

thm_2ECoder_2Ewf_coder_bool
(TMb1XJDiaC3eYLDfjFYUiY4hsvXyoaRUn3J)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 4 We define `c_2Ebool_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27$

Definition 6 We define `c_2EEncode_2Ewf_pred` to be $\lambda A.27a : \iota.\lambda V0p \in (2^{A-27a}).(ap (c_2Ebool_2E_3F A.2$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (2)$$

Let `ty_2Eoption_2Eoption` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (3)$$

Let `c_2ECoder_2Ewf_coder` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c_2ECoder_2Ewf_coder A.27a \in (2^{(ty_2Epair_2Eprod (2^{A-27a}) (ty_2Epair_2Eprod ((ty_2Elist_2Elist 2)^{A-27a}) ((ty_2Eoption_2Eoption (ty_2Epair_2Eprod A.27a))))} \quad (4)$$

Definition 7 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}})$$
 (5)

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a)$$
 (6)

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a})$$
 (7)

Definition 10 We define $c_2Eencode_2Eencode_bool$ to be $\lambda V0x \in 2.(ap (ap (c_2Elist_2ECONS 2) V0x)$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone$$
 (8)

Definition 11 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone.$

Definition 12 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t)).$

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1)$$
 (9)

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2})$$
 (10)

Definition 14 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)})$$
 (11)

Definition 15 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap (c_2Eoption_2Eoption_ABS A_27a) (c_2Eoption_2ENONE A_27a))$.
Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EAPPEND A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(ty_2Elist_2Elist A_27a)}) \quad (12)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (13)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (14)$$

Definition 16 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a})$

Definition 17 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS A_27a) (ap (c_2Esum_2EABS A_27b) V0e))$

Definition 18 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap (c_2Eoption_2Eoption_ABS A_27a) V0x)$

Definition 19 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Ebool_2ECOND A_27a) V1t1 V2t2))))$

Definition 20 We define $c_2EDecode_2Eenc2dec$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1e \in ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})$

Definition 21 We define $c_2EDecode_2Edec_bool$ to be $\lambda V0p \in (2^2).(ap (ap (c_2EDecode_2Eenc2dec A_27a) V0p) (ap (c_2Ebool_2ECOND A_27a) V0p)))$

Definition 22 We define $c_2ECoder_2Ebool_coder$ to be $\lambda V0p \in (2^2).(ap (ap (c_2Epair_2E_2C A_27a) (ty_2Elist_2Elist A_27a)) (ap (c_2Ebool_2ECOND A_27a) V0p)))$

Let $c_2Elist_2EisPREFIX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EisPREFIX A_27a \in ((2^{(ty_2Elist_2Elist A_27a)})^{(ty_2Elist_2Elist A_27a)}) \quad (15)$$

Definition 23 We define $c_2EEncode_2Ewf_encoder$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1e \in ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0p \in (2^{A_27a}).(\forall V1e \in \\ & ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)}).(\forall V2d \in ((ty_2Eoption_2Eoption \\ & (ty_2Epair_2Eprod A_27a (ty_2Elist_2Elist A_27a))^{(ty_2Elist_2Elist A_27a)}). \\ & ((p (ap (c_2ECoder_2Ewf_coder A_27a) (ap (ap (c_2Epair_2E_2C \\ & (2^{A_27a}) (ty_2Epair_2Eprod ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)} \\ & (ty_2Eoption_2Eoption (ty_2Epair_2Eprod A_27a (ty_2Elist_2Elist A_27a))^{(ty_2Elist_2Elist A_27a)})) \\ & V0p) (ap (ap (c_2Epair_2E_2C ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)} \\ & (ty_2Eoption_2Eoption (ty_2Epair_2Eprod A_27a (ty_2Elist_2Elist A_27a))^{(ty_2Elist_2Elist A_27a)})) \\ & V1e) V2d)))) \Leftrightarrow ((p (ap (c_2EEncode_2Ewf_pred \\ & A_27a) V0p)) \wedge ((p (ap (ap (c_2EEncode_2Ewf_encoder A_27a) V0p) \\ & V1e)) \wedge (V2d = (ap (ap (c_2EDecode_2Eenc2dec A_27a) V0p) V1e)))))) \end{aligned} \quad (16)$$

Assume the following.

$$(\forall V0p \in (2^2).(p (ap (ap (c_2EEncode_2Ewf_encoder 2) V0p) c_2EEncode_2Eencode_bool))) \quad (17)$$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (23)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (24)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (25)$$

Theorem 1

$$(\forall V0p \in (2^2).(p (ap (c_2EEncode_2Ewf_pred 2) V0p) \Rightarrow (p (ap (c_2ECoder_2Ewf_coder 2) (ap c_2ECoder_2Ebool_coder V0p))))))$$