

thm\_2EASCIIInumbers\_2Enum\_hex\_string  
 (TMNi1GFsqFVtvD8kir3AMzhdTp93czrzDfT)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \quad (2)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (3)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (4)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (6)$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num\ m)$

Let  $c_2$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

**Definition 7** We define `c_2Earithmetic_2EBIT1` to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2EBIT1\ n)\ V)$

**Definition 8** We define  $c_2\text{Earthmetic\_ENUMERAL}$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 9** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 10** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A.\_27a : \iota.\lambda A.\_27b : \iota.\lambda A.\_27c : \iota.(\lambda V0f \in ((A.\_27c^{A.\_27b})^{A.\_27a}))$

**Definition 11** We define  $\mathbf{c} \in \text{Ecombin } 2\text{EI}$  to be  $\lambda A \ 27a : \iota.(ap \ (ap \ (\mathbf{c} \in \text{Ecombin } 2\text{ES} \ 27a) \ (A \ 27a^A)^{A-27a}) \ A)$

**Definition 12** We define  $c$ -2Earthmetic-2EBIT2 to be  $\lambda V0n \in tu\ 2Enum\_2Enum\_ap\ (ap\ c\ 2Earthmetic\_2EBIT2)$

Let  $t$  be a string of characters. Assume the following:

$$nonempty \; tu \; ?Estring \; ?Echar \quad (8)$$

Let  $c \in Estring(2ECHR)$  be given. Assume the following.

$$c\_2Estring\_2ECHR \in (ty\_2Estring\_2Echar^{ty\_2Enum\_2Enum}) \quad (9)$$

**Definition 13** We define  $c \in \text{bool} \rightarrow \text{EF}$  to be  $(\lambda p. (c \in \text{bool} \rightarrow E \ 21 \ 2)) (\lambda V0t \in \text{V0t})$ .

**Definition 14** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 15** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\_2)\ (\lambda V2t \in$

**Definition 16** We define  $c_2Emin_2E_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\lambda x.x \in A \wedge$  of type  $\iota \rightarrow \iota$ .

**Definition 17** We define  $c_2\text{-Ebool-2ECOND}$  to be  $\lambda A.\lambda 27a:\iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

**Definition 18** We define  $c_{\mathcal{E}\text{bool}} : \mathcal{E}\text{literal\_case}$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.(\lambda V0f \in (A.27b)^{A.27a}).(\lambda V1x$

**Definition 19** We define  $c_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin\_3D\_3D\_3E\ V0t)\ c_2Ebool\_2E))$

**Definition 20** We define  $c \in \text{Bool}$  to be  $\lambda A. \exists T a : \nu. (\forall V O P \in (2^A \rightarrow T a), (ap\; V O P) \in (ap\; c \in \text{Bool}) \rightarrow 2^T)$

**Definition 21** We define a 2Erelation 2EWF to be  $\lambda A \cdot 27a : \lambda V0B \in ((2^{A \cdot 27a})^A)^{A \cdot 27a}$  (an (c 2Ebool 2E 21

**Definition 22.** We define a 2Frelation 2FRESTRICT to be  $\lambda A. \exists^2 a : A. \exists^2 b : A. \forall f \in (A. \exists^2 b^{A \rightarrow 27a}). \forall k$

**D. Setting 23.** We take  $\epsilon = 25$ , lattice size  $25T \times 25$ ,  $t \in [1, 4, 27]$ ,  $NVOR \in \{24, 27\}$ ,  $NV1 \in \{4, 27\}$ ,  $NV2 \in \{4, 27\}$ .

