

# thm\_2EConseqConv\_2ECOND\_\_CLAUSES\_\_CT (TMYMQ1AVxm6ji23TJ914Jtip1PCU6Z5kGZR)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (\lambda V1t1 \in 2.V1t1)) (\lambda V0t1 \in 2.V0t1)))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p \ P \Rightarrow p \ Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. inj\_o (p \ V2t \Rightarrow p \ V1t2))))$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x)) \text{ else } (\lambda x. x \in A \wedge \neg p x)$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. (ap (c\_2Ebool\_2E\_21 2) (\lambda V3t3 \in A\_27a. (p (ap V0t V3t3)) \wedge (p (ap V1t1 V2t2) = V3t3)))))))$

Assume the following.

$$\begin{aligned} & \forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in A\_27a. ((ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) \\ & V0t1) V1t2) = V1t2))) \end{aligned} \tag{1}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in (2^{A\_27a}). ((\forall V2x \in A\_27a. ((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow \\ & ((\forall V3x \in A\_27a. (p (ap V0P V3x))) \wedge (\forall V4x \in A\_27a. (p (ap V1Q V4x))))))) \end{aligned} \tag{2}$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in A\_27a. ((ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\ & V1t2) = V0t1))) \end{aligned}$$