thm_2ECoder_2Ewf__coder__op (TMQsDkffSfQvSrbEQyUWGFfVqZu5ZiHteGj)

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Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{3}$$

Let $c\_2ECoder\_2Ewf\_\_coder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2ECoder\_2Ewf\_\_coder\ A\_27a \in (2^{(ty\_2Epair\_2Eprod\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ ((ty\_2Elist\_2Elist\ 2)^{A\_27a})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_} \tag{4}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{5}$$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in 2$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{6}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{7}$$

Definition 10 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{8}$$

Definition 11 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{9}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{10}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{11}$$

Definition 12 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Definition 13 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 14 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 15 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 16 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 17 *We define* c_2EDecode_2Eenc2dec *to be* $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1e \in ((ty\_2Elist\_2El$

Definition 18 *We define* c_2EEncode_2Ewf__pred *to be* $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_3F A

Let $c\_2Elist\_2EisPREFIX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EisPREFIX\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27} \tag{12}$$

Definition 19 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 20 *We define* c_2EEncode_2Ebiprefix *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Elist\_2Elist\ A\_27a).\lambda V1b \in$

Definition 21 *We define* c_2EEncode_2Ewf__encoder *to be* $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1e \in ((ty\_2Elist\_$

Definition 22 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 23 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 24 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap$ (c_2Ecombin_2ES $A\_27a\ (A\_27a^{A\_27a})\ A$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{13}$$

Definition 25 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (2^{A\_27a}).(\forall V1e \in ((ty\_2Elist\_2Elist\ 2)^{A\_27a}).(\forall V2d \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)}).((p\ (ap\ (c\_2ECoder\_2Ewf\_\_coder\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ ((ty\_2Elist\_2Elist\ 2)^{A\_27a})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)})))\ V0p)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ((ty\_2Elist\_2Elist\ 2)^{A\_27a})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)}))\ V1e)\ V2d)))) \Leftrightarrow ((p\ (ap\ (\mathsf{c\_2EEncode\_2Ewf\_\_pred}\ A\_27a)\ V0p)) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Ewf\_\_encoder}\ A\_27a)\ V0p)\ V1e)) \wedge (V2d = (ap\ (ap\ (\mathsf{c\_2EDecode\_2Eenc2dec}\ A\_27a)\ V0p)\ V1e)))))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (2^{A\_27a}).(\forall V1e \in ((ty\_2Elist\_2Elist\ 2)^{A\_27a}).(\forall V2l \in (ty\_2Elist\_2Elist\ 2).(((ap\ (ap\ (ap\ (\mathsf{c\_2EDecode\_2Eenc2dec}\ A\_27a)\ V0p)\ V1e)\ V2l) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))) \Leftrightarrow (\forall V3x \in A\_27a.(\forall V4t \in (ty\_2Elist\_2Elist\ 2).((p\ (ap\ V0p\ V3x)) \Rightarrow (\neg(V2l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ 2)\ (ap\ V1e\ V3x))\ V4t))))))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (2^{A\_27a}).(\forall V1e \in ((ty\_2Elist\_2Elist\ 2)^{A\_27a}).(\forall V2l \in (ty\_2Elist\_2Elist\ 2).(\forall V3x \in (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)).((p\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Ewf\_\_encoder}\ A\_27a)\ V0p)\ V1e)) \Rightarrow (((ap\ (ap\ (ap\ (\mathsf{c\_2EDecode\_2Eenc2dec}\ A\_27a)\ V0p)\ V1e)\ V2l) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))\ V3x)) \Leftrightarrow ((p\ (ap\ V0p\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ (ty\_2Elist\_2Elist\ 2))\ V3x))) \wedge (V2l = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ 2)\ (ap\ V1e\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ (ty\_2Elist\_2Elist\ 2))\ V3x)))\ (ap\ (c\_2Epair\_2ESND\ A\_27a\ (ty\_2Elist\_2Elist\ 2))\ V3x)))))))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Ebiprefix}\ A\_27a)\ V0x)\ V0x))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1b \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2c \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3d \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Ebiprefix}\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0a)\ V1b))\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V2c)\ V3d))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Ebiprefix}\ A\_27a)\ V0a)\ V2c))))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (2^{A\_27a}).(\forall V1e \in ((ty\_2Elist\_2Elist\ 2)^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Ewf\_\_encoder}\ A\_27a)\ V0p)\ V1e)) \Leftrightarrow (\forall V2x \in A\_27a.(\forall V3y \in A\_27a.(((p\ (ap\ V0p\ V2x)) \wedge ((p\ (ap\ V0p\ V3y)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Ebiprefix}\ 2)\ (ap\ V1e\ V2x))\ (ap\ V1e\ V3y))))) \Rightarrow (V2x = V3y))))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (2^{A\_27a}).(\forall V1e \in ((ty\_2Elist\_2Elist\ 2)^{A\_27a}).(\forall V2f \in ((ty\_2Elist\_2Elist\ 2)^{A\_27a}).(((p\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Ewf\_\_encoder}\ A\_27a)\ V0p)\ V1e)) \wedge (\forall V3x \in A\_27a.((p\ (ap\ V0p\ V3x)) \Rightarrow ((ap\ V1e\ V3x) = (ap\ V2f\ V3x))))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Ewf\_\_encoder}\ A\_27a)\ V0p)\ V2f)))))) \tag{20}$$

