

thm_2EASCIInumbers_2ESUB__num__to__bin__string
(TMLqm-
Jev92TnHTEEx9UCqNvg4FMHKP6VeZ1r)

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Let $ty_2Estring_2Echar : \iota$ be given. Assume the following.

$$nonempty\ ty_2Estring_2Echar \tag{1}$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Ebool_2EARB\ A.27a \in A.27a \tag{2}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{4}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{5}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{6}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{7}$$

Definition 4 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 6 We define `c_2Enum_2ESUC` to be $\lambda V0m \in \text{ty_2Enum_2Enum}.(\text{ap } \text{c_2Enum_2EABS_num } (2^{A-27a}))$

Let `c_2Earithmetic_2E_2B` : ι be given. Assume the following.

$$\text{c_2Earithmetic_2E_2B} \in ((\text{ty_2Enum_2Enum}^{\text{ty_2Enum_2Enum}})^{\text{ty_2Enum_2Enum}}) \quad (8)$$

Definition 7 We define `c_2Earithmetic_2EBIT1` to be $\lambda V0n \in \text{ty_2Enum_2Enum}.(\text{ap } (\text{ap } \text{c_2Earithmetic_2E_2B } (V0n)))$

Definition 8 We define `c_2Earithmetic_2EBIT2` to be $\lambda V0n \in \text{ty_2Enum_2Enum}.(\text{ap } (\text{ap } \text{c_2Earithmetic_2E_2B } (V0n)))$

Definition 9 We define `c_2Earithmetic_2ENUMERAL` to be $\lambda V0x \in \text{ty_2Enum_2Enum}.V0x$.

Let `c_2Estring_2ECHR` : ι be given. Assume the following.

$$\text{c_2Estring_2ECHR} \in (\text{ty_2Estring_2Echar}^{\text{ty_2Enum_2Enum}}) \quad (9)$$

Definition 10 We define `c_2Ecombin_2EK` to be $\lambda A.27a : \iota.\lambda A.27b : \iota.(\lambda V0x \in A.27a.(\lambda V1y \in A.27b.V0x))$

Definition 11 We define `c_2Ecombin_2ES` to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.(\lambda V0f \in ((A.27c^{A-27b})^{A-27a}))$

Definition 12 We define `c_2Ecombin_2EI` to be $\lambda A.27a : \iota.(\text{ap } (\text{ap } (\text{c_2Ecombin_2ES } A.27a (A.27a^{A-27a}))))$

Definition 13 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2.V0t))$.

Definition 14 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.\text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 15 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2.V2t))))$

Definition 16 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (\text{ap } P x)) \text{ then } (\lambda x.x \in A \wedge P x)$ of type $\iota \Rightarrow \iota$.

Definition 17 We define `c_2Ebool_2ECOND` to be $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.(\lambda V3t3 \in A.27a.(\lambda V4t4 \in A.27a.V4t4))))$

Definition 18 We define `c_2Ebool_2Eliteral_case` to be $\lambda A.27a : \iota.\lambda A.27b : \iota.(\lambda V0f \in (A.27b^{A-27a}).(\lambda V1x \in A.27a.V1x))$

Definition 19 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(\text{ap } (\text{ap } \text{c_2Emin_2E_3D_3D_3E } V0t) \text{c_2Ebool_2E_21 } (V0t)))$

Definition 20 We define `c_2Ebool_2E_3F` to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } (V0P))))$

Definition 21 We define `c_2Erelation_2EWF` to be $\lambda A.27a : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).(\text{ap } (\text{c_2Ebool_2E_21 } (V0R)))$

Definition 22 We define `c_2Erelation_2ERESTRICT` to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0f \in (A.27b^{A-27a}).\lambda V1x \in A.27a.V1x$

Definition 23 We define `c_2Erelation_2ETC` to be $\lambda A.27a : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).\lambda V1a \in A.27a.\lambda V2b \in A.27a.V2b$

Definition 24 We define `c_2Erelation_2Eapprox` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda V0R \in ((2^{A_{.27a}})^{A_{.27a}}). \lambda V1M$

Definition 25 We define `c_2Erelation_2Ethe_fun` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda V0R \in ((2^{A_{.27a}})^{A_{.27a}}). \lambda V1M$

Definition 26 We define `c_2Erelation_2EWFREC` to be $\lambda A_{.27a} : \iota. \lambda A_{.27b} : \iota. \lambda V0R \in ((2^{A_{.27a}})^{A_{.27a}}). \lambda V1M$

Definition 27 We define `c_2EASCIInumbers_2EHEX` to be $(ap (ap (c_2Erelation_2EWFREC ty_2Enum_2E$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (10)$$

Let `c_2Enumposrep_2En2l` : ι be given. Assume the following.

