $\forall XYz.z \in X \setminus Y \Rightarrow z \notin Y$.

Proof. Use Axiom 1.13. ☐

Proposition 3.22. $\forall X : \iota.X \subseteq X$.

Proof. Use Definition 1.1. ☐

Lemma 3.7. $\forall XYZ : \iota.X \subseteq Y \Rightarrow Y \subseteq Z \Rightarrow X \subseteq Z$.

Proof. Use Definition 1.1. ☐

Proposition 3.23. $\forall XYz : \iota.X \subseteq Y \Rightarrow z \notin Y \Rightarrow z \notin X$.

Proof. Use Definition 1.1. ☐

Proposition 3.24. $\forall X : \iota.0 \subseteq X$.

Proof. Use Axiom 1.4 and Definition 1.1. ☐

Proposition 3.25. $\forall X : \iota.X \subseteq 0 \Rightarrow X = 0$.

Proof. Use Proposition 3.24 and Axiom 1.1. ☐

Proposition 3.26. $\forall X : \iota.0 \in \wp\, X$.

Proof. Use Proposition 3.24 and Proposition 3.11. ☐

Proposition 3.27. $\forall X : \iota.X \in \wp\, X$.

Proof. Use Proposition 3.22 and Proposition 3.11. ☐

Proposition 3.28. $\forall XY : \iota.X \subseteq X \cup Y$.

Proof. Use Proposition 3.15 and Definition 1.1. ☐

Lemma 3.8. $\forall XY : \iota.Y \subseteq X \cup Y$.

Proof. Use Lemma 3.5 and Definition 1.1. ☐

Lemma 3.9. $\forall XYZ : \iota.X \subseteq Z \Rightarrow Y \subseteq Z \Rightarrow X \cup Y \subseteq Z$.

Proof. Use Proposition 3.16 and Definition 1.1. ☐

Lemma 3.10. $\forall XYZ.Y \subseteq Z \Rightarrow X \cup Y \subseteq X \cup Z.$

Proof. Use Lemma 3.8, Lemma 3.7, Proposition 3.28 and Lemma 3.9. ☐

Proposition 3.29. $\forall XYZ : \iota.X \cup (Y \cup Z) \subseteq (X \cup Y) \cup Z.$

Proof. Use Lemma 3.5, Proposition 3.15, Proposition 3.16 and Definition 1.1. ☐

Proposition 3.30. $\forall XYZ : \iota.(X \cup Y) \cup Z \subseteq X \cup (Y \cup Z).$

Proof. Use Lemma 3.5, Proposition 3.15, Proposition 3.16 and Definition 1.1. ☐

Lemma 3.11. $\forall XYZ : \iota.X \cup (Y \cup Z) = (X \cup Y) \cup Z.$

Proof. Use Proposition 3.30, Proposition 3.29 and Axiom 1.1. ☐

Proposition 3.31. $\forall XY : \iota.X \cup Y \subseteq Y \cup X.$

Proof. Use Proposition 3.15, Lemma 3.5, Proposition 3.16 and Definition 1.1. ☐

Proposition 3.32. $\forall XY : \iota.X \cup Y = Y \cup X.$

Proof. Use Proposition 3.31 and Axiom 1.1. ☐

Proposition 3.33. $\forall XY : \iota.X \cap Y \subseteq Y.$

Proof. Use Proposition 3.19 and Definition 1.1. ☐

Proposition 3.34. $\forall XYZ : \iota.Z \subseteq X \Rightarrow Z \subseteq Y \Rightarrow Z \subseteq X \cap Y.$

Proof. Use Proposition 3.17 and Definition 1.1. ☐

Proposition 3.35. $\forall XY : \iota.X \setminus Y \subseteq X.$

Proof. Use Proposition 3.21 and Definition 1.1. ☐

Lemma 3.12. $\forall XYZ : \iota.Z \subseteq Y \Rightarrow X \setminus Y \subseteq X \setminus Z.$

Proof. Use Lemma 3.6, Proposition 3.23, Proposition 3.21, Proposition 3.20 and Definition 1.1. ☐

Proposition 3.36. $\forall XYZ.(\forall x \in X.x \in Y \Rightarrow x \in Z) \Rightarrow X \cap Y \subseteq Z.$

Proof. Use Proposition 3.19, Proposition 3.18 and Definition 1.1. ☐

Proposition 3.37. $\forall XYZ.(\forall x \in X.x \in Y \Rightarrow x \in Z) \Rightarrow Z \subseteq X \Rightarrow Z \subseteq Y \Rightarrow X \cap Y = Z.$

Proof. Use Proposition 3.34, Proposition 3.36 and Axiom 1.1. ☐

Lemma 3.13. $\forall XYZ.(\forall x \in X.x \notin Y \Rightarrow x \in Z) \Rightarrow X \setminus Y \subseteq Z.$

Proof. Use Lemma 3.6, Proposition 3.21 and Definition 1.1. □

Proposition 3.38. $\forall XY.(\forall x \in X.x \notin Y) \Rightarrow \mathsf{disj}\ X\ Y.$

Proof. Use Proposition 3.19, Proposition 3.18, Proposition 3.25, Definition 1.1 and Definition 1.2. □

Lemma 3.14. $\forall XYx.\mathsf{disj}\ X\ Y \Rightarrow x \in X \Rightarrow x \notin Y.$

Proof. Use Proposition 3.17, Axiom 1.4 and Definition 1.2. □

Proposition 3.39. $\forall XY.\mathsf{disj}\ X\ Y \Rightarrow \mathsf{disj}\ Y\ X.$

Proof. Use Lemma 3.14 and Proposition 3.38. □

Proposition 3.40. $\forall XYZ.\mathsf{disj}\ X\ Y \Rightarrow Z \subseteq Y \Rightarrow \mathsf{disj}\ X\ Z.$

Proof. Use Lemma 3.14, Proposition 3.38 and Definition 1.1. □

Proposition 3.41. $\forall XYZ.\mathsf{disj}\ X\ Y \Rightarrow X \cup Y \subseteq X \cup Z \Rightarrow Y \subseteq Z.$

Proof. Use Lemma 3.5, Lemma 3.14, Proposition 3.16 and Definition 1.1. □

Proposition 3.42. $\forall XY.\mathsf{disj}\ (X \setminus Y)\ (X \cap Y).$

Proof. Use Proposition 3.19, Lemma 3.6 and Proposition 3.38. □

Proposition 3.43. $\forall X.\forall x \in X.X \nsubseteq X \setminus \{x\}.$

Proof. Use Proposition 3.13, Lemma 3.6 and Definition 1.1. □

Lemma 3.15. $\forall xy.y \neq x \Rightarrow y \notin \{x\}.$

Proof. Use Proposition 3.14. □

Proposition 3.44. $\forall xy.y \notin \{x\} \Rightarrow y \neq x.$

Proof. Use Proposition 3.13. □

Proposition 3.45. $\forall XYz.z \notin X \Rightarrow z \notin Y \Rightarrow z \notin X \cup Y.$

Proof. Use Proposition 3.16. □

Lemma 3.16. $\forall XYz.z \notin X \Rightarrow z \notin X \setminus Y.$

Proof. Use Proposition 3.21. □

Lemma 3.17. $\forall XYz.z \in Y \Rightarrow z \notin X \setminus Y.$

Proof. Use Lemma 3.6. □

Lemma 3.18. $\forall XY.X \in Y \Rightarrow \mathsf{atleast2}\ (\wp\ Y).$

Proof. Use Proposition 3.12, Axiom 1.4, Proposition 3.27, Proposition 3.26, Definition 1.3 and Definition 1.1. □

Proposition 3.46. $\forall XYZ.Y \in X \Rightarrow Z \in X \Rightarrow Z \not\subseteq Y \Rightarrow \textsf{atleast2}\ X.$

Proof. Use Proposition 3.12, Definition 1.3 and Definition 1.1. □

Proposition 3.47. $\forall XYZ.Y \in X \Rightarrow Z \in X \Rightarrow Y \neq Z \Rightarrow \textsf{atleast2}\ X.$

Proof. Use Proposition 3.46 and Axiom 1.1. □

Lemma 3.19. $\forall xy.x \neq y \Rightarrow \textsf{atleast2}\ (\{x\} \cup \{y\}).$

Proof. Use Proposition 3.15, Proposition 3.13, Lemma 3.5, Proposition 3.46 and Axiom 1.1. □

Lemma 3.20. $\forall Xx.x \notin X \Rightarrow \textsf{atleast2}\ X \Rightarrow \textsf{atleast3}\ (X \cup \{x\}).$

Proof. Use Proposition 3.13, Lemma 3.5, Proposition 3.28, Definition 1.4 and Definition 1.1. □

Proposition 3.48. $\forall X.\forall x \in X.\{x\} \subseteq X.$

Proof. Use Proposition 3.14 and Definition 1.1. □

Proposition 3.49. $\forall X.\forall x \in X.\neg\textsf{atleast4}\ X \Rightarrow \neg\textsf{atleast3}\ (X \setminus \{x\}).$

Proof. Use Lemma 3.6, Proposition 3.13, Proposition 3.35, Definition 1.5 and Definition 1.1. □

Lemma 3.21. $\forall X.\forall x \in X.\neg\textsf{atleast5}\ X \Rightarrow \neg\textsf{atleast4}\ (X \setminus \{x\}).$

Proof. Use Lemma 3.6, Proposition 3.13, Proposition 3.35, Definition 1.6 and Definition 1.1. □

Proposition 3.50. $\forall X.\forall x \in X.\neg\textsf{atleast6}\ X \Rightarrow \neg\textsf{atleast5}\ (X \setminus \{x\}).$

Proof. Use Lemma 3.6, Proposition 3.13, Proposition 3.35, Definition 1.7 and Definition 1.1. □

Proposition 3.51. $\forall X.\forall x \in X.\neg\textsf{atleast7}\ X \Rightarrow \neg\textsf{atleast6}\ (X \setminus \{x\}).$

Proof. Use Lemma 3.6, Proposition 3.13, Proposition 3.35, Definition 1.8 and Definition 1.1. □

Proposition 3.52. $1 \subseteq \{0\}.$

Proof. Use Proposition 3.13, Proposition 3.2 and Definition 1.1. □

Proposition 3.53. $\{0\} \subseteq 1.$

Proof. Use Proposition 3.1, Proposition 3.14 and Definition 1.1. ☐

Proposition 3.54. $1 = \{0\}$.

Proof. Use Proposition 3.53, Proposition 3.52 and Axiom 1.1. ☐

Lemma 3.22. $\forall XY.Y \subseteq X \Rightarrow X \not\subseteq Y \Rightarrow \exists x \in X.Y \subseteq X \setminus \{x\}$.

Proof. Use Proposition 3.14, Proposition 3.20 and Definition 1.1. ☐

Proposition 3.55. $\forall XY.X \subseteq Y \Rightarrow \mathsf{atleast2}\ X \Rightarrow \mathsf{atleast2}\ Y$.

Proof. Use Lemma 3.7, Definition 1.3 and Definition 1.1. ☐

Proposition 3.56. $\forall XY.X \subseteq Y \Rightarrow \mathsf{atleast3}\ X \Rightarrow \mathsf{atleast3}\ Y$.

Proof. Use Lemma 3.7 and Definition 1.4. ☐

Lemma 3.23. $\forall XY.X \subseteq Y \Rightarrow \mathsf{atleast4}\ X \Rightarrow \mathsf{atleast4}\ Y$.

Proof. Use Lemma 3.7 and Definition 1.5. ☐

Proposition 3.57. $\forall XY.X \subseteq Y \Rightarrow \mathsf{atleast5}\ X \Rightarrow \mathsf{atleast5}\ Y$.

Proof. Use Lemma 3.7 and Definition 1.6. ☐

Proposition 3.58. $\forall XY.X \subseteq Y \Rightarrow \mathsf{atleast6}\ X \Rightarrow \mathsf{atleast6}\ Y$.

Proof. Use Lemma 3.7 and Definition 1.7. ☐

Proposition 3.59. $\forall X.\mathsf{atleast4}\ X \Rightarrow \mathsf{atleast3}\ X$.

Proof. Use Proposition 3.56 and Definition 1.5. ☐

Lemma 3.24. $\forall z.\neg\mathsf{atleast2}\ \{z\}$.

Proof. Use Proposition 3.27, Proposition 3.14, Definition 1.3 and Definition 1.1. ☐

Lemma 3.25. $\forall XY.\mathsf{disj}\ X\ Y \Rightarrow \mathsf{atleast2}\ Y \Rightarrow \forall y \in Y.X \cup Y \not\subseteq X \cup \{y\}$.

Proof. Use Proposition 3.41, Proposition 3.55 and Lemma 3.24. ☐

Proposition 3.60. $\forall XY.\mathsf{disj}\ X\ Y \Rightarrow \mathsf{atleast2}\ X \Rightarrow \forall y \in Y.\mathsf{atleast3}\ (X \cup \{y\})$.

Proof. Use Lemma 3.14 and Lemma 3.20. ☐

Lemma 3.26. $\forall XY.\mathsf{disj}\ X\ Y \Rightarrow \mathsf{atleast2}\ X \Rightarrow \mathsf{atleast2}\ Y \Rightarrow \mathsf{atleast4}\ (X \cup Y)$.

Proof. Use Proposition 3.60, Lemma 3.25, Proposition 3.48, Lemma 3.10, Definition 1.3 and Definition 1.5. ☐

Lemma 3.27. $\forall XY.\mathsf{disj}\ X\ Y \Rightarrow \mathsf{atleast2}\ X \Rightarrow \mathsf{atleast3}\ Y \Rightarrow \mathsf{atleast5}\ (X \cup Y)$.

Proof. Use Proposition 3.40, Lemma 3.26, Proposition 3.41, Lemma 3.10, Definition 1.4 and Definition 1.6. □

Proposition 3.61. $\forall XY.\mathsf{disj}\ X\ Y \Rightarrow \mathsf{atleast2}\ X \Rightarrow \mathsf{atleast4}\ Y \Rightarrow \mathsf{atleast6}\ (X \cup Y)$.

Proof. Use Proposition 3.40, Lemma 3.27, Proposition 3.41, Lemma 3.10, Definition 1.5 and Definition 1.7. □

Proposition 3.62. $\forall XY.\mathsf{disj}\ X\ Y \Rightarrow \mathsf{atleast4}\ X \Rightarrow \mathsf{atleast2}\ Y \Rightarrow \mathsf{atleast6}\ (X \cup Y)$.

Proof. Use Proposition 3.39, Proposition 3.61 and Proposition 3.32. □

Proposition 3.63. $\forall XY.X \subseteq (X \setminus Y) \cup (X \cap Y)$.

Proof. Use Proposition 3.20, Proposition 3.15, Proposition 3.17, Lemma 3.5 and Definition 1.1. □

Proposition 3.64. $\forall XY.(X \setminus Y) \cup (X \cap Y) \subseteq X$.

Proof. Use Proposition 3.18, Proposition 3.21, Proposition 3.16 and Definition 1.1. □

Proposition 3.65. $\forall XY.X = (X \setminus Y) \cup (X \cap Y)$.

Proof. Use Proposition 3.64, Proposition 3.63 and Axiom 1.1. □

Proposition 3.66. $\forall XY.\neg\mathsf{atleast6}\ X \Rightarrow \mathsf{atleast2}\ (X \cap Y) \Rightarrow \neg\mathsf{atleast4}\ (X \setminus Y)$.

Proof. Use Proposition 3.42, Proposition 3.62 and Proposition 3.65. □

Lemma 3.28. $\forall Xz.X \in \wp\ \{z\} \Rightarrow X = 0 \vee X = \{z\}$.

Proof. Use Proposition 3.12, Proposition 3.14, Proposition 3.25, Proposition 3.48, Axiom 1.1 and Definition 1.1. □

Proposition 3.67. $\forall Yz.Y \subseteq \wp\ \{z\} \Rightarrow \mathsf{atleast2}\ Y \Rightarrow \wp\ \{z\} \subseteq Y$.

Proof. Use Proposition 3.27, Proposition 3.26, Lemma 3.28, Definition 1.3 and Definition 1.1. □

Lemma 3.29. $\forall z.\neg\mathsf{atleast3}\ (\wp\ \{z\})$.

Proof. Use Proposition 3.67 and Definition 1.4. □

Proposition 3.68. $\forall z.\mathsf{exactly2}\ (\wp\ \{z\})$.

Proof. Use Lemma 3.29, Proposition 3.13, Lemma 3.18 and Definition 1.9. □

Proposition 3.69. $\forall XYZ.Z \subseteq X \cup Y \Rightarrow Z \not\subseteq X \Rightarrow Z \not\subseteq Y \Rightarrow X \cup Y \not\subseteq Z \Rightarrow \mathsf{atleast2}\ X \vee \mathsf{atleast2}\ Y$.

Proof. Use Proposition 3.47, Proposition 3.16 and Definition 1.1. □

Proposition 3.70. $\forall XY.\mathsf{atleast3}\ (X \cup Y) \Rightarrow \mathsf{atleast2}\ X \vee \mathsf{atleast2}\ Y.$

Proof. Use Proposition 3.69, Proposition 3.55 and Definition 1.4. ☐

Proposition 3.71. $\forall xy.\neg\mathsf{atleast3}\ (\{x\} \cup \{y\}).$

Proof. Use Lemma 3.24 and Proposition 3.70. ☐

Proposition 3.72. $\forall xy.x \neq y \Rightarrow \mathsf{exactly2}\ (\{x\} \cup \{y\}).$

Proof. Use Proposition 3.71, Lemma 3.19 and Definition 1.9. ☐

Proposition 3.73. $\forall X.\forall x \in X.X \cap \{x\} = \{x\}.$

Proof. Use Proposition 3.13, Proposition 3.17, Proposition 3.48, Proposition 3.33 and Axiom 1.1. ☐

Proposition 3.74. $\forall X.\forall x \in X.\mathsf{atleast3}\ X \Rightarrow \mathsf{atleast2}\ (X \setminus \{x\}).$

Proof. Use Proposition 3.65, Lemma 3.24, Proposition 3.73 and Proposition 3.70. ☐

Proposition 3.75. $\forall X.\forall x \in X.\mathsf{exactly3}\ X \Rightarrow \mathsf{exactly2}\ (X \setminus \{x\}).$

Proof. Use Proposition 3.49, Proposition 3.74, Definition 1.10 and Definition 1.9. ☐

Proposition 3.76. $\forall XYZ.Z \subseteq X \cup Y \Rightarrow \forall w.w \notin Z \Rightarrow Z \subseteq (X \setminus \{w\}) \cup Y.$

Proof. Use Lemma 3.5, Lemma 3.15, Proposition 3.20, Proposition 3.15, Proposition 3.16 and Definition 1.1. ☐

Lemma 3.30. $\forall XYZ.Z \subseteq X \cup Y \Rightarrow \forall w \in X.w \notin Z \Rightarrow \mathsf{atleast3}\ Z \Rightarrow \mathsf{atleast3}\ ((X \setminus \{w\}) \cup Y).$

Proof. Use Proposition 3.76 and Proposition 3.56. ☐

Proposition 3.77. $\forall XYZ.Z \subseteq X \cup Y \Rightarrow \forall w \in X.w \notin Z \Rightarrow \mathsf{atleast3}\ Z \Rightarrow \mathsf{atleast3}\ X \vee \mathsf{atleast2}\ Y.$

Proof. Use Lemma 3.30, Proposition 3.43, Proposition 3.35, Proposition 3.70 and Definition 1.4. ☐

Proposition 3.78. $\forall XY.\mathsf{atleast4}\ (X \cup Y) \Rightarrow \mathsf{atleast3}\ X \vee \mathsf{atleast2}\ Y.$

Proof. Use Proposition 3.47, Proposition 3.77, Proposition 3.16, Proposition 3.56, Definition 1.5 and Definition 1.1. ☐

Lemma 3.31. $\forall X.\forall x \in X.\mathsf{atleast4}\ X \Rightarrow \mathsf{atleast3}\ (X \setminus \{x\}).$

Proof. Use Proposition 3.65, Lemma 3.24, Proposition 3.73 and Proposition 3.78. ☐

Lemma 3.32. $\forall X.\forall x \in X.\mathsf{exactly4}\ X \Rightarrow \mathsf{exactly3}\ (X \setminus \{x\}).$

Proof. Use Lemma 3.21, Lemma 3.31, Definition 1.11 and Definition 1.10. ☐

Proposition 3.79. $\forall X x. \neg \mathsf{atleast3}\ X \Rightarrow \neg \mathsf{atleast4}\ (X \cup \{x\})$.

Proof. Use Lemma 3.24 and Proposition 3.78. □

Proposition 3.80. $\forall XY. \mathsf{atleast4}\ (X \cup Y) \Rightarrow \mathsf{atleast2}\ X \vee \mathsf{atleast3}\ Y$.

Proof. Use Proposition 3.78 and Proposition 3.32. □

Proposition 3.81. $\forall XYZ. Z \subseteq X \cup Y \Rightarrow \forall w \in X. w \notin Z \Rightarrow \mathsf{atleast4}\ Z \Rightarrow \mathsf{atleast4}\ ((X \setminus \{w\}) \cup Y)$.

Proof. Use Proposition 3.76 and Lemma 3.23. □

Proposition 3.82. $\forall XYZ. Z \subseteq X \cup Y \Rightarrow \forall w \in X. w \notin Z \Rightarrow \mathsf{atleast4}\ Z \Rightarrow \mathsf{atleast4}\ X \vee \mathsf{atleast2}\ Y$.

Proof. Use Proposition 3.81, Proposition 3.43, Proposition 3.35, Proposition 3.78 and Definition 1.5. □

Proposition 3.83. $\forall XY. \mathsf{atleast5}\ (X \cup Y) \Rightarrow \mathsf{atleast4}\ X \vee \mathsf{atleast2}\ Y$.

Proof. Use Proposition 3.47, Proposition 3.82, Proposition 3.16, Lemma 3.23, Definition 1.6 and Definition 1.1. □

Lemma 3.33. $\forall X. \forall x \in X. \mathsf{atleast5}\ X \Rightarrow \mathsf{atleast4}\ (X \setminus \{x\})$.

Proof. Use Proposition 3.65, Lemma 3.24, Proposition 3.73 and Proposition 3.83. □

Proposition 3.84. $\forall X. \forall x \in X. \mathsf{exactly5}\ X \Rightarrow \mathsf{exactly4}\ (X \setminus \{x\})$.

Proof. Use Proposition 3.50, Lemma 3.33, Definition 1.12 and Definition 1.11. □

Proposition 3.85. $\forall X x. \neg \mathsf{atleast4}\ X \Rightarrow \neg \mathsf{atleast5}\ (X \cup \{x\})$.

Proof. Use Lemma 3.24 and Proposition 3.83. □

Proposition 3.86. $\forall XY. \mathsf{atleast5}\ (X \cup Y) \Rightarrow \mathsf{atleast2}\ X \vee \mathsf{atleast4}\ Y$.

Proof. Use Proposition 3.83 and Proposition 3.32. □

Proposition 3.87. $\forall XYZ. Z \subseteq X \cup Y \Rightarrow \forall w \in X. w \notin Z \Rightarrow \mathsf{atleast4}\ Z \Rightarrow \mathsf{atleast3}\ X \vee \mathsf{atleast3}\ Y$.

Proof. Use Proposition 3.81, Proposition 3.43, Proposition 3.35, Proposition 3.80 and Definition 1.4. □

Proposition 3.88. $\forall XYZ. Z \subseteq X \cup Y \Rightarrow \forall w \in Y. w \notin Z \Rightarrow \mathsf{atleast4}\ Z \Rightarrow \mathsf{atleast3}\ X \vee \mathsf{atleast3}\ Y$.

Proof. Use Proposition 3.87 and Proposition 3.32. □

Proposition 3.89. $\forall XY. \mathsf{atleast5}\ (X \cup Y) \Rightarrow \mathsf{atleast3}\ X \vee \mathsf{atleast3}\ Y$.

Proof. Use Proposition 3.88, Proposition 3.87, Proposition 3.16, Lemma 3.23, Proposition 3.59, Definition 1.6 and Definition 1.1. □

Proposition 3.90. $\forall XYZ.Z \subseteq X \cup Y \Rightarrow \forall w \in X.w \notin Z \Rightarrow \mathsf{atleast5}\ Z \Rightarrow \mathsf{atleast5}\ ((X \setminus \{w\}) \cup Y)$.

Proof. Use Proposition 3.76 and Proposition 3.57. □

Lemma 3.34. $\forall XYZ.Z \subseteq X \cup Y \Rightarrow \forall w \in X.w \notin Z \Rightarrow \mathsf{atleast5}\ Z \Rightarrow \mathsf{atleast5}\ X \vee \mathsf{atleast2}\ Y$.

Proof. Use Proposition 3.90, Proposition 3.43, Proposition 3.35, Proposition 3.83 and Definition 1.6. □

Proposition 3.91. $\forall XY.\mathsf{atleast6}\ (X \cup Y) \Rightarrow \mathsf{atleast5}\ X \vee \mathsf{atleast2}\ Y$.

Proof. Use Proposition 3.47, Lemma 3.34, Proposition 3.16, Proposition 3.57, Definition 1.7 and Definition 1.1. □

Proposition 3.92. $\forall X.\forall x \in X.\mathsf{atleast6}\ X \Rightarrow \mathsf{atleast5}\ (X \setminus \{x\})$.

Proof. Use Proposition 3.65, Lemma 3.24, Proposition 3.73 and Proposition 3.91. □

Proposition 3.93. $\forall X.\forall x \in X.\mathsf{exactly6}\ X \Rightarrow \mathsf{exactly5}\ (X \setminus \{x\})$.

Proof. Use Proposition 3.51, Proposition 3.92, Definition 1.13 and Definition 1.12. □

Proposition 3.94. $\forall Xx.\neg\mathsf{atleast5}\ X \Rightarrow \neg\mathsf{atleast6}\ (X \cup \{x\})$.

Proof. Use Lemma 3.24 and Proposition 3.91. □

Proposition 3.95. $\forall XYZ.Z \subseteq X \cup Y \Rightarrow \forall w \in X.w \notin Z \Rightarrow \mathsf{atleast6}\ Z \Rightarrow \mathsf{atleast6}\ ((X \setminus \{w\}) \cup Y)$.

Proof. Use Proposition 3.76 and Proposition 3.58. □

Proposition 3.96. $\forall XYZ.Z \subseteq X \cup Y \Rightarrow \forall w \in X.w \notin Z \Rightarrow \mathsf{atleast6}\ Z \Rightarrow \mathsf{atleast6}\ X \vee \mathsf{atleast2}\ Y$.

Proof. Use Proposition 3.95, Proposition 3.43, Proposition 3.35, Proposition 3.91 and Definition 1.7. □

Proposition 3.97. $\forall XY.\mathsf{atleast7}\ (X \cup Y) \Rightarrow \mathsf{atleast6}\ X \vee \mathsf{atleast2}\ Y$.

Proof. Use Proposition 3.47, Proposition 3.96, Proposition 3.16, Proposition 3.58, Definition 1.8 and Definition 1.1. □

Proposition 3.98. $\forall Xx.\neg\mathsf{atleast6}\ X \Rightarrow \neg\mathsf{atleast7}\ (X \cup \{x\})$.

Proof. Use Lemma 3.24 and Proposition 3.97. □

Lemma 3.35. $\forall X.\forall x \in X.\mathsf{atleast3}\ (X \setminus \{x\}) \Rightarrow \mathsf{atleast4}\ X$.

Proof. Use Proposition 3.43, Proposition 3.35 and Definition 1.5. □

Lemma 3.36. $\forall X.\forall x \in X.\mathsf{atleast4}\ (X \setminus \{x\}) \Rightarrow \mathsf{atleast5}\ X$.

Proof. Use Proposition 3.43, Proposition 3.35 and Definition 1.6. □

Proposition 3.99. $\forall XY.\mathsf{disj}\ X\ Y \Rightarrow \mathsf{exactly2}\ X \Rightarrow \mathsf{exactly2}\ Y \Rightarrow \mathsf{exactly4}\ (X \cup Y)$.

Proof. Use Proposition 3.89, Lemma 3.26, Definition 1.9 and Definition 1.11. □

Proposition 3.100. $\forall XY.\forall z \in X \cup Y.\mathsf{atleast3}\ ((X \cup Y) \setminus \{z\}) \Rightarrow \mathsf{atleast3}\ X \vee \mathsf{atleast2}\ Y$.

Proof. Use Lemma 3.35 and Proposition 3.78. □

Proposition 3.101. $\forall XY.\forall z \in X \cup Y.\mathsf{atleast4}\ ((X \cup Y) \setminus \{z\}) \Rightarrow \mathsf{atleast2}\ X \vee \mathsf{atleast4}\ Y$.

Proof. Use Lemma 3.36 and Proposition 3.86. □

Lemma 3.37. $\forall XY.\forall z \in X \cup Y.\mathsf{atleast4}\ ((X \cup Y) \setminus \{z\}) \Rightarrow \mathsf{atleast3}\ X \vee \mathsf{atleast3}\ Y$.

Proof. Use Lemma 3.36 and Proposition 3.89. □

Proposition 3.102. $\forall XY.\mathsf{atleast5}\ X \Rightarrow \neg\mathsf{atleast3}\ (X \cap Y) \Rightarrow \mathsf{atleast3}\ (X \setminus Y)$.