**Definition 24** We define  $\mathcal{C}$ -relation  $\approx_{\text{Lapprox}}$  to be  $X_1 \approx_{\text{Lapprox}} X_2 : \forall x \in X_1 \exists y \in X_2 : \mathcal{C}(x, y)$

**Definition 25** We define  $c\_2Erelation\_2Ethe\_fun$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 26** We define  $c\_2Erelation\_2EWFREC$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 27** We define  $c\_2EASCIInumbers\_2EUNHEX$  to be  $(ap (ap (c\_2Erelation\_2EWFREC ty\_2Estring)))$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (10)$$

Let  $c\_2Elist\_2EREVERSE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EREVERSE A\_27a &\in ((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)}) \\ &\quad (11) \end{aligned}$$

Let  $c\_2Elist\_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Elist\_2EMAP \\ A\_27a A\_27b &\in (((ty\_2Elist\_2Elist A\_27b)^{(ty\_2Elist\_2Elist A\_27a)}))^{(A\_27b^{A\_27a})} \quad (12) \end{aligned}$$

Let  $c\_2Enumposrep\_2El2n : \iota$  be given. Assume the following.

$$c\_2Enumposrep\_2El2n \in ((ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist ty\_2Enum\_2Enum)})^{ty\_2Enum\_2Enum}) \quad (13)$$

**Definition 28** We define  $c\_2EASCIInumbers\_2Es2n$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1f \in (ty\_2Enum\_2Enum)$

**Definition 29** We define  $c\_2EASCIInumbers\_2Enum\_from\_hex\_string$  to be  $(ap (ap c\_2EASCIInumbers\_2Es2n))$

**Definition 30** We define  $c\_2EASCIInumbers\_2EHEX$  to be  $(ap (ap (c\_2Erelation\_2EWFREC ty\_2Enum\_2Enum)))$

Let  $c\_2Enumposrep\_2En2l : \iota$  be given. Assume the following.

$$c\_2Enumposrep\_2En2l \in (((ty\_2Elist\_2Elist ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (14)$$

**Definition 31** We define  $c\_2EASCIInumbers\_2En2s$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1f \in (ty\_2Estring)$

**Definition 32** We define  $c\_2EASCIInumbers\_2Enum\_to\_hex\_string$  to be  $(ap (ap c\_2EASCIInumbers\_2En2s))$

**Definition 33** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in (A\_27b^{A\_27c}).\lambda V1f$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (15)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (16)$$

**Definition 34** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 35** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 36** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

**Definition 37** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 38** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 39** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ebool\_2E$

Let  $c\_2Earithmetic\_2EEEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (17)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (18)$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (19)$$

**Definition 40** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x.$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\ V0n) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 \\ (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2EBIT1 \\ c\_2Earithmetic\_2EZERO))))))) \Rightarrow ((ap c\_2EASCIInumbers\_2EUNHEX \\ (ap c\_2EASCIInumbers\_2EHEX V0n)) = V0n))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0c2n \in (ty\_2Enum\_2Enum^{ty\_2Estring\_2Echar}). (\forall V1n2c \in \\ (ty\_2Estring\_2Echar ty\_2Enum\_2Enum). (\forall V2b \in ty\_2Enum\_2Enum. \\ (\forall V3n \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\ (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \\ V2b)) \wedge (\forall V4x \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\ (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \\ V4x) V2b)) \Rightarrow ((ap V0c2n (ap V1n2c V4x)) = V4x)))) \Rightarrow ((ap (ap (ap c\_2EASCIInumbers\_2Es2n \\ V2b) V0c2n) (ap (ap (ap c\_2EASCIInumbers\_2En2s V2b) V1n2c) V3n)) = \\ V3n))))))) \end{aligned} \quad (21)$$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_{27a}. (p V0t)) \Leftrightarrow (p V0t))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \end{aligned} \quad (25)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True)))))) \quad (26)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0x \in A_{27a}. ((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0x \in A_{27a}. (\forall V1y \in A_{27a}. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow \forall A_{27b}. \text{nonempty } A_{27b} \Rightarrow \\ & \forall V0f \in (A_{27b}^{A_{27a}}). (\forall V1g \in (A_{27b}^{A_{27a}}). ((V0f = \\ & V1g) \Leftrightarrow (\forall V2x \in A_{27a}. ((ap V0f V2x) = (ap V1g V2x)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & (p V0t))))))) \end{aligned} \quad (30)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_{27} \in 2. (\forall V2y \in 2. (\forall V3y_{27} \in \\ & 2. (((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A_{\_27a}.nonempty\ A_{\_27a} &\Rightarrow \forall A_{\_27b}.nonempty\ A_{\_27b} \Rightarrow \forall A_{\_27c}. \\ nonempty\ A_{\_27c} &\Rightarrow (\forall V0f \in (A_{\_27b}^{A_{\_27a}}).(\forall V1g \in (A_{\_27a}^{A_{\_27c}}). \\ (\forall V2x \in A_{\_27c}.((ap\ (ap\ (ap\ (c\_2Ecombin\_2Eo\ A_{\_27c}\ A_{\_27b}\ A_{\_27a}) \\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x))))))) \end{aligned} \quad (33)$$