$$c_2Enumposrep_2En2l \in (((ty_2Elist_2Elist ty_2Enum_2EEnum)^{ty_2Enum_2EEnum})^{ty_2Enum_2EEnum}) \quad (11)$$

Let `c_2Elist_2EMAP` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow c_2Elist_2EMAP A_{.27a} A_{.27b} \in (((ty_2Elist_2Elist A_{.27b})^{(ty_2Elist_2Elist A_{.27a})})^{(A_{.27b}^{A_{.27a}})}) \quad (12)$$

Let `c_2Elist_2EREVERSE` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow c_2Elist_2EREVERSE A_{.27a} \in ((ty_2Elist_2Elist A_{.27a})^{(ty_2Elist_2Elist A_{.27a})}) \quad (13)$$

Definition 28 We define `c_2EASCIInumbers_2En2s` to be $\lambda V0b \in ty_2Enum_2EEnum. \lambda V1f \in (ty_2Estring$

Definition 29 We define `c_2EASCIInumbers_2Enum_to_bin_string` to be $(ap (ap c_2EASCIInumbers_2En2s$

Let `c_2Earithmetic_2EEXP` : ι be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2EEnum)^{ty_2Enum_2EEnum})^{ty_2Enum_2EEnum} \quad (14)$$

Let `c_2Earithmetic_2EDIV` : ι be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2EEnum)^{ty_2Enum_2EEnum})^{ty_2Enum_2EEnum} \quad (15)$$

Definition 30 We define `c_2Ebit_2EDIV_2EXP` to be $\lambda V0x \in ty_2Enum_2EEnum. \lambda V1n \in ty_2Enum_2EEnum$

Let `c_2Earithmetic_2E_2D` : ι be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2EEnum)^{ty_2Enum_2EEnum})^{ty_2Enum_2EEnum} \quad (16)$$

Let `c_2Earithmetic_2EMOD` : ι be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2EEnum)^{ty_2Enum_2EEnum})^{ty_2Enum_2EEnum} \quad (17)$$

Definition 31 We define $c_Ebit_EMOD_EXP$ to be $\lambda V0x \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Definition 32 We define c_Ebit_EBITS to be $\lambda V0h \in ty_Enum_Enum.\lambda V1l \in ty_Enum_Enum.\lambda V$

Definition 33 We define c_Ebit_EBITV to be $\lambda V0n \in ty_Enum_Enum.\lambda V1b \in ty_Enum_Enum.(ap$

Let $c_Earithmetic_EEVEN : \iota$ be given. Assume the following.

$$c_Earithmetic_EEVEN \in (2^{ty_Enum_Enum}) \quad (18)$$

Let $c_Earithmetic_EODD : \iota$ be given. Assume the following.

$$c_Earithmetic_EODD \in (2^{ty_Enum_Enum}) \quad (19)$$

Definition 34 We define $c_Eprim_rec_E_3C$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Definition 35 We define $c_Earithmetic_E_3E$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Definition 36 We define $c_Ebool_E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E_21 2) (\lambda V2t \in$

Definition 37 We define $c_Earithmetic_E_3E_3D$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Definition 38 We define $c_Eprim_rec_EPRE$ to be $\lambda V0m \in ty_Enum_Enum.(ap (ap (ap (c_Ebool_E$

Let $c_Earithmetic_E_2A : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2A \in ((ty_Enum_Enum)^{ty_Enum_Enum})^{ty_Enum_Enum} \quad (20)$$

Definition 39 We define $c_Enumeral_EiZ$ to be $\lambda V0x \in ty_Enum_Enum.V0x$.

Definition 40 We define $c_Earithmetic_E_3C_3D$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Let $c_Elogroot_ELOG : \iota$ be given. Assume the following.

$$c_Elogroot_ELOG \in ((ty_Enum_Enum)^{ty_Enum_Enum})^{ty_Enum_Enum} \quad (21)$$

Let $c_Elist_ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Elist_ELENGTH A_27a \in (ty_Enum_Enum)^{(ty_Elist_Elist A_27a)} \quad (22)$$

Let $c_Elist_EEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Elist_EEL A_27a \in ((A_27a)^{(ty_Elist_Elist A_27a)})^{ty_Enum_Enum} \quad (23)$$

Let $ty_Epair_Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_Epair_Eprod A0 A1) \quad (24)$$

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_EABS_prod A_27a A_27b \in ((ty_Epair_Eprod A_27a A_27b))^{((2^{A_27b})^{A_27a})} \quad (25)$$

Definition 41 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2Estring_2ESUB : \iota$ be given. Assume the following.

