# thm_2EDecode_2Edec2enc__decode__sum (TMSYtWiCHrG8GosVuceHuWeisEFeUMbuMst)

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Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

**Definition 1** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* $\iota$.

**Definition 3** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

**Definition 4** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

**Definition 5** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{3}$$

**Definition 6** *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{4}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{5}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \quad (6)$$

**Definition 7** *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS\_$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (7)$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (8)$$

**Definition 8** *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_A$

**Definition 9** *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

**Definition 10** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (9)$$

**Definition 11** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

**Definition 12** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

**Definition 13** *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 14** *We define* c_2EDecode_2Ewf__decoder *to be* $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1d \in ((ty\_2Eopti$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (10)$$

**Definition 15** *We define* c_2EDecode_2Edec2enc *to be* $\lambda A\_27a : \iota.\lambda V0d \in ((ty\_2Eoption\_2Eoption\ (ty\_2Ep$

**Definition 16** *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

**Definition 17** *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

**Definition 18** *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (11)$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (12)$$

**Definition 19** *We define* $\mathsf{c\_2Epair\_2EUNCURRY}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

**Definition 20** *We define* $\mathsf{c\_2EDecode\_2Eenc2dec}$ *to be* $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1e \in ((ty\_2Elist\_2El$

Let $c\_2EEncode\_2Eencode\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2EEncode\_2Eencode\_\_sum\ A\_27a\ A\_27b \in ((((ty\_2Elist\_2Elist\ 2)^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)})^{((ty\_2Elist\_2Elist\ 2)^{A\_27b})})^{((ty\_2Elist\_2Elis} \quad (13)$$

**Definition 21** *We define* $\mathsf{c\_2EDecode\_2Edecode\_\_sum}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (2^{(ty\_2Esum\_2Esum}$

Let $c\_2Esum\_2Esum\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow c\_2Esum\_2Esum\_\_CASE\ A\_27a\ A\_27b\ A\_27c \in (((A\_27c^{(A\_27c^{A\_27b})})^{(A\_27c^{A\_27a})})^{(ty\_2E} \quad (14)$$

**Definition 22** *We define* $\mathsf{c\_2EEncode\_2Elift\_\_sum}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p1 \in (2^{A\_27a}).\lambda V1p2 \in (2$

Let $c\_2Elist\_2EisPREFIX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EisPREFIX\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27} \quad (15)$$

**Definition 23** *We define* $\mathsf{c\_2EEncode\_2Ewf\_\_encoder}$ *to be* $\lambda A\_27a : \iota.\lambda V0p \in (2^{A\_27a}).\lambda V1e \in ((ty\_2Elist\_$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (2^{A\_27a}).(\forall V1d \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)}).((p\ (ap\ (ap\ (\mathsf{c\_2EDecode\_2Ewf\_\_decoder}\ A\_27a)\ V0p)\ V1d)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Ewf\_\_encoder}\ A\_27a)\ V0p)\ (ap\ (\mathsf{c\_2EDecode\_2Edec2enc}\ A\_27a)\ V1d)))))) \quad (16)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0p \in (2^{A\_27a}).(\forall V1e \in ((ty\_2Elist\_2Elist\ 2)^{A\_27a}).(\forall V2x \in A\_27a.(((p\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Ewf\_\_encoder}\ A\_27a)\ V0p)\ V1e)) \wedge (p\ (ap\ V0p\ V2x))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2EDecode\_2Edec2enc}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2EDecode\_2Eenc2dec}\ A\_27a)\ V0p)\ V1e))\ V2x) = (ap\ V1e\ V2x)))))) \quad (17)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0p1 \in (2^{A\_27a}).(\forall V1p2 \in (2^{A\_27b}).(\forall V2e1 \in ((ty\_2Elist\_2Elist\ 2)^{A\_27a}).(\forall V3e2 \in ((ty\_2Elist\_2Elist\ 2)^{A\_27b}).(((p\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Ewf\_\_encoder}\ A\_27a)\ V0p1)\ V2e1)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Ewf\_\_encoder}\ A\_27b)\ V1p2)\ V3e2))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Ewf\_\_encoder}\ (ty\_2Esum\_2Esum\ A\_27a\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Elift\_\_sum}\ A\_27a\ A\_27b)\ V0p1)\ V1p2))\ (ap\ ( ap\ (c\_2EEncode\_2Eencode\_\_sum\ A\_27a\ A\_27b)\ V2e1)\ V3e2)))))))) \tag{18}$$

Assume the following.

$$True \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg ( p\ V0t))))))) \tag{24}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{25}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{26}$$

**Theorem 1**

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0p1 \in (2^{A\_27a}).(\forall V1p2 \in (2^{A\_27b}).(\forall V2d1 \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)}).(\forall V3d2 \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27b\ (ty\_2Elist\_2Elist\ 2)))^{(ty\_2Elist\_2Elist\ 2)}). (\forall V4x \in (ty\_2Esum\_2Esum\ A\_27a\ A\_27b).(((p\ (ap\ (ap\ (\mathsf{c\_2EDecode\_2Ewf\_\_decoder}\ A\_27a)\ V0p1)\ V2d1)) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2EDecode\_2Ewf\_\_decoder}\ A\_27b)\ V1p2)\ V3d2)) \wedge (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Elift\_\_sum}\ A\_27a\ A\_27b)\ V0p1)\ V1p2)\ V4x)))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2EDecode\_2Edec2enc}\ (ty\_2Esum\_2Esum\ A\_27a\ A\_27b))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EDecode\_2Edecode\_\_sum}\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2EEncode\_2Elift\_\_sum}\ A\_27a\ A\_27b)\ V0p1)\ V1p2))\ V2d1)\ V3d2))\ V4x) = (ap\ (ap\ (ap\ (c\_2EEncode\_2Eencode\_\_sum\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2EDecode\_2Edec2enc}\ A\_27a)\ V2d1))\ (ap\ (\mathsf{c\_2EDecode\_2Edec2enc}\ A\_27b)\ V3d2))\ V4x))))))))$$