Proof. Use Proposition 3.65 and Proposition 3.89. □

Proposition 3.103. $\forall XY.\mathsf{exactly5}\ X \Rightarrow \mathsf{exactly2}\ (X \cap Y) \Rightarrow \mathsf{exactly3}\ (X \setminus Y)$.

Proof. Use Proposition 3.66, Proposition 3.102, Definition 1.12, Definition 1.9 and Definition 1.10. □

Lemma 3.38. $0 \notin 0$.

Proof. Use Axiom 1.4. □

Proposition 3.104. $1 \notin 0$.

Proof. Use Axiom 1.4. □

Proposition 3.105. $2 \notin 0$.

Proof. Use Axiom 1.4. □

Proposition 3.106. $1 \notin 1$.

Proof. Use Axiom 1.2. □

Proposition 3.107. $2 \notin 1$.

Proof. Use Lemma 3.1 and Axiom 1.3. □

Lemma 3.39. $2 \notin 2$.

Proof. Use Axiom 1.2. □

Lemma 3.40. $3 \notin 2$.

Proof. Use Proposition 3.5 and Axiom 1.3. □

Proposition 3.108. $3 \notin 3$.

Proof. Use Axiom 1.2. □

Proposition 3.109. $1 \subseteq 2$.

Proof. Use Proposition 3.3, Proposition 3.2 and Definition 1.1. □

Lemma 3.41. $2 \nsubseteq 1$.

Proof. Use Lemma 3.1, Proposition 3.106 and Definition 1.1. □

Proposition 3.110. $2 \subseteq 3$.

Proof. Use Proposition 3.4, Lemma 3.3, Lemma 3.2 and Definition 1.1. □

Proposition 3.111. $3 \nsubseteq 2$.

Proof. Use Proposition 3.5, Axiom 1.2 and Definition 1.1. □

Proposition 3.112. $3 \subseteq 4$.

Proof. Use Proposition 3.8, Proposition 3.7, Lemma 3.4, Proposition 3.6 and Definition 1.1. □

Lemma 3.42. $4 \nsubseteq 3$.

Proof. Use Proposition 3.9, Axiom 1.2 and Definition 1.1. □

Proposition 3.113. $0 \neq 1$.

Proof. Use Proposition 3.1 and Axiom 1.4. □

Lemma 3.43. $0 \neq 2$.

Proof. Use Proposition 3.3 and Axiom 1.4. □

Proposition 3.114. $1 \neq 2$.

Proof. Use Lemma 3.1 and Axiom 1.2. □

Lemma 3.44. $0 \neq 3$.

Proof. Use Lemma 3.3 and Axiom 1.4. □

Proposition 3.115. $1 \neq 3$.

Proof. Use Proposition 3.5, Lemma 3.1 and Axiom 1.3. □

Lemma 3.45. $2 \neq 3$.

Proof. Use Proposition 3.5 and Axiom 1.2. □

Proposition 3.116. $1 \not\subseteq 0$.

Proof. Use Proposition 3.1, Axiom 1.4 and Definition 1.1. □

Proposition 3.117. $2 \not\subseteq 0$.

Proof. Use Proposition 3.3, Axiom 1.4 and Definition 1.1. □

Proposition 3.118. $\forall X \subseteq 2.0 \notin X \Rightarrow 1 \notin X \Rightarrow X = 0$.

Proof. Use Lemma 3.2, Proposition 3.25 and Definition 1.1. □

Proposition 3.119. $\forall X \subseteq 2.1 \notin X \Rightarrow X \subseteq 1$.

Proof. Use Proposition 3.1, Lemma 3.2 and Definition 1.1. □

Proposition 3.120. $\forall X.0 \in X \Rightarrow 1 \subseteq X$.

Proof. Use Proposition 3.2 and Definition 1.1. □

Proposition 3.121. $\forall X \subseteq 2.0 \in X \Rightarrow 1 \notin X \Rightarrow X = 1$.

Proof. Use Proposition 3.120, Proposition 3.119 and Axiom 1.1. □

Proposition 3.122. $\forall X \subseteq 2.0 \notin X \Rightarrow X \subseteq \{1\}$.

Proof. Use Proposition 3.13, Lemma 3.2 and Definition 1.1. □

Proposition 3.123. $\forall X \subseteq 2.0 \notin X \Rightarrow 1 \in X \Rightarrow X = \{1\}$.

Proof. Use Proposition 3.48, Proposition 3.122 and Axiom 1.1. □

Proposition 3.124. $\forall X \subseteq 2.0 \in X \Rightarrow 1 \in X \Rightarrow X = 2$.

Proof. Use Lemma 3.2, Axiom 1.1 and Definition 1.1. □

Proposition 3.125. $\forall X \subseteq 2.X = 0 \vee X = 1 \vee X = \{1\} \vee X = 2$.

Proof. Use Proposition 3.118, Proposition 3.121, Proposition 3.123 and Proposition 3.124. □

Proposition 3.126. $\forall x.\forall X \subseteq \{x\}.X = 0 \vee X = \{x\}$.

Proof. Use Proposition 3.25, Proposition 3.14, Axiom 1.1 and Definition 1.1. □

Lemma 3.46. $\forall X \subseteq \{1\}.X = 0 \vee X = \{1\}$.

Proof. Use Proposition 3.126. □

Lemma 3.47. $\forall X \subseteq \{\{1\}\}.X = 0 \vee X = \{\{1\}\}$.

Proof. Use Proposition 3.126. □

Proposition 3.127. $\{1\} \notin 0$.

Proof. Use Axiom 1.4. □

Proposition 3.128. $1 \in \{1\}$.

Proof. Use Proposition 3.13. □

Proposition 3.129. $\{1\} \in \{\{1\}\}$.

Proof. Use Proposition 3.13. □

Lemma 3.48. $0 \in (\wp\ \{1\})$.

Proof. Use Proposition 3.26. □

Proposition 3.130. $1 \in (\{1\} \cup \{\{1\}\})$.

Proof. Use Proposition 3.128 and Proposition 3.15. □

Lemma 3.49. $0 \in (\wp\ 2)$.

Proof. Use Proposition 3.26. □

Lemma 3.50. $0 \notin \{2\}$.

Proof. Use Lemma 3.43 and Lemma 3.15. □

Proposition 3.131. $0 \in ((\wp\ 2) \setminus \{2\})$.

Proof. Use Lemma 3.50, Lemma 3.49 and Proposition 3.20. □

Proposition 3.132. $2 \in \{2\}$.

Proof. Use Proposition 3.13. □

Proposition 3.133. $0 \notin \{1\}$.

Proof. Use Proposition 3.113 and Lemma 3.15. □

Proposition 3.134. $0 \in (3 \setminus \{1\})$.

Proof. Use Proposition 3.133, Lemma 3.3 and Proposition 3.20. □

Proposition 3.135. $1 \in (\wp\ 2)$.

Proof. Use Proposition 3.109 and Proposition 3.11. □

Lemma 3.51. $1 \not\subseteq \{1\}$.

Proof. Use Proposition 3.1, Proposition 3.133 and Definition 1.1. □

Lemma 3.52. $1 \notin (\wp\ \{1\})$.

Proof. Use Proposition 3.12 and Lemma 3.51. ☐

Proposition 3.136. $1 \in ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Lemma 3.52, Proposition 3.135 and Proposition 3.20. ☐

Proposition 3.137. $\forall x \in \{1\}.x = 1$.

Proof. Use Proposition 3.14. ☐

Lemma 3.53. $\{1\} \subseteq 2$.

Proof. Use Proposition 3.137, Lemma 3.1 and Definition 1.1. ☐

Proposition 3.138. $\{1\} \in (\wp\ 2)$.

Proof. Use Lemma 3.53 and Proposition 3.11. ☐

Proposition 3.139. $0 \neq \{1\}$.

Proof. Use Proposition 3.128 and Proposition 3.104. ☐

Proposition 3.140. $1 \neq \{1\}$.

Proof. Use Proposition 3.1 and Proposition 3.133. ☐

Proposition 3.141. $\{1\} \notin 2$.

Proof. Use Proposition 3.140, Proposition 3.139 and Lemma 3.2. ☐

Lemma 3.54. $\{1\} \in ((\wp\ 2) \setminus 2)$.

Proof. Use Proposition 3.141, Proposition 3.138 and Proposition 3.20. ☐

Proposition 3.142. $0 \in ((\wp\ 2) \setminus \{1\})$.

Proof. Use Proposition 3.133, Lemma 3.49 and Proposition 3.20. ☐

Proposition 3.143. $1 \not\subseteq \{2\}$.

Proof. Use Proposition 3.1, Lemma 3.50 and Definition 1.1. ☐

Proposition 3.144. $1 \notin (\wp\ \{2\})$.

Proof. Use Proposition 3.12 and Proposition 3.143. ☐

Proposition 3.145. $1 \in ((\wp\ 2) \setminus (\wp\ \{2\}))$.

Proof. Use Proposition 3.144, Proposition 3.135 and Proposition 3.20. ☐

Proposition 3.146. $\{1\} \notin 1$.

Proof. Use Proposition 3.128 and Axiom 1.3. ☐

Proposition 3.147. $0 \neq \{\{1\}\}$.

Proof. Use Proposition 3.129 and Proposition 3.127. ☐

Proposition 3.148. $0 \neq (\wp \{1\})$.

Proof. Use Lemma 3.48 and Lemma 3.38. ☐

Proposition 3.149. $1 \notin \{2\}$.

Proof. Use Proposition 3.114 and Lemma 3.15. ☐

Lemma 3.55. $1 \in ((\wp 2) \setminus \{2\})$.

Proof. Use Proposition 3.149, Proposition 3.135 and Proposition 3.20. ☐

Proposition 3.150. $0 \neq \{2\}$.

Proof. Use Proposition 3.132 and Proposition 3.105. ☐

Proposition 3.151. $\{2\} \notin 1$.

Proof. Use Proposition 3.2 and Proposition 3.150. ☐

Lemma 3.56. $0 \neq (3 \setminus \{1\})$.

Proof. Use Proposition 3.134 and Lemma 3.38. ☐

Proposition 3.152. $0 \neq ((\wp 2) \setminus 2)$.

Proof. Use Lemma 3.54 and Proposition 3.127. ☐

Proposition 3.153. $0 \notin \{\{1\}\}$.

Proof. Use Proposition 3.139 and Lemma 3.15. ☐

Proposition 3.154. $\{1\} \in (\wp \{1\})$.

Proof. Use Proposition 3.27. ☐

Proposition 3.155. $1 \subseteq (\wp \{1\})$.

Proof. Use Proposition 3.2, Lemma 3.48 and Definition 1.1. ☐

Lemma 3.57. $\{1\} \in (\{1\} \cup \{\{1\}\})$.

Proof. Use Proposition 3.129 and Lemma 3.5. ☐

Lemma 3.58. $0 \notin (\{1\} \cup \{\{1\}\})$.

Proof. Use Proposition 3.153, Proposition 3.133 and Proposition 3.45. ☐

Proposition 3.156. $\{1\} \neq 2$.

Proof. Use Proposition 3.3 and Proposition 3.133. □

Proposition 3.157. $\{1\} \notin \{2\}$.

Proof. Use Proposition 3.156 and Lemma 3.15. □

Lemma 3.59. $\{1\} \in ((\wp\ 2) \setminus \{2\})$.

Proof. Use Proposition 3.157, Proposition 3.138 and Proposition 3.20. □

Proposition 3.158. $2 \notin \{1\}$.

Proof. Use Proposition 3.114 and Lemma 3.15. □

Proposition 3.159. $2 \in (3 \setminus \{1\})$.

Proof. Use Proposition 3.158, Proposition 3.5 and Proposition 3.20. □

Proposition 3.160. $1 \subseteq (3 \setminus \{1\})$.

Proof. Use Proposition 3.2, Proposition 3.134 and Definition 1.1. □

Proposition 3.161. $2 \in (\wp\ 2)$.

Proof. Use Proposition 3.27. □

Proposition 3.162. $2 \not\subseteq \{1\}$.

Proof. Use Proposition 3.3, Proposition 3.133 and Definition 1.1. □

Proposition 3.163. $2 \notin (\wp\ \{1\})$.

Proof. Use Proposition 3.12 and Proposition 3.162. □

Proposition 3.164. $2 \in ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Proposition 3.163, Proposition 3.161 and Proposition 3.20. □

Proposition 3.165. $2 \in ((\wp\ 2) \setminus 2)$.

Proof. Use Lemma 3.39, Proposition 3.161 and Proposition 3.20. □

Proposition 3.166. $0 \notin ((\wp\ 2) \setminus 2)$.

Proof. Use Proposition 3.3 and Lemma 3.17. □

Proposition 3.167. $2 \in ((\wp\ 2) \setminus \{1\})$.

Proof. Use Proposition 3.158, Proposition 3.161 and Proposition 3.20. □

Proposition 3.168. $2 \not\subseteq \{2\}$.

Proof. Use Lemma 3.1, Proposition 3.149 and Definition 1.1. □

Proposition 3.169. $2 \notin (\wp\ \{2\})$.

Proof. Use Proposition 3.12 and Proposition 3.168. □

Proposition 3.170. $2 \in ((\wp\ 2) \setminus (\wp\ \{2\}))$.

Proof. Use Proposition 3.169, Proposition 3.161 and Proposition 3.20. □

Proposition 3.171. $0 \in (\wp\ \{2\})$.

Proof. Use Proposition 3.26. □

Proposition 3.172. $0 \notin ((\wp\ 2) \setminus (\wp\ \{2\}))$.

Proof. Use Proposition 3.171 and Lemma 3.17. □

Proposition 3.173. $\{1\} \notin \{1\}$.

Proof. Use Axiom 1.2. □

Proposition 3.174. $1 \neq \{2\}$.

Proof. Use Proposition 3.1 and Lemma 3.50. □

Proposition 3.175. $1 \neq (3 \setminus \{1\})$.

Proof. Use Proposition 3.159 and Proposition 3.107. □

Lemma 3.60. $3 \notin \{1\}$.

Proof. Use Proposition 3.115 and Lemma 3.15. □

Proposition 3.176. $\{1\} \in ((\wp\ 2) \setminus \{1\})$.

Proof. Use Proposition 3.173, Proposition 3.138 and Proposition 3.20. □

Lemma 3.61. $\{1\} \nsubseteq \{2\}$.

Proof. Use Proposition 3.128, Proposition 3.149 and Definition 1.1. □

Proposition 3.177. $\{1\} \notin (\wp\ \{2\})$.

Proof. Use Proposition 3.12 and Lemma 3.61. □

Lemma 3.62. $\{1\} \in ((\wp\ 2) \setminus (\wp\ \{2\}))$.

Proof. Use Proposition 3.177, Proposition 3.138 and Proposition 3.20. □

Proposition 3.178. $1 \notin \{\{1\}\}$.

Proof. Use Proposition 3.140 and Lemma 3.15. □

Proposition 3.179. $\{1\} \nsubseteq \{\{1\}\}$.

Proof. Use Proposition 3.128, Proposition 3.178 and Definition 1.1. ◻

Proposition 3.180. $\{1\} \subseteq (\{1\} \cup \{\{1\}\})$.

Proof. Use Proposition 3.28. ◻

Lemma 3.63. $\{1\} \subseteq ((\wp\ 2) \setminus \{2\})$.

Proof. Use Proposition 3.137, Lemma 3.55 and Definition 1.1. ◻

Proposition 3.181. $1 \notin (3 \setminus \{1\})$.

Proof. Use Proposition 3.128 and Lemma 3.17. ◻

Proposition 3.182. $1 \notin ((\wp\ 2) \setminus 2)$.

Proof. Use Lemma 3.1 and Lemma 3.17. ◻

Proposition 3.183. $1 \notin ((\wp\ 2) \setminus \{1\})$.

Proof. Use Proposition 3.128 and Lemma 3.17. ◻

Proposition 3.184. $1 \neq \{\{1\}\}$.

Proof. Use Proposition 3.1 and Proposition 3.153. ◻

Proposition 3.185. $\{\{1\}\} \notin 2$.

Proof. Use Proposition 3.184, Proposition 3.147 and Lemma 3.2. ◻

Proposition 3.186. $1 \neq (\wp\ \{1\})$.

Proof. Use Proposition 3.154 and Proposition 3.146. ◻

Proposition 3.187. $0 \neq (\{1\} \cup \{\{1\}\})$.

Proof. Use Proposition 3.130 and Proposition 3.104. ◻

Proposition 3.188. $\{2\} \notin 2$.

Proof. Use Proposition 3.132 and Axiom 1.3. ◻

Lemma 3.64. $(3 \setminus \{1\}) \notin 2$.

Proof. Use Proposition 3.159 and Axiom 1.3. ◻

Proposition 3.189. $(\wp\ 2) \notin 2$.

Proof. Use Proposition 3.161 and Axiom 1.3. ◻

Proposition 3.190. $\{2\} \not\subseteq 2$.

Proof. Use Proposition 3.132, Lemma 3.39 and Definition 1.1. ◻

Proposition 3.191. $(3 \setminus \{1\}) \not\subseteq 2$.

Proof. Use Proposition 3.159, Lemma 3.39 and Definition 1.1. □

Proposition 3.192. $2 \not\subseteq (3 \setminus \{1\})$.

Proof. Use Lemma 3.1, Proposition 3.181 and Definition 1.1. □

Lemma 3.65. $((\wp\ 2) \setminus 2) \not\subseteq 2$.

Proof. Use Proposition 3.165, Lemma 3.39 and Definition 1.1. □

Proposition 3.193. $((\wp\ 2) \setminus (\wp\ \{2\})) \not\subseteq 2$.

Proof. Use Proposition 3.170, Lemma 3.39 and Definition 1.1. □

Lemma 3.66. $2 \notin \{\{1\}\}$.

Proof. Use Proposition 3.156 and Lemma 3.15. □

Lemma 3.67. $\{1\} \neq (\{1\} \cup \{\{1\}\})$.

Proof. Use Lemma 3.57 and Proposition 3.173. □

Proposition 3.194. $\{1\} \neq \{2\}$.

Proof. Use Proposition 3.128 and Proposition 3.149. □

Proposition 3.195. $\{2\} \notin \{\{1\}\}$.

Proof. Use Proposition 3.194 and Lemma 3.15. □

Proposition 3.196. $\{1\} \neq (3 \setminus \{1\})$.

Proof. Use Proposition 3.128 and Proposition 3.181. □

Proposition 3.197. $(3 \setminus \{1\}) \notin \{\{1\}\}$.

Proof. Use Proposition 3.196 and Lemma 3.15. □

Proposition 3.198. $\{1\} \neq 3$.

Proof. Use Proposition 3.5 and Proposition 3.158. □

Proposition 3.199. $\{1\} \neq ((\wp\ 2) \setminus 2)$.

Proof. Use Proposition 3.128 and Proposition 3.182. □

Proposition 3.200. $((\wp\ 2) \setminus 2) \notin \{\{1\}\}$.

Proof. Use Proposition 3.199 and Lemma 3.15. □

Lemma 3.68. $\{1\} \neq (\wp\ 2)$.

Proof. Use Proposition 3.161 and Proposition 3.158. □

Proposition 3.201. $(\wp\ 2) \notin \{\{1\}\}$.

Proof. Use Lemma 3.68 and Lemma 3.15. □

Proposition 3.202. $\{1\} \neq \{\{1\}\}$.

Proof. Use Proposition 3.128 and Proposition 3.178. □

Proposition 3.203. $2 \neq \{\{1\}\}$.

Proof. Use Lemma 3.1 and Proposition 3.178. □

Proposition 3.204. $\forall x \in \{\{1\}\}.x = \{1\}$.

Proof. Use Proposition 3.14. □

Proposition 3.205. $\{\{1\}\} \subseteq (\wp\ \{1\})$.

Proof. Use Proposition 3.204, Proposition 3.154 and Definition 1.1. □

Proposition 3.206. $\{\{1\}\} \subseteq (\{1\} \cup \{\{1\}\})$.

Proof. Use Lemma 3.8. □

Lemma 3.69. $\{\{1\}\} \subseteq ((\wp\ 2) \setminus \{2\})$.

Proof. Use Proposition 3.204, Lemma 3.59 and Definition 1.1. □

Proposition 3.207. $\{1\} \notin 3$.

Proof. Use Proposition 3.156, Proposition 3.140, Proposition 3.139 and Proposition 3.6. □

Proposition 3.208. $\{1\} \notin (3 \setminus \{1\})$.

Proof. Use Proposition 3.207 and Lemma 3.16. □

Lemma 3.70. $\{1\} \notin ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Proposition 3.154 and Lemma 3.17. □

Proposition 3.209. $(\wp\ 2) \not\subseteq \{\{1\}\}$.

Proof. Use Proposition 3.161, Lemma 3.66 and Definition 1.1. □

Proposition 3.210. $2 \neq (\wp\ \{1\})$.

Proof. Use Lemma 3.1 and Lemma 3.52. □

Proposition 3.211. $\{\{1\}\} \neq (\wp\ \{1\})$.

Proof. Use Lemma 3.48 and Proposition 3.153. ☐

Proposition 3.212. $\forall x \in (\wp\ \{1\}).x = 0 \vee x = \{1\}$.

Proof. Use Proposition 3.12 and Lemma 3.46. ☐

Lemma 3.71. $\forall X.\{1\} \in X \Rightarrow 0 \in X \Rightarrow (\wp\ \{1\}) \subseteq X$.

Proof. Use Proposition 3.212 and Definition 1.1. ☐

Proposition 3.213. $\forall X \subseteq (\wp\ \{1\}).\{1\} \in X \Rightarrow 0 \in X \Rightarrow X = (\wp\ \{1\})$.

Proof. Use Lemma 3.71 and Axiom 1.1. ☐

Proposition 3.214. $\forall X \subseteq (\wp\ \{1\}).\{1\} \in X \Rightarrow 0 \in X \Rightarrow X = 0 \vee X = 1 \vee X = \{\{1\}\} \vee X = (\wp\ \{1\})$.

Proof. Use Proposition 3.213. ☐

Lemma 3.72. $\forall X \subseteq (\wp\ \{1\}).0 \notin X \Rightarrow X \subseteq \{\{1\}\}$.

Proof. Use Proposition 3.13, Proposition 3.212 and Definition 1.1. ☐

Lemma 3.73. $\forall X \subseteq (\wp\ \{1\}).\{1\} \in X \Rightarrow 0 \notin X \Rightarrow X = \{\{1\}\}$.

Proof. Use Proposition 3.48, Lemma 3.72 and Axiom 1.1. ☐

Proposition 3.215. $\forall X \subseteq (\wp\ \{1\}).\{1\} \in X \Rightarrow 0 \notin X \Rightarrow X = 0 \vee X = 1 \vee X = \{\{1\}\} \vee X = (\wp\ \{1\})$.

Proof. Use Lemma 3.73. ☐

Lemma 3.74. $\forall X \subseteq (\wp\ \{1\}).\{1\} \notin X \Rightarrow X \subseteq 1$.

Proof. Use Proposition 3.1, Proposition 3.212 and Definition 1.1. ☐

Proposition 3.216. $\forall X \subseteq (\wp\ \{1\}).\{1\} \notin X \Rightarrow 0 \in X \Rightarrow X = 1$.

Proof. Use Proposition 3.120, Lemma 3.74 and Axiom 1.1. ☐

Proposition 3.217. $\forall X \subseteq (\wp\ \{1\}).\{1\} \notin X \Rightarrow 0 \in X \Rightarrow X = 0 \vee X = 1 \vee X = \{\{1\}\} \vee X = (\wp\ \{1\})$.

Proof. Use Proposition 3.216. ☐

Proposition 3.218. $\forall X \subseteq (\wp\ \{1\}).\{1\} \notin X \Rightarrow 0 \notin X \Rightarrow X = 0$.

Proof. Use Proposition 3.212, Proposition 3.25 and Definition 1.1. ☐

Proposition 3.219. $\forall X \subseteq (\wp\ \{1\}).\{1\} \notin X \Rightarrow 0 \notin X \Rightarrow X = 0 \vee X = 1 \vee X = \{\{1\}\} \vee X = (\wp\ \{1\})$.

Proof. Use Proposition 3.218. ☐

Proposition 3.220. $\forall X \subseteq (\wp\ \{1\}).\{1\} \in X \Rightarrow X = 0 \vee X = 1 \vee X = \{\{1\}\} \vee X = (\wp\ \{1\})$.

Proof. Use Proposition 3.215 and Proposition 3.214. ☐

Lemma 3.75. $\forall X \subseteq (\wp\ \{1\}).\{1\} \notin X \Rightarrow X = 0 \vee X = 1 \vee X = \{\{1\}\} \vee X = (\wp\ \{1\})$.

Proof. Use Proposition 3.219 and Proposition 3.217. □

Lemma 3.76. $\forall X \subseteq (\wp\ \{1\}).X = 0 \vee X = 1 \vee X = \{\{1\}\} \vee X = (\wp\ \{1\})$.

Proof. Use Lemma 3.75 and Proposition 3.220. □

Lemma 3.77. $(\wp\ 2) \nsubseteq (\wp\ \{1\})$.

Proof. Use Proposition 3.161, Proposition 3.163 and Definition 1.1. □

Proposition 3.221. $2 \notin (\{1\} \cup \{\{1\}\})$.

Proof. Use Lemma 3.66, Proposition 3.158 and Proposition 3.45. □

Proposition 3.222. $\{\{1\}\} \neq (\{1\} \cup \{\{1\}\})$.

Proof. Use Proposition 3.130 and Proposition 3.178. □

Proposition 3.223. $((\wp\ 2) \setminus \{2\}) \nsubseteq (\{1\} \cup \{\{1\}\})$.

Proof. Use Proposition 3.131, Lemma 3.58 and Definition 1.1. □

Proposition 3.224. $(\{1\} \cup \{\{1\}\}) \subseteq ((\wp\ 2) \setminus \{2\})$.

Proof. Use Lemma 3.69, Lemma 3.63 and Lemma 3.9. □

Proposition 3.225. $\{2\} \notin (\wp\ 2)$.

Proof. Use Proposition 3.12 and Proposition 3.190. □

Proposition 3.226. $(3 \setminus \{1\}) \notin (\wp\ 2)$.

Proof. Use Proposition 3.12 and Proposition 3.191. □

Lemma 3.78. $3 \notin (\wp\ 2)$.

Proof. Use Proposition 3.12 and Proposition 3.111. □

Proposition 3.227. $((\wp\ 2) \setminus 2) \notin (\wp\ 2)$.

Proof. Use Proposition 3.12 and Lemma 3.65. □

Proposition 3.228. $((\wp\ 2) \setminus (\wp\ \{2\})) \notin (\wp\ 2)$.

Proof. Use Proposition 3.12 and Proposition 3.193. □

Proposition 3.229. $((\wp\ 2) \setminus (\wp\ \{2\})) \notin ((\wp\ 2) \setminus \{2\})$.

Proof. Use Proposition 3.228 and Lemma 3.16. □

Lemma 3.79. $(\wp\ 2) \notin (\wp\ 2)$.

Proof. Use Axiom 1.2. ☐

Proposition 3.230. $2 \neq (3 \setminus \{1\})$.

Proof. Use Lemma 3.1 and Proposition 3.181. ☐

Proposition 3.231. $(3 \setminus \{1\}) \notin \{2\}$.

Proof. Use Proposition 3.230 and Lemma 3.15. ☐

Proposition 3.232. $3 \notin \{2\}$.

Proof. Use Lemma 3.45 and Lemma 3.15. ☐

Proposition 3.233. $2 \neq ((\wp\ 2) \setminus 2)$.

Proof. Use Lemma 3.1 and Proposition 3.182. ☐

Proposition 3.234. $2 \neq (\wp\ 2)$.

Proof. Use Proposition 3.161 and Lemma 3.39. ☐

Proposition 3.235. $(\wp\ 2) \notin \{2\}$.

Proof. Use Proposition 3.234 and Lemma 3.15. ☐

Lemma 3.80. $2 \neq \{2\}$.

Proof. Use Lemma 3.1 and Proposition 3.149. ☐

Proposition 3.236. $(3 \setminus \{1\}) \nsubseteq \{2\}$.

Proof. Use Proposition 3.134, Lemma 3.50 and Definition 1.1. ☐

Proposition 3.237. $\forall x \in \{2\}.x = 2$.

Proof. Use Proposition 3.14. ☐

Proposition 3.238. $3 \nsubseteq \{2\}$.

Proof. Use Proposition 3.4, Proposition 3.149 and Definition 1.1. ☐

Proposition 3.239. $\{2\} \subseteq ((\wp\ 2) \setminus (\wp\ \{2\}))$.

Proof. Use Proposition 3.237, Proposition 3.170 and Definition 1.1. ☐

Proposition 3.240. $(\wp\ 2) \nsubseteq \{2\}$.

Proof. Use Proposition 3.138, Proposition 3.157 and Definition 1.1. ☐

Proposition 3.241. $\{\{1\}\} \notin 3$.

Proof. Use Proposition 3.203, Proposition 3.184, Proposition 3.147 and Proposition 3.6. □

Proposition 3.242. $(\wp\ \{1\}) \notin 3$.

Proof. Use Proposition 3.210, Proposition 3.186, Proposition 3.148 and Proposition 3.6. □

Proposition 3.243. $\{2\} \notin 3$.

Proof. Use Lemma 3.80, Proposition 3.174, Proposition 3.150 and Proposition 3.6. □

Proposition 3.244. $\{2\} \notin (3 \setminus \{1\})$.

