

thm_2EDecode_2Edec2enc__decode__prod (TMT9brqjZaKEtQbapxV9gv6qvSW9rJ5v8m7)

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Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (1)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o\ (p \Rightarrow q)$ of type ι .

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2. V0x))\ (\lambda V1x \in 2. V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a})))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (2)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (3)$$

Definition 6 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Ebool_2E_21\ 2)\ (c_2Epair_2E_2C\ A_27a\ A_27b\ V0x\ V1y))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (4)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (5)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (6)$$

Definition 7 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (7)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (8)$$

Definition 8 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Definition 9 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E21))$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (9)$$

Definition 11 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A)\ P)$ of type $\iota \Rightarrow \iota$.

Definition 12 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ A_27a)\ V0P)))$

Definition 13 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ (c_2Emin_2E40\ A_27a)\ V2t2))))$

Definition 14 We define $c_2EDecode_2Ewf_decoder$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1d \in ((ty_2Eoption_2Eoption\ A_27a)\ V1d)$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (10)$$

Definition 15 We define $c_2EDecode_2Edec2enc$ to be $\lambda A_27a : \iota.\lambda V0d \in ((ty_2Eoption_2Eoption\ (ty_2Eoption_2Eoption\ A_27a)\ V0d))$

Definition 16 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone.V0x))$

Definition 17 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 18 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (ty_2Eoption_2Eoption\ A_27a))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (11)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (12)$$

Definition 19 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})$

Definition 20 We define $c_2EDecode_2Eenc2dec$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1e \in ((ty_2Elist_2Elist$

Let $c_2EEncode_2Eencode_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2EEncode_2Eencode_prod \\ A_27a\ A_27b \in (((ty_2Elist_2Elist\ 2)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})^{(ty_2Elist_2Elist\ 2)^{A_27b}})^{(ty_2Elist_2Elist\ 2)} \end{aligned} \quad (13)$$

Definition 21 We define $c_2EDecode_2Edec_prod$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (2^{(ty_2Epair_2Eprod$

Definition 22 We define $c_2EEncode_2Elift_prod$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p1 \in (2^{A_27a}).\lambda V1p2 \in (2$

Let $c_2Elist_2EisPREFIX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EisPREFIX\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (14)$$

Definition 23 We define $c_2EEncode_2Ewf_encoder$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1e \in ((ty_2Elist_2Elist$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0p \in (2^{A_27a}).(\forall V1d \in \\ ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a\ (ty_2Elist_2Elist \\ 2)))^{(ty_2Elist_2Elist\ 2)}).((p\ (ap\ (ap\ (c_2EDecode_2Ewf_decoder \\ A_27a)\ V0p)\ V1d)) \Rightarrow (p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder\ A_27a) \\ V0p)\ (ap\ (c_2EDecode_2Edec2enc\ A_27a)\ V1d)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0p \in (2^{A_27a}).(\forall V1e \in \\ ((ty_2Elist_2Elist\ 2)^{A_27a}).(\forall V2x \in A_27a.(((p\ (ap\ (ap \\ (c_2EEncode_2Ewf_encoder\ A_27a)\ V0p)\ V1e)) \wedge (p\ (ap\ V0p\ V2x))) \Rightarrow \\ ((ap\ (ap\ (c_2EDecode_2Edec2enc\ A_27a)\ (ap\ (ap\ (c_2EDecode_2Eenc2dec \\ A_27a)\ V0p)\ V1e))\ V2x) = (ap\ V1e\ V2x)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0p1 \in (2^{A_27a}). (\forall V1p2 \in (2^{A_27b}). (\forall V2e1 \in \\
& \quad ((ty_2Elist_2Elist\ 2)^{A_27a}). (\forall V3e2 \in ((ty_2Elist_2Elist \\
& \quad 2)^{A_27b}). (((p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder\ A_27a)\ V0p1) \\
& \quad V2e1)) \wedge (p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder\ A_27b)\ V1p2)\ V3e2))) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder\ (ty_2Epair_2Eprod\ A_27a \\
& \quad A_27b))\ (ap\ (ap\ (c_2EEncode_2Elift_prod\ A_27a\ A_27b)\ V0p1)\ V1p2)) \\
& \quad (ap\ (ap\ (c_2EEncode_2Eencode_prod\ A_27a\ A_27b)\ V2e1)\ V3e2))))))))) \\
& \hspace{15em} (17)
\end{aligned}$$

Assume the following.

$$True \hspace{15em} (18)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\
& (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& \quad (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \\
& \hspace{15em} (19)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\
& \quad (p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \\
& \hspace{15em} (20)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \\
& \quad True)) \\
& \hspace{15em} (21)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
& \quad A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \\
& \hspace{15em} (22)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& \quad p\ V0t)))))) \\
& \hspace{15em} (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\
& (p\ V1t2) \Rightarrow (p\ V2t3)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \\
& \hspace{15em} (24)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\
& 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\
& \quad (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \\
& \hspace{15em} (25)
\end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0p1 \in (2^{A_27a}). (\forall V1p2 \in (2^{A_27b}). (\forall V2d1 \in \\ & \quad ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a\ (ty_2Elist_2Elist \\ & \quad 2))) (ty_2Elist_2Elist\ 2)). (\forall V3d2 \in ((ty_2Eoption_2Eoption \\ & \quad (ty_2Epair_2Eprod\ A_27b\ (ty_2Elist_2Elist\ 2))) (ty_2Elist_2Elist\ 2)). \\ & (\forall V4x \in (ty_2Epair_2Eprod\ A_27a\ A_27b). (((p\ (ap\ (ap\ (c_2EDecode_2Ewf_decoder \\ & \quad A_27a\ V0p1)\ V2d1)) \wedge ((p\ (ap\ (ap\ (c_2EDecode_2Ewf_decoder\ A_27b) \\ & \quad V1p2)\ V3d2)) \wedge (p\ (ap\ (ap\ (ap\ (c_2EEncode_2Elift_prod\ A_27a\ A_27b) \\ & \quad V0p1)\ V1p2)\ V4x)))) \Rightarrow ((ap\ (ap\ (c_2EDecode_2Edec2enc\ (ty_2Epair_2Eprod \\ & \quad A_27a\ A_27b))\ (ap\ (ap\ (ap\ (c_2EDecode_2Edecode_prod\ A_27a\ A_27b) \\ & \quad (ap\ (ap\ (c_2EEncode_2Elift_prod\ A_27a\ A_27b)\ V0p1)\ V1p2))\ V2d1 \\ & \quad V3d2))\ V4x) = (ap\ (ap\ (ap\ (c_2EEncode_2Eencode_prod\ A_27a\ A_27b) \\ & \quad (ap\ (c_2EDecode_2Edec2enc\ A_27a)\ V2d1))\ (ap\ (c_2EDecode_2Edec2enc \\ & \quad A_27b)\ V3d2))\ V4x))))))))) \end{aligned}$$