Assume the following.

$$\forall A_{\_27a}.nonempty\ A_{\_27a} \Rightarrow (\forall V0x \in A_{\_27a}.((ap\ (c\_2Ecombin\_2El\ A_{\_27a})\ V0x) = V0x)) \quad (34)$$

Assume the following.

$(\forall V0n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2B c_2Enum\_2E0) V0n) = V0n)) \wedge (\forall V1n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2B V1n) c_2Enum\_2E0) = V1n)) \wedge (\forall V2n \in ty\_2Enum\_2Enum. (\forall V3m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2B V3m) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Enum\_2EiZ (ap (ap c_2Earithmetic\_2E\_2B V2n) V3m))))))) \wedge (\forall V4n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2A c_2Enum\_2E0) V4n) = c_2Enum\_2E0)) \wedge (\forall V5n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2A V5n) c_2Enum\_2E0) = c_2Enum\_2E0)) \wedge (\forall V6n \in ty\_2Enum\_2Enum. (\forall V7m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2A (ap c_2Earithmetic\_2ENUMERAL V6n)) (ap c_2Earithmetic\_2ENUMERAL V7m)) = (ap c_2Earithmetic\_2ENUMERAL (ap (ap c_2Earithmetic\_2E\_2A V6n) V7m)))))) \wedge (\forall V8n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2D c_2Enum\_2E0) V8n) = c_2Enum\_2E0)) \wedge (\forall V9n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2D V9n) c_2Enum\_2E0) = V9n)) \wedge (\forall V10n \in ty\_2Enum\_2Enum. (\forall V11m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2D (ap c_2Earithmetic\_2ENUMERAL V10n)) (ap c_2Earithmetic\_2ENUMERAL V11m)) = (ap c_2Earithmetic\_2ENUMERAL (ap (ap c_2Earithmetic\_2E\_2D V10n) V11m)))))) \wedge (\forall V12n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2EEEXP c_2Enum\_2E0) (ap c_2Earithmetic\_2ENUMERAL (ap c_2Earithmetic\_2EBIT1 V12n)))) = c_2Enum\_2E0)) \wedge (\forall V13n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2EEEXP c_2Enum\_2E0) (ap c_2Earithmetic\_2ENUMERAL (ap c_2Earithmetic\_2EBIT2 V13n)))) = c_2Enum\_2E0)) \wedge (\forall V14n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2EEEXP V14n) c_2Enum\_2E0) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Earithmetic\_2EBIT1 c_2Earithmetic\_2EZERO)))))) \wedge (\forall V15n \in ty\_2Enum\_2Enum. (\forall V16m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2EEEXP (ap c_2Earithmetic\_2ENUMERAL V15n)) (ap c_2Earithmetic\_2ENUMERAL V16m)) = (ap c_2Earithmetic\_2ENUMERAL (ap (ap c_2Earithmetic\_2EEEXP V15n) V16m)))))) \wedge (((ap c_2Enum\_2ESUC c_2Enum\_2E0) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Earithmetic\_2EBIT1 c_2Earithmetic\_2EZERO)))) \wedge (\forall V17n \in ty\_2Enum\_2Enum. ((ap c_2Enum\_2ESUC (ap c_2Earithmetic\_2ENUMERAL V17n)) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Enum\_2ESUC V17n)))))) \wedge (((ap c_2Eprim\_rec\_2EPRE c_2Enum\_2E0) = c_2Enum\_2E0) \wedge (\forall V18n \in ty\_2Enum\_2Enum. ((ap