Proof. Use Proposition 3.243 and Lemma 3.16. □

Proposition 3.245. $3 \notin (3 \setminus \{1\})$.

Proof. Use Proposition 3.108 and Lemma 3.16. □

Proposition 3.246. $1 \neq ((\wp\ 2) \setminus 2)$.

Proof. Use Proposition 3.1 and Proposition 3.166. □

Proposition 3.247. $((\wp\ 2) \setminus 2) \notin 3$.

Proof. Use Proposition 3.233, Proposition 3.246, Proposition 3.152 and Proposition 3.6. □

Proposition 3.248. $0 \neq (\wp\ 2)$.

Proof. Use Lemma 3.49 and Lemma 3.38. □

Proposition 3.249. $1 \neq (\wp\ 2)$.

Proof. Use Proposition 3.161 and Proposition 3.107. □

Lemma 3.81. $(\wp\ 2) \notin 3$.

Proof. Use Proposition 3.234, Proposition 3.249, Proposition 3.248 and Proposition 3.6. □

Proposition 3.250. $\{2\} \neq (3 \setminus \{1\})$.

Proof. Use Proposition 3.134 and Lemma 3.50. □

Proposition 3.251. $\forall x \in (3 \setminus \{1\}).x = 0 \vee x = 2$.

Proof. Use Proposition 3.21, Lemma 3.6, Proposition 3.128 and Proposition 3.6. □

Proposition 3.252. $(3 \setminus \{1\}) \subseteq ((\wp\ 2) \setminus \{1\})$.

Proof. Use Proposition 3.167, Proposition 3.142, Proposition 3.251 and Definition 1.1. □

Lemma 3.82. $(3 \setminus \{1\}) \subseteq (\wp\ 2)$.

Proof. Use Proposition 3.161, Lemma 3.49, Proposition 3.251 and Definition 1.1. □

Lemma 3.83. $\forall x \in (\wp\ 2).x = 0 \vee x = 1 \vee x = \{1\} \vee x = 2$.

Proof. Use Proposition 3.12 and Proposition 3.125. □

Lemma 3.84. $\forall x \in ((\wp\ 2) \setminus (\wp\ \{1\})).x = 1 \vee x = 2$.

Proof. Use Proposition 3.21, Lemma 3.6, Proposition 3.154, Lemma 3.48 and Lemma 3.83. □

Proposition 3.253. $((\wp\ 2) \setminus (\wp\ \{1\})) \subseteq ((\wp\ 2) \setminus (\wp\ \{2\}))$.

Proof. Use Proposition 3.170, Proposition 3.145, Lemma 3.84 and Definition 1.1. □

Proposition 3.254. $\{\{1\}\} \neq 3$.

Proof. Use Proposition 3.129 and Proposition 3.207. □

Proposition 3.255. $(\wp\ \{1\}) \neq 3$.

Proof. Use Proposition 3.154 and Proposition 3.207. □

Proposition 3.256. $\{2\} \neq 3$.

Proof. Use Proposition 3.4 and Proposition 3.149. □

Proposition 3.257. $(3 \setminus \{1\}) \neq 3$.

Proof. Use Proposition 3.4 and Proposition 3.181. □

Proposition 3.258. $3 \subseteq (\wp\ 2)$.

Proof. Use Proposition 3.161, Proposition 3.135, Lemma 3.49, Proposition 3.6 and Definition 1.1. □

Proposition 3.259. $\{2\} \notin ((\wp\ 2) \setminus 2)$.

Proof. Use Proposition 3.225 and Lemma 3.16. □

Lemma 3.85. $\mathsf{disj}\ ((\wp\ 2) \setminus 2)\ (\wp\ \{2\})$.

Proof. Use Proposition 3.259, Proposition 3.166, Lemma 3.28 and Proposition 3.38. □

Proposition 3.260. $\{2\} \neq ((\wp\ 2) \setminus 2)$.

Proof. Use Lemma 3.54 and Proposition 3.157. □

Lemma 3.86. $3 \neq ((\wp\ 2) \setminus 2)$.

Proof. Use Proposition 3.4 and Proposition 3.182. □

Proposition 3.261. $((\wp\ 2) \setminus \{1\}) \not\subseteq ((\wp\ 2) \setminus 2)$.

Proof. Use Proposition 3.142, Proposition 3.166 and Definition 1.1. □

Proposition 3.262. $((\wp\ 2) \setminus 2) \subseteq ((\wp\ 2) \setminus \{1\})$.

Proof. Use Lemma 3.53 and Lemma 3.12. □

Proposition 3.263. $((\wp\ 2) \setminus (\wp\ \{2\})) \not\subseteq ((\wp\ 2) \setminus 2)$.

Proof. Use Proposition 3.145, Proposition 3.182 and Definition 1.1. □

Proposition 3.264. $\forall x \in ((\wp\ 2) \setminus 2).x = \{1\} \vee x = 2$.

Proof. Use Proposition 3.21, Lemma 3.6, Lemma 3.1, Proposition 3.3 and Lemma 3.83. □

Lemma 3.87. $((\wp\ 2) \setminus 2) \subseteq ((\wp\ 2) \setminus (\wp\ \{2\}))$.

Proof. Use Proposition 3.170, Lemma 3.62, Proposition 3.264 and Definition 1.1. □

Proposition 3.265. $\{2\} \notin ((\wp\ 2) \setminus \{1\})$.

Proof. Use Proposition 3.225 and Lemma 3.16. □

Lemma 3.88. $3 \notin ((\wp\ 2) \setminus \{1\})$.

Proof. Use Lemma 3.78 and Lemma 3.16. □

Proposition 3.266. $((\wp\ 2) \setminus (\wp\ \{2\})) \notin ((\wp\ 2) \setminus \{1\})$.

Proof. Use Proposition 3.228 and Lemma 3.16. □

Proposition 3.267. $(3 \setminus \{1\}) \notin ((\wp\ 2) \setminus (\wp\ \{2\}))$.

Proof. Use Proposition 3.226 and Lemma 3.16. □

Proposition 3.268. $(\wp\ 2) \not\subseteq ((\wp\ 2) \setminus (\wp\ \{2\}))$.

Proof. Use Lemma 3.49, Proposition 3.172 and Definition 1.1. □

Proposition 3.269. $((\wp\ 2) \setminus (\wp\ \{2\})) \subseteq (\wp\ 2)$.

Proof. Use Proposition 3.35. □

Proposition 3.270. $\{2\} \neq (\wp\ 2)$.

Proof. Use Proposition 3.138 and Proposition 3.157. □

Proposition 3.271. $(3 \setminus \{1\}) \neq (\wp\ 2)$.

Proof. Use Proposition 3.138 and Proposition 3.208. ☐

Lemma 3.89. $3 \neq (\wp\ 2)$.

Proof. Use Proposition 3.138 and Proposition 3.207. ☐

Proposition 3.272. $((\wp\ 2) \setminus 2) \neq (\wp\ 2)$.

Proof. Use Proposition 3.135 and Proposition 3.182. ☐

Proposition 3.273. $((\wp\ 2) \setminus (\wp\ \{2\})) \neq (\wp\ 2)$.

Proof. Use Lemma 3.49 and Proposition 3.172. ☐

Lemma 3.90. $\{\{1\}\} \in \{\{\{1\}\}\}$.

Proof. Use Proposition 3.13. ☐

Proposition 3.274. $0 \in (\wp\ \{\{1\}\})$.

Proof. Use Proposition 3.26. ☐

Proposition 3.275. $\{1\} \notin (\wp\ \{\{1\}\})$.

Proof. Use Proposition 3.12 and Proposition 3.179. ☐

Proposition 3.276. $\{\{1\}\} \in (\wp\ \{\{1\}\})$.

Proof. Use Proposition 3.27. ☐

Proposition 3.277. $(\wp\ 2) \notin (\wp\ \{\{1\}\})$.

Proof. Use Proposition 3.12 and Proposition 3.209. ☐

Proposition 3.278. $0 \in (\wp\ (\wp\ \{1\}))$.

Proof. Use Proposition 3.26. ☐

Proposition 3.279. $0 \notin \{(\wp\ \{1\})\}$.

Proof. Use Proposition 3.148 and Lemma 3.15. ☐

Proposition 3.280. $0 \in ((\wp\ (\wp\ \{1\})) \setminus \{(\wp\ \{1\})\})$.

Proof. Use Proposition 3.279, Proposition 3.278 and Proposition 3.20. ☐

Lemma 3.91. $1 \in (\wp\ (\wp\ \{1\}))$.

Proof. Use Proposition 3.155 and Proposition 3.11. ☐

Proposition 3.281. $1 \notin \{(\wp\ \{1\})\}$.

Proof. Use Proposition 3.186 and Lemma 3.15. ☐

Proposition 3.282. $1 \in ((\wp\ (\wp\ \{1\})) \setminus \{(\wp\ \{1\})\})$.

Proof. Use Proposition 3.281, Lemma 3.91 and Proposition 3.20. ☐

Lemma 3.92. $\{\{1\}\} \in (\wp\ (\wp\ \{1\}))$.

Proof. Use Proposition 3.205 and Proposition 3.11. ☐

Proposition 3.283. $\{\{1\}\} \notin \{(\wp\ \{1\})\}$.

Proof. Use Proposition 3.211 and Lemma 3.15. ☐

Proposition 3.284. $\{\{1\}\} \in ((\wp\ (\wp\ \{1\})) \setminus \{(\wp\ \{1\})\})$.

Proof. Use Proposition 3.283, Lemma 3.92 and Proposition 3.20. ☐

Proposition 3.285. $(\wp\ \{1\}) \in \{(\wp\ \{1\})\}$.

Proof. Use Proposition 3.13. ☐

Proposition 3.286. $(\wp\ 2) \notin (\wp\ (\wp\ \{1\}))$.

Proof. Use Proposition 3.12 and Lemma 3.77. ☐

Proposition 3.287. $(\wp\ 2) \notin ((\wp\ (\wp\ \{1\})) \setminus \{(\wp\ \{1\})\})$.

Proof. Use Proposition 3.286 and Lemma 3.16. ☐

Proposition 3.288. $\{1\} \in (\{\{1\}\} \cup \{\{\{1\}\}\})$.

Proof. Use Proposition 3.129 and Proposition 3.15. ☐

Lemma 3.93. $\{\{1\}\} \in (\{\{1\}\} \cup \{\{\{1\}\}\})$.

Proof. Use Lemma 3.90 and Lemma 3.5. ☐

Proposition 3.289. $0 \in (\wp\ (\{1\} \cup \{\{1\}\}))$.

Proof. Use Proposition 3.26. ☐

Proposition 3.290. $0 \notin \{(\{1\} \cup \{\{1\}\})\}$.

Proof. Use Proposition 3.187 and Lemma 3.15. ☐

Proposition 3.291. $0 \in ((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\})$.

Proof. Use Proposition 3.290, Proposition 3.289 and Proposition 3.20. ☐

Proposition 3.292. $\{1\} \in (\wp\ (\{1\} \cup \{\{1\}\}))$.

Proof. Use Proposition 3.180 and Proposition 3.11. ☐

Proposition 3.293. $\{1\} \notin \{(\{1\} \cup \{\{1\}\})\}$.

Proof. Use Lemma 3.67 and Lemma 3.15. ☐

Proposition 3.294. $\{1\} \in ((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\})$.

Proof. Use Proposition 3.293, Proposition 3.292 and Proposition 3.20. ☐

Lemma 3.94. $\{\{1\}\} \in (\wp\ (\{1\} \cup \{\{1\}\}))$.

Proof. Use Proposition 3.206 and Proposition 3.11. ☐

Proposition 3.295. $\{\{1\}\} \notin \{(\{1\} \cup \{\{1\}\})\}$.

Proof. Use Proposition 3.222 and Lemma 3.15. ☐

Lemma 3.95. $\{\{1\}\} \in ((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\})$.

Proof. Use Proposition 3.295, Lemma 3.94 and Proposition 3.20. ☐

Proposition 3.296. $(\{1\} \cup \{\{1\}\}) \in \{(\{1\} \cup \{\{1\}\})\}$.

Proof. Use Proposition 3.13. ☐

Proposition 3.297. $0 \in ((\{1\} \cup \{\{1\}\}) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.274 and Lemma 3.5. ☐

Lemma 3.96. $1 \in ((\{1\} \cup \{\{1\}\}) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.130 and Proposition 3.15. ☐

Proposition 3.298. $\{1\} \in ((\{1\} \cup \{\{1\}\}) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Lemma 3.57 and Proposition 3.15. ☐

Lemma 3.97. $\{\{1\}\} \in ((\{1\} \cup \{\{1\}\}) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.276 and Lemma 3.5. ☐

Proposition 3.299. $0 \in (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.274 and Lemma 3.5. ☐

Proposition 3.300. $1 \in (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.136 and Proposition 3.15. ☐

Proposition 3.301. $\{1\} \notin (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.275, Lemma 3.70 and Proposition 3.45. ☐

Proposition 3.302. $2 \in (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.164 and Proposition 3.15. □

Proposition 3.303. $\{\{1\}\} \in (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.276 and Lemma 3.5. □

Proposition 3.304. $0 \in (((\wp\ 2) \setminus 2) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.274 and Lemma 3.5. □

Proposition 3.305. $\{1\} \in (((\wp\ 2) \setminus 2) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Lemma 3.54 and Proposition 3.15. □

Proposition 3.306. $2 \in (((\wp\ 2) \setminus 2) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.165 and Proposition 3.15. □

Proposition 3.307. $\{\{1\}\} \in (((\wp\ 2) \setminus 2) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.276 and Lemma 3.5. □

Proposition 3.308. $1 \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup \{\{\{1\}\}\})$.

Proof. Use Proposition 3.145 and Proposition 3.15. □

Proposition 3.309. $\{1\} \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup \{\{\{1\}\}\})$.

Proof. Use Lemma 3.62 and Proposition 3.15. □

Proposition 3.310. $2 \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup \{\{\{1\}\}\})$.

Proof. Use Proposition 3.170 and Proposition 3.15. □

Proposition 3.311. $\{\{1\}\} \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup \{\{\{1\}\}\})$.

Proof. Use Lemma 3.90 and Lemma 3.5. □

Proposition 3.312. $0 \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.274 and Lemma 3.5. □

Proposition 3.313. $1 \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.145 and Proposition 3.15. □

Proposition 3.314. $\{1\} \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Lemma 3.62 and Proposition 3.15. □

Proposition 3.315. $2 \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.170 and Proposition 3.15. □

Proposition 3.316. $\{\{1\}\} \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.276 and Lemma 3.5. ☐

Proposition 3.317. $(\wp\ \{1\}) \in (3 \cup \{(\wp\ \{1\})\})$.

Proof. Use Proposition 3.285 and Lemma 3.5. ☐

Proposition 3.318. $\{1\} \notin \{\{2\}\}$.

Proof. Use Proposition 3.194 and Lemma 3.15. ☐

Proposition 3.319. $\{2\} \in \{\{2\}\}$.

Proof. Use Proposition 3.13. ☐

Proposition 3.320. $(3 \setminus \{1\}) \notin \{\{2\}\}$.

Proof. Use Proposition 3.250 and Lemma 3.15. ☐

Proposition 3.321. $3 \notin \{\{2\}\}$.

Proof. Use Proposition 3.256 and Lemma 3.15. ☐

Proposition 3.322. $((\wp\ 2) \setminus 2) \notin \{\{2\}\}$.

Proof. Use Proposition 3.260 and Lemma 3.15. ☐

Proposition 3.323. $(\wp\ 2) \notin \{\{2\}\}$.

Proof. Use Proposition 3.270 and Lemma 3.15. ☐

Proposition 3.324. $\{2\} \in (\wp\ \{2\})$.

Proof. Use Proposition 3.27. ☐

Proposition 3.325. $(3 \setminus \{1\}) \notin (\wp\ \{2\})$.

Proof. Use Proposition 3.12 and Proposition 3.236. ☐

Proposition 3.326. $3 \notin (\wp\ \{2\})$.

Proof. Use Proposition 3.12 and Proposition 3.238. ☐

Proposition 3.327. $(\wp\ 2) \notin (\wp\ \{2\})$.

Proof. Use Proposition 3.12 and Proposition 3.240. ☐

Proposition 3.328. $0 \in (\{1\} \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.171 and Lemma 3.5. ☐

Proposition 3.329. $1 \in (\{1\} \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.128 and Proposition 3.15. □

Proposition 3.330. $\{2\} \in (\{1\} \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.324 and Lemma 3.5. □

Proposition 3.331. $3 \notin (\{1\} \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.326, Lemma 3.60 and Proposition 3.45. □

Proposition 3.332. $0 \in (4 \setminus \{1\})$.

Proof. Use Proposition 3.133, Lemma 3.4 and Proposition 3.20. □

Proposition 3.333. $\{1\} \notin 4$.

Proof. Use Proposition 3.198, Proposition 3.156, Proposition 3.140, Proposition 3.139 and Proposition 3.10. □

Lemma 3.98. $2 \in (4 \setminus \{1\})$.

Proof. Use Proposition 3.158, Proposition 3.8 and Proposition 3.20. □

Proposition 3.334. $\{2\} \in ((\wp\ 2) \cup \{\{2\}\})$.

Proof. Use Proposition 3.319 and Lemma 3.5. □

Lemma 3.99. $\{2\} \notin 4$.

Proof. Use Proposition 3.256, Lemma 3.80, Proposition 3.174, Proposition 3.150 and Proposition 3.10. □

Proposition 3.335. $(3 \setminus \{1\}) \notin ((\wp\ 2) \cup \{\{2\}\})$.

Proof. Use Proposition 3.320, Proposition 3.226 and Proposition 3.45. □

Lemma 3.100. $3 \notin ((\wp\ 2) \cup \{\{2\}\})$.

Proof. Use Proposition 3.321, Lemma 3.78 and Proposition 3.45. □

Proposition 3.336. $((\wp\ 2) \setminus 2) \notin ((\wp\ 2) \cup \{\{2\}\})$.

Proof. Use Proposition 3.322, Proposition 3.227 and Proposition 3.45. □

Proposition 3.337. $(\wp\ 2) \notin ((\wp\ 2) \cup \{\{2\}\})$.

Proof. Use Proposition 3.323, Lemma 3.79 and Proposition 3.45. □

Proposition 3.338. $0 \in (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.171 and Lemma 3.5. □

Proposition 3.339. $1 \notin (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.144, Proposition 3.149 and Proposition 3.45. □

Proposition 3.340. $\{1\} \notin (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.177, Proposition 3.157 and Proposition 3.45. □

Proposition 3.341. $2 \in (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.132 and Proposition 3.15. □

Proposition 3.342. $\{2\} \in (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.324 and Lemma 3.5. □

Lemma 3.101. $3 \notin (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.326, Proposition 3.232 and Proposition 3.45. □

Lemma 3.102. $(\wp\ 2) \notin (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.327, Proposition 3.235 and Proposition 3.45. □

Proposition 3.343. $0 \in (3 \cup \{\{2\}\})$.

Proof. Use Lemma 3.3 and Proposition 3.15. □

Proposition 3.344. $1 \in (3 \cup \{\{2\}\})$.

Proof. Use Proposition 3.4 and Proposition 3.15. □

Proposition 3.345. $2 \in (3 \cup \{\{2\}\})$.

Proof. Use Proposition 3.5 and Proposition 3.15. □

Lemma 3.103. $\{2\} \in (3 \cup \{\{2\}\})$.

Proof. Use Proposition 3.319 and Lemma 3.5. □

Proposition 3.346. $(\wp\ 2) \notin (3 \cup \{\{2\}\})$.

Proof. Use Proposition 3.323, Lemma 3.81 and Proposition 3.45. □

Lemma 3.104. $((\wp\ 2) \setminus 2) \in \{((\wp\ 2) \setminus 2)\}$.

Proof. Use Proposition 3.13. □

Lemma 3.105. $(3 \setminus \{1\}) \in \{(3 \setminus \{1\})\}$.

Proof. Use Proposition 3.13. □

Lemma 3.106. $0 \in (\wp\ (3 \setminus \{1\}))$.

Proof. Use Proposition 3.26. ☐

Proposition 3.347. $2 \notin (\wp\,(3 \setminus \{1\}))$.

Proof. Use Proposition 3.12 and Proposition 3.192. ☐

Proposition 3.348. $(3 \setminus \{1\}) \in (\wp\,(3 \setminus \{1\}))$.

Proof. Use Proposition 3.27. ☐

Proposition 3.349. $(3 \setminus \{1\}) \in (3 \cup \{(3 \setminus \{1\})\})$.

Proof. Use Lemma 3.105 and Lemma 3.5. ☐

Lemma 3.107. $1 \in (\wp\,(3 \setminus \{1\}))$.

Proof. Use Proposition 3.160 and Proposition 3.11. ☐

Proposition 3.350. $0 \notin \{3\}$.

Proof. Use Lemma 3.44 and Lemma 3.15. ☐

Lemma 3.108. $1 \notin \{3\}$.

Proof. Use Proposition 3.115 and Lemma 3.15. ☐

Lemma 3.109. $2 \notin \{3\}$.

Proof. Use Lemma 3.45 and Lemma 3.15. ☐

Proposition 3.351. $\{2\} \notin \{3\}$.

Proof. Use Proposition 3.256 and Lemma 3.15. ☐

Proposition 3.352. $3 \in \{3\}$.

Proof. Use Proposition 3.13. ☐

Proposition 3.353. $0 \in (1 \cup \{3\})$.

Proof. Use Proposition 3.1 and Proposition 3.15. ☐

Lemma 3.110. $2 \notin (1 \cup \{3\})$.

Proof. Use Lemma 3.109, Proposition 3.107 and Proposition 3.45. ☐

Lemma 3.111. $\{2\} \notin (1 \cup \{3\})$.

Proof. Use Proposition 3.351, Proposition 3.151 and Proposition 3.45. ☐

Proposition 3.354. $\mathsf{disj}\ (\{2\} \cup \{\{2\}\})\ (1 \cup \{3\})$.

Proof. Use Lemma 3.111, Lemma 3.110, Proposition 3.14, Proposition 3.16 and Proposition 3.38. □

Proposition 3.355. $3 \in (1 \cup \{3\})$.

Proof. Use Proposition 3.352 and Lemma 3.5. □

Proposition 3.356. $1 \in (\{1\} \cup \{3\})$.

Proof. Use Proposition 3.128 and Proposition 3.15. □

Lemma 3.112. $3 \in (\{1\} \cup \{3\})$.

Proof. Use Proposition 3.352 and Lemma 3.5. □

Proposition 3.357. $0 \in (4 \setminus \{2\})$.

Proof. Use Lemma 3.50, Lemma 3.4 and Proposition 3.20. □

Lemma 3.113. $1 \in (4 \setminus \{2\})$.

Proof. Use Proposition 3.149, Proposition 3.7 and Proposition 3.20. □

Proposition 3.358. $\{\{1\}\} \notin 4$.

Proof. Use Proposition 3.254, Proposition 3.203, Proposition 3.184, Proposition 3.147 and Proposition 3.10. □

Proposition 3.359. $(\wp\ \{1\}) \notin 4$.

Proof. Use Proposition 3.255, Proposition 3.210, Proposition 3.186, Proposition 3.148 and Proposition 3.10. □

Proposition 3.360. $(3 \setminus \{1\}) \notin 4$.

Proof. Use Proposition 3.257, Proposition 3.230, Proposition 3.175, Lemma 3.56 and Proposition 3.10. □

Proposition 3.361. $3 \in (4 \setminus \{2\})$.

Proof. Use Proposition 3.232, Proposition 3.9 and Proposition 3.20. □

Proposition 3.362. $((\wp\ 2) \setminus 2) \notin 4$.

Proof. Use Lemma 3.86, Proposition 3.233, Proposition 3.246, Proposition 3.152 and Proposition 3.10. □

Proposition 3.363. $(\wp\ 2) \notin 4$.

Proof. Use Lemma 3.89, Proposition 3.234, Proposition 3.249, Proposition 3.248 and Proposition 3.10. □

Lemma 3.114. $0 \notin (\{\{1\}\} \cup \{3\})$.

Proof. Use Proposition 3.350, Proposition 3.153 and Proposition 3.45. ☐

Lemma 3.115. $1 \notin (\{\{1\}\} \cup \{3\})$.

Proof. Use Lemma 3.108, Proposition 3.178 and Proposition 3.45. ☐

Proposition 3.364. $\mathsf{disj}\ 2\ (\{\{1\}\} \cup \{3\})$.

Proof. Use Lemma 3.115, Lemma 3.114, Lemma 3.2 and Proposition 3.38. ☐

Proposition 3.365. $\{1\} \in (\{\{1\}\} \cup 4)$.

Proof. Use Proposition 3.129 and Proposition 3.15. ☐

Lemma 3.116. $0 \notin (4 \setminus 2)$.

Proof. Use Proposition 3.3 and Lemma 3.17. ☐

Lemma 3.117. $\{1\} \notin (4 \setminus 2)$.

Proof. Use Proposition 3.333 and Lemma 3.16. ☐

Proposition 3.366. $\mathsf{disj}\ (\wp\ \{1\})\ (4 \setminus 2)$.

Proof. Use Lemma 3.117, Lemma 3.116, Proposition 3.212 and Proposition 3.38. ☐

Proposition 3.367. $2 \in (4 \setminus 2)$.

Proof. Use Lemma 3.39, Proposition 3.8 and Proposition 3.20. ☐

Proposition 3.368. $3 \in (4 \setminus 2)$.

Proof. Use Lemma 3.40, Proposition 3.9 and Proposition 3.20. ☐

Proposition 3.369. $3 \in (4 \setminus \{1\})$.

Proof. Use Lemma 3.60, Proposition 3.9 and Proposition 3.20. ☐

Proposition 3.370. $1 \in (4 \setminus (\wp\ \{2\}))$.

Proof. Use Proposition 3.144, Proposition 3.7 and Proposition 3.20. ☐

Lemma 3.118. $2 \in (4 \setminus (\wp\ \{2\}))$.

Proof. Use Proposition 3.169, Proposition 3.8 and Proposition 3.20. ☐

Proposition 3.371. $3 \in (4 \setminus (\wp\ \{2\}))$.

Proof. Use Proposition 3.326, Proposition 3.9 and Proposition 3.20. ☐

Lemma 3.119. $\{\{1\}\} \in (\{\{\{1\}\}\} \cup 4)$.

Proof. Use Lemma 3.90 and Proposition 3.15. □

Proposition 3.372. $(\wp\ \{1\}) \in (\{(\wp\ \{1\})\} \cup 4)$.

Proof. Use Proposition 3.285 and Proposition 3.15. □

Proposition 3.373. $0 \in ((\wp\ \{2\}) \cup \{3\})$.

Proof. Use Proposition 3.171 and Proposition 3.15. □

Proposition 3.374. $2 \notin ((\wp\ \{2\}) \cup \{3\})$.

Proof. Use Lemma 3.109, Proposition 3.169 and Proposition 3.45. □

Proposition 3.375. $\{2\} \in ((\wp\ \{2\}) \cup \{3\})$.

Proof. Use Proposition 3.324 and Proposition 3.15. □

Proposition 3.376. $3 \in ((\wp\ \{2\}) \cup \{3\})$.

Proof. Use Proposition 3.352 and Lemma 3.5. □

Lemma 3.120. $1 \in (\{\{2\}\} \cup 4)$.

Proof. Use Proposition 3.7 and Lemma 3.5. □

Proposition 3.377. $1 \in ((\{\{2\}\} \cup 4) \setminus ((\wp\ 2) \setminus \{1\}))$.

Proof. Use Proposition 3.183, Lemma 3.120 and Proposition 3.20. □

Proposition 3.378. $\{1\} \notin (\{\{2\}\} \cup 4)$.

Proof. Use Proposition 3.333, Proposition 3.318 and Proposition 3.45. □

Lemma 3.121. $\{2\} \in (\{\{2\}\} \cup 4)$.

Proof. Use Proposition 3.319 and Proposition 3.15. □

Proposition 3.379. $\{2\} \in ((\{\{2\}\} \cup 4) \setminus ((\wp\ 2) \setminus \{1\}))$.

Proof. Use Proposition 3.265, Lemma 3.121 and Proposition 3.20. □

Proposition 3.380. $(3 \setminus \{1\}) \notin (\{\{2\}\} \cup 4)$.

Proof. Use Proposition 3.360, Proposition 3.320 and Proposition 3.45. □

Proposition 3.381. $3 \in (\{\{2\}\} \cup 4)$.

Proof. Use Proposition 3.9 and Lemma 3.5. □

Proposition 3.382. $3 \in ((\{\{2\}\} \cup 4) \setminus ((\wp\ 2) \setminus \{1\}))$.

Proof. Use Lemma 3.88, Proposition 3.381 and Proposition 3.20. □

Lemma 3.122. $((\wp\ 2) \setminus 2) \notin (\{\{2\}\} \cup 4)$.

Proof. Use Proposition 3.362, Proposition 3.322 and Proposition 3.45. ☐

Proposition 3.383. $(\wp\ 2) \notin (\{\{2\}\} \cup 4)$.

Proof. Use Proposition 3.363, Proposition 3.323 and Proposition 3.45. ☐

Proposition 3.384. $2 \notin ((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\}))$.

Proof. Use Proposition 3.374, Proposition 3.221 and Proposition 3.45. ☐

Proposition 3.385. $3 \in ((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\}))$.

Proof. Use Proposition 3.376 and Lemma 3.5. ☐

Proposition 3.386. $0 \in (\{\{2\}\} \cup 4)$.

