

thm_2ECoder_2Ewf__coder__bnum
(TMHN3fFgFh1P6amzD19fGX4TEKcqEdqPMgL)

October 26, 2020

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (1)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (2)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (3)$$

Let $c_2ECoder_2Ewf_coder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2ECoder_2Ewf_coder\ A_27a \in (2^{(ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ ((ty_2Elist_2Elist\ 2)^{A_27a}))\ ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a\ A_27b))^{A_27a}))} \quad (4)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1t \in 2.V1t))\ (\lambda V2t \in 2.V2t))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (5)$$

Definition 6 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$
 Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \quad (6)$$

Let $c_2EEncode_2Eencode_bnum : \iota$ be given. Assume the following.

$$c_2EEncode_2Eencode_bnum \in (((ty_2Elist_2Elist\ 2)^{ty_2Eenum_2Eenum})^{ty_2Eenum_2Eenum}) \quad (7)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (8)$$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap\ P\ x)) \mathbf{then}$ (the $(\lambda x.x \in A \wedge p$
 of type $\iota \Rightarrow \iota$).

Definition 8 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40\ ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone$

Definition 9 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (9)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (10)$$

Definition 11 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (11)$$

Definition 12 We define $c_2Eoption_2E_NONE$ to be $\lambda A_27a : \iota.(ap (c_2Eoption_2Eoption_ABS\ A_27a) ($

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (12)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (13)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (14)$$

Definition 13 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c)^{A_27a})$

Definition 14 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS$

Definition 15 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_2$

Definition 16 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 18 We define $c_2EDecode_2Eenc2dec$ to be $\lambda A_27a : \iota. \lambda V0p \in (2^{A_27a}). \lambda V1e \in ((ty_2Elist_2EEl$

Definition 19 We define $c_2EDecode_2Edecode_bnum$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1p \in (2^{ty_2Enum$

Definition 20 We define $c_2ECoder_2Ebnum_coder$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1p \in (2^{ty_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (15)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (16)$$

Definition 21 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 22 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega)^{ty_2Enum_2Enum} \quad (17)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega)^{\omega} \quad (18)$$

Definition 23 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (19)$$

Definition 24 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B))$

Definition 25 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Definition 26 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. V0m$

Definition 27 We define $c_2EEncode_2Ewf_pred$ to be $\lambda A_27a : \iota. \lambda V0p \in (2^{A_27a}). (ap (c_2Ebool_2E_3F A_27a))$

Let $c_2Elist_2EisPREFIX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2EisPREFIX A_27a \in ((2^{(ty_2Elist_2Elist A_27a)})^{(ty_2Elist_2Elist A_27a)}) \quad (21)$$

Definition 28 We define $c_2EEncode_2Ewf_encoder$ to be $\lambda A_27a : \iota. \lambda V0p \in (2^{A_27a}). \lambda V1e \in ((ty_2Elist_2Elist A_27a))$

Definition 29 We define $c_2EEncode_2Ewf_pred_bnum$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1p \in (2^{ty_2Enum_2Enum})$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0p \in (2^{A_27a}). (\forall V1e \in \\ & ((ty_2Elist_2Elist 2)^{A_27a}). (\forall V2d \in ((ty_2Eoption_2Eoption \\ & (ty_2Epair_2Eprod A_27a (ty_2Elist_2Elist 2))^{(ty_2Elist_2Elist 2)}), \\ & ((p (ap (c_2ECoder_2Ewf_coder A_27a)) (ap (ap (c_2Epair_2E_2C \\ & (2^{A_27a}) (ty_2Epair_2Eprod ((ty_2Elist_2Elist 2)^{A_27a}) ((\\ & ty_2Eoption_2Eoption (ty_2Epair_2Eprod A_27a (ty_2Elist_2Elist \\ & 2))^{(ty_2Elist_2Elist 2)}))) V0p) (ap (ap (c_2Epair_2E_2C ((ty_2Elist_2Elist \\ & 2)^{A_27a}) ((ty_2Eoption_2Eoption (ty_2Epair_2Eprod A_27a (ty_2Elist_2Elist \\ & 2))^{(ty_2Elist_2Elist 2)})) V1e) V2d)))) \Leftrightarrow ((p (ap (c_2EEncode_2Ewf_pred \\ & A_27a) V0p)) \wedge ((p (ap (ap (c_2EEncode_2Ewf_encoder A_27a) V0p) \\ & V1e)) \wedge (V2d = (ap (ap (c_2EDecode_2Eenc2dec A_27a) V0p) V1e)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1p \in (2^{ty_2Enum_2Enum}). \\ & ((p (ap (ap (c_2EEncode_2Ewf_pred_bnum V0m) V1p)) \Rightarrow (p (ap (ap (\\ & c_2EEncode_2Ewf_encoder ty_2Enum_2Enum) V1p) (ap c_2EEncode_2Eencode_bnum \\ & V0m)))))) \end{aligned} \quad (23)$$

Assume the following.

$$True \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (26)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p \ V0t)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\ & ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in \\ & 2.(((p \ V0x) \Leftrightarrow (p \ V1x.27)) \wedge ((p \ V1x.27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y.27)))) \Rightarrow \\ & (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x.27) \Rightarrow (p \ V3y.27)))))) \end{aligned} \quad (30)$$

Theorem 1

$$\begin{aligned} & (\forall V0m \in ty.2Enum.2Enum.(\forall V1p \in (2^{ty.2Enum.2Enum}). \\ & ((p \ (ap \ (ap \ c.2EEncode.2Ewf_pred_bnum \ V0m) \ V1p)) \Rightarrow (p \ (ap \ (c.2ECoder.2Ewf_coder \\ & ty.2Enum.2Enum) \ (ap \ (ap \ c.2ECoder.2Ebnm_coder \ V0m) \ V1p)))))) \end{aligned}$$