

# thm\_2ECoder\_2Edecode\_\_encode (TMMr- CpX64yxGUV78qJnHrDT2giUnmCQDp3i)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty \ A0 \Rightarrow nonempty \ (ty\_2Elist\_2Elist \ A0) \quad (1)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty \ A0 \Rightarrow \forall A1. nonempty \ A1 \Rightarrow nonempty \ (ty\_2Epair\_2Eprod \ A0 \ A1) \quad (2)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty \ A\_27a \Rightarrow \forall A\_27b. nonempty \ A\_27b \Rightarrow c\_2Epair\_2ESND \ A\_27a \ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod \ A\_27a \ A\_27b)}) \quad (3)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty \ A\_27a \Rightarrow \forall A\_27b. nonempty \ A\_27b \Rightarrow c\_2Epair\_2EFST \ A\_27a \ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod \ A\_27a \ A\_27b)}) \quad (4)$$

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o \ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $(ap \ (ap \ (c\_2Emin\_2E\_3D \ (2^2)) \ (\lambda V0x \in 2. V0x)) \ (\lambda V1x \in 2. V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap \ (ap \ (c\_2Emin\_2E\_3D \ (2^{A\_27a})))$

**Definition 5** We define  $c\_2Epair\_2Epair\_CASE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0p \in (ty\_2Epair\_2Eprod \ A\_27a \ A\_27b)$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty \ A0 \Rightarrow nonempty \ (ty\_2Eoption\_2Eoption \ A0) \quad (5)$$

Let  $c\_2Eoption\_2Eoption\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (6)$$

**Definition 6** We define  $c\_2ECoder\_2Edecode$  to be  $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1d \in ((ty\_2Eoption\_2Eoption\ A\_27a))$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (7)$$

**Definition 7** We define  $c\_2Eone\_2Eone$  to be  $(ap\ (c\_2Emin\_2E40\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2Eone))$

**Definition 8** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 9** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 10** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF))$

**Definition 11** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (8)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \quad (9)$$

**Definition 12** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b)\ V0e)$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (10)$$

**Definition 13** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_ABS\ A\_27a)\ 0)$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (11)$$

**Definition 14** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 15** We define  $c\_Esum\_2EINL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap (c\_Esum\_2EABS$

**Definition 16** We define  $c\_Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap (c\_Eoption\_2Eoption\_2$

**Definition 17** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_Emin\_2E\_40$

**Definition 18** We define  $c\_Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(\lambda$

**Definition 19** We define  $c\_EDecode\_2Eenc2dec$  to be  $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1e \in ((ty\_2Elist\_2El$

Let  $c\_2Elist\_2EisPREFIX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EisPREFIX A\_27a \in ((2^{(ty\_2Elist\_2Elist A\_27a)})(ty\_2Elist\_2Elist A\_27a)) \quad (12)$$

**Definition 20** We define  $c\_2EEncode\_2Ewf\_encoder$  to be  $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1e \in ((ty\_2Elist\_2El$

**Definition 21** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x$

**Definition 22** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 23** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a}) A$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (13)$$

**Definition 24** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (14)$$

**Definition 25** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0p \in (2^{A\_27a}).(\forall V1e \in \\ & ((ty\_2Elist\_2Elist 2)^{A\_27a}).(\forall V2l \in (ty\_2Elist\_2Elist \\ & 2).(((ap (ap (ap (c\_2EDecode\_2Eenc2dec A\_27a) V0p) V1e) V2l) = \\ & (c\_2Eoption\_2ENONE (ty\_2Epair\_2Eprod A\_27a (ty\_2Elist\_2Elist \\ & 2)))) \Leftrightarrow (\forall V3x \in A\_27a.(\forall V4t \in (ty\_2Elist\_2Elist 2). \\ & ((p (ap V0p V3x)) \Rightarrow (\neg (V2l = (ap (ap (c\_2Elist\_2EAPPEND 2) (ap V1e \\ & V3x)) V4t)))))))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (2^{A\_27a}). (\forall V1e \in \\
& ((ty\_2Elist\_2Elist\ 2)^{A\_27a}). (\forall V2l \in (ty\_2Elist\_2Elist \\
& 2). (\forall V3x \in A\_27a. (\forall V4t \in (ty\_2Elist\_2Elist\ 2). \\
& ((p\ (ap\ (ap\ (c\_2EEncode\_2Ewf\_encoder\ A\_27a)\ V0p)\ V1e)) \Rightarrow (((ap \\
& (ap\ (ap\ (c\_2EDecode\_2Enc2dec\ A\_27a)\ V0p)\ V1e)\ V2l) = (ap\ (c\_2Eoption\_2ESOME \\
& (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& A\_27a\ (ty\_2Elist\_2Elist\ 2))\ V3x)\ V4t))) \Leftrightarrow ((p\ (ap\ V0p\ V3x)) \wedge (V2l = \\
& (ap\ (ap\ (c\_2Elist\_2EAPPEND\ 2)\ (ap\ V1e\ V3x))\ V4t)))))))))
\end{aligned} \tag{16}$$