Proof. Use Lemma 3.4 and Lemma 3.5. ☐

Lemma 3.123. $2 \in (\{\{2\}\} \cup 4)$.

Proof. Use Proposition 3.8 and Lemma 3.5. ☐

Proposition 3.387. $0 \in (\{\{1\}\} \cup (\{\{2\}\} \cup 4))$.

Proof. Use Proposition 3.386 and Lemma 3.5. ☐

Proposition 3.388. $\{1\} \in (\{\{1\}\} \cup (\{\{2\}\} \cup 4))$.

Proof. Use Proposition 3.129 and Proposition 3.15. ☐

Proposition 3.389. $2 \in (\{\{1\}\} \cup (\{\{2\}\} \cup 4))$.

Proof. Use Lemma 3.123 and Lemma 3.5. ☐

Proposition 3.390. $(3 \setminus \{1\}) \notin (\{\{1\}\} \cup (\{\{2\}\} \cup 4))$.

Proof. Use Proposition 3.380, Proposition 3.197 and Proposition 3.45. ☐

Proposition 3.391. $((\wp\ 2) \setminus 2) \notin (\{\{1\}\} \cup (\{\{2\}\} \cup 4))$.

Proof. Use Lemma 3.122, Proposition 3.200 and Proposition 3.45. ☐

Proposition 3.392. $(\wp\ 2) \notin (\{\{1\}\} \cup (\{\{2\}\} \cup 4))$.

Proof. Use Proposition 3.383, Proposition 3.201 and Proposition 3.45. ☐

Proposition 3.393. $((\wp\ 2) \setminus 2) \in (3 \cup \{((\wp\ 2) \setminus 2)\})$.

Proof. Use Lemma 3.104 and Lemma 3.5. ☐

Lemma 3.124. $((\wp\ 2) \setminus 2) \in (4 \cup \{((\wp\ 2) \setminus 2)\})$.

Proof. Use Lemma 3.104 and Lemma 3.5. □

Proposition 3.394. $((\wp\ 2) \setminus (\wp\ \{2\})) \in \{((\wp\ 2) \setminus (\wp\ \{2\}))\}$.

Proof. Use Proposition 3.13. □

Lemma 3.125. $((\wp\ 2) \setminus (\wp\ \{2\})) \in (((\wp\ 2) \setminus \{2\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\})$.

Proof. Use Proposition 3.394 and Lemma 3.5. □

Lemma 3.126. $((\wp\ 2) \setminus (\wp\ \{2\})) \in (((\wp\ 2) \setminus \{1\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\})$.

Proof. Use Proposition 3.394 and Lemma 3.5. □

Proposition 3.395. $0 \notin \{(\wp\ 2)\}$.

Proof. Use Proposition 3.248 and Lemma 3.15. □

Proposition 3.396. $1 \notin \{(\wp\ 2)\}$.

Proof. Use Proposition 3.249 and Lemma 3.15. □

Proposition 3.397. $\{1\} \notin \{(\wp\ 2)\}$.

Proof. Use Lemma 3.68 and Lemma 3.15. □

Lemma 3.127. $\{2\} \notin \{(\wp\ 2)\}$.

Proof. Use Proposition 3.270 and Lemma 3.15. □

Proposition 3.398. $(3 \setminus \{1\}) \notin \{(\wp\ 2)\}$.

Proof. Use Proposition 3.271 and Lemma 3.15. □

Proposition 3.399. $3 \notin \{(\wp\ 2)\}$.

Proof. Use Lemma 3.89 and Lemma 3.15. □

Proposition 3.400. $((\wp\ 2) \setminus 2) \notin \{(\wp\ 2)\}$.

Proof. Use Proposition 3.272 and Lemma 3.15. □

Proposition 3.401. $(\wp\ 2) \in \{(\wp\ 2)\}$.

Proof. Use Proposition 3.13. □

Proposition 3.402. $0 \in (1 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.1 and Proposition 3.15. □

Lemma 3.128. $(\wp\ 2) \in (1 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.401 and Lemma 3.5. □

Proposition 3.403. $0 \in (2 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.3 and Proposition 3.15. ☐

Proposition 3.404. $1 \in (2 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 3.1 and Proposition 3.15. ☐

Proposition 3.405. $(\wp\ 2) \in (2 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.401 and Lemma 3.5. ☐

Proposition 3.406. $3 \notin ((\wp\ 2) \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.399, Lemma 3.78 and Proposition 3.45. ☐

Proposition 3.407. $(\wp\ 2) \in ((\wp\ 2) \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.401 and Lemma 3.5. ☐

Lemma 3.129. $0 \in (3 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 3.3 and Proposition 3.15. ☐

Proposition 3.408. $1 \in (3 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.4 and Proposition 3.15. ☐

Proposition 3.409. $2 \in (3 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.5 and Proposition 3.15. ☐

Proposition 3.410. $(\wp\ 2) \in (3 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.401 and Lemma 3.5. ☐

Lemma 3.130. $0 \in (\{\{\{1\}\}\} \cup (1 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.402 and Lemma 3.5. ☐

Lemma 3.131. $\{\{1\}\} \in (\{\{\{1\}\}\} \cup (1 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 3.90 and Proposition 3.15. ☐

Proposition 3.411. $(\wp\ 2) \in (\{\{\{1\}\}\} \cup (1 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 3.128 and Lemma 3.5. ☐

Proposition 3.412. $0 \in (\{\{\{1\}\}\} \cup (2 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.403 and Lemma 3.5. ☐

Proposition 3.413. $1 \in (\{\{\{1\}\}\} \cup (2 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.404 and Lemma 3.5. ☐

Lemma 3.132. $\{\{1\}\} \in (\{\{\{1\}\}\} \cup (2 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 3.90 and Proposition 3.15. ☐

Lemma 3.133. $(\wp\ 2) \in (\{\{\{1\}\}\} \cup (2 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.405 and Lemma 3.5. ☐

Lemma 3.134. $\{2\} \in (\{\{2\}\} \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.319 and Proposition 3.15. ☐

Proposition 3.414. $(\wp\ 2) \in (\{\{2\}\} \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.401 and Lemma 3.5. ☐

Proposition 3.415. $0 \in ((\wp\ \{2\}) \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.171 and Proposition 3.15. ☐

Proposition 3.416. $(3 \setminus \{1\}) \notin ((\wp\ \{2\}) \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.398, Proposition 3.325 and Proposition 3.45. ☐

Proposition 3.417. $(\wp\ 2) \in ((\wp\ \{2\}) \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.401 and Lemma 3.5. ☐

Proposition 3.418. $1 \in (\{1\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.128 and Proposition 3.15. ☐

Proposition 3.419. $2 \in (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.132 and Proposition 3.15. ☐

Proposition 3.420. $(3 \setminus \{1\}) \notin (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.416, Proposition 3.231 and Proposition 3.45. ☐

Proposition 3.421. $1 \in (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\{\{2\}\} \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.136 and Proposition 3.15. ☐

Proposition 3.422. $2 \in (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\{\{2\}\} \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.164 and Proposition 3.15. ☐

Proposition 3.423. $\{2\} \in (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\{\{2\}\} \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 3.134 and Lemma 3.5. ☐

Proposition 3.424. $(\wp\ 2) \in (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\{\{2\}\} \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.414 and Lemma 3.5. □

Lemma 3.135. $0 \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.415 and Lemma 3.5. □

Proposition 3.425. $1 \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.145 and Proposition 3.15. □

Proposition 3.426. $2 \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.170 and Proposition 3.15. □

Proposition 3.427. $(3 \setminus \{1\}) \notin (((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.416, Proposition 3.267 and Proposition 3.45. □

Proposition 3.428. $(\wp\ 2) \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.417 and Lemma 3.5. □

Lemma 3.136. $0 \notin (\{3\} \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.395, Proposition 3.350 and Proposition 3.45. □

Lemma 3.137. $1 \notin (\{3\} \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.396, Lemma 3.108 and Proposition 3.45. □

Proposition 3.429. $\mathsf{disj}\ 2\ (\{3\} \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 3.137, Lemma 3.136, Lemma 3.2 and Proposition 3.38. □

Proposition 3.430. $0 \in (4 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 3.4 and Proposition 3.15. □

Proposition 3.431. $\{1\} \notin (4 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.397, Proposition 3.333 and Proposition 3.45. □

Proposition 3.432. $\{2\} \notin (4 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 3.127, Lemma 3.99 and Proposition 3.45. □

Proposition 3.433. $3 \in (4 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.9 and Proposition 3.15. □

Proposition 3.434. $((\wp\ 2) \setminus 2) \notin (4 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.400, Proposition 3.362 and Proposition 3.45. ☐

Proposition 3.435. $(\wp\ 2) \in (4 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.401 and Lemma 3.5. ☐

Proposition 3.436. $1 \in (4 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.7 and Proposition 3.15. ☐

Proposition 3.437. $2 \in (4 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.8 and Proposition 3.15. ☐

Proposition 3.438. $0 \in (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.430 and Lemma 3.5. ☐

Proposition 3.439. $1 \in (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.436 and Lemma 3.5. ☐

Proposition 3.440. $\{1\} \in (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.129 and Proposition 3.15. ☐

Lemma 3.138. $2 \in (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.437 and Lemma 3.5. ☐

Lemma 3.139. $\{2\} \notin (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.432, Proposition 3.195 and Proposition 3.45. ☐

Proposition 3.441. $3 \in (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.433 and Lemma 3.5. ☐

Proposition 3.442. $((\wp\ 2) \setminus 2) \notin (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.434, Proposition 3.200 and Proposition 3.45. ☐

Lemma 3.140. $0 \in (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.430 and Lemma 3.5. ☐

Lemma 3.141. $1 \in (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.436 and Lemma 3.5. ☐

Proposition 3.443. $\{1\} \notin (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.431, Proposition 3.318 and Proposition 3.45. ☐

Lemma 3.142. $2 \in (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.437 and Lemma 3.5. □

Proposition 3.444. $\{2\} \in (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.319 and Proposition 3.15. □

Proposition 3.445. $3 \in (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.433 and Lemma 3.5. □

Proposition 3.446. $(\wp\ 2) \in (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.435 and Lemma 3.5. □

Proposition 3.447. $\forall x \in (\{1\} \cup \{\{1\}\}).x = 1 \vee x = \{1\}$.

Proof. Use Proposition 3.204, Proposition 3.137 and Proposition 3.16. □

Lemma 3.143. $\forall X.\{1\} \in X \Rightarrow 1 \in X \Rightarrow (\{1\} \cup \{\{1\}\}) \subseteq X$.

Proof. Use Proposition 3.447 and Definition 1.1. □

Proposition 3.448. $\forall X \subseteq (\{1\} \cup \{\{1\}\}).\{1\} \in X \Rightarrow 1 \in X \Rightarrow X = (\{1\} \cup \{\{1\}\})$.

Proof. Use Lemma 3.143 and Axiom 1.1. □

Proposition 3.449. $\forall X \subseteq (\{1\} \cup \{\{1\}\}).\{1\} \in X \Rightarrow 1 \in X \Rightarrow X = 0 \vee X = \{1\} \vee X = \{\{1\}\} \vee X =$

Proof. Use Proposition 3.448. □

Proposition 3.450. $\forall X \subseteq (\{1\} \cup \{\{1\}\}).1 \notin X \Rightarrow X \subseteq \{\{1\}\}$.

Proof. Use Proposition 3.13, Proposition 3.447 and Definition 1.1. □

Lemma 3.144. $\forall X \subseteq (\{1\} \cup \{\{1\}\}).\{1\} \in X \Rightarrow 1 \notin X \Rightarrow X = \{\{1\}\}$.

Proof. Use Proposition 3.48, Proposition 3.450 and Axiom 1.1. □

Proposition 3.451. $\forall X \subseteq (\{1\} \cup \{\{1\}\}).\{1\} \in X \Rightarrow 1 \notin X \Rightarrow X = 0 \vee X = \{1\} \vee X = \{\{1\}\} \vee X =$

Proof. Use Lemma 3.144. □

Proposition 3.452. $\forall X \subseteq (\{1\} \cup \{\{1\}\}).\{1\} \notin X \Rightarrow X \subseteq \{1\}$.

Proof. Use Proposition 3.13, Proposition 3.447 and Definition 1.1. □

Proposition 3.453. $\forall X \subseteq (\{1\} \cup \{\{1\}\}).\{1\} \notin X \Rightarrow 1 \in X \Rightarrow X = \{1\}$.

Proof. Use Proposition 3.48, Proposition 3.452 and Axiom 1.1. □

Proposition 3.454. $\forall X \subseteq (\{1\} \cup \{\{1\}\}).\{1\} \notin X \Rightarrow 1 \in X \Rightarrow X = 0 \vee X = \{1\} \vee X = \{\{1\}\} \vee X =$

Proof. Use Proposition 3.453. ☐

Proposition 3.455. $\forall X \subseteq (\{1\} \cup \{\{1\}\}).\{1\} \notin X \Rightarrow 1 \notin X \Rightarrow X = 0.$

Proof. Use Proposition 3.447, Proposition 3.25 and Definition 1.1. ☐

Proposition 3.456. $\forall X \subseteq (\{1\} \cup \{\{1\}\}).\{1\} \notin X \Rightarrow 1 \notin X \Rightarrow X = 0 \vee X = \{1\} \vee X = \{\{1\}\} \vee X =$

Proof. Use Proposition 3.455. ☐

Proposition 3.457. $\forall X \subseteq (\{1\} \cup \{\{1\}\}).\{1\} \in X \Rightarrow X = 0 \vee X = \{1\} \vee X = \{\{1\}\} \vee X = (\{1\} \cup \{\{1$

Proof. Use Proposition 3.451 and Proposition 3.449. ☐

Proposition 3.458. $\forall X \subseteq (\{1\} \cup \{\{1\}\}).\{1\} \notin X \Rightarrow X = 0 \vee X = \{1\} \vee X = \{\{1\}\} \vee X = (\{1\} \cup \{\{1$

Proof. Use Proposition 3.456 and Proposition 3.454. ☐

Proposition 3.459. $\forall X \subseteq (\{1\} \cup \{\{1\}\}).X = 0 \vee X = \{1\} \vee X = \{\{1\}\} \vee X = (\{1\} \cup \{\{1\}\}).$

Proof. Use Proposition 3.458 and Proposition 3.457. ☐

Lemma 3.145. $\forall x \in ((\wp\ 2) \setminus \{2\}).x = 0 \vee x = 1 \vee x = \{1\}.$

Proof. Use Proposition 3.21, Lemma 3.6, Proposition 3.132 and Lemma 3.83. ☐

Proposition 3.460. $\forall x \in ((\wp\ 2) \setminus \{1\}).x = 0 \vee x = \{1\} \vee x = 2.$

Proof. Use Proposition 3.21, Lemma 3.6, Proposition 3.128 and Lemma 3.83. ☐

Proposition 3.461. $\forall x \in ((\wp\ 2) \setminus (\wp\ \{2\})).x = 1 \vee x = \{1\} \vee x = 2.$

Proof. Use Proposition 3.21, Lemma 3.6, Proposition 3.171 and Lemma 3.83. ☐

Proposition 3.462. $\forall x \in \{\{\{1\}\}\}.x = \{\{1\}\}.$

Proof. Use Proposition 3.14. ☐

Proposition 3.463. $\forall x \in (\wp\ \{\{1\}\}).x = 0 \vee x = \{\{1\}\}.$

Proof. Use Proposition 3.12 and Lemma 3.47. ☐

Proposition 3.464. $\forall x \in (\wp\ (\wp\ \{1\})).x = 0 \vee x = 1 \vee x = \{\{1\}\} \vee x = (\wp\ \{1\}).$

Proof. Use Proposition 3.12 and Lemma 3.76. ☐

Proposition 3.465. $\forall x \in ((\wp\ (\wp\ \{1\})) \setminus \{(\wp\ \{1\})\}).x = 0 \vee x = 1 \vee x = \{\{1\}\}.$

Proof. Use Proposition 3.21, Lemma 3.6, Proposition 3.285 and Proposition 3.464. ☐

Proposition 3.466. $\forall x \in (\{\{1\}\} \cup \{\{\{1\}\}\}).x = \{1\} \vee x = \{\{1\}\}.$

Proof. Use Proposition 3.462, Proposition 3.204 and Proposition 3.16. ☐

Proposition 3.467. $\forall x \in (\wp\ (\{1\} \cup \{\{1\}\})).x = 0 \vee x = \{1\} \vee x = \{\{1\}\} \vee x = (\{1\} \cup \{\{1\}\})$.

Proof. Use Proposition 3.12 and Proposition 3.459. □

Proposition 3.468. $\forall x \in ((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}).x = 0 \vee x = \{1\} \vee x = \{\{1\}\}$.

Proof. Use Proposition 3.21, Lemma 3.6, Proposition 3.296 and Proposition 3.467. □

Proposition 3.469. $\forall x \in ((\{1\} \cup \{\{1\}\}) \cup (\wp\ \{\{1\}\})).x = 0 \vee x = 1 \vee x = \{1\} \vee x = \{\{1\}\}$.

Proof. Use Proposition 3.463, Proposition 3.447 and Proposition 3.16. □

Proposition 3.470. $\forall x \in (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\wp\ \{\{1\}\})).x = 0 \vee x = 1 \vee x = 2 \vee x = \{\{1\}\}$.

Proof. Use Proposition 3.463, Lemma 3.84 and Proposition 3.16. □

Proposition 3.471. $\forall x \in (((\wp\ 2) \setminus 2) \cup (\wp\ \{\{1\}\})).x = 0 \vee x = \{1\} \vee x = 2 \vee x = \{\{1\}\}$.

Proof. Use Proposition 3.463, Proposition 3.264 and Proposition 3.16. □

Lemma 3.146. $\forall x \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup \{\{\{1\}\}\}).x = 1 \vee x = \{1\} \vee x = 2 \vee x = \{\{1\}\}$.

Proof. Use Proposition 3.462, Proposition 3.461 and Proposition 3.16. □

Proposition 3.472. $\forall x \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})).x = 0 \vee x = 1 \vee x = \{1\} \vee x = 2 \vee x = \{\{1\}\}$

Proof. Use Proposition 3.463, Proposition 3.461 and Proposition 3.16. □

Proposition 3.473. $\forall x \in 4.x \neq 2 \Rightarrow x = 0 \vee x = 1 \vee x = 3$.

Proof. Use Proposition 3.10. □

Proposition 3.474. $\forall x \in 4 \setminus \{2\}.x = 0 \vee x = 1 \vee x = 3$.

Proof. Use Lemma 3.6, Proposition 3.44, Proposition 3.21 and Proposition 3.473. □

Proposition 3.475. $\forall x \in 4.x \notin 2 \Rightarrow x = 2 \vee x = 3$.

Proof. Use Lemma 3.1, Proposition 3.3 and Proposition 3.10. □

Proposition 3.476. $\forall x \in 4 \setminus 2.x = 2 \vee x = 3$.

Proof. Use Lemma 3.6, Proposition 3.21 and Proposition 3.475. □

Lemma 3.147. $\forall x \in 4.x \neq 1 \Rightarrow x = 0 \vee x = 2 \vee x = 3$.

Proof. Use Proposition 3.10. □

Proposition 3.477. $\forall x \in 4 \setminus \{1\}.x = 0 \vee x = 2 \vee x = 3$.

Proof. Use Lemma 3.6, Proposition 3.44, Proposition 3.21 and Lemma 3.147. □

Proposition 3.478. $\forall x \in 4.x \notin \wp\ \{2\} \Rightarrow x = 1 \vee x = 2 \vee x = 3$.

Proof. Use Proposition 3.26 and Proposition 3.10. □

Proposition 3.479. $\forall x \in 4 \setminus \wp\ \{2\}. x = 1 \vee x = 2 \vee x = 3.$

Proof. Use Lemma 3.6, Proposition 3.21 and Proposition 3.478. □

Lemma 3.148. $2 \subseteq ((\wp\ (\wp\ \{1\})) \setminus \{(\wp\ \{1\})\}).$

Proof. Use Proposition 3.282, Proposition 3.280, Lemma 3.2 and Definition 1.1. □

Proposition 3.480. $((\wp\ (\wp\ \{1\})) \setminus \{(\wp\ \{1\})\}) \not\subseteq 2.$

Proof. Use Proposition 3.284, Proposition 3.185 and Definition 1.1. □

Proposition 3.481. $2 \subseteq (\{1\} \cup (\wp\ \{2\})).$

Proof. Use Proposition 3.329, Proposition 3.328, Lemma 3.2 and Definition 1.1. □

Proposition 3.482. $(\{1\} \cup (\wp\ \{2\})) \not\subseteq 2.$

Proof. Use Proposition 3.330, Proposition 3.188 and Definition 1.1. □

Proposition 3.483. $2 \subseteq (4 \setminus \{2\}).$

Proof. Use Lemma 3.113, Proposition 3.357, Lemma 3.2 and Definition 1.1. □

Lemma 3.149. $(4 \setminus \{2\}) \not\subseteq 2.$

Proof. Use Proposition 3.361, Lemma 3.40 and Definition 1.1. □

Proposition 3.484. $2 \subseteq (2 \cup \{(\wp\ 2)\}).$

Proof. Use Proposition 3.28. □

Lemma 3.150. $(2 \cup \{(\wp\ 2)\}) \not\subseteq 2.$

Proof. Use Proposition 3.405, Proposition 3.189 and Definition 1.1. □

Proposition 3.485. $3 \subseteq (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\wp\ \{\{1\}\})).$

Proof. Use Proposition 3.302, Proposition 3.300, Proposition 3.299, Proposition 3.6 and Definition 1.1. □

Proposition 3.486. $(((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\wp\ \{\{1\}\})) \not\subseteq 3.$

Proof. Use Proposition 3.303, Proposition 3.241 and Definition 1.1. □

Proposition 3.487. $3 \subseteq (3 \cup \{(\wp\ \{1\})\}).$

Proof. Use Proposition 3.28. □

Proposition 3.488. $(3 \cup \{(\wp\ \{1\})\}) \not\subseteq 3.$

Proof. Use Proposition 3.317, Proposition 3.242 and Definition 1.1. □

Proposition 3.489. $3 \subseteq (3 \cup \{\{2\}\})$.

Proof. Use Proposition 3.28. □

Proposition 3.490. $(3 \cup \{\{2\}\}) \not\subseteq 3$.

Proof. Use Lemma 3.103, Proposition 3.243 and Definition 1.1. □

Proposition 3.491. $3 \subseteq (3 \cup \{((\wp\ 2) \setminus 2)\})$.

Proof. Use Proposition 3.28. □

Proposition 3.492. $(3 \cup \{((\wp\ 2) \setminus 2)\}) \not\subseteq 3$.

Proof. Use Proposition 3.393, Proposition 3.247 and Definition 1.1. □

Proposition 3.493. $3 \subseteq (3 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.28. □

Proposition 3.494. $(3 \cup \{(\wp\ 2)\}) \not\subseteq 3$.

Proof. Use Proposition 3.410, Lemma 3.81 and Definition 1.1. □

Proposition 3.495. $3 \subseteq ((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.426, Proposition 3.425, Lemma 3.135, Proposition 3.6 and Definition 1.1. □

Proposition 3.496. $\{\{2\}\} \subseteq ((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.27, Proposition 3.15, Lemma 3.5 and Proposition 3.48. □

Proposition 3.497. $3 \cup \{\{2\}\} \subseteq ((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.496, Proposition 3.495 and Lemma 3.9. □

Proposition 3.498. $4 \subseteq (\{\{1\}\} \cup 4)$.

Proof. Use Lemma 3.8. □

Proposition 3.499. $(\{\{1\}\} \cup 4) \not\subseteq 4$.

Proof. Use Proposition 3.365, Proposition 3.333 and Definition 1.1. □

Proposition 3.500. $4 \subseteq (\{\{\{1\}\}\} \cup 4)$.

Proof. Use Lemma 3.8. □

Proposition 3.501. $(\{\{\{1\}\}\} \cup 4) \not\subseteq 4$.

Proof. Use Lemma 3.119, Proposition 3.358 and Definition 1.1. ☐

Proposition 3.502. $4 \subseteq (\{(\wp\ \{1\})\} \cup 4)$.

Proof. Use Lemma 3.8. ☐

Proposition 3.503. $(\{(\wp\ \{1\})\} \cup 4) \not\subseteq 4$.

Proof. Use Proposition 3.372, Proposition 3.359 and Definition 1.1. ☐

Proposition 3.504. $4 \subseteq (\{\{2\}\} \cup 4)$.

Proof. Use Lemma 3.8. ☐

Proposition 3.505. $(\{\{2\}\} \cup 4) \not\subseteq 4$.

Proof. Use Lemma 3.121, Lemma 3.99 and Definition 1.1. ☐

Lemma 3.151. $4 \subseteq \{\{1\}\} \cup (\{\{2\}\} \cup 4)$.

Proof. Use Lemma 3.8 and Lemma 3.7. ☐

Proposition 3.506. $4 \subseteq (4 \cup \{((\wp\ 2) \setminus 2)\})$.

Proof. Use Proposition 3.28. ☐

Lemma 3.152. $(4 \cup \{((\wp\ 2) \setminus 2)\}) \not\subseteq 4$.

Proof. Use Lemma 3.124, Proposition 3.362 and Definition 1.1. ☐

Proposition 3.507. $4 \subseteq (4 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.28. ☐

Proposition 3.508. $(4 \cup \{(\wp\ 2)\}) \not\subseteq 4$.

Proof. Use Proposition 3.435, Proposition 3.363 and Definition 1.1. ☐

Proposition 3.509. $4 \subseteq \{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 3.8, Proposition 3.28 and Lemma 3.7. ☐

Lemma 3.153. $4 \subseteq \{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 3.8, Proposition 3.28 and Lemma 3.7. ☐

Proposition 3.510. $3 \subseteq \{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 3.153, Proposition 3.112 and Lemma 3.7. ☐

Proposition 3.511. $3 \cup \{\{2\}\} \subseteq \{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.28, Proposition 3.510 and Lemma 3.9. ☐

Lemma 3.154. $(\wp\ \{\{1\}\}) \subseteq (\{\{\{1\}\}\} \cup (1 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 3.131, Lemma 3.130, Proposition 3.463 and Definition 1.1. □

Proposition 3.512. $(\{\{\{1\}\}\} \cup (1 \cup \{(\wp\ 2)\})) \not\subseteq (\wp\ \{\{1\}\})$.

Proof. Use Proposition 3.411, Proposition 3.277 and Definition 1.1. □

Proposition 3.513. $(3 \setminus \{1\}) \subseteq (4 \setminus \{1\})$.

Proof. Use Lemma 3.98, Proposition 3.332, Proposition 3.251 and Definition 1.1. □

Lemma 3.155. $(4 \setminus \{1\}) \not\subseteq (3 \setminus \{1\})$.

Proof. Use Proposition 3.369, Proposition 3.245 and Definition 1.1. □

Proposition 3.514. $(3 \setminus \{1\}) \subseteq (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.341, Proposition 3.338, Proposition 3.251 and Definition 1.1. □

Proposition 3.515. $(\{2\} \cup (\wp\ \{2\})) \not\subseteq (3 \setminus \{1\})$.

Proof. Use Proposition 3.342, Proposition 3.244 and Definition 1.1. □

Proposition 3.516. $(\wp\ \{\{1\}\}) \subseteq ((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\})$.

Proof. Use Lemma 3.95, Proposition 3.291, Proposition 3.463 and Definition 1.1. □

Proposition 3.517. $((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}) \not\subseteq (\wp\ \{\{1\}\})$.

Proof. Use Proposition 3.294, Proposition 3.275 and Definition 1.1. □

Proposition 3.518. $((\wp\ (\wp\ \{1\})) \setminus \{(\wp\ \{1\})\}) \subseteq (\{\{\{1\}\}\} \cup (2 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 3.132, Proposition 3.413, Proposition 3.412, Proposition 3.465 and Definition 1.1. □

Proposition 3.519. $(\{\{\{1\}\}\} \cup (2 \cup \{(\wp\ 2)\})) \not\subseteq ((\wp\ (\wp\ \{1\})) \setminus \{(\wp\ \{1\})\})$.

Proof. Use Lemma 3.133, Proposition 3.287 and Definition 1.1. □

Proposition 3.520. $((\wp\ 2) \setminus \{1\}) \subseteq (((\wp\ 2) \setminus \{1\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\})$.

Proof. Use Proposition 3.28. □

Proposition 3.521. $(((\wp\ 2) \setminus \{1\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\}) \not\subseteq ((\wp\ 2) \setminus \{1\})$.

Proof. Use Lemma 3.126, Proposition 3.266 and Definition 1.1. □

Proposition 3.522. $((\wp\ 2) \setminus \{2\}) \subseteq (((\wp\ 2) \setminus \{2\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\})$.

Proof. Use Proposition 3.28. ☐

Proposition 3.523. $(((\wp\ 2) \setminus \{2\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\}) \not\subseteq ((\wp\ 2) \setminus \{2\})$.

Proof. Use Lemma 3.125, Proposition 3.229 and Definition 1.1. ☐

Lemma 3.156. $(\wp\ 2) \subseteq ((\wp\ 2) \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.28. ☐

Lemma 3.157. $((\wp\ 2) \cup \{(\wp\ 2)\}) \not\subseteq (\wp\ 2)$.

Proof. Use Proposition 3.407, Lemma 3.79 and Definition 1.1. ☐

Proposition 3.524. $(\wp\ 2) \subseteq ((\wp\ 2) \cup \{\{2\}\})$.