$$c_2Estring_2ESUB \in (ty_2Estring_2Echar^{(ty_2Epair_2Eprod (ty_2Elist_2Elist ty_2Estring_2Echar) ty_2Enum_2Eenum_2Eenum)})) \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\ & ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\ & (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\ & V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\ & V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n)))))))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\ & V1n) V0m)))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\ & (ap c_2Enum_2ESUC V0m)) V1n)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D \\ & c_2Enum_2E0) V0n))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (\neg (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\ & V1n) V0m)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. ((ap c_2Eprim_rec_2EPRE V0m) = \\ & (ap (ap c_2Earithmetic_2E_2D V0m) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
& ((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge (\\
& ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
& (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
& (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
& V0m) V1n))))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& V1n)) V0m))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\
& c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))) V0n)))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D (ap \\
& (ap c_2Earithmetic_2E_2D V0m) V1n)) V2p) = (ap (ap c_2Earithmetic_2E_2D \\
& \quad V0m) (ap (ap c_2Earithmetic_2E_2B V1n) V2p))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C (\\
& ap (ap c_2Earithmetic_2E_2D V0m) V1n)) V2p)) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0m) (ap (ap c_2Earithmetic_2E_2B V1n) V2p))) \wedge (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad c_2Enum_2E0) V2p))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}). (\forall V1a \in ty_2Enum_2Enum. \\
& (\forall V2b \in ty_2Enum_2Enum. ((p (ap V0P (ap (ap c_2Earithmetic_2E_2D \\
& V1a) V2b))) \Leftrightarrow (\forall V3d \in ty_2Enum_2Enum. (((V2b = (ap (ap c_2Earithmetic_2E_2B \\
& V1a) V3d)) \Rightarrow (p (ap V0P c_2Enum_2E0))) \wedge ((V1a = (ap (ap c_2Earithmetic_2E_2B \\
& \quad V2b) V3d)) \Rightarrow (p (ap V0P V3d))))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2EEXP \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \\
& V0n) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((ap (ap c_2Earithmetic_2EEXP V0n) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = \\
& \quad V0n)))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D (\\
& ap c_2Enum_2ESUC V0a)) V0a) = (ap c_2Earithmetic_2ENUMERAL (ap \\
& \quad c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty_2Enum_2Enum. (\forall V1l \in ty_2Enum_2Enum. (\\
& \quad \forall V2n \in ty_2Enum_2Enum. ((ap (ap (ap c_2Ebit_2EBITS V0h) V1l) \\
& V2n) = (ap (ap c_2Earithmetic_2EMOD (ap (ap c_2Earithmetic_2EDIV \\
& V2n) (ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V1l))) (\\
& \quad ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL (ap \\
& c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) (ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Enum_2ESUC V0h)) V1l))))))
\end{aligned} \tag{44}$$

Assume the following.

$$True \quad (45)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2.((p \ V0t) \vee (\neg(p \ V0t)))) \quad (48)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (49)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \wedge ((p \ V1t2) \wedge (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \wedge (p \ V2t3)))))) \quad (50)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (51)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (52)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (53)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (54)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (55)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (56)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t))))) \quad (57)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (58)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C))))) \quad (59)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B))))) \quad (60)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A) \vee (p\ V1B))))) \quad (61)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (62)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3))))) \quad (63)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Leftrightarrow (p\ V1t2)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \vee ((\neg(p\ V0t1) \wedge \neg(p\ V1t2))))) \quad (64)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1b \in 2. (\forall V2x \in A_27a. (\forall V3y \in A_27a. ((ap\ V0f\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1b)\ (ap\ V0f\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ V1b)\ (ap\ V0f\ V2x))\ (ap\ V0f\ V3y))))))) \quad (65)$$

Assume the following.