Assume the following.

$$True \tag{17}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee \neg(p\ V0t))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Rightarrow \neg(p\ V0t))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\
& (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\
& (p\ V0t)) \Leftrightarrow (p\ V0t))))))
\end{aligned} \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (26)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (27)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (28)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (31)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.(((\forall V2x \in A.27a.(p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in A.27a.((p (ap V0P V3x)) \wedge (p V1Q)))))) \quad (32)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p V0P) \vee (\exists V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A.27a.((p V0P) \vee (p (ap V1Q V3x)))))) \quad (33)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).((\exists V2x \in A.27a.((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A.27a.(p (ap V1Q V3x)))))) \quad (34)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A\_27a}). ((\forall V2x \in A\_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a. (p\ (ap\ V1Q\ V3x))))))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B)))))) \quad (38)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (39)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (40)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0P \in ((2^{A\_27b})^{A\_27a}). ((\forall V1x \in A\_27a. (\exists V2y \in A\_27b. (p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A\_27b)^{A\_27a}. (\forall V4x \in A\_27a. (p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \quad (41)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap\ (c\_2Ecombin\_2EI\ A\_27a)\ V0x) = V0x)) \quad (42)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption\ A\_27a). ((V0opt = (c\_2Eoption\_2ENONE\ A\_27a)) \vee (\exists V1x \in A\_27a. (V0opt = (ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V1x)))))) \quad (43)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & (\forall V0v \in A\_27b. (\forall V1f \in (A\_27b^{A\_27a}). ((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_CASE \\ & A\_27a\ A\_27b)\ (c\_2Eoption\_2ENONE\ A\_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\ & A\_27a. (\forall V3v \in A\_27b. (\forall V4f \in (A\_27b^{A\_27a}). ((ap\ (ap \\ & (ap\ (c\_2Eoption\_2Eoption\_CASE\ A\_27a\ A\_27b)\ (ap\ (c\_2Eoption\_2ESOME \\ & A\_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b). (\exists V1q \in A\_27a. \\ & (\exists V2r \in A\_27b. (V0x = (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b) \\ & V1q)\ V2r)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow (\forall V0x \in A\_27b. (\forall V1y \in A\_27c. (\forall V2f \in \\ & ((A\_27a^{A\_27c})^{A\_27b}). ((ap\ (ap\ (c\_2Epair\_2Epair\_CASE\ A\_27a\ A\_27b \\ & A\_27c)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27b\ A\_27c)\ V0x)\ V1y))\ V2f) = (ap \\ & (ap\ V2f\ V0x)\ V1y)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a). ((ap\ (ap\ (c\_2Elist\_2EAPPEND \\ & A\_27a)\ V0l)\ (c\_2Elist\_2ENIL\ A\_27a)) = V0l)) \wedge (\forall V1l \in (ty\_2Elist\_2Elist \\ & A\_27b). ((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27b)\ (c\_2Elist\_2ENIL\ A\_27b)) \\ & V1l) = V1l))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist \\ & A\_27a). (\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a). ((p\ (ap\ (ap\ (c\_2Elist\_2EisPREFIX \\ & A\_27a)\ V1l2)\ V0l1)) \Leftrightarrow (\exists V2l \in (ty\_2Elist\_2Elist\ A\_27a). ( \\ & V0l1 = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l2)\ V2l)))))) \end{aligned} \quad (48)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (50)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (58)$$

### Theorem 1

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0p \in (2^{A\_27a}).(\forall V1e \in ((ty\_2Elist\_2Elist 2)^{A\_27a}).(\forall V2x \in A\_27a.(((p (ap (ap (c\_2EEncode\_2Ewf\_encoder A\_27a) V0p) V1e)) \wedge (p (ap V0p V2x))) \Rightarrow ((ap (ap (ap (c\_2ECoder\_2Edecode A\_27a) V0p) (ap (ap (c\_2EDecode\_2Eenc2dec A\_27a) V0p) V1e)) (ap V1e V2x)) = V2x))))))$$