Proof. Use Proposition 3.28. ☐

Proposition 3.525. $((\wp\ 2) \cup \{\{2\}\}) \not\subseteq (\wp\ 2)$.

Proof. Use Proposition 3.334, Proposition 3.225 and Definition 1.1. ☐

Lemma 3.158. $((\wp\ 2) \setminus (\wp\ \{1\})) \subseteq (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.28, Proposition 3.253 and Lemma 3.7. ☐

Proposition 3.526. $(\wp\ \{\{1\}\}) \subseteq (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Lemma 3.8. ☐

Proposition 3.527. $(((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\wp\ \{\{1\}\})) \subseteq (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.526, Lemma 3.158 and Lemma 3.9. ☐

Proposition 3.528. $(((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \not\subseteq (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.314, Proposition 3.301 and Definition 1.1. ☐

Proposition 3.529. $\wp\ 2 \subseteq \{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 3.138, Proposition 3.440, Proposition 3.439, Proposition 3.438, Lemma 3.83 and Definition 1.1. ☐

Proposition 3.530. $(4 \cup \{(\wp\ 2)\}) \subseteq (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 3.8. ☐

Proposition 3.531. $(\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})) \not\subseteq (4 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.444, Proposition 3.432 and Definition 1.1. ☐

Lemma 3.159. $(4 \cup \{(\wp\ 2)\}) \subseteq (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 3.8. ☐

Proposition 3.532. $(\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})) \not\subseteq (4 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.440, Proposition 3.431 and Definition 1.1. ☐

Lemma 3.160. $(\{\{2\}\} \cup 4) \subseteq (\{\{1\}\} \cup (\{\{2\}\} \cup 4))$.

Proof. Use Lemma 3.8. ☐

Proposition 3.533. $(\{\{1\}\} \cup (\{\{2\}\} \cup 4)) \not\subseteq (\{\{2\}\} \cup 4)$.

Proof. Use Proposition 3.388, Proposition 3.378 and Definition 1.1. ☐

Lemma 3.161. $2 \setminus \{0\} \subseteq \{1\}$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.128, Lemma 3.2 and Definition 1.1. ☐

Lemma 3.162. $\{1\} \subseteq 2 \setminus \{0\}$.

Proof. Use Proposition 3.113, Lemma 3.15, Lemma 3.1, Proposition 3.20, Proposition 3.137 and Definition 1.1. ☐

Lemma 3.163. $2 \setminus \{0\} = \{1\}$.

Proof. Use Lemma 3.162, Lemma 3.161 and Axiom 1.1. ☐

Proposition 3.534. $2 \setminus \{1\} \subseteq 1$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.1, Lemma 3.2 and Definition 1.1. ☐

Proposition 3.535. $1 \subseteq 2 \setminus \{1\}$.

Proof. Use Proposition 3.113, Lemma 3.15, Proposition 3.3, Proposition 3.20, Proposition 3.2 and Definition 1.1. ☐

Lemma 3.164. $2 \setminus \{1\} = 1$.

Proof. Use Proposition 3.535, Proposition 3.534 and Axiom 1.1. ☐

Proposition 3.536. $\forall x \in 3.x \neq 0 \Rightarrow x \in ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Proposition 3.164, Proposition 3.136 and Proposition 3.6. ☐

Proposition 3.537. $3 \setminus \{0\} \subseteq ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.536 and Definition 1.1. ☐

Proposition 3.538. $((\wp\ 2) \setminus (\wp\ \{1\})) \subseteq 3 \setminus \{0\}$.

Proof. Use Lemma 3.43, Proposition 3.5, Proposition 3.113, Lemma 3.15, Proposition 3.4, Proposition 3.20, Lemma 3.84 and Definition 1.1. □

Lemma 3.165. $3 \setminus \{0\} = ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Proposition 3.538, Proposition 3.537 and Axiom 1.1. □

Proposition 3.539. $\forall x \in 3.x \neq 2 \Rightarrow x \in 2$.

Proof. Use Lemma 3.1, Proposition 3.3 and Proposition 3.6. □

Proposition 3.540. $3 \setminus \{2\} \subseteq 2$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.539 and Definition 1.1. □

Proposition 3.541. $2 \subseteq 3 \setminus \{2\}$.

Proof. Use Proposition 3.114, Proposition 3.4, Lemma 3.43, Lemma 3.15, Lemma 3.3, Proposition 3.20, Lemma 3.2 and Definition 1.1. □

Proposition 3.542. $3 \setminus \{2\} = 2$.

Proof. Use Proposition 3.541, Proposition 3.540 and Axiom 1.1. □

Lemma 3.166. $\forall x \in ((\wp\ 2) \setminus \{2\}).x \neq 0 \Rightarrow x \in (\{1\} \cup \{\{1\}\})$.

Proof. Use Lemma 3.57, Proposition 3.130 and Lemma 3.145. □

Proposition 3.543. $((\wp\ 2) \setminus \{2\}) \setminus \{0\} \subseteq (\{1\} \cup \{\{1\}\})$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Lemma 3.166 and Definition 1.1. □

Proposition 3.544. $(\{1\} \cup \{\{1\}\}) \subseteq ((\wp\ 2) \setminus \{2\}) \setminus \{0\}$.

Proof. Use Proposition 3.139, Lemma 3.59, Proposition 3.113, Lemma 3.15, Lemma 3.55, Proposition 3.20, Proposition 3.447 and Definition 1.1. □

Proposition 3.545. $((\wp\ 2) \setminus \{2\}) \setminus \{0\} = (\{1\} \cup \{\{1\}\})$.

Proof. Use Proposition 3.544, Proposition 3.543 and Axiom 1.1. □

Proposition 3.546. $\forall x \in ((\wp\ 2) \setminus \{2\}).x \neq 1 \Rightarrow x \in (\wp\ \{1\})$.

Proof. Use Proposition 3.154, Lemma 3.48 and Lemma 3.145. □

Lemma 3.167. $((\wp\ 2) \setminus \{2\}) \setminus \{1\} \subseteq (\wp\ \{1\})$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.546 and Definition 1.1. □

Proposition 3.547. $(\wp\ \{1\}) \subseteq ((\wp\ 2) \setminus \{2\}) \setminus \{1\}$.

Proof. Use Proposition 3.140, Lemma 3.59, Proposition 3.113, Lemma 3.15, Proposition 3.131, Proposition 3.20, Proposition 3.212 and Definition 1.1. □

Proposition 3.548. $((\wp\ 2) \setminus \{2\}) \setminus \{1\} = (\wp\ \{1\})$.

Proof. Use Proposition 3.547, Lemma 3.167 and Axiom 1.1. □

Proposition 3.549. $\forall x \in ((\wp\ 2) \setminus \{2\}).x \neq \{1\} \Rightarrow x \in 2$.

Proof. Use Lemma 3.1, Proposition 3.3 and Lemma 3.145. □

Proposition 3.550. $((\wp\ 2) \setminus \{2\}) \setminus \{\{1\}\} \subseteq 2$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.549 and Definition 1.1. □

Proposition 3.551. $2 \subseteq ((\wp\ 2) \setminus \{2\}) \setminus \{\{1\}\}$.

Proof. Use Proposition 3.140, Lemma 3.55, Proposition 3.139, Lemma 3.15, Proposition 3.131, Proposition 3.20, Lemma 3.2 and Definition 1.1. □

Proposition 3.552. $((\wp\ 2) \setminus \{2\}) \setminus \{\{1\}\} = 2$.

Proof. Use Proposition 3.551, Proposition 3.550 and Axiom 1.1. □

Lemma 3.168. $(3 \setminus \{1\}) \setminus \{0\} \subseteq \{2\}$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.132, Proposition 3.251 and Definition 1.1. □

Proposition 3.553. $\{2\} \subseteq (3 \setminus \{1\}) \setminus \{0\}$.

Proof. Use Lemma 3.43, Lemma 3.15, Proposition 3.159, Proposition 3.20, Proposition 3.237 and Definition 1.1. □

Lemma 3.169. $(3 \setminus \{1\}) \setminus \{0\} = \{2\}$.

Proof. Use Proposition 3.553, Lemma 3.168 and Axiom 1.1. □

Proposition 3.554. $(3 \setminus \{1\}) \setminus \{2\} \subseteq 1$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.1, Proposition 3.251 and Definition 1.1. □

Proposition 3.555. $1 \subseteq (3 \setminus \{1\}) \setminus \{2\}$.

Proof. Use Lemma 3.43, Lemma 3.15, Proposition 3.134, Proposition 3.20, Proposition 3.2 and Definition 1.1. □

Proposition 3.556. $(3 \setminus \{1\}) \setminus \{2\} = 1$.

Proof. Use Proposition 3.555, Proposition 3.554 and Axiom 1.1. □

Proposition 3.557. $((\wp\ 2) \setminus (\wp\ \{1\})) \setminus \{1\} \subseteq \{2\}$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.132, Lemma 3.84 and Definition 1.1. □

Proposition 3.558. $\{2\} \subseteq ((\wp\ 2) \setminus (\wp\ \{1\})) \setminus \{1\}$.

Proof. Use Proposition 3.114, Lemma 3.15, Proposition 3.164, Proposition 3.20, Proposition 3.237 and Definition 1.1. □

Proposition 3.559. $((\wp\ 2) \setminus (\wp\ \{1\})) \setminus \{1\} = \{2\}$.

Proof. Use Proposition 3.558, Proposition 3.557 and Axiom 1.1. □

Proposition 3.560. $((\wp\ 2) \setminus (\wp\ \{1\})) \setminus \{2\} \subseteq \{1\}$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.128, Lemma 3.84 and Definition 1.1. □

Proposition 3.561. $\{1\} \subseteq ((\wp\ 2) \setminus (\wp\ \{1\})) \setminus \{2\}$.

Proof. Use Proposition 3.114, Lemma 3.15, Proposition 3.136, Proposition 3.20, Proposition 3.137 and Definition 1.1. □

Lemma 3.170. $((\wp\ 2) \setminus (\wp\ \{1\})) \setminus \{2\} = \{1\}$.

Proof. Use Proposition 3.561, Proposition 3.560 and Axiom 1.1. □

Proposition 3.562. $((\wp\ 2) \setminus 2) \setminus \{\{1\}\} \subseteq \{2\}$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.132, Proposition 3.264 and Definition 1.1. □

Proposition 3.563. $\{2\} \subseteq ((\wp\ 2) \setminus 2) \setminus \{\{1\}\}$.

Proof. Use Proposition 3.156, Lemma 3.15, Proposition 3.165, Proposition 3.20, Proposition 3.237 and Definition 1.1. □

Proposition 3.564. $((\wp\ 2) \setminus 2) \setminus \{\{1\}\} = \{2\}$.

Proof. Use Proposition 3.563, Proposition 3.562 and Axiom 1.1. □

Proposition 3.565. $((\wp\ 2) \setminus 2) \setminus \{2\} \subseteq \{\{1\}\}$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.129, Proposition 3.264 and Definition 1.1. □

Proposition 3.566. $\{\{1\}\} \subseteq ((\wp\ 2) \setminus 2) \setminus \{2\}$.

Proof. Use Proposition 3.156, Lemma 3.15, Lemma 3.54, Proposition 3.20, Proposition 3.204 and Definition 1.1. □

Proposition 3.567. $((\wp\ 2) \setminus 2) \setminus \{2\} = \{\{1\}\}$.

Proof. Use Proposition 3.566, Proposition 3.565 and Axiom 1.1. □

Lemma 3.171. $\forall x \in ((\wp\ 2) \setminus \{1\}).x \neq 0 \Rightarrow x \in ((\wp\ 2) \setminus 2)$.

Proof. Use Proposition 3.165, Lemma 3.54 and Proposition 3.460. □

Proposition 3.568. $((\wp\ 2) \setminus \{1\}) \setminus \{0\} \subseteq ((\wp\ 2) \setminus 2)$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Lemma 3.171 and Definition 1.1. □

Proposition 3.569. $((\wp\ 2) \setminus 2) \subseteq ((\wp\ 2) \setminus \{1\}) \setminus \{0\}$.

Proof. Use Lemma 3.43, Proposition 3.167, Proposition 3.139, Lemma 3.15, Proposition 3.176, Proposition 3.20, Proposition 3.264 and Definition 1.1. □

Proposition 3.570. $((\wp\ 2) \setminus \{1\}) \setminus \{0\} = ((\wp\ 2) \setminus 2)$.

Proof. Use Proposition 3.569, Proposition 3.568 and Axiom 1.1. □

Proposition 3.571. $\forall x \in ((\wp\ 2) \setminus \{1\}).x \neq \{1\} \Rightarrow x \in (3 \setminus \{1\})$.

Proof. Use Proposition 3.159, Proposition 3.134 and Proposition 3.460. □

Lemma 3.172. $((\wp\ 2) \setminus \{1\}) \setminus \{\{1\}\} \subseteq (3 \setminus \{1\})$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.571 and Definition 1.1. □

Proposition 3.572. $(3 \setminus \{1\}) \subseteq ((\wp\ 2) \setminus \{1\}) \setminus \{\{1\}\}$.

Proof. Use Proposition 3.156, Proposition 3.167, Proposition 3.139, Lemma 3.15, Proposition 3.142, Proposition 3.20, Proposition 3.251 and Definition 1.1. □

Proposition 3.573. $((\wp\ 2) \setminus \{1\}) \setminus \{\{1\}\} = (3 \setminus \{1\})$.

Proof. Use Proposition 3.572, Lemma 3.172 and Axiom 1.1. □

Proposition 3.574. $\forall x \in ((\wp\ 2) \setminus \{1\}).x \neq 2 \Rightarrow x \in (\wp\ \{1\})$.

Proof. Use Proposition 3.154, Lemma 3.48 and Proposition 3.460. □

Proposition 3.575. $((\wp\ 2) \setminus \{1\}) \setminus \{2\} \subseteq (\wp\ \{1\})$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.574 and Definition 1.1. □

Proposition 3.576. $(\wp\ \{1\}) \subseteq ((\wp\ 2) \setminus \{1\}) \setminus \{2\}$.

Proof. Use Proposition 3.156, Proposition 3.176, Lemma 3.43, Lemma 3.15, Proposition 3.142, Proposition 3.20, Proposition 3.212 and Definition 1.1. □

Proposition 3.577. $((\wp\ 2) \setminus \{1\}) \setminus \{2\} = (\wp\ \{1\})$.

Proof. Use Proposition 3.576, Proposition 3.575 and Axiom 1.1. □

Proposition 3.578. $\forall x \in ((\wp\ 2) \setminus (\wp\ \{2\})).x \neq 1 \Rightarrow x \in ((\wp\ 2) \setminus 2)$.

Proof. Use Proposition 3.165, Lemma 3.54 and Proposition 3.461. □

Proposition 3.579. $((\wp\ 2) \setminus (\wp\ \{2\})) \setminus \{1\} \subseteq ((\wp\ 2) \setminus 2)$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.578 and Definition 1.1. □

Proposition 3.580. $((\wp\ 2) \setminus 2) \subseteq ((\wp\ 2) \setminus (\wp\ \{2\})) \setminus \{1\}$.

Proof. Use Proposition 3.114, Proposition 3.170, Proposition 3.140, Lemma 3.15, Lemma 3.62, Proposition 3.20, Proposition 3.264 and Definition 1.1. □

Proposition 3.581. $((\wp\ 2) \setminus (\wp\ \{2\})) \setminus \{1\} = ((\wp\ 2) \setminus 2)$.

Proof. Use Proposition 3.580, Proposition 3.579 and Axiom 1.1. □

Lemma 3.173. $\forall x \in ((\wp\ 2) \setminus (\wp\ \{2\})).x \neq \{1\} \Rightarrow x \in ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Proposition 3.164, Proposition 3.136 and Proposition 3.461. □

Proposition 3.582. $((\wp\ 2) \setminus (\wp\ \{2\})) \setminus \{\{1\}\} \subseteq ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Lemma 3.173 and Definition 1.1. □

Proposition 3.583. $((\wp\ 2) \setminus (\wp\ \{1\})) \subseteq ((\wp\ 2) \setminus (\wp\ \{2\})) \setminus \{\{1\}\}$.

Proof. Use Proposition 3.156, Proposition 3.170, Proposition 3.140, Lemma 3.15, Proposition 3.145, Proposition 3.20, Lemma 3.84 and Definition 1.1. □

Proposition 3.584. $((\wp\ 2) \setminus (\wp\ \{2\})) \setminus \{\{1\}\} = ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Proposition 3.583, Proposition 3.582 and Axiom 1.1. □

Lemma 3.174. $\forall x \in ((\wp\ 2) \setminus (\wp\ \{2\})).x \neq 2 \Rightarrow x \in (\{1\} \cup \{\{1\}\})$.

Proof. Use Lemma 3.57, Proposition 3.130 and Proposition 3.461. □

Lemma 3.175. $((\wp\ 2) \setminus (\wp\ \{2\})) \setminus \{2\} \subseteq (\{1\} \cup \{\{1\}\})$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Lemma 3.174 and Definition 1.1. □

Proposition 3.585. $(\{1\} \cup \{\{1\}\}) \subseteq ((\wp\ 2) \setminus (\wp\ \{2\})) \setminus \{2\}$.

Proof. Use Proposition 3.156, Lemma 3.62, Proposition 3.114, Lemma 3.15, Proposition 3.145, Proposition 3.20, Proposition 3.447 and Definition 1.1. □

Lemma 3.176. $((\wp\ 2) \setminus (\wp\ \{2\})) \setminus \{2\} = (\{1\} \cup \{\{1\}\})$.

Proof. Use Proposition 3.585, Lemma 3.175 and Axiom 1.1. □

Proposition 3.586. $\forall x \in (\wp\ 2).x \neq 0 \Rightarrow x \in ((\wp\ 2) \setminus (\wp\ \{2\}))$.

Proof. Use Proposition 3.170, Lemma 3.62, Proposition 3.145 and Lemma 3.83. □

Proposition 3.587. $(\wp\ 2) \setminus \{0\} \subseteq ((\wp\ 2) \setminus (\wp\ \{2\}))$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.586 and Definition 1.1. □

Proposition 3.588. $((\wp\ 2) \setminus (\wp\ \{2\})) \subseteq (\wp\ 2) \setminus \{0\}$.

Proof. Use Lemma 3.43, Proposition 3.161, Proposition 3.139, Proposition 3.138, Proposition 3.113, Lemma 3.15, Proposition 3.135, Proposition 3.20, Proposition 3.461 and Definition 1.1. □

Proposition 3.589. $(\wp\ 2) \setminus \{0\} = ((\wp\ 2) \setminus (\wp\ \{2\}))$.

Proof. Use Proposition 3.588, Proposition 3.587 and Axiom 1.1. □

Proposition 3.590. $\forall x \in (\wp\ 2).x \neq \{1\} \Rightarrow x \in 3$.

Proof. Use Proposition 3.5, Proposition 3.4, Lemma 3.3 and Lemma 3.83. □

Lemma 3.177. $(\wp\ 2) \setminus \{\{1\}\} \subseteq 3$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.590 and Definition 1.1. □

Lemma 3.178. $3 \subseteq (\wp\ 2) \setminus \{\{1\}\}$.

Proof. Use Proposition 3.156, Proposition 3.161, Proposition 3.140, Proposition 3.135, Proposition 3.139, Lemma 3.15, Lemma 3.49, Proposition 3.20, Proposition 3.6 and Definition 1.1. □

Proposition 3.591. $(\wp\ 2) \setminus \{\{1\}\} = 3$.

Proof. Use Lemma 3.178, Lemma 3.177 and Axiom 1.1. □

Lemma 3.179. $\forall x \in ((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}).x \neq 0 \Rightarrow x \in (\{\{1\}\} \cup \{\{\{1\}\}\})$.

Proof. Use Lemma 3.93, Proposition 3.288 and Proposition 3.468. □

Proposition 3.592. $((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}) \setminus \{0\} \subseteq (\{\{1\}\} \cup \{\{\{1\}\}\})$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Lemma 3.179 and Definition 1.1. □

Proposition 3.593. $(\{\{1\}\} \cup \{\{\{1\}\}\}) \subseteq ((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}) \setminus \{0\}$.

Proof. Use Proposition 3.147, Lemma 3.95, Proposition 3.139, Lemma 3.15, Proposition 3.294, Proposition 3.20, Proposition 3.466 and Definition 1.1. □

Proposition 3.594. $((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}) \setminus \{0\} = (\{\{1\}\} \cup \{\{\{1\}\}\})$.

Proof. Use Proposition 3.593, Proposition 3.592 and Axiom 1.1. □

Proposition 3.595. $\forall x \in ((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}).x \neq \{1\} \Rightarrow x \in (\wp\ \{\{1\}\})$.

Proof. Use Proposition 3.276, Proposition 3.274 and Proposition 3.468. □

Proposition 3.596. $((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}) \setminus \{\{1\}\} \subseteq (\wp\ \{\{1\}\})$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.595 and Definition 1.1. □

Proposition 3.597. $(\wp\ \{\{1\}\}) \subseteq ((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}) \setminus \{\{1\}\}$.

Proof. Use Proposition 3.202, Lemma 3.95, Proposition 3.139, Lemma 3.15, Proposition 3.291, Proposition 3.20, Proposition 3.463 and Definition 1.1. □

Lemma 3.180. $((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}) \setminus \{\{1\}\} = (\wp\ \{\{1\}\})$.

Proof. Use Proposition 3.597, Proposition 3.596 and Axiom 1.1. □

Proposition 3.598. $\forall x \in ((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}).x \neq \{\{1\}\} \Rightarrow x \in (\wp\ \{1\})$.

Proof. Use Proposition 3.154, Lemma 3.48 and Proposition 3.468. □

Proposition 3.599. $((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}) \setminus \{\{\{1\}\}\} \subseteq (\wp\ \{1\})$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.598 and Definition 1.1. □

Proposition 3.600. $(\wp\ \{1\}) \subseteq ((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}) \setminus \{\{\{1\}\}\}$.

Proof. Use Proposition 3.202, Proposition 3.294, Proposition 3.147, Lemma 3.15, Proposition 3.291, Proposition 3.20, Proposition 3.212 and Definition 1.1. □

Proposition 3.601. $((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}) \setminus \{\{\{1\}\}\} = (\wp\ \{1\})$.

Proof. Use Proposition 3.600, Proposition 3.599 and Axiom 1.1. □

Proposition 3.602. $\forall x \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})).x \neq 0 \Rightarrow x \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup \{\{\{1\}\}\})$.

Proof. Use Proposition 3.311, Proposition 3.310, Proposition 3.309, Proposition 3.308 and Proposition 3.472. □

Proposition 3.603. $(((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{0\} \subseteq (((\wp\ 2) \setminus (\wp\ \{2\})) \cup \{\{\{1\}\}\})$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.602 and Definition 1.1. □

Lemma 3.181. $(((\wp\ 2) \setminus (\wp\ \{2\})) \cup \{\{\{1\}\}\}) \subseteq (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{0\}$.

Proof. Use Proposition 3.147, Proposition 3.316, Lemma 3.43, Proposition 3.315, Proposition 3.139, Proposition 3.314, Proposition 3.113, Lemma 3.15, Proposition 3.313, Proposition 3.20, Lemma 3.146 and Definition 1.1. □

Proposition 3.604. $(((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{0\} = (((\wp\ 2) \setminus (\wp\ \{2\})) \cup \{\{\{1\}\}\})$.

Proof. Use Lemma 3.181, Proposition 3.603 and Axiom 1.1. □

Lemma 3.182. $\forall x \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})).x \neq 1 \Rightarrow x \in (((\wp\ 2) \setminus 2) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.307, Proposition 3.306, Proposition 3.305, Proposition 3.304 and Proposition 3.472. □

Lemma 3.183. $(((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{1\} \subseteq (((\wp\ 2) \setminus 2) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Lemma 3.182 and Definition 1.1. □

Proposition 3.605. $(((\wp\ 2) \setminus 2) \cup (\wp\ \{\{1\}\})) \subseteq (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{1\}$.

Proof. Use Proposition 3.184, Proposition 3.316, Proposition 3.114, Proposition 3.315, Proposition 3.140, Proposition 3.314, Proposition 3.113, Lemma 3.15, Proposition 3.312, Proposition 3.20, Proposition 3.471 and Definition 1.1. □

Proposition 3.606. $(((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{1\} = (((\wp\ 2) \setminus 2) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.605, Lemma 3.183 and Axiom 1.1. □

Proposition 3.607. $\forall x \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})).x \neq \{1\} \Rightarrow x \in (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\wp\ \{\{1\}$

Proof. Use Proposition 3.303, Proposition 3.302, Proposition 3.300, Proposition 3.299 and Proposition 3.472. □

Proposition 3.608. $(((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{\{1\}\} \subseteq (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.607 and Definition 1.1. □

Proposition 3.609. $(((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\wp\ \{\{1\}\})) \subseteq (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{\{1\}\}$.

Proof. Use Proposition 3.202, Proposition 3.316, Proposition 3.156, Proposition 3.315, Proposition 3.140, Proposition 3.313, Proposition 3.139, Lemma 3.15, Proposition 3.312, Proposition 3.20, Proposition 3.470 and Definition 1.1. □

Lemma 3.184. $(((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{\{1\}\} = (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.609, Proposition 3.608 and Axiom 1.1. □

Proposition 3.610. $\forall x \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})).x \neq 2 \Rightarrow x \in ((\{1\} \cup \{\{1\}\}) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Lemma 3.97, Proposition 3.298, Lemma 3.96, Proposition 3.297 and Proposition 3.472. □

Proposition 3.611. $(((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{2\} \subseteq ((\{1\} \cup \{\{1\}\}) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.610 and Definition 1.1. □

Proposition 3.612. $((\{1\} \cup \{\{1\}\}) \cup (\wp\ \{\{1\}\})) \subseteq (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{2\}$.

Proof. Use Proposition 3.203, Proposition 3.316, Proposition 3.156, Proposition 3.314, Proposition 3.114, Proposition 3.313, Lemma 3.43, Lemma 3.15, Proposition 3.312, Proposition 3.20, Proposition 3.469 and Definition 1.1. □

Proposition 3.613. $(((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{2\} = ((\{1\} \cup \{\{1\}\}) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.612, Proposition 3.611 and Axiom 1.1. □

Lemma 3.185. $\forall x \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})).x \neq \{\{1\}\} \Rightarrow x \in (\wp\ 2)$.

Proof. Use Proposition 3.161, Proposition 3.138, Proposition 3.135, Lemma 3.49 and Proposition 3.472. □

Proposition 3.614. $(((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{\{\{1\}\}\} \subseteq (\wp\ 2)$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Lemma 3.185 and Definition 1.1. □

Proposition 3.615. $(\wp\ 2) \subseteq (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{\{\{1\}\}\}$.

Proof. Use Proposition 3.203, Proposition 3.315, Proposition 3.202, Proposition 3.314, Proposition 3.184, Proposition 3.313, Proposition 3.147, Lemma 3.15, Proposition 3.312, Proposition 3.20, Lemma 3.83 and Definition 1.1. □

Proposition 3.616. $(((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{\{\{1\}\}\} = (\wp\ 2)$.

Proof. Use Proposition 3.615, Proposition 3.614 and Axiom 1.1. □

Proposition 3.617. $\forall x \in (4 \setminus \{2\}).x \neq 0 \Rightarrow x \in (\{1\} \cup \{3\})$.

Proof. Use Lemma 3.112, Proposition 3.356 and Proposition 3.474. □

Proposition 3.618. $(4 \setminus \{2\}) \setminus \{0\} \subseteq (\{1\} \cup \{3\})$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.617 and Definition 1.1. □

Proposition 3.619. $(\{1\} \cup \{3\}) \subseteq (4 \setminus \{2\}) \setminus \{0\}$.

Proof. Use Lemma 3.44, Proposition 3.361, Proposition 3.113, Lemma 3.15, Lemma 3.113, Proposition 3.20, Proposition 3.14, Proposition 3.16 and Definition 1.1. □

Proposition 3.620. $(4 \setminus \{2\}) \setminus \{0\} = (\{1\} \cup \{3\})$.

Proof. Use Proposition 3.619, Proposition 3.618 and Axiom 1.1. □

Proposition 3.621. $\forall x \in (4 \setminus \{2\}).x \neq 1 \Rightarrow x \in (1 \cup \{3\})$.

Proof. Use Proposition 3.355, Proposition 3.353 and Proposition 3.474. □

Proposition 3.622. $(4 \setminus \{2\}) \setminus \{1\} \subseteq (1 \cup \{3\})$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.621 and Definition 1.1. □

Lemma 3.186. $(1 \cup \{3\}) \subseteq (4 \setminus \{2\}) \setminus \{1\}$.

Proof. Use Proposition 3.115, Proposition 3.361, Proposition 3.14, Proposition 3.113, Lemma 3.15, Proposition 3.357, Proposition 3.20, Proposition 3.2, Proposition 3.16 and Definition 1.1. □

Proposition 3.623. $(4 \setminus \{2\}) \setminus \{1\} = (1 \cup \{3\})$.

Proof. Use Lemma 3.186, Proposition 3.622 and Axiom 1.1. □

Proposition 3.624. $\forall x \in (4 \setminus \{2\}).x \neq 3 \Rightarrow x \in 2$.