$$2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))) \Rightarrow \\ (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27)))) \quad (66)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ \forall V0l \in (ty_2Elist_2Elist \ A_27a).(\forall V1f \in (A_27b^{A_27a}). \\ ((ap \ (c_2Elist_2ELENGTH \ A_27b) \ (ap \ (ap \ (c_2Elist_2EMAP \ A_27a \ A_27b) \\ V1f) \ V0l)) = (ap \ (c_2Elist_2ELENGTH \ A_27a) \ V0l)))) \quad (67)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ \forall V0n \in ty_2Enum_2Enum.(\forall V1l \in (ty_2Elist_2Elist \\ A_27a).((p \ (ap \ (ap \ c_2Eprim_rec_2E_3C \ V0n) \ (ap \ (c_2Elist_2ELENGTH \\ A_27a) \ V1l))) \Rightarrow (\forall V2f \in (A_27b^{A_27a}).((ap \ (ap \ (c_2Elist_2EEL \\ A_27b) \ V0n) \ (ap \ (ap \ (c_2Elist_2EMAP \ A_27a \ A_27b) \ V2f) \ V1l)) = (ap \\ V2f \ (ap \ (ap \ (c_2Elist_2EEL \ A_27a) \ V0n) \ V1l))))))) \quad (68)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ A_27a).((ap \ (c_2Elist_2ELENGTH \ A_27a) \ (ap \ (c_2Elist_2EREVERSE \\ A_27a) \ V0l)) = (ap \ (c_2Elist_2ELENGTH \ A_27a) \ V0l))) \quad (69)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\ \forall V1l \in (ty_2Elist_2Elist \ A_27a).((p \ (ap \ (ap \ c_2Eprim_rec_2E_3C \\ V0n) \ (ap \ (c_2Elist_2ELENGTH \ A_27a) \ V1l))) \Rightarrow ((ap \ (ap \ (c_2Elist_2EEL \\ A_27a) \ V0n) \ (ap \ (c_2Elist_2EREVERSE \ A_27a) \ V1l)) = (ap \ (ap \ (c_2Elist_2EEL \\ A_27a) \ (ap \ c_2Eprim_rec_2EPRE \ (ap \ (ap \ c_2Earithmetic_2E_2D \ (\\ ap \ (c_2Elist_2ELENGTH \ A_27a) \ V1l)) \ V0n))) \ V1l)))) \quad (70)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) (ap c_2Earithmic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) \\
& (ap c_2Earithmic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmic_2E_3C_3D c_2Earithmic_2EZERO) V0n))) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D \\
& (ap c_2Earithmic_2EBIT2 V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmic_2ENUMERAL (\\
& ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO))) V0b)) \Rightarrow ((\\
& ap (c_2Elist_2ELENGTH ty_2Enum_2Enum) (ap (ap c_2Enumposrep_2En2l \\
& V0b) V1n)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap \\
& (c_2Emin_2E_3D ty_2Enum_2Enum) V1n) c_2Enum_2E0)) (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO))) (ap c_2Enum_2ESUC \\
& (ap (ap c_2Elogroot_2ELOG V0b) V1n))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty_2Enum_2Enum. (\forall V1x \in ty_2Enum_2Enum. (\\
& \quad \forall V2n \in ty_2Enum_2Enum. (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \\
& \quad V0b)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C V1x) (ap (c_2Elist_2ELENGTH \\
& \quad ty_2Enum_2Enum) (ap (ap c_2Enumposrep_2En2l V0b) V2n)))))) \Rightarrow ((\\
& \quad ap (ap (c_2Elist_2EEL ty_2Enum_2Enum) V1x) (ap (ap c_2Enumposrep_2En2l \\
& \quad V0b) V2n)) = (ap (ap c_2Earithmetic_2EMOD (ap (ap c_2Earithmetic_2EDIV \\
& \quad V2n) (ap (ap c_2Earithmetic_2EEXP V0b) V1x))) V0b))))))
\end{aligned} \tag{75}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{76}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{77}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{78}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{79}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{80}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& \quad (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& \quad p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& \quad ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& \quad (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& \quad (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))
\end{aligned} \tag{82}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& \quad (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& \quad ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))
\end{aligned} \tag{83}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (84)$$

Assume the following.

$$(\forall V0s \in (ty_2Elist_2Elist\ ty_2Estring_2Echar). (\forall V1n \in ty_2Enum_2Enum. ((ap\ c_2Estring_2ESUB\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Elist_2Elist\ ty_2Estring_2Echar)\ ty_2Enum_2Enum)\ V0s)\ V1n)) = (ap\ (ap\ (c_2Elist_2EEL\ ty_2Estring_2Echar)\ V1n)\ V0s)))) \quad (85)$$

Theorem 1

$$(\forall V0x \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Eprim_rec_2E3C\ V0x)\ (ap\ (c_2Elist_2ELENGTH\ ty_2Estring_2Echar)\ (ap\ c_2EASCIInumbers_2Enum_to_bin_string\ V1n)))) \Rightarrow ((ap\ c_2Estring_2ESUB\ (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Elist_2Elist\ ty_2Estring_2Echar)\ ty_2Enum_2Enum)\ (ap\ c_2EASCIInumbers_2Enum_to_bin_string\ V1n))\ V0x)) = (ap\ c_2EASCIInumbers_2EHEX\ (ap\ (ap\ c_2Ebit_2EBITV\ V1n)\ (ap\ c_2Eprim_rec_2EPRE\ (ap\ (ap\ c_2Earithmetic_2E2D\ (ap\ (c_2Elist_2ELENGTH\ ty_2Estring_2Echar)\ (ap\ c_2EASCIInumbers_2Enum_to_bin_string\ V1n))))\ V0x))))))$$