thm_2ECoder_2Ewf__coder__closed (TMFuxQB-Dugm4JfEQE58niYqyVBanuHvmZkA)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{3}$$

Let $c\_2ECoder\_2Ewf\_\_coder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2ECoder\_2Ewf\_\_coder\ A\_27a \in (2^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ ((ty\_2Elist\_2Elist\ 2)^{A\_27a})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_} \tag{4}$$

Let $c\_2ECoder\_2Edomain : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2ECoder\_2Edomain\ A\_27a \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2E} \tag{5}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{6}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{7}$$

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Epair_2Epair__CASE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0p \in (ty\_2Epair\_2$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (8)$$

Definition 6 *We define* c_2ECoder_2Edecode *to be* $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1d \in ((ty\_2Eoption\_2Eo$

Let $c\_2ECoder\_2Edecoder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2ECoder\_2Edecoder\ A\_27a \in ((A\_27a^{(ty\_2Elist\_2Elist\ 2)})^{(ty\_2Epair\_2Eprod\ (2^{A\text{-}} \quad (9)$$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (10)$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (11)$$

Definition 7 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

Definition 8 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 9 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 10 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (12)$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \quad (13)$$

Definition 12 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (14)$$

Definition 13 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 14 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Definition 15 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 16 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 17 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 18 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 19 *We define* c_2EDecode_2Eenc2dec *to be* $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1e \in ((ty\_2Elist\_2El$

Let $c\_2Elist\_2EisPREFIX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EisPREFIX\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27} \quad (15)$$

Definition 20 *We define* c_2EEncode_2Ewf__encoder *to be* $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1e \in ((ty\_2Elist\_$

Definition 21 *We define* c_2EEncode_2Ewf__pred *to be* $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ A\_$

Definition 22 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 23 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 24 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (16)$$

Definition 25 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 26 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (2^{A\_27a}).(\forall V1e \in ((ty\_2Elist\_2Elist\ 2)^{A\_27a}).(\forall V2d \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)}).((p\ (ap\ (c\_2ECoder\_2Ewf\_\_coder\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ ((ty\_2Elist\_2Elist\ 2)^{A\_27a})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)})))\ V0p)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ((ty\_2Elist\_2Elist\ 2)^{A\_27a})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)}))\ V1e)\ V2d)))) \Leftrightarrow ((p\ (ap\ (\mathsf{c\_2EEncode\_2Ewf\_\_pred}\ A\_27a)\ V0p)) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Ewf\_\_encoder}\ A\_27a)\ V0p)\ V1e)) \wedge (V2d = (ap\ (ap\ (\mathsf{c\_2EDecode\_2Eenc2dec}\ A\_27a)\ V0p)\ V1e)))))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (2^{A\_27a}).(\forall V1e \in ((ty\_2Elist\_2Elist\ 2)^{A\_27a}).(\forall V2d \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)}).((ap\ (c\_2ECoder\_2Edomain\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ ((ty\_2Elist\_2Elist\ 2)^{A\_27a})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)})))\ V0p)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ((ty\_2Elist\_2Elist\ 2)^{A\_27a})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)}))\ V1e)\ V2d))) = V0p)))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (2^{A\_27a}).(\forall V1e \in ((ty\_2Elist\_2Elist\ 2)^{A\_27a}).(\forall V2d \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)}).((ap\ (c\_2ECoder\_2Edecoder\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ ((ty\_2Elist\_2Elist\ 2)^{A\_27a})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)})))\ V0p)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ((ty\_2Elist\_2Elist\ 2)^{A\_27a})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)}))\ V1e)\ V2d))) = (ap\ (ap\ (\mathsf{c\_2ECoder\_2Edecode}\ A\_27a)\ V0p)\ V2d))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (2^{A\_27a}).(\forall V1e \in ((ty\_2Elist\_2Elist\ 2)^{A\_27a}).(\forall V2l \in (ty\_2Elist\_2Elist\ 2).(\forall V3x \in A\_27a.(\forall V4t \in (ty\_2Elist\_2Elist\ 2).((p\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Ewf\_\_encoder}\ A\_27a)\ V0p)\ V1e)) \Rightarrow (((ap\ (ap\ (ap\ (\mathsf{c\_2EDecode\_2Eenc2dec}\ A\_27a)\ V0p)\ V1e)\ V2l) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ (ty\_2Elist\_2Elist\ 2))\ V3x)\ V4t))) \Leftrightarrow ((p\ (ap\ V0p\ V3x)) \wedge (V2l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ 2)\ (ap\ V1e\ V3x))\ V4t)))))))))) \tag{20}$$

Assume the following.

$$True \tag{21}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg (p\ V0t)))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{26}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{29}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg (\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg (p\ (ap\ V0P\ V2x)))))) \tag{31}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{32}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption\ A\_27a).((V0opt = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \vee (\exists V1x \in A\_27a.(V0opt = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0v \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in A\_27a.(\forall V3v \in A\_27b.(\forall V4f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\exists V1q \in A\_27a.(\exists V2r \in A\_27b.(V0x = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1q)\ V2r))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0x \in A\_27b.(\forall V1y \in A\_27c.(\forall V2f \in ((A\_27a^{A\_27c})^{A\_27b}).((ap\ (ap\ (\mathsf{c\_2Epair\_2Epair\_\_CASE}\ A\_27a\ A\_27b\ A\_27c)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ A\_27c)\ V0x)\ V1y))\ V2f) = (ap\ (ap\ V2f\ V0x)\ V1y))))) \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{39}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{40}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{41}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{42}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{43}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \tag{44}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \tag{45}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))) \tag{46}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in (ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ ((ty\_2Elist\_2Elist\ 2)^{A\_27a})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)}))).((p\ (ap\ (c\_2ECoder\_2Ewf\_\_coder\ A\_27a)\ V0c)) \Rightarrow (\forall V1l \in (ty\_2Elist\_2Elist\ 2).(p\ (ap\ (ap\ (c\_2ECoder\_2Edomain\ A\_27a)\ V0c)\ (ap\ (ap\ (c\_2ECoder\_2Edecoder\ A\_27a)\ V0c)\ V1l))))))$$