Proof. Use Lemma 3.1, Proposition 3.3 and Proposition 3.474. □

Lemma 3.187. $(4 \setminus \{2\}) \setminus \{3\} \subseteq 2$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.624 and Definition 1.1. □

Proposition 3.625. $2 \subseteq (4 \setminus \{2\}) \setminus \{3\}$.

Proof. Use Proposition 3.115, Lemma 3.113, Lemma 3.44, Lemma 3.15, Proposition 3.357, Proposition 3.20, Lemma 3.2 and Definition 1.1. □

Proposition 3.626. $(4 \setminus \{2\}) \setminus \{3\} = 2$.

Proof. Use Proposition 3.625, Lemma 3.187 and Axiom 1.1. □

Proposition 3.627. $\forall x \in (4 \setminus 2).x \neq 2 \Rightarrow x \in \{3\}$.

Proof. Use Proposition 3.352 and Proposition 3.476. □

Lemma 3.188. $(4 \setminus 2) \setminus \{2\} \subseteq \{3\}$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.627 and Definition 1.1. □

Proposition 3.628. $\{3\} \subseteq (4 \setminus 2) \setminus \{2\}$.

Proof. Use Lemma 3.45, Lemma 3.15, Proposition 3.368, Proposition 3.20, Proposition 3.14 and Definition 1.1. □

Proposition 3.629. $(4 \setminus 2) \setminus \{2\} = \{3\}$.

Proof. Use Proposition 3.628, Lemma 3.188 and Axiom 1.1. □

Lemma 3.189. $\forall x \in (4 \setminus 2).x \neq 3 \Rightarrow x \in \{2\}$.

Proof. Use Proposition 3.132 and Proposition 3.476. □

Proposition 3.630. $(4 \setminus 2) \setminus \{3\} \subseteq \{2\}$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Lemma 3.189 and Definition 1.1. □

Proposition 3.631. $\{2\} \subseteq (4 \setminus 2) \setminus \{3\}$.

Proof. Use Lemma 3.45, Lemma 3.15, Proposition 3.367, Proposition 3.20, Proposition 3.237 and Definition 1.1. □

Lemma 3.190. $(4 \setminus 2) \setminus \{3\} = \{2\}$.

Proof. Use Proposition 3.631, Proposition 3.630 and Axiom 1.1. □

Lemma 3.191. $\forall x \in (4 \setminus \{1\}).x \neq 0 \Rightarrow x \in (4 \setminus 2)$.

Proof. Use Proposition 3.368, Proposition 3.367 and Proposition 3.477. □

Proposition 3.632. $(4 \setminus \{1\}) \setminus \{0\} \subseteq (4 \setminus 2)$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Lemma 3.191 and Definition 1.1. □

Proposition 3.633. $(4 \setminus 2) \subseteq (4 \setminus \{1\}) \setminus \{0\}$.

Proof. Use Lemma 3.44, Proposition 3.369, Lemma 3.43, Lemma 3.15, Lemma 3.98, Proposition 3.20, Proposition 3.476 and Definition 1.1. □

Proposition 3.634. $(4 \setminus \{1\}) \setminus \{0\} = (4 \setminus 2)$.

Proof. Use Proposition 3.633, Proposition 3.632 and Axiom 1.1. □

Lemma 3.192. $\forall x \in (4 \setminus \{1\}).x \neq 2 \Rightarrow x \in (1 \cup \{3\})$.

Proof. Use Proposition 3.355, Proposition 3.353 and Proposition 3.477. □

Proposition 3.635. $(4 \setminus \{1\}) \setminus \{2\} \subseteq (1 \cup \{3\})$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Lemma 3.192 and Definition 1.1. □

Proposition 3.636. $(1 \cup \{3\}) \subseteq (4 \setminus \{1\}) \setminus \{2\}$.

Proof. Use Lemma 3.45, Proposition 3.369, Proposition 3.14, Lemma 3.43, Lemma 3.15, Proposition 3.332, Proposition 3.20, Proposition 3.2, Proposition 3.16 and Definition 1.1. □

Proposition 3.637. $(4 \setminus \{1\}) \setminus \{2\} = (1 \cup \{3\})$.

Proof. Use Proposition 3.636, Proposition 3.635 and Axiom 1.1. □

Proposition 3.638. $\forall x \in (4 \setminus \{1\}).x \neq 3 \Rightarrow x \in (3 \setminus \{1\})$.

Proof. Use Proposition 3.159, Proposition 3.134 and Proposition 3.477. □

Proposition 3.639. $(4 \setminus \{1\}) \setminus \{3\} \subseteq (3 \setminus \{1\})$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.638 and Definition 1.1. □

Proposition 3.640. $(3 \setminus \{1\}) \subseteq (4 \setminus \{1\}) \setminus \{3\}$.

Proof. Use Lemma 3.45, Lemma 3.98, Lemma 3.44, Lemma 3.15, Proposition 3.332, Proposition 3.20, Proposition 3.251 and Definition 1.1. □

Lemma 3.193. $(4 \setminus \{1\}) \setminus \{3\} = (3 \setminus \{1\})$.

Proof. Use Proposition 3.640, Proposition 3.639 and Axiom 1.1. □

Proposition 3.641. $\forall x \in (4 \setminus (\wp\ \{2\})).x \neq 1 \Rightarrow x \in (4 \setminus 2)$.

Proof. Use Proposition 3.368, Proposition 3.367 and Proposition 3.479. □

Lemma 3.194. $(4 \setminus (\wp\ \{2\})) \setminus \{1\} \subseteq (4 \setminus 2)$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.641 and Definition 1.1. □

Lemma 3.195. $(4 \setminus 2) \subseteq (4 \setminus (\wp\ \{2\})) \setminus \{1\}$.

Proof. Use Proposition 3.115, Proposition 3.371, Proposition 3.114, Lemma 3.15, Lemma 3.118, Proposition 3.20, Proposition 3.476 and Definition 1.1. □

Proposition 3.642. $(4 \setminus (\wp\ \{2\})) \setminus \{1\} = (4 \setminus 2)$.

Proof. Use Lemma 3.195, Lemma 3.194 and Axiom 1.1. □

Proposition 3.643. $\forall x \in (4 \setminus (\wp\ \{2\})).x \neq 2 \Rightarrow x \in (\{1\} \cup \{3\})$.

Proof. Use Lemma 3.112, Proposition 3.356 and Proposition 3.479. □

Proposition 3.644. $(4 \setminus (\wp\ \{2\})) \setminus \{2\} \subseteq (\{1\} \cup \{3\})$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.643 and Definition 1.1. □

Proposition 3.645. $(\{1\} \cup \{3\}) \subseteq (4 \setminus (\wp\ \{2\})) \setminus \{2\}$.

Proof. Use Lemma 3.45, Proposition 3.371, Proposition 3.114, Lemma 3.15, Proposition 3.370, Proposition 3.20, Proposition 3.14, Proposition 3.16 and Definition 1.1. □

Proposition 3.646. $(4 \setminus (\wp\ \{2\})) \setminus \{2\} = (\{1\} \cup \{3\})$.

Proof. Use Proposition 3.645, Proposition 3.644 and Axiom 1.1. □

Proposition 3.647. $\forall x \in (4 \setminus (\wp\ \{2\})).x \neq 3 \Rightarrow x \in ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Proposition 3.164, Proposition 3.136 and Proposition 3.479. □

Proposition 3.648. $(4 \setminus (\wp\ \{2\})) \setminus \{3\} \subseteq ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.647 and Definition 1.1. □

Proposition 3.649. $((\wp\ 2) \setminus (\wp\ \{1\})) \subseteq (4 \setminus (\wp\ \{2\})) \setminus \{3\}$.

Proof. Use Lemma 3.45, Lemma 3.118, Proposition 3.115, Lemma 3.15, Proposition 3.370, Proposition 3.20, Lemma 3.84 and Definition 1.1. □

Proposition 3.650. $(4 \setminus (\wp\ \{2\})) \setminus \{3\} = ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Proposition 3.649, Proposition 3.648 and Axiom 1.1. □

Proposition 3.651. $\forall x \in 4.x \neq 0 \Rightarrow x \in (4 \setminus (\wp\ \{2\}))$.

Proof. Use Proposition 3.371, Lemma 3.118, Proposition 3.370 and Proposition 3.10. □

Lemma 3.196. $4 \setminus \{0\} \subseteq (4 \setminus (\wp \{2\}))$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Proposition 3.651 and Definition 1.1. □

Proposition 3.652. $(4 \setminus (\wp \{2\})) \subseteq 4 \setminus \{0\}$.

Proof. Use Lemma 3.44, Proposition 3.9, Lemma 3.43, Proposition 3.8, Proposition 3.113, Lemma 3.15, Proposition 3.7, Proposition 3.20, Proposition 3.479 and Definition 1.1. □

Proposition 3.653. $4 \setminus \{0\} = (4 \setminus (\wp \{2\}))$.

Proof. Use Proposition 3.652, Lemma 3.196 and Axiom 1.1. □

Lemma 3.197. $\forall x \in 4 . x \neq 3 \Rightarrow x \in 3$.

Proof. Use Proposition 3.5, Proposition 3.4, Lemma 3.3 and Proposition 3.10. □

Proposition 3.654. $4 \setminus \{3\} \subseteq 3$.

Proof. Use Proposition 3.21, Proposition 3.13, Lemma 3.6, Lemma 3.197 and Definition 1.1. □

Lemma 3.198. $3 \subseteq 4 \setminus \{3\}$.

Proof. Use Lemma 3.45, Proposition 3.8, Proposition 3.115, Proposition 3.7, Lemma 3.44, Lemma 3.15, Lemma 3.4, Proposition 3.20, Proposition 3.6 and Definition 1.1. □

Lemma 3.199. $4 \setminus \{3\} = 3$.

Proof. Use Lemma 3.198, Proposition 3.654 and Axiom 1.1. □

Proposition 3.655. $3 \cup \{\{2\}\} \subseteq \wp\, 2 \cup \{\{2\}\}$.

Proof. Use Lemma 3.8, Proposition 3.28, Proposition 3.258, Lemma 3.7 and Lemma 3.9. □

Proposition 3.656. $\{2\} \cup \wp \{2\} \subseteq \{2\} \cup (\wp \{2\} \cup \{\wp\, 2\})$.

Proof. Use Proposition 3.28 and Lemma 3.11. □

Proposition 3.657. $3 \subseteq \wp\, 2 \cup \{\{2\}\}$.

Proof. Use Proposition 3.28, Proposition 3.258 and Lemma 3.7. □

Proposition 3.658. $3 \subseteq 4 \cup \{((\wp\, 2) \setminus 2)\}$.

Proof. Use Proposition 3.28, Proposition 3.112 and Lemma 3.7. ☐

Proposition 3.659. $\{2\} \cup \wp\,\{2\} \subseteq \wp\,2 \cup \{\{2\}\}$.

Proof. Use Proposition 3.13, Lemma 3.5, Proposition 3.26, Lemma 3.28, Proposition 3.27, Proposition 3.15, Proposition 3.14, Proposition 3.16 and Definition 1.1. ☐

Lemma 3.200. $\{2\} \cup \wp\,\{2\} \subseteq (\{2\} \cup (\wp\,\{2\})) \cup \{(3 \setminus \{1\})\}$.

Proof. Use Proposition 3.28. ☐

Lemma 3.201. $2 \subseteq 3 \cup \{(3 \setminus \{1\})\}$.

Proof. Use Proposition 3.28, Proposition 3.110 and Lemma 3.7. ☐

Lemma 3.202. $2 \subseteq \wp\,(3 \setminus \{1\})$.

Proof. Use Lemma 3.107, Lemma 3.106, Lemma 3.2 and Definition 1.1. ☐

Proposition 3.660. $3 \setminus \{1\} \subseteq 4 \cup \{(\wp\,2)\}$.

Proof. Use Proposition 3.28, Proposition 3.112, Proposition 3.35 and Lemma 3.7. ☐

Proposition 3.661. $\{1\} \cup \wp\,\{2\} \subseteq (\{1\} \cup \{\{1\}\}) \cup \wp\,\{2\}$.

Proof. Use Lemma 3.8, Lemma 3.11 and Proposition 3.32. ☐

Proposition 3.662. $(\{1\} \cup \{\{1\}\}) \cup \wp\,\{2\} \subseteq (\{1\} \cup \{\{1\}\}) \cup ((\wp\,\{2\}) \cup \{3\})$.

Proof. Use Proposition 3.28 and Lemma 3.11. ☐

Proposition 3.663. $\{1\} \cup \wp\,\{2\} \subseteq (\{1\} \cup \{\{1\}\}) \cup ((\wp\,\{2\}) \cup \{3\})$.

Proof. Use Proposition 3.662, Proposition 3.661 and Lemma 3.7. ☐

Proposition 3.664. $\wp\,\{2\} \subseteq \{\{2\}\} \cup 4$.

Proof. Use Proposition 3.13, Proposition 3.15, Lemma 3.4, Lemma 3.5, Lemma 3.28 and Definition 1.1. ☐

Lemma 3.203. $\{1\} \cup \wp\,\{2\} \subseteq \{\{2\}\} \cup 4$.

Proof. Use Proposition 3.664, Lemma 3.120, Proposition 3.48 and Lemma 3.9. ☐

Proposition 3.665. $\{2\} \cup \wp\,\{2\} \subseteq \{\{2\}\} \cup 4$.

Proof. Use Proposition 3.664, Proposition 3.8, Lemma 3.5, Proposition 3.48 and Lemma 3.9. ☐

Proposition 3.666. $\{2\} \cup \wp\,\{2\} \subseteq \{\{1\}\} \cup (\{\{2\}\} \cup 4)$.

Proof. Use Lemma 3.8, Proposition 3.665 and Lemma 3.7. ☐

Proposition 3.667. $\{2\} \subseteq ((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{\wp\ 2\})$.

Proof. Use Proposition 3.28, Proposition 3.239 and Lemma 3.7. □

Proposition 3.668. $\wp\ \{2\} \subseteq ((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{\wp\ 2\})$.

Proof. Use Lemma 3.8, Proposition 3.28 and Lemma 3.7. □

Proposition 3.669. $\{2\} \cup \wp\ \{2\} \subseteq ((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{\wp\ 2\})$.

Proof. Use Proposition 3.668, Proposition 3.667 and Lemma 3.9. □

Proposition 3.670. $(3 \setminus \{1\}) \subseteq ((\wp\ 2) \cup \{\{2\}\})$.

Proof. Use Proposition 3.28, Lemma 3.82 and Lemma 3.7. □

Lemma 3.204. $(3 \setminus \{1\}) \subseteq 4$.

Proof. Use Proposition 3.8, Lemma 3.4, Proposition 3.251 and Definition 1.1. □

Lemma 3.205. $(3 \setminus \{1\}) \subseteq (\{\{1\}\} \cup 4)$.

Proof. Use Lemma 3.8, Lemma 3.204 and Lemma 3.7. □

Proposition 3.671. $(3 \setminus \{1\}) \subseteq (\{\{2\}\} \cup 4)$.

Proof. Use Lemma 3.8, Lemma 3.204 and Lemma 3.7. □

Proposition 3.672. $(3 \setminus \{1\}) \subseteq ((\wp\ 2) \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.28, Lemma 3.82 and Lemma 3.7. □

Proposition 3.673. $((\wp\ 2) \setminus (\wp\ \{1\})) \subseteq 4$.

Proof. Use Proposition 3.8, Proposition 3.7, Lemma 3.84 and Definition 1.1. □

Lemma 3.206. $((\wp\ 2) \setminus (\wp\ \{1\})) \subseteq (4 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.28, Proposition 3.673 and Lemma 3.7. □

Lemma 3.207. $\forall u \in 3.u \in \wp\ (3 \setminus \{1\}) \Rightarrow u \in 2$.

Proof. Use Proposition 3.347, Lemma 3.1, Proposition 3.3 and Proposition 3.6. □

Proposition 3.674. $(((3 \cup \{(3 \setminus \{1\})\}) \cap (\wp\ (3 \setminus \{1\}))) \setminus \{3 \setminus \{1\}\}) \subseteq 2$.

Proof. Use Proposition 3.18, Proposition 3.19, Lemma 3.207, Proposition 3.16 and Lemma 3.13. □

Proposition 3.675. $(\wp\ \{2\}) \subseteq (3 \cup \{\{2\}\})$.

Proof. Use Lemma 3.103, Proposition 3.343, Lemma 3.28 and Definition 1.1. □

Proposition 3.676. $((\wp\ 2) \cup \{\{2\}\}) \cap ((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\}) = (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Lemma 3.200, Proposition 3.659, Proposition 3.14, Proposition 3.335, Proposition 3.16 and Proposition 3.37. □

Lemma 3.208. $\forall x \in 4.x \in 3 \vee x = 3$.

Proof. Use Proposition 3.5, Proposition 3.4, Lemma 3.3 and Proposition 3.10. □

Proposition 3.677. $((\wp\ 2) \cup \{\{2\}\}) \cap (4 \cup \{((\wp\ 2) \setminus 2)\}) = 3$.

Proof. Use Proposition 3.658, Proposition 3.657, Proposition 3.14, Proposition 3.336, Lemma 3.100, Lemma 3.208, Proposition 3.16 and Proposition 3.37. □

Lemma 3.209. $\forall x \in (\wp\ 2) \cup \{\{2\}\}.x \in (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \Rightarrow x \notin \{\wp\ 2\}$.

Proof. Use Proposition 3.14 and Proposition 3.337. □

Proposition 3.678. $\forall x \in (\wp\ 2) \cup \{\{2\}\}.x \in (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \Rightarrow x \in (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Lemma 3.11, Lemma 3.209 and Proposition 3.16. □

Proposition 3.679. $((\wp\ 2) \cup \{\{2\}\}) \cap (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) = (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.656, Proposition 3.659, Proposition 3.678 and Proposition 3.37. □

Proposition 3.680. $((\wp\ 2) \cup \{\{2\}\}) \cap (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})) = (\wp\ 2)$.

Proof. Use Proposition 3.529, Proposition 3.524, Lemma 3.139, Proposition 3.14, Proposition 3.16 and Proposition 3.37. □

Proposition 3.681. $\forall x \in \wp\ 2.x \in 3 \vee x = \{1\}$.

Proof. Use Proposition 3.5, Proposition 3.4, Lemma 3.3 and Lemma 3.83. □

Lemma 3.210. $((\wp\ 2) \cup \{\{2\}\}) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})) = (3 \cup \{\{2\}\})$.

Proof. Use Proposition 3.511, Proposition 3.655, Lemma 3.5, Proposition 3.443, Proposition 3.15, Proposition 3.681, Proposition 3.16 and Proposition 3.37. □

Proposition 3.682. $((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\}) \cap (\{\{1\}\} \cup (\{\{2\}\} \cup 4)) = (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.666, Lemma 3.200, Proposition 3.390, Proposition 3.14, Proposition 3.16 and Proposition 3.37. □

Lemma 3.211. $((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\}) \cap (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) = (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.656, Lemma 3.200, Proposition 3.420, Proposition 3.14, Proposition 3.16 and Proposition 3.37. □

Lemma 3.212. $((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\}) \cap (((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) = (\{2\} \cup (\wp$

Proof. Use Proposition 3.669, Lemma 3.200, Proposition 3.427, Proposition 3.14, Proposition 3.16 and Proposition 3.37. □

Proposition 3.683. $(\{\{1\}\} \cup (\{\{2\}\} \cup 4)) \cap (4 \cup \{((\wp\ 2) \setminus 2)\}) = 4.$

Proof. Use Proposition 3.506, Lemma 3.151, Proposition 3.391, Proposition 3.14, Proposition 3.16 and Proposition 3.37. □

Proposition 3.684. $\forall x \in \{\{1\}\} \cup (\{\{2\}\} \cup 4).x \in \{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}) \Rightarrow x \notin \{\wp\ 2\}.$

Proof. Use Proposition 3.392 and Proposition 3.14. □

Lemma 3.213. $\forall x \in \{\{1\}\} \cup (\{\{2\}\} \cup 4).x \in \{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}) \Rightarrow x \in \{2\} \cup (\wp\ \{2\}).$

Proof. Use Lemma 3.11, Proposition 3.684 and Proposition 3.16. □

Proposition 3.685. $(\{\{1\}\} \cup (\{\{2\}\} \cup 4)) \cap (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) = (\{2\} \cup (\wp\ \{2\})).$

Proof. Use Proposition 3.656, Proposition 3.666, Lemma 3.213 and Proposition 3.37. □

Proposition 3.686. $(4 \cup \{((\wp\ 2) \setminus 2)\}) \cap (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})) = 4.$

Proof. Use Proposition 3.509, Proposition 3.506, Proposition 3.442, Proposition 3.14, Proposition 3.16 and Proposition 3.37. □

Lemma 3.214. $(3 \setminus \{1\}) \subseteq ((\wp\ 2) \cup \{\{2\}\}) \cap (4 \setminus \{1\}).$

Proof. Use Proposition 3.513, Proposition 3.670 and Proposition 3.34. □

Proposition 3.687. $((\wp\ 2) \cup \{\{2\}\}) \cap (4 \setminus \{1\}) \subseteq (3 \setminus \{1\}).$

Proof. Use Proposition 3.21, Lemma 3.100, Lemma 3.6, Proposition 3.20, Lemma 3.208 and Proposition 3.36. □

Proposition 3.688. $((\wp\ 2) \cup \{\{2\}\}) \cap (4 \setminus \{1\}) = (3 \setminus \{1\}).$

Proof. Use Lemma 3.214, Proposition 3.687 and Axiom 1.1. □

Proposition 3.689. $(\{\{1\}\} \cup 4) \cap (\{2\} \cup (\wp\ \{2\})) \subseteq (3 \setminus \{1\}).$

Proof. Use Proposition 3.339, Lemma 3.101, Proposition 3.20, Lemma 3.208, Proposition 3.340, Proposition 3.14, Proposition 3.16 and Proposition 3.36. □

Proposition 3.690. $(3 \setminus \{1\}) \subseteq (\{\{1\}\} \cup 4) \cap (\{2\} \cup (\wp\ \{2\})).$

Proof. Use Proposition 3.514, Lemma 3.205 and Proposition 3.34. □

Lemma 3.215. $(\{\{1\}\} \cup 4) \cap (\{2\} \cup (\wp\ \{2\})) = (3 \setminus \{1\}).$

Proof. Use Proposition 3.690, Proposition 3.689 and Axiom 1.1. □

Proposition 3.691. $(\{\{2\} \cup 4) \cap ((\wp\ 2) \setminus \{1\}) \subseteq (3 \setminus \{1\})$.

Proof. Use Proposition 3.183, Lemma 3.88, Proposition 3.20, Lemma 3.208, Proposition 3.265, Proposition 3.14, Proposition 3.16 and Proposition 3.36. □

Proposition 3.692. $(3 \setminus \{1\}) \subseteq (\{\{2\} \cup 4) \cap ((\wp\ 2) \setminus \{1\})$.

Proof. Use Proposition 3.252, Proposition 3.671 and Proposition 3.34. □

Proposition 3.693. $(\{\{2\} \cup 4) \cap ((\wp\ 2) \setminus \{1\}) = (3 \setminus \{1\})$.

Proof. Use Proposition 3.692, Proposition 3.691 and Axiom 1.1. □

Proposition 3.694. $((\wp\ 2) \cup \{(\wp\ 2)\}) \cap (4 \setminus \{1\}) \subseteq (3 \setminus \{1\})$.

Proof. Use Proposition 3.21, Proposition 3.406, Lemma 3.6, Proposition 3.20, Lemma 3.208 and Proposition 3.36. □

Proposition 3.695. $(3 \setminus \{1\}) \subseteq ((\wp\ 2) \cup \{(\wp\ 2)\}) \cap (4 \setminus \{1\})$.

Proof. Use Proposition 3.513, Proposition 3.672 and Proposition 3.34. □

Lemma 3.216. $((\wp\ 2) \cup \{(\wp\ 2)\}) \cap (4 \setminus \{1\}) = (3 \setminus \{1\})$.

Proof. Use Proposition 3.695, Proposition 3.694 and Axiom 1.1. □

Proposition 3.696. $(4 \cup \{(\wp\ 2)\}) \cap ((\wp\ 2) \setminus (\wp\ \{2\})) \subseteq ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Proposition 3.431, Proposition 3.172, Lemma 3.28, Proposition 3.21, Proposition 3.20 and Proposition 3.36. □

Proposition 3.697. $((\wp\ 2) \setminus (\wp\ \{1\})) \subseteq (4 \cup \{(\wp\ 2)\}) \cap ((\wp\ 2) \setminus (\wp\ \{2\}))$.

Proof. Use Proposition 3.253, Lemma 3.206 and Proposition 3.34. □

Proposition 3.698. $(4 \cup \{(\wp\ 2)\}) \cap ((\wp\ 2) \setminus (\wp\ \{2\})) = ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Proposition 3.697, Proposition 3.696 and Axiom 1.1. □

Proposition 3.699. $(4 \cup \{(\wp\ 2)\}) \cap (\{2\} \cup (\wp\ \{2\})) \subseteq (3 \setminus \{1\})$.

Proof. Use Proposition 3.339, Lemma 3.102, Proposition 3.14, Lemma 3.101, Proposition 3.20, Lemma 3.208, Proposition 3.16 and Proposition 3.36. □

Proposition 3.700. $(3 \setminus \{1\}) \subseteq (4 \cup \{(\wp\ 2)\}) \cap (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.514, Proposition 3.660 and Proposition 3.34. □

Proposition 3.701. $(4 \cup \{(\wp\ 2)\}) \cap (\{2\} \cup (\wp\ \{2\})) = (3 \setminus \{1\})$.

Proof. Use Proposition 3.700, Proposition 3.699 and Axiom 1.1. ☐

Lemma 3.217. $\neg$atleast2 1.

Proof. Use Proposition 3.27, Proposition 3.2, Definition 1.3 and Definition 1.1. ☐

Proposition 3.702. $\neg$atleast2 $\{1\}$.

Proof. Use Lemma 3.24. ☐

Proposition 3.703. $\neg$atleast2 $\{\{1\}\}$.

Proof. Use Lemma 3.24. ☐

Proposition 3.704. $\neg$atleast2 $\{2\}$.

Proof. Use Lemma 3.24. ☐

Proposition 3.705. $\neg$atleast2 $\{3\}$.

Proof. Use Lemma 3.24. ☐

Proposition 3.706. $\neg$atleast2 $\{(\wp\ 2)\}$.

Proof. Use Lemma 3.24. ☐

Proposition 3.707. $\forall x \in 2.\neg$atleast2 $(2 \setminus \{x\})$.

Proof. Use Lemma 3.164, Lemma 3.217, Lemma 3.163, Proposition 3.702, Lemma 3.2 and Definition 1.3. ☐

Proposition 3.708. $\neg$atleast3 2.

Proof. Use Lemma 3.22, Proposition 3.55, Proposition 3.707 and Definition 1.4. ☐

Proposition 3.709. $\neg$atleast3 $(\wp\ \{1\})$.

Proof. Use Lemma 3.29. ☐

Proposition 3.710. $\neg$atleast3 $(\{1\} \cup \{\{1\}\})$.

Proof. Use Proposition 3.71. ☐

Proposition 3.711. $\forall x \in (3 \setminus \{1\}).\neg$atleast2 $((3 \setminus \{1\}) \setminus \{x\})$.

Proof. Use Proposition 3.556, Lemma 3.217, Lemma 3.169, Proposition 3.704, Proposition 3.251 and Definition 1.3. ☐

Proposition 3.712. $\neg$atleast3 $(3 \setminus \{1\})$.

Proof. Use Lemma 3.22, Proposition 3.55, Proposition 3.711 and Definition 1.4. ☐

Proposition 3.713. $\forall x \in ((\wp\ 2) \setminus (\wp\ \{1\})).\neg \mathsf{atleast2}\ (((\wp\ 2) \setminus (\wp\ \{1\})) \setminus \{x\})$.

Proof. Use Lemma 3.170, Proposition 3.702, Proposition 3.559, Proposition 3.704, Lemma 3.84 and Definition 1.3. □

Proposition 3.714. $\neg \mathsf{atleast3}\ ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Lemma 3.22, Proposition 3.55, Proposition 3.713 and Definition 1.4. □

Proposition 3.715. $\forall x \in ((\wp\ 2) \setminus 2).\neg \mathsf{atleast2}\ (((\wp\ 2) \setminus 2) \setminus \{x\})$.

Proof. Use Proposition 3.567, Proposition 3.703, Proposition 3.564, Proposition 3.704, Proposition 3.264 and Definition 1.3. □

Proposition 3.716. $\neg \mathsf{atleast3}\ ((\wp\ 2) \setminus 2)$.

Proof. Use Lemma 3.22, Proposition 3.55, Proposition 3.715 and Definition 1.4. □

Proposition 3.717. $\neg \mathsf{atleast3}\ (\wp\ \{\{1\}\})$.

Proof. Use Lemma 3.29. □

Proposition 3.718. $\neg \mathsf{atleast3}\ (\{\{1\}\} \cup \{\{\{1\}\}\})$.

Proof. Use Proposition 3.71. □

Lemma 3.218. $\neg \mathsf{atleast3}\ (\wp\ \{2\})$.

Proof. Use Lemma 3.29. □

Lemma 3.219. $\neg \mathsf{atleast3}\ (1 \cup \{3\})$.

Proof. Use Proposition 3.71 and Proposition 3.54. □

Proposition 3.719. $\neg \mathsf{atleast3}\ (\{1\} \cup \{3\})$.