Assume the following.

$$True \tag{21}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{25}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg (p\ V0t)))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{29}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{31}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((\forall V2x \in A\_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x))) \Rightarrow (V0f = V1g)))) \tag{35}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg (\forall V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg (p\ (ap\ V0P\ V2x)))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg (\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg (p\ (ap\ V0P\ V2x)))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P) \wedge (\forall V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P) \vee (\exists V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V3x))))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.((\exists V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \wedge (p\ V1Q)))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\exists V2x \in A\_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \wedge (\exists V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{43}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.((\neg((p\ V0A) \Rightarrow (p\ V1B))) \Leftrightarrow ((p\ V0A) \wedge (\neg(p\ V1B)))))) \tag{44}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \tag{45}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A))))) \tag{46}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \tag{47}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \tag{48}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{49}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{50}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0P \in ((2^{A\_27b})^{A\_27a}).((\forall V1x \in A\_27a.(\exists V2y \in A\_27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A\_27b^{A\_27a}).(\forall V4x \in A\_27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \tag{51}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{52}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l3 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ V1l2) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V0l1)\ V2l3)) \Leftrightarrow (V1l2 = V2l3))))) \wedge (\forall V3l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V4l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V5l3 \in (ty\_2Elist\_2Elist\ A\_27a).(((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V4l2)\ V3l1) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V5l3)\ V3l1)) \Leftrightarrow (V4l2 = V5l3)))))) \tag{53}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption\ A\_27a).((V0opt = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \vee (\exists V1x \in A\_27a.(V0opt = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))))) \tag{54}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \tag{55}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg((\mathsf{c\_2Eoption\_2ENONE}\ A\_27a) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x)))) \tag{56}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \tag{57}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}(\forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\exists V1q \in A\_27a. (\exists V2r \in A\_27b.(V0x = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1q)\ V2r))))) \tag{58}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V0x))) \tag{59}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V1y))) \tag{60}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))){\Leftrightarrow}(p\ V0t))) \tag{61}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A){\Rightarrow}((\neg(p\ V0A)){\Rightarrow}False))) \tag{62}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B))){\Rightarrow}False){\Leftrightarrow} (((p\ V0A){\Rightarrow}False){\Rightarrow}((\neg(p\ V1B)){\Rightarrow}False))))) \tag{63}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B))){\Rightarrow}False){\Leftrightarrow} ((p\ V0A){\Rightarrow}((\neg(p\ V1B)){\Rightarrow}False))))) \tag{64}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)){\Rightarrow}False){\Rightarrow}(((p\ V0A){\Rightarrow}False){\Rightarrow}False))) \tag{65}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p){\Leftrightarrow}((p\ V1q){\Leftrightarrow}(p\ V2r))){\Leftrightarrow}(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee ((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))))) \tag{66}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg (p\ V1q)) \vee (\neg (p\ V2r)))) \wedge (((p\ V1q) \vee (\neg (p\ V0p))) \wedge ((p\ V2r) \vee (\neg (p\ V0p))))))))) \quad (67)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg (p\ V1q))) \wedge (((p\ V0p) \vee (\neg (p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg (p\ V0p))))))))) \quad (68)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg (p\ V2r))) \wedge ((\neg (p\ V1q)) \vee ((p\ V2r) \vee (\neg (p\ V0p))))))))) \quad (69)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg (p\ V1q))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge ((\neg (p\ V1q)) \vee (\neg (p\ V0p))))))) \quad (70)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg ((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \quad (71)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg ((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg (p\ V1q))))) \quad (72)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg ((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg (p\ V0p))))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg ((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg (p\ V1q))))) \quad (74)$$

Assume the following.

$$(\forall V0p \in 2.((\neg (\neg (p\ V0p))) \Rightarrow (p\ V0p))) \quad (75)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (2^{A\_27a}).(\forall V1e \in ((ty\_2Elist\_2Elist\ 2)^{A\_27a}).(\forall V2f \in ((ty\_2Elist\_2Elist\ 2)^{A\_27a}).(((\exists V3x \in A\_27a.(p\ (ap\ V0p\ V3x))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Ewf\_\_encoder}\ A\_27a)\ V0p)\ V1e)) \wedge (\forall V4x \in A\_27a.((p\ (ap\ V0p\ V4x)) \Rightarrow ((ap\ V1e\ V4x) = (ap\ V2f\ V4x)))))) \Rightarrow (p\ (ap\ (c\_2ECoder\_2Ewf\_\_coder\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (2^{A\_27a})\ (ty\_2Epair\_2Eprod\ ((ty\_2Elist\_2Elist\ 2)^{A\_27a})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)}))))\ V0p)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ((ty\_2Elist\_2Elist\ 2)^{A\_27a})\ ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)}))\ V1e)\ (ap\ (ap\ (\mathsf{c\_2EDecode\_2Eenc2dec}\ A\_27a)\ V0p)\ V2f)))))))))$$