c_2Eprim\_rec\_2EPRE (ap c_2Earithmetic\_2ENUMERAL V18n)) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Eprim\_rec\_2EPRE V18n)))))) \wedge (\forall V19n \in ty\_2Enum\_2Enum. (((ap c_2Earithmetic\_2ENUMERAL V19n) = c_2Enum\_2E0) \Leftrightarrow (V19n = c_2Earithmetic\_2EZERO))) \wedge (\forall V20n \in ty\_2Enum\_2Enum. ((c_2Enum\_2E0 = (ap c_2Earithmetic\_2ENUMERAL V20n)) \Leftrightarrow (V20n = c_2Earithmetic\_2EZERO))) \wedge (\forall V21n \in ty\_2Enum\_2Enum. ((\forall V22m \in ty\_2Enum\_2Enum. (((ap c_2Earithmetic\_2ENUMERAL V21n) = (ap c_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge ((\forall V23n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Eprim\_rec\_2E\_3C V23n) c_2Enum\_2E0)) \Leftrightarrow False))) \wedge (\forall V24n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Eprim\_rec\_2E\_3C c_2Enum\_2E0) (ap c_2Earithmetic\_2ENUMERAL V24n)) \Leftrightarrow (p (ap (ap c_2Eprim\_rec\_2E\_3C c_2Earithmetic\_2EZERO) V24n)))) \wedge (\forall V25n \in ty\_2Enum\_2Enum. (\forall V26m \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Eprim\_rec\_2E\_3C c_2Earithmetic\_2ENUMERAL V25n) (ap c_2Earithmetic\_2ENUMERAL V26m)))) \Leftrightarrow (p (ap (ap c_2Eprim\_rec\_2E\_3C V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3E c_2Enum\_2E0) V27n)) \Leftrightarrow False))) \wedge (\forall V28n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3E c_2Enum\_2E0) (ap c_2Earithmetic\_2ENUMERAL V28n)) \Leftrightarrow (p (ap (ap c_2Eprim\_rec\_2E\_3C c_2Earithmetic\_2EZERO) V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3E c_2Enum\_2E0) V29n)) \Leftrightarrow True))) \wedge (\forall V30m \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3E c_2Enum\_2E0) V30m)) \Leftrightarrow (p (ap (ap c_2Earithmetic\_2E\_3D c_2Enum\_2E0) V30m)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3D c_2Enum\_2E0) V31n)) \Leftrightarrow True))) \wedge (\forall V32n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3D c_2Enum\_2E0) V32n)) \Leftrightarrow True)))$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) (ap c\_2Earithmetic\_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& (ap c\_2Earithmetic\_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0n) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2EBIT1 V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2EBIT1 V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg(p (ap (ap c\_2Eprim\_rec\_2E\_3C V1m) V0n))) \wedge ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))))))))))) \\
\end{aligned} \tag{36}$$

### Theorem 1

$$((ap (ap (c\_2Ecombin\_2Eo ty\_2Enum\_2Enum ty\_2Enum\_2Enum (ty\_2Elist\_2Elist \\
ty\_2Estring\_2Echar)) c\_2EASCIInumbers\_2Enum\_from\_hex\_string) \\
c\_2EASCIInumbers\_2Enum\_to\_hex\_string) = (c\_2Ecombin\_2Ei \\
ty\_2Enum\_2Enum))$$