Proof. Use Proposition 3.71. □

Proposition 3.720. $\forall x \in 4.x \notin 2 \Rightarrow \neg \mathsf{atleast2}\ ((4 \setminus 2) \setminus \{x\})$.

Proof. Use Lemma 3.190, Proposition 3.704, Proposition 3.629, Proposition 3.705, Lemma 3.1, Proposition 3.3, Proposition 3.10 and Definition 1.3. □

Proposition 3.721. $\neg \mathsf{atleast3}\ (4 \setminus 2)$.

Proof. Use Lemma 3.22, Proposition 3.55, Lemma 3.6, Proposition 3.21, Proposition 3.720 and Definition 1.4. □

Proposition 3.722. $\neg \mathsf{atleast3}\ (1 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.71 and Proposition 3.54. □

Proposition 3.723. $\neg\mathsf{atleast3}\ (\{\{2\}\} \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.71. □

Proposition 3.724. $\forall x \in ((\wp\ 2) \setminus \{2\}).\neg\mathsf{atleast3}\ (((\wp\ 2) \setminus \{2\}) \setminus \{x\})$.

Proof. Use Proposition 3.552, Proposition 3.708, Proposition 3.548, Proposition 3.709, Proposition 3.545, Proposition 3.710, Lemma 3.145 and Definition 1.4. □

Proposition 3.725. $\neg\mathsf{atleast4}\ ((\wp\ 2) \setminus \{2\})$.

Proof. Use Lemma 3.22, Proposition 3.56, Proposition 3.724 and Definition 1.5. □

Proposition 3.726. $\forall x \in 3.\neg\mathsf{atleast3}\ (3 \setminus \{x\})$.

Proof. Use Proposition 3.542, Proposition 3.708, Proposition 3.712, Lemma 3.165, Proposition 3.714, Proposition 3.6 and Definition 1.4. □

Proposition 3.727. $\neg\mathsf{atleast4}\ 3$.

Proof. Use Lemma 3.22, Proposition 3.56, Proposition 3.726 and Definition 1.5. □

Proposition 3.728. $\forall x \in ((\wp\ 2) \setminus \{1\}).\neg\mathsf{atleast3}\ (((\wp\ 2) \setminus \{1\}) \setminus \{x\})$.

Proof. Use Proposition 3.577, Proposition 3.709, Proposition 3.573, Proposition 3.712, Proposition 3.570, Proposition 3.716, Proposition 3.460 and Definition 1.4. □

Proposition 3.729. $\neg\mathsf{atleast4}\ ((\wp\ 2) \setminus \{1\})$.

Proof. Use Lemma 3.22, Proposition 3.56, Proposition 3.728 and Definition 1.5. □

Proposition 3.730. $\forall x \in ((\wp\ 2) \setminus (\wp\ \{2\})).\neg\mathsf{atleast3}\ (((\wp\ 2) \setminus (\wp\ \{2\})) \setminus \{x\})$.

Proof. Use Lemma 3.176, Proposition 3.710, Proposition 3.584, Proposition 3.714, Proposition 3.581, Proposition 3.716, Proposition 3.461 and Definition 1.4. □

Proposition 3.731. $\neg\mathsf{atleast4}\ ((\wp\ 2) \setminus (\wp\ \{2\}))$.

Proof. Use Lemma 3.22, Proposition 3.56, Proposition 3.730 and Definition 1.5. □

Proposition 3.732. $\forall x \in ((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}).\neg\mathsf{atleast3}\ (((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\}$

Proof. Use Proposition 3.601, Proposition 3.709, Lemma 3.180, Proposition 3.717, Proposition 3.594, Proposition 3.718, Proposition 3.468 and Definition 1.4. □

Proposition 3.733. $\neg\mathsf{atleast4}\ ((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\})$.

Proof. Use Lemma 3.22, Proposition 3.56, Proposition 3.732 and Definition 1.5. □

Proposition 3.734. $\neg\mathsf{atleast4}\ (\{1\} \cup (\wp\ \{2\}))$.

Proof. Use Lemma 3.218, Proposition 3.79 and Proposition 3.32. ☐

Lemma 3.220. $\neg\mathsf{atleast4}\ (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Lemma 3.218, Proposition 3.79 and Proposition 3.32. ☐

Proposition 3.735. $\forall x \in 4.x \neq 2 \Rightarrow \neg\mathsf{atleast3}\ ((4 \setminus \{2\}) \setminus \{x\})$.

Proof. Use Proposition 3.626, Proposition 3.708, Proposition 3.623, Lemma 3.219, Proposition 3.620, Proposition 3.719, Proposition 3.10 and Definition 1.4. ☐

Proposition 3.736. $\neg\mathsf{atleast4}\ (4 \setminus \{2\})$.

Proof. Use Lemma 3.22, Proposition 3.56, Lemma 3.6, Proposition 3.44, Proposition 3.21, Proposition 3.735 and Definition 1.5. ☐

Proposition 3.737. $\forall x \in 4.x \neq 1 \Rightarrow \neg\mathsf{atleast3}\ ((4 \setminus \{1\}) \setminus \{x\})$.

Proof. Use Lemma 3.193, Proposition 3.712, Proposition 3.637, Lemma 3.219, Proposition 3.634, Proposition 3.721, Proposition 3.10 and Definition 1.4. ☐

Proposition 3.738. $\neg\mathsf{atleast4}\ (4 \setminus \{1\})$.

Proof. Use Lemma 3.22, Proposition 3.56, Lemma 3.6, Proposition 3.44, Proposition 3.21, Proposition 3.737 and Definition 1.5. ☐

Proposition 3.739. $\forall x \in 4.x \notin \wp\ \{2\} \Rightarrow \neg\mathsf{atleast3}\ ((4 \setminus (\wp\ \{2\})) \setminus \{x\})$.

Proof. Use Proposition 3.650, Proposition 3.714, Proposition 3.646, Proposition 3.719, Proposition 3.642, Proposition 3.721, Proposition 3.171, Proposition 3.10 and Definition 1.4. ☐

Proposition 3.740. $\neg\mathsf{atleast4}\ (4 \setminus (\wp\ \{2\}))$.

Proof. Use Lemma 3.22, Proposition 3.56, Lemma 3.6, Proposition 3.21, Proposition 3.739 and Definition 1.5. ☐

Proposition 3.741. $\forall x \in (\wp\ \{2\}) \cup \{3\}.\neg\mathsf{atleast3}\ (((\wp\ \{2\}) \cup \{3\}) \setminus \{x\})$.

Proof. Use Proposition 3.705, Lemma 3.218 and Proposition 3.100. ☐

Proposition 3.742. $\neg\mathsf{atleast4}\ ((\wp\ \{2\}) \cup \{3\})$.

Proof. Use Lemma 3.22, Proposition 3.56, Proposition 3.741 and Definition 1.5. ☐

Lemma 3.221. $\neg\mathsf{atleast4}\ (2 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.708 and Proposition 3.79. ☐

Proposition 3.743. $\neg\mathsf{atleast4}\ (\{\{\{1\}\}\} \cup (1 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.722, Proposition 3.79 and Proposition 3.32. □

Proposition 3.744. $\forall x \in (\wp\ \{2\}) \cup \{(\wp\ 2)\}. \neg \mathsf{atleast3}\ (((\wp\ \{2\}) \cup \{(\wp\ 2)\}) \setminus \{x\})$.

Proof. Use Proposition 3.706, Lemma 3.218 and Proposition 3.100. □

Proposition 3.745. $\neg \mathsf{atleast4}\ ((\wp\ \{2\}) \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 3.22, Proposition 3.56, Proposition 3.744 and Definition 1.5. □

Proposition 3.746. $\forall x \in (\wp\ 2). \neg \mathsf{atleast4}\ ((\wp\ 2) \setminus \{x\})$.

Proof. Use Proposition 3.725, Proposition 3.591, Proposition 3.727, Proposition 3.729, Proposition 3.589, Proposition 3.731, Lemma 3.83 and Definition 1.5. □

Proposition 3.747. $\neg \mathsf{atleast5}\ (\wp\ 2)$.

Proof. Use Lemma 3.22, Lemma 3.23, Proposition 3.746 and Definition 1.6. □

Proposition 3.748. $\neg \mathsf{atleast5}\ ((\{1\} \cup \{\{1\}\}) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.717, Proposition 3.710 and Proposition 3.89. □

Proposition 3.749. $\neg \mathsf{atleast5}\ (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.717, Proposition 3.714 and Proposition 3.89. □

Proposition 3.750. $\neg \mathsf{atleast5}\ (((\wp\ 2) \setminus 2) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.717, Proposition 3.716 and Proposition 3.89. □

Lemma 3.222. $\neg \mathsf{atleast5}\ (((\wp\ 2) \setminus (\wp\ \{2\})) \cup \{\{\{1\}\}\})$.

Proof. Use Proposition 3.731 and Proposition 3.85. □

Proposition 3.751. $\neg \mathsf{atleast5}\ (3 \cup \{(\wp\ \{1\})\})$.

Proof. Use Proposition 3.727 and Proposition 3.85. □

Proposition 3.752. $\neg \mathsf{atleast5}\ (3 \cup \{\{2\}\})$.

Proof. Use Proposition 3.727 and Proposition 3.85. □

Proposition 3.753. $\forall x \in 4. \neg \mathsf{atleast4}\ (4 \setminus \{x\})$.

Proof. Use Lemma 3.199, Proposition 3.727, Proposition 3.736, Proposition 3.738, Proposition 3.653, Proposition 3.740, Proposition 3.10 and Definition 1.5. □

Proposition 3.754. $\neg \mathsf{atleast5}\ 4$.

Proof. Use Lemma 3.22, Lemma 3.23, Proposition 3.753 and Definition 1.6. □

Proposition 3.755. $\neg\mathsf{atleast5}\ (3 \cup \{((\wp\ 2) \setminus 2)\})$.

Proof. Use Proposition 3.727 and Proposition 3.85. ☐

Proposition 3.756. $\neg\mathsf{atleast5}\ (((\wp\ 2) \setminus \{2\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\})$.

Proof. Use Proposition 3.725 and Proposition 3.85. ☐

Lemma 3.223. $\neg\mathsf{atleast5}\ (((\wp\ 2) \setminus \{1\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\})$.

Proof. Use Proposition 3.729 and Proposition 3.85. ☐

Proposition 3.757. $\neg\mathsf{atleast5}\ (3 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.727 and Proposition 3.85. ☐

Proposition 3.758. $\neg\mathsf{atleast5}\ (\{\{\{1\}\}\} \cup (2 \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 3.221, Proposition 3.85 and Proposition 3.32. ☐

Proposition 3.759. $\forall x \in \{1\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}).\neg\mathsf{atleast4}\ ((\{1\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \setminus \{x\})$.

Proof. Use Proposition 3.745, Proposition 3.702 and Proposition 3.101. ☐

Proposition 3.760. $\neg\mathsf{atleast5}\ (\{1\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 3.22, Lemma 3.23, Proposition 3.759 and Definition 1.6. ☐

Lemma 3.224. $\forall x \in \{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}).\neg\mathsf{atleast4}\ ((\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \setminus \{x\})$.

Proof. Use Proposition 3.745, Proposition 3.704 and Proposition 3.101. ☐

Lemma 3.225. $\neg\mathsf{atleast5}\ (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 3.22, Lemma 3.23, Lemma 3.224 and Definition 1.6. ☐

Proposition 3.761. $\forall x \in ((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\{\{2\}\} \cup \{(\wp\ 2)\}).\neg\mathsf{atleast4}\ ((((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\{\{2\}\}$

Proof. Use Proposition 3.723, Proposition 3.714 and Lemma 3.37. ☐

Proposition 3.762. $\neg\mathsf{atleast5}\ (((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\{\{2\}\} \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 3.22, Lemma 3.23, Proposition 3.761 and Definition 1.6. ☐

Proposition 3.763. $\forall x.\mathsf{atleast5}\ ((((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{x\}) \Rightarrow x \neq 0$.

Proof. Use Proposition 3.604, Lemma 3.222 and Definition 1.6. ☐

Proposition 3.764. $\forall x.\mathsf{atleast5}\ ((((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{x\}) \Rightarrow x \neq 1$.

Proof. Use Proposition 3.606, Proposition 3.750 and Definition 1.6. ☐

Proposition 3.765. $\forall x.\mathsf{atleast5}\ ((((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{x\}) \Rightarrow x \neq \{1\}$.

Proof. Use Lemma 3.184, Proposition 3.749 and Definition 1.6. □

Lemma 3.226. $\forall x.\mathsf{atleast5}\ ((((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{x\}) \Rightarrow x \neq 2.$

Proof. Use Proposition 3.613, Proposition 3.748 and Definition 1.6. □

Proposition 3.766. $\forall x.\mathsf{atleast5}\ ((((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{x\}) \Rightarrow x \neq \{\{1\}\}.$

Proof. Use Proposition 3.616, Proposition 3.747 and Definition 1.6. □

Proposition 3.767. $\forall x \in (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})).\neg\mathsf{atleast5}\ ((((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{$

Proof. Use Proposition 3.766, Lemma 3.226, Proposition 3.765, Proposition 3.764, Proposition 3.763 and Proposition 3.472. □

Proposition 3.768. $\neg\mathsf{atleast6}\ (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})).$

Proof. Use Lemma 3.22, Proposition 3.57, Proposition 3.767 and Definition 1.7. □

Proposition 3.769. $\neg\mathsf{atleast6}\ ((\wp\ 2) \cup \{\{2\}\}).$

Proof. Use Proposition 3.747 and Proposition 3.94. □

Proposition 3.770. $\neg\mathsf{atleast6}\ (\{\{1\}\} \cup 4).$

Proof. Use Proposition 3.754, Proposition 3.94 and Proposition 3.32. □

Lemma 3.227. $\neg\mathsf{atleast6}\ (\{\{\{1\}\}\} \cup 4).$

Proof. Use Proposition 3.754, Proposition 3.94 and Proposition 3.32. □

Proposition 3.771. $\neg\mathsf{atleast6}\ (\{(\wp\ \{1\})\} \cup 4).$

Proof. Use Proposition 3.754, Proposition 3.94 and Proposition 3.32. □

Proposition 3.772. $\neg\mathsf{atleast6}\ (\{\{2\}\} \cup 4).$

Proof. Use Proposition 3.754, Proposition 3.94 and Proposition 3.32. □

Proposition 3.773. $\neg\mathsf{atleast6}\ (4 \cup \{((\wp\ 2) \setminus 2)\}).$

Proof. Use Proposition 3.754 and Proposition 3.94. □

Lemma 3.228. $\neg\mathsf{atleast6}\ ((\wp\ 2) \cup \{(\wp\ 2)\}).$

Proof. Use Proposition 3.747 and Proposition 3.94. □

Proposition 3.774. $\neg\mathsf{atleast6}\ (4 \cup \{(\wp\ 2)\}).$

Proof. Use Proposition 3.754 and Proposition 3.94. □

Proposition 3.775. $\neg\mathsf{atleast7}\ (\{\{1\}\} \cup (\{\{2\}\} \cup 4)).$

Proof. Use Proposition 3.772, Proposition 3.98 and Proposition 3.32. □

Proposition 3.776. $\neg\mathsf{atleast7}\ (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.774, Proposition 3.98 and Proposition 3.32. □

Proposition 3.777. $\neg\mathsf{atleast7}\ (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.774, Proposition 3.98 and Proposition 3.32. □

Proposition 3.778. $\mathsf{atleast2}\ 2$.

Proof. Use Proposition 3.116, Lemma 3.1, Proposition 3.3 and Proposition 3.46. □

Proposition 3.779. $\mathsf{atleast2}\ (\{1\} \cup \{\{1\}\})$.

Proof. Use Proposition 3.140 and Lemma 3.19. □

Lemma 3.229. $\mathsf{atleast2}\ (3 \setminus \{1\})$.

Proof. Use Proposition 3.117, Proposition 3.159, Proposition 3.134 and Proposition 3.46. □

Proposition 3.780. $\mathsf{atleast2}\ ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Lemma 3.41, Proposition 3.164, Proposition 3.136 and Proposition 3.46. □

Proposition 3.781. $\mathsf{atleast2}\ ((\wp\ 2) \setminus 2)$.

Proof. Use Proposition 3.162, Proposition 3.165, Lemma 3.54 and Proposition 3.46. □

Proposition 3.782. $\mathsf{atleast2}\ (\wp\ \{\{1\}\})$.

Proof. Use Proposition 3.129 and Lemma 3.18. □

Lemma 3.230. $\mathsf{atleast2}\ (4 \setminus 2)$.

Proof. Use Proposition 3.111, Proposition 3.368, Proposition 3.367 and Proposition 3.46. □

Proposition 3.783. $\mathsf{atleast3}\ ((\wp\ 2) \setminus \{2\})$.

Proof. Use Proposition 3.779, Proposition 3.223, Proposition 3.224 and Definition 1.4. □

Proposition 3.784. $\mathsf{atleast3}\ 3$.

Proof. Use Proposition 3.778, Proposition 3.111, Proposition 3.110 and Definition 1.4. □

Proposition 3.785. $\mathsf{atleast3}\ ((\wp\ 2) \setminus \{1\})$.

Proof. Use Proposition 3.781, Proposition 3.261, Proposition 3.262 and Definition 1.4. □

Proposition 3.786. atleast3 $((\wp\ 2) \setminus (\wp\ \{2\}))$.

Proof. Use Proposition 3.781, Proposition 3.263, Lemma 3.87 and Definition 1.4. □

Proposition 3.787. atleast3 $((\wp\ (\wp\ \{1\})) \setminus \{(\wp\ \{1\})\})$.

Proof. Use Proposition 3.778, Proposition 3.480, Lemma 3.148 and Definition 1.4. □

Proposition 3.788. atleast3 $((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\})$.

Proof. Use Proposition 3.782, Proposition 3.517, Proposition 3.516 and Definition 1.4. □

Proposition 3.789. atleast3 $(\{1\} \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.778, Proposition 3.482, Proposition 3.481 and Definition 1.4. □

Proposition 3.790. atleast3 $(\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Lemma 3.229, Proposition 3.515, Proposition 3.514 and Definition 1.4. □

Proposition 3.791. atleast3 $(4 \setminus \{2\})$.

Proof. Use Proposition 3.778, Lemma 3.149, Proposition 3.483 and Definition 1.4. □

Proposition 3.792. atleast3 $(4 \setminus \{1\})$.

Proof. Use Lemma 3.229, Lemma 3.155, Proposition 3.513 and Definition 1.4. □

Lemma 3.231. atleast3 $(2 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.778, Lemma 3.150, Proposition 3.484 and Definition 1.4. □

Proposition 3.793. atleast3 $(\{\{\{1\}\}\} \cup (1 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.782, Proposition 3.512, Lemma 3.154 and Definition 1.4. □

Lemma 3.232. atleast4 $(\wp\ 2)$.

Proof. Use Proposition 3.786, Proposition 3.268, Proposition 3.269 and Definition 1.5. □

Proposition 3.794. atleast4 $(((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.784, Proposition 3.486, Proposition 3.485 and Definition 1.5. □

Proposition 3.795. atleast4 $(3 \cup \{(\wp\ \{1\})\})$.

Proof. Use Proposition 3.784, Proposition 3.488, Proposition 3.487 and Definition 1.5. □

Proposition 3.796. atleast4 $(3 \cup \{\{2\}\})$.

Proof. Use Proposition 3.784, Proposition 3.490, Proposition 3.489 and Definition 1.5. □

Proposition 3.797. atleast4 4.

Proof. Use Proposition 3.784, Lemma 3.42, Proposition 3.112 and Definition 1.5. □

Lemma 3.233. atleast4 $(3 \cup \{((\wp\ 2) \setminus 2)\})$.

Proof. Use Proposition 3.784, Proposition 3.492, Proposition 3.491 and Definition 1.5. □

Proposition 3.798. atleast4 $(((\wp\ 2) \setminus \{2\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\})$.

Proof. Use Proposition 3.783, Proposition 3.523, Proposition 3.522 and Definition 1.5. □

Proposition 3.799. atleast4 $(((\wp\ 2) \setminus \{1\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\})$.

Proof. Use Proposition 3.785, Proposition 3.521, Proposition 3.520 and Definition 1.5. □

Proposition 3.800. atleast4 $(3 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.784, Proposition 3.494, Proposition 3.493 and Definition 1.5. □

Proposition 3.801. atleast4 $(\{\{\{1\}\}\} \cup (2 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.787, Proposition 3.519, Proposition 3.518 and Definition 1.5. □

Lemma 3.234. atleast5 $(((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.794, Proposition 3.528, Proposition 3.527 and Definition 1.6. □

Lemma 3.235. atleast5 $((\wp\ 2) \cup \{\{2\}\})$.

Proof. Use Lemma 3.232, Proposition 3.525, Proposition 3.524 and Definition 1.6. □

Lemma 3.236. atleast5 $(\{\{1\}\} \cup 4)$.

Proof. Use Proposition 3.797, Proposition 3.499, Proposition 3.498 and Definition 1.6. ☐

Proposition 3.802. atleast5 $(\{\{\{1\}\}\} \cup 4)$.

Proof. Use Proposition 3.797, Proposition 3.501, Proposition 3.500 and Definition 1.6. ☐

Proposition 3.803. atleast5 $(\{(\wp\ \{1\})\} \cup 4)$.

Proof. Use Proposition 3.797, Proposition 3.503, Proposition 3.502 and Definition 1.6. ☐

Lemma 3.237. atleast5 $(\{\{2\}\} \cup 4)$.

Proof. Use Proposition 3.797, Proposition 3.505, Proposition 3.504 and Definition 1.6. ☐

Proposition 3.804. atleast5 $(4 \cup \{((\wp\ 2) \setminus 2)\})$.

Proof. Use Proposition 3.797, Lemma 3.152, Proposition 3.506 and Definition 1.6. ☐

Proposition 3.805. atleast5 $((\wp\ 2) \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 3.232, Lemma 3.157, Lemma 3.156 and Definition 1.6. ☐

Proposition 3.806. atleast5 $(4 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.797, Proposition 3.508, Proposition 3.507 and Definition 1.6. ☐

Proposition 3.807. atleast6 $(\{\{1\}\} \cup (\{\{2\}\} \cup 4))$.

Proof. Use Lemma 3.237, Proposition 3.533, Lemma 3.160 and Definition 1.7. ☐

Proposition 3.808. atleast6 $(\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.806, Proposition 3.532, Lemma 3.159 and Definition 1.7. ☐

Lemma 3.238. atleast6 $(\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.806, Proposition 3.531, Proposition 3.530 and Definition 1.7. ☐

Proposition 3.809. atleast3 $((3 \cup \{(3 \setminus \{1\})\}) \cap (\wp\ (3 \setminus \{1\})))$.

Proof. Use Proposition 3.778, Proposition 3.348, Proposition 3.349, Proposition 3.17, Lemma 3.64, Lemma 3.202, Lemma 3.201, Proposition 3.34, Definition 1.4 and Definition 1.1. ☐

Lemma 3.239. atleast5 $((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})))$.

Proof. Use Proposition 3.796, Proposition 3.446, Proposition 3.428, Proposition 3.17, Proposition 3.346, Proposition 3.511, Proposition 3.497, Proposition 3.34, Definition 1.6 and Definition 1.1. □

Theorem 3.1. $\mathsf{exactly2}\ 2$.

Proof. Use Proposition 3.708, Proposition 3.778 and Definition 1.9. □

Proposition 3.810. $\mathsf{exactly2}\ (\wp\ \{1\})$.

Proof. Use Proposition 3.68. □

Theorem 3.2. $\mathsf{exactly2}\ (\{1\} \cup \{\{1\}\})$.

Proof. Use Proposition 3.140 and Proposition 3.72. □

Theorem 3.3. $\mathsf{exactly2}\ (3 \setminus \{1\})$.

Proof. Use Proposition 3.712, Lemma 3.229 and Definition 1.9. □

Theorem 3.4. $\mathsf{exactly2}\ (((\wp\ 2) \cup \{\{2\}\}) \cap (4 \setminus \{1\}))$.

Proof. Use Theorem 3.3 and Proposition 3.688. □

Theorem 3.5. $\mathsf{exactly2}\ ((\{\{2\}\} \cup 4) \cap ((\wp\ 2) \setminus \{1\}))$.

Proof. Use Theorem 3.3 and Proposition 3.693. □

Theorem 3.6. $\mathsf{exactly2}\ (((\wp\ 2) \cup \{(\wp\ 2)\}) \cap (4 \setminus \{1\}))$.

Proof. Use Theorem 3.3 and Lemma 3.216. □

Theorem 3.7. $\mathsf{exactly2}\ ((4 \cup \{(\wp\ 2)\}) \cap (\{2\} \cup (\wp\ \{2\})))$.

Proof. Use Theorem 3.3 and Proposition 3.701. □

Theorem 3.8. $\mathsf{exactly2}\ ((\wp\ 2) \setminus (\wp\ \{1\}))$.

Proof. Use Proposition 3.714, Proposition 3.780 and Definition 1.9. □

Theorem 3.9. $\mathsf{exactly2}\ ((\wp\ 2) \setminus 2)$.

Proof. Use Proposition 3.716, Proposition 3.781 and Definition 1.9. □

Proposition 3.811. $\mathsf{exactly2}\ (\wp\ \{2\})$.

Proof. Use Proposition 3.68. □

Proposition 3.812. $\mathsf{exactly2}\ (\{2\} \cup \{\{2\}\})$.

Proof. Use Lemma 3.80 and Proposition 3.72. □

Theorem 3.10. $\mathsf{exactly2}\ (1 \cup \{3\})$.

Proof. Use Lemma 3.44, Proposition 3.72 and Proposition 3.54. □

Proposition 3.813. $\mathsf{exactly2}\ (\{\{1\}\} \cup \{3\})$.

Proof. Use Proposition 3.198 and Proposition 3.72. □

Theorem 3.11. $\mathsf{exactly2}\ (4 \setminus 2)$.

Proof. Use Proposition 3.721, Lemma 3.230 and Definition 1.9. □

Theorem 3.12. $\mathsf{exactly2}\ ((\{\{1\}\} \cup 4) \cap (\{2\} \cup (\wp\ \{2\})))$.

Proof. Use Theorem 3.3 and Lemma 3.215. □

Theorem 3.13. $\mathsf{exactly2}\ (1 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.248, Proposition 3.72 and Proposition 3.54. □

Proposition 3.814. $\mathsf{exactly2}\ (\{3\} \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 3.89 and Proposition 3.72. □

Theorem 3.14. $\mathsf{exactly2}\ (\{((\wp\ 2) \setminus (\wp\ \{2\}))\} \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.273 and Proposition 3.72. □

Theorem 3.15. $\mathsf{exactly2}\ ((4 \cup \{(\wp\ 2)\}) \cap ((\wp\ 2) \setminus (\wp\ \{2\})))$.

Proof. Use Theorem 3.8 and Proposition 3.698. □

Theorem 3.16. $\mathsf{exactly3}\ ((\wp\ 2) \setminus \{2\})$.

Proof. Use Proposition 3.725, Proposition 3.783 and Definition 1.10. □

Theorem 3.17. $\mathsf{exactly3}\ 3$.

Proof. Use Proposition 3.727, Proposition 3.784 and Definition 1.10. □

Theorem 3.18. $\mathsf{exactly2}\ (3 \setminus \{2\})$.

Proof. Use Theorem 3.17, Proposition 3.5 and Proposition 3.75. □

Theorem 3.19. $\mathsf{exactly3}\ ((\wp\ 2) \setminus \{1\})$.

Proof. Use Proposition 3.729, Proposition 3.785 and Definition 1.10. □

Proposition 3.815. $\mathsf{exactly3}\ ((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\})$.

Proof. Use Proposition 3.733, Proposition 3.788 and Definition 1.10. □

Theorem 3.20. $\mathsf{exactly2}\ (((\wp\ (\{1\} \cup \{\{1\}\})) \setminus \{(\{1\} \cup \{\{1\}\})\}) \setminus \{0\})$.

Proof. Use Proposition 3.815, Proposition 3.291 and Proposition 3.75. ☐

Theorem 3.21. $\mathsf{exactly3}\ (\{2\} \cup (\wp\ \{2\}))$.

Proof. Use Lemma 3.220, Proposition 3.790 and Definition 1.10. ☐

Theorem 3.22. $\mathsf{exactly3}\ (((\wp\ 2) \cup \{\{2\}\}) \cap ((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\}))$.

Proof. Use Theorem 3.21, Proposition 3.676 and Definition 1.10. ☐

Theorem 3.23. $\mathsf{exactly3}\ (4 \setminus \{2\})$.

Proof. Use Proposition 3.736, Proposition 3.791 and Definition 1.10. ☐

Theorem 3.24. $\mathsf{exactly2}\ ((4 \setminus \{2\}) \setminus \{1\})$.

Proof. Use Theorem 3.23, Lemma 3.113 and Proposition 3.75. ☐

Theorem 3.25. $\mathsf{exactly2}\ ((4 \setminus \{2\}) \setminus \{3\})$.

Proof. Use Theorem 3.23, Proposition 3.361 and Proposition 3.75. ☐

Theorem 3.26. $\mathsf{exactly3}\ (4 \setminus \{1\})$.

Proof. Use Proposition 3.738, Proposition 3.792 and Definition 1.10. ☐

Theorem 3.27. $\mathsf{exactly2}\ ((4 \setminus \{1\}) \setminus \{0\})$.

Proof. Use Theorem 3.26, Proposition 3.332 and Proposition 3.75. ☐

Theorem 3.28. $\mathsf{exactly2}\ ((4 \setminus \{1\}) \setminus \{2\})$.

Proof. Use Theorem 3.26, Lemma 3.98 and Proposition 3.75. ☐

Theorem 3.29. $\mathsf{exactly2}\ ((4 \setminus \{1\}) \setminus \{3\})$.

Proof. Use Theorem 3.26, Proposition 3.369 and Proposition 3.75. ☐

Theorem 3.30. $\mathsf{exactly3}\ (2 \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 3.221, Lemma 3.231 and Definition 1.10. ☐

Theorem 3.31. $\mathsf{exactly3}\ (\{\{\{1\}\}\} \cup (1 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.743, Proposition 3.793 and Definition 1.10. ☐

Theorem 3.32. $\mathsf{exactly3}\ (((\wp\ 2) \cup \{\{2\}\}) \cap (4 \cup \{((\wp\ 2) \setminus 2)\}))$.

Proof. Use Theorem 3.17, Proposition 3.677 and Definition 1.10. ☐

Theorem 3.33. $\mathsf{exactly3}\ (((\wp\ 2) \cup \{\{2\}\}) \cap (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})))$.

Proof. Use Theorem 3.21, Proposition 3.679 and Definition 1.10. ☐

Theorem 3.34. $\mathsf{exactly3}\ (((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\}) \cap (\{\{1\}\} \cup (\{\{2\}\} \cup 4)))$.

Proof. Use Theorem 3.21, Proposition 3.682 and Definition 1.10. □

Theorem 3.35. $\mathsf{exactly3}\ (((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\}) \cap (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})))$.

Proof. Use Theorem 3.21, Lemma 3.211 and Definition 1.10. □

Theorem 3.36. $\mathsf{exactly3}\ (((\{2\} \cup (\wp\ \{2\})) \cup \{(3 \setminus \{1\})\}) \cap (((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})))$.

Proof. Use Theorem 3.21, Lemma 3.212 and Definition 1.10. □

Lemma 3.240. $\forall u \in 3.u \in \wp\ (3 \setminus \{1\}) \Rightarrow u \notin \{0\} \Rightarrow u \in \{1\} \cup \{3 \setminus \{1\}\}$.

Proof. Use Proposition 3.347, Proposition 3.15, Proposition 3.13 and Proposition 3.6. □

Proposition 3.816. $(((3 \cup \{(3 \setminus \{1\})\}) \cap (\wp\ (3 \setminus \{1\}))) \setminus \{0\}) \subseteq \{1\} \cup \{3 \setminus \{1\}\}$.

Proof. Use Lemma 3.5, Proposition 3.19, Lemma 3.240, Proposition 3.18, Proposition 3.16 and Lemma 3.13. □

Proposition 3.817. $\neg\mathsf{atleast3}\ (((3 \cup \{(3 \setminus \{1\})\}) \cap (\wp\ (3 \setminus \{1\}))) \setminus \{0\})$.

Proof. Use Proposition 3.816, Proposition 3.56 and Proposition 3.71. □

Lemma 3.241. $\forall u \in 3.u \in \wp\ (3 \setminus \{1\}) \Rightarrow u \notin \{1\} \Rightarrow u \in \{0\} \cup \{3 \setminus \{1\}\}$.

Proof. Use Proposition 3.347, Proposition 3.13, Proposition 3.15 and Proposition 3.6. □

Proposition 3.818. $(((3 \cup \{(3 \setminus \{1\})\}) \cap (\wp\ (3 \setminus \{1\}))) \setminus \{1\}) \subseteq \{0\} \cup \{3 \setminus \{1\}\}$.

Proof. Use Lemma 3.5, Proposition 3.19, Lemma 3.241, Proposition 3.18, Proposition 3.16 and Lemma 3.13. □

Proposition 3.819. $\neg\mathsf{atleast3}\ (((3 \cup \{(3 \setminus \{1\})\}) \cap (\wp\ (3 \setminus \{1\}))) \setminus \{1\})$.

Proof. Use Proposition 3.818, Proposition 3.56 and Proposition 3.71. □

Proposition 3.820. $\forall u \in 3.u \in \wp\ (3 \setminus \{1\}) \Rightarrow \neg\mathsf{atleast3}\ (((3 \cup \{(3 \setminus \{1\})\}) \cap (\wp\ (3 \setminus \{1\}))) \setminus \{u\})$.

Proof. Use Proposition 3.347, Proposition 3.819, Proposition 3.817 and Proposition 3.6. □

Lemma 3.242. $\neg\mathsf{atleast3}\ (((3 \cup \{(3 \setminus \{1\})\}) \cap (\wp\ (3 \setminus \{1\}))) \setminus \{3 \setminus \{1\}\})$.

Proof. Use Proposition 3.674, Proposition 3.56 and Proposition 3.708. □

Proposition 3.821. $\neg\mathsf{atleast4}\ ((3 \cup \{(3 \setminus \{1\})\}) \cap (\wp\ (3 \setminus \{1\})))$.

Proof. Use Proposition 3.18, Proposition 3.19, Lemma 3.242, Proposition 3.14, Proposition 3.820, Proposition 3.56, Proposition 3.16, Lemma 3.22 and Definition 1.5. ☐

Theorem 3.37. $\mathsf{exactly3}\ ((3 \cup \{(3 \setminus \{1\})\}) \cap (\wp\ (3 \setminus \{1\})))$.

Proof. Use Proposition 3.821, Proposition 3.809 and Definition 1.10. ☐

Theorem 3.38. $\mathsf{exactly3}\ ((\{\{1\}\} \cup (\{\{2\}\} \cup 4)) \cap (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})))$.

Proof. Use Theorem 3.21, Proposition 3.685 and Definition 1.10. ☐

Theorem 3.39. $\mathsf{exactly4}\ (\wp\ 2)$.

Proof. Use Proposition 3.747, Lemma 3.232 and Definition 1.11. ☐

Theorem 3.40. $\mathsf{exactly3}\ ((\wp\ 2) \setminus \{0\})$.

Proof. Use Theorem 3.39, Lemma 3.49 and Lemma 3.32. ☐

Theorem 3.41. $\mathsf{exactly3}\ ((\wp\ 2) \setminus \{\{1\}\})$.

Proof. Use Theorem 3.39, Proposition 3.138 and Lemma 3.32. ☐

Theorem 3.42. $\mathsf{exactly4}\ (3 \cup \{(\wp\ \{1\})\})$.

Proof. Use Proposition 3.751, Proposition 3.795 and Definition 1.11. ☐

Theorem 3.43. $\mathsf{exactly4}\ (3 \cup \{\{2\}\})$.

Proof. Use Proposition 3.752, Proposition 3.796 and Definition 1.11. ☐

Theorem 3.44. $\mathsf{exactly3}\ ((3 \cup \{\{2\}\}) \setminus \{2\})$.

Proof. Use Theorem 3.43, Proposition 3.345 and Lemma 3.32. ☐

Theorem 3.45. $\mathsf{exactly4}\ 4$.

Proof. Use Proposition 3.754, Proposition 3.797 and Definition 1.11. ☐

Theorem 3.46. $\mathsf{exactly3}\ (4 \setminus \{0\})$.

Proof. Use Theorem 3.45, Lemma 3.4 and Lemma 3.32. ☐

Theorem 3.47. $\mathsf{exactly3}\ (4 \setminus \{3\})$.

Proof. Use Theorem 3.45, Proposition 3.9 and Lemma 3.32. ☐

Theorem 3.48. $\mathsf{exactly4}\ ((\{2\} \cup \{\{2\}\}) \cup (1 \cup \{3\}))$.

Proof. Use Theorem 3.10, Proposition 3.812, Proposition 3.354 and Proposition 3.99. ☐

Theorem 3.49. $\mathsf{exactly4}\ (2 \cup (\{\{1\}\} \cup \{3\}))$.

Proof. Use Proposition 3.813, Theorem 3.1, Proposition 3.364 and Proposition 3.99. □

Theorem 3.50. $\mathsf{exactly4}\ (2 \cup (\{3\} \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.814, Theorem 3.1, Proposition 3.429 and Proposition 3.99. □

Theorem 3.51. $\mathsf{exactly4}\ ((\wp\ \{1\}) \cup (4 \setminus 2))$.

Proof. Use Theorem 3.11, Proposition 3.810, Proposition 3.366 and Proposition 3.99. □

Theorem 3.52. $\mathsf{exactly4}\ (((\wp\ 2) \setminus 2) \cup (\wp\ \{2\}))$.

Proof. Use Proposition 3.811, Theorem 3.9, Lemma 3.85 and Proposition 3.99. □

Theorem 3.53. $\mathsf{exactly4}\ (3 \cup \{((\wp\ 2) \setminus 2)\})$.

Proof. Use Proposition 3.755, Lemma 3.233 and Definition 1.11. □

Theorem 3.54. $\mathsf{exactly4}\ (((\wp\ 2) \setminus \{2\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\})$.

Proof. Use Proposition 3.756, Proposition 3.798 and Definition 1.11. □

Theorem 3.55. $\mathsf{exactly4}\ (((\wp\ 2) \setminus \{1\}) \cup \{((\wp\ 2) \setminus (\wp\ \{2\}))\})$.

Proof. Use Lemma 3.223, Proposition 3.799 and Definition 1.11. □

Theorem 3.56. $\mathsf{exactly4}\ (3 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.757, Proposition 3.800 and Definition 1.11. □

Proposition 3.822. $\mathsf{exactly4}\ (\{\{\{1\}\}\} \cup (2 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.758, Proposition 3.801 and Definition 1.11. □

Theorem 3.57. $\mathsf{exactly4}\ (((\wp\ 2) \cup \{\{2\}\}) \cap (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})))$.

Proof. Use Theorem 3.39, Proposition 3.680 and Definition 1.11. □

Theorem 3.58. $\mathsf{exactly4}\ (((\wp\ 2) \cup \{\{2\}\}) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})))$.

Proof. Use Theorem 3.43, Lemma 3.210 and Definition 1.11. □

Theorem 3.59. $\mathsf{exactly4}\ ((\{\{1\}\} \cup (\{\{2\}\} \cup 4)) \cap (4 \cup \{((\wp\ 2) \setminus 2)\}))$.

Proof. Use Theorem 3.45, Proposition 3.683 and Definition 1.11. □

Theorem 3.60. $\mathsf{exactly4}\ ((4 \cup \{((\wp\ 2) \setminus 2)\}) \cap (\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})))$.

Proof. Use Theorem 3.45, Proposition 3.686 and Definition 1.11. □

Theorem 3.61. $\mathsf{exactly3}\ ((3 \cup \{(\wp\ 2)\}) \setminus \{0\})$.

Proof. Use Theorem 3.56, Lemma 3.129 and Lemma 3.32. ☐

Theorem 3.62. $\mathsf{exactly3}\ ((3 \cup \{(\wp\ 2)\}) \setminus \{1\})$.

Proof. Use Theorem 3.56, Proposition 3.408 and Lemma 3.32. ☐

Theorem 3.63. $\mathsf{exactly3}\ ((3 \cup \{(\wp\ 2)\}) \setminus \{2\})$.

Proof. Use Theorem 3.56, Proposition 3.409 and Lemma 3.32. ☐

Theorem 3.64. $\mathsf{exactly3}\ ((\{\{\{1\}\}\} \cup (2 \cup \{(\wp\ 2)\})) \setminus \{0\})$.

Proof. Use Proposition 3.822, Proposition 3.412 and Lemma 3.32. ☐

Proposition 3.823. $\mathsf{exactly5}\ (((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\}))$.

Proof. Use Proposition 3.768, Lemma 3.234 and Definition 1.12. ☐

Theorem 3.65. $\mathsf{exactly4}\ ((((\wp\ 2) \setminus (\wp\ \{2\})) \cup (\wp\ \{\{1\}\})) \setminus \{0\})$.

Proof. Use Proposition 3.823, Proposition 3.312 and Proposition 3.84. ☐

Theorem 3.66. $\mathsf{exactly5}\ ((\wp\ 2) \cup \{\{2\}\})$.

Proof. Use Proposition 3.769, Lemma 3.235 and Definition 1.12. ☐

Theorem 3.67. $\mathsf{exactly5}\ (\{\{1\}\} \cup 4)$.

Proof. Use Proposition 3.770, Lemma 3.236 and Definition 1.12. ☐

Theorem 3.68. $\mathsf{exactly5}\ (\{\{\{1\}\}\} \cup 4)$.

Proof. Use Lemma 3.227, Proposition 3.802 and Definition 1.12. ☐

Theorem 3.69. $\mathsf{exactly5}\ (\{(\wp\ \{1\})\} \cup 4)$.

Proof. Use Proposition 3.771, Proposition 3.803 and Definition 1.12. ☐

Theorem 3.70. $\mathsf{exactly5}\ (\{\{2\}\} \cup 4)$.

Proof. Use Proposition 3.772, Lemma 3.237 and Definition 1.12. ☐

Theorem 3.71. $\mathsf{exactly4}\ ((\{\{2\}\} \cup 4) \setminus \{1\})$.

Proof. Use Theorem 3.70, Lemma 3.120 and Proposition 3.84. ☐

Theorem 3.72. $\mathsf{exactly4}\ ((\{\{2\}\} \cup 4) \setminus \{2\})$.

Proof. Use Theorem 3.70, Lemma 3.123 and Proposition 3.84. ☐

Theorem 3.73. $\mathsf{exactly4}\ ((\{\{2\}\} \cup 4) \setminus \{3\})$.

Proof. Use Theorem 3.70, Proposition 3.381 and Proposition 3.84. □

Theorem 3.74. exactly5 $(4 \cup \{((\wp\ 2) \setminus 2)\})$.

Proof. Use Proposition 3.773, Proposition 3.804 and Definition 1.12. □

Theorem 3.75. exactly5 $((\wp\ 2) \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 3.228, Proposition 3.805 and Definition 1.12. □

Theorem 3.76. exactly5 $(4 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.774, Proposition 3.806 and Definition 1.12. □

Theorem 3.77. exactly4 $((4 \cup \{(\wp\ 2)\}) \setminus \{0\})$.

Proof. Use Theorem 3.76, Proposition 3.430 and Proposition 3.84. □

Theorem 3.78. exactly4 $((4 \cup \{(\wp\ 2)\}) \setminus \{1\})$.

Proof. Use Theorem 3.76, Proposition 3.436 and Proposition 3.84. □

Theorem 3.79. exactly4 $((4 \cup \{(\wp\ 2)\}) \setminus \{2\})$.

Proof. Use Theorem 3.76, Proposition 3.437 and Proposition 3.84. □

Theorem 3.80. exactly4 $((4 \cup \{(\wp\ 2)\}) \setminus \{3\})$.

Proof. Use Theorem 3.76, Proposition 3.433 and Proposition 3.84. □

Theorem 3.81. exactly6 $(\{\{1\}\} \cup (\{\{2\}\} \cup 4))$.

Proof. Use Proposition 3.775, Proposition 3.807 and Definition 1.13. □

Theorem 3.82. exactly6 $(\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.776, Proposition 3.808 and Definition 1.13. □

Theorem 3.83. exactly6 $(\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))$.

Proof. Use Proposition 3.777, Lemma 3.238 and Definition 1.13. □

Theorem 3.84. exactly3 $(((\wp\ 2) \cup \{\{2\}\}) \setminus (4 \setminus \{1\}))$.

Proof. Use Theorem 3.4, Theorem 3.66 and Proposition 3.103. □

Theorem 3.85. exactly3 $((\{\{1\}\} \cup 4) \setminus (\{2\} \cup (\wp\ \{2\})))$.

Proof. Use Theorem 3.12, Theorem 3.67 and Proposition 3.103. □

Theorem 3.86. exactly3 $((\{\{2\}\} \cup 4) \setminus ((\wp\ 2) \setminus \{1\}))$.

Proof. Use Theorem 3.5, Theorem 3.70 and Proposition 3.103. □

Theorem 3.87. $\mathsf{exactly3}\ (((\wp\ 2) \cup \{(\wp\ 2)\}) \setminus (4 \setminus \{1\}))$.

Proof. Use Theorem 3.6, Theorem 3.75 and Proposition 3.103. □

Theorem 3.88. $\mathsf{exactly3}\ ((4 \cup \{(\wp\ 2)\}) \setminus ((\wp\ 2) \setminus (\wp\ \{2\})))$.

Proof. Use Theorem 3.15, Theorem 3.76 and Proposition 3.103. □

Theorem 3.89. $\mathsf{exactly3}\ ((4 \cup \{(\wp\ 2)\}) \setminus (\{2\} \cup (\wp\ \{2\})))$.

Proof. Use Theorem 3.7, Theorem 3.76 and Proposition 3.103. □

Proposition 3.824. $\mathsf{atleast4}\ (((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\})) \cap (\{\{2\}\} \cup 4))$.

Proof. Use Proposition 3.789, Proposition 3.381, Proposition 3.385, Proposition 3.17, Proposition 3.331, Lemma 3.203, Proposition 3.663, Proposition 3.34, Definition 1.5 and Definition 1.1. □

Lemma 3.243. $(((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\})) \cap (\{\{2\}\} \cup 4)) \setminus \{\{2\}\} \subseteq 4 \setminus \{2\}$.

Proof. Use Proposition 3.21, Lemma 3.6, Proposition 3.18, Proposition 3.19, Proposition 3.384, Proposition 3.14, Proposition 3.20, Proposition 3.16 and Definition 1.1. □

Proposition 3.825. $\neg\mathsf{atleast4}\ ((((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\})) \cap (\{\{2\}\} \cup 4)) \setminus \{\{2\}\})$.

Proof. Use Lemma 3.243, Lemma 3.23 and Proposition 3.736. □

Proposition 3.826. $\forall u \in 4. u \notin \{0\} \Rightarrow u \in (\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\}) \Rightarrow u \in (\{\{2\}\} \cup 4) \setminus ((\wp\ 2)$

Proof. Use Proposition 3.382, Proposition 3.384, Proposition 3.377, Proposition 3.13 and Proposition 3.10. □

Proposition 3.827. $(((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\})) \cap (\{\{2\}\} \cup 4)) \setminus \{0\} \subseteq (\{\{2\}\} \cup 4) \setminus ((\wp\ 2) \setminus \{1$

Proof. Use Proposition 3.18, Proposition 3.826, Proposition 3.379, Proposition 3.14, Proposition 3.19, Proposition 3.16 and Lemma 3.13. □

Proposition 3.828. $\forall u \in 4. u \notin \{1\} \Rightarrow u \in (\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\}) \Rightarrow u \in \wp\ \{2\} \cup \{3\}$.

Proof. Use Proposition 3.376, Proposition 3.384, Proposition 3.13, Proposition 3.373 and Proposition 3.10. □

Proposition 3.829. $(((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\})) \cap (\{\{2\}\} \cup 4)) \setminus \{1\} \subseteq \wp\ \{2\} \cup \{3\}$.

Proof. Use Proposition 3.18, Proposition 3.828, Proposition 3.375, Proposition 3.14, Proposition 3.19, Proposition 3.16 and Lemma 3.13. □

Proposition 3.830. $\forall u \in 4. u \notin \{3\} \Rightarrow u \in (\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\}) \Rightarrow u \in \{1\} \cup \wp\ \{2\}$.

Proof. Use Proposition 3.13, Proposition 3.384, Proposition 3.329, Proposition 3.328 and Proposition 3.10. □

Proposition 3.831. $(((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\})) \cap (\{\{2\}\} \cup 4)) \setminus \{3\} \subseteq \{1\} \cup \wp\ \{2\}$.

Proof. Use Proposition 3.18, Proposition 3.830, Proposition 3.330, Proposition 3.14, Proposition 3.19, Proposition 3.16 and Lemma 3.13. □

Proposition 3.832. $\neg\mathsf{atleast4}\ ((\{\{2\}\} \cup 4) \setminus ((\wp\ 2) \setminus \{1\}))$.

Proof. Use Theorem 3.86 and Definition 1.10. □

Proposition 3.833. $\neg\mathsf{atleast4}\ ((((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\})) \cap (\{\{2\}\} \cup 4)) \setminus \{0\})$.

Proof. Use Proposition 3.827, Lemma 3.23 and Proposition 3.832. □

Proposition 3.834. $\neg\mathsf{atleast4}\ ((((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\})) \cap (\{\{2\}\} \cup 4)) \setminus \{1\})$.

Proof. Use Proposition 3.829, Lemma 3.23 and Proposition 3.742. □

Proposition 3.835. $\neg\mathsf{atleast4}\ ((((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\})) \cap (\{\{2\}\} \cup 4)) \setminus \{3\})$.

Proof. Use Proposition 3.831, Lemma 3.23 and Proposition 3.734. □

Proposition 3.836. $\forall u \in 4. u \in ((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\})) \Rightarrow \neg\mathsf{atleast4}\ ((((\{1\} \cup \{\{1\}\}) \cup ((\wp$

Proof. Use Proposition 3.835, Proposition 3.384, Proposition 3.834, Proposition 3.833 and Proposition 3.10. □

Lemma 3.244. $\neg\mathsf{atleast5}\ (((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\})) \cap (\{\{2\}\} \cup 4))$.

Proof. Use Proposition 3.18, Proposition 3.19, Proposition 3.836, Proposition 3.825, Proposition 3.14, Lemma 3.23, Proposition 3.16, Lemma 3.22 and Definition 1.6. □

Theorem 3.90. $\mathsf{exactly4}\ (((\{1\} \cup \{\{1\}\}) \cup ((\wp\ \{2\}) \cup \{3\})) \cap (\{\{2\}\} \cup 4))$.

Proof. Use Lemma 3.244, Proposition 3.824 and Definition 1.11. □

Theorem 3.91. $\mathsf{exactly5}\ ((\{\{1\}\} \cup (\{\{2\}\} \cup 4)) \setminus \{0\})$.

Proof. Use Theorem 3.81, Proposition 3.387 and Proposition 3.93. □

Theorem 3.92. $\mathsf{exactly5}\ ((\{\{1\}\} \cup (\{\{2\}\} \cup 4)) \setminus \{2\})$.

Proof. Use Theorem 3.81, Proposition 3.389 and Proposition 3.93. □

Theorem 3.93. $\mathsf{exactly5}\ ((\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{1\})$.

Proof. Use Theorem 3.82, Proposition 3.439 and Proposition 3.93. □

Theorem 3.94. $\mathsf{exactly5}\ ((\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{2\})$.

Proof. Use Theorem 3.82, Lemma 3.138 and Proposition 3.93. □

Theorem 3.95. $\mathsf{exactly5}\ ((\{\{1\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{3\})$.

Proof. Use Theorem 3.82, Proposition 3.441 and Proposition 3.93. □

Theorem 3.96. exactly5 $((\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{0\})$.

Proof. Use Theorem 3.83, Lemma 3.140 and Proposition 3.93. □

Theorem 3.97. exactly5 $((\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{1\})$.

Proof. Use Theorem 3.83, Lemma 3.141 and Proposition 3.93. □

Theorem 3.98. exactly5 $((\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{2\})$.

Proof. Use Theorem 3.83, Lemma 3.142 and Proposition 3.93. □

Theorem 3.99. exactly5 $((\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})) \setminus \{3\})$.

Proof. Use Theorem 3.83, Proposition 3.445 and Proposition 3.93. □

Lemma 3.245. $\forall u \in (\wp\ 2) \setminus (\wp\ \{2\}).u \in \{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}) \Rightarrow u \in ((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\{\{2\}\} \cup \{(\wp$

Proof. Use Proposition 3.422, Proposition 3.443, Proposition 3.421 and Proposition 3.461. □

Proposition 3.837. $\forall u \in \wp\ \{2\}.u \notin \{0\} \Rightarrow u \in ((\wp\ 2) \setminus (\wp\ \{1\})) \cup (\{\{2\}\} \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.423, Proposition 3.13 and Lemma 3.28. □

Proposition 3.838. $(((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))) \setminus \{0\}) \subseteq ((\wp$

Proof. Use Proposition 3.424, Proposition 3.14, Proposition 3.837, Proposition 3.19, Lemma 3.245, Proposition 3.18, Proposition 3.16 and Lemma 3.13. □

Proposition 3.839. $\forall u \in (\wp\ 2) \setminus (\wp\ \{2\}).u \in \{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}) \Rightarrow u \notin \{1\} \Rightarrow u \in (\{2\} \cup ((\wp\ \{2\})$

Proof. Use Proposition 3.419, Proposition 3.443, Proposition 3.128 and Proposition 3.461. □

Proposition 3.840. $\wp\ \{2\} \cup \{(\wp\ 2)\} \subseteq (\{2\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\}))$.

Proof. Use Lemma 3.8. □

Proposition 3.841. $(((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))) \setminus \{1\}) \subseteq \{2$

Proof. Use Proposition 3.840, Proposition 3.19, Proposition 3.839, Proposition 3.18, Proposition 3.16, Lemma 3.13 and Definition 1.1. □

Proposition 3.842. $\forall u \in (\wp\ 2) \setminus (\wp\ \{2\}).u \in \{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}) \Rightarrow u \notin \{2\} \Rightarrow u \in (\{1\} \cup ((\wp\ \{2\})$

Proof. Use Proposition 3.132, Proposition 3.443, Proposition 3.418 and Proposition 3.461. □

Proposition 3.843. $\wp\ \{2\} \cup \{(\wp\ 2)\} \subseteq \{1\} \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})$.

Proof. Use Lemma 3.8. □

Proposition 3.844. $(((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))) \setminus \{2\}) \subseteq \{1$

Proof. Use Proposition 3.843, Proposition 3.19, Proposition 3.842, Proposition 3.18, Proposition 3.16, Lemma 3.13 and Definition 1.1. □

Proposition 3.845. $\forall u \in (\wp\ 2) \setminus (\wp\ \{2\}).u \in \{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}) \Rightarrow u \in (3 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.409, Proposition 3.443, Proposition 3.408 and Proposition 3.461. □

Proposition 3.846. $\forall u \in \wp\ \{2\}.u \notin \{\{2\}\} \Rightarrow u \in (3 \cup \{(\wp\ 2)\})$.

Proof. Use Proposition 3.319, Lemma 3.129 and Lemma 3.28. □

Proposition 3.847. $(((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))) \setminus \{\{2\}\}) \subseteq 3$

Proof. Use Proposition 3.410, Proposition 3.14, Proposition 3.846, Proposition 3.19, Proposition 3.845, Proposition 3.18, Proposition 3.16 and Lemma 3.13. □

Proposition 3.848. $\forall u \in (\wp\ 2) \setminus (\wp\ \{2\}).u \in \{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}) \Rightarrow u \in (3 \cup \{\{2\}\})$.

Proof. Use Proposition 3.345, Proposition 3.443, Proposition 3.344 and Proposition 3.461. □

Lemma 3.246. $(((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))) \setminus \{\wp\ 2\}) \subseteq 3 \cup \{$

Proof. Use Proposition 3.675, Proposition 3.19, Proposition 3.848, Proposition 3.18, Proposition 3.16, Lemma 3.13 and Definition 1.1. □

Proposition 3.849. $\neg\mathsf{atleast5}\ (((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))) \setminus$

Proof. Use Proposition 3.841, Proposition 3.57 and Lemma 3.225. □

Proposition 3.850. $\neg\mathsf{atleast5}\ (((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))) \setminus$

Proof. Use Proposition 3.844, Proposition 3.57 and Proposition 3.760. □

Proposition 3.851. $\forall u \in (\wp\ 2) \setminus (\wp\ \{2\}).u \in \{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}) \Rightarrow \neg\mathsf{atleast5}\ (((((\wp\ 2) \setminus (\wp\ \{2\})) \cup$

Proof. Use Proposition 3.850, Proposition 3.443, Proposition 3.849 and Proposition 3.461. □

Proposition 3.852. $\neg\mathsf{atleast5}\ (((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))) \setminus$

Proof. Use Proposition 3.838, Proposition 3.57 and Proposition 3.762. □

Proposition 3.853. $\neg\mathsf{atleast5}\ (((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))) \setminus$

Proof. Use Proposition 3.847, Proposition 3.57 and Proposition 3.757. □

Proposition 3.854. $\forall u \in (\wp\ \{2\}).\neg\mathsf{atleast5}\ (((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{2\}\} \cup (4 \cup$

Proof. Use Proposition 3.853, Proposition 3.852 and Lemma 3.28. □

Proposition 3.855. $\neg\mathsf{atleast5}\ (((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\}))) \setminus$

Proof. Use Lemma 3.246, Proposition 3.57 and Proposition 3.752. □

Proposition 3.856. $\forall u \in (\wp\ \{2\}) \cup \{(\wp\ 2)\}.\neg\mathsf{atleast5}\ (((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{$

Proof. Use Proposition 3.855, Proposition 3.14, Proposition 3.854 and Proposition 3.16. □

Proposition 3.857. $\neg\mathsf{atleast6}\ ((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})))$.

Proof. Use Proposition 3.18, Proposition 3.19, Proposition 3.856, Proposition 3.851, Proposition 3.57, Proposition 3.16, Lemma 3.22 and Definition 1.7. □

Theorem 3.100. $\mathsf{exactly5}\ ((((\wp\ 2) \setminus (\wp\ \{2\})) \cup ((\wp\ \{2\}) \cup \{(\wp\ 2)\})) \cap (\{\{2\}\} \cup (4 \cup \{(\wp\ 2)\})))$.

Proof. Use Proposition 3.857, Lemma 3.239 and Definition 1.12. □