

thm_2EDecode_2Edec2enc__decode__unit
(TMGX6cSuRK5JVobFFeSQoKLWgBcVvkTV9Ww)

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Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (1)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Elist_2ENIL\ A.27a \in (ty_2Elist_2Elist\ A.27a) \quad (2)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 3 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define c_2Ebool_2E21 to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a})))$

Definition 5 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2))\ (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (3)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2EABS_prod\ A.27a\ A.27b \in ((ty_2Epair_2Eprod\ A.27a\ A.27b)^{(2^{A-27b})^{A-27a}}) \quad (4)$$

Definition 6 We define c_2Epair_2E2C to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap\ (c_2Ebool_2E21\ 2))\ (ty_2Epair_2Eprod\ x\ y))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (5)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (6)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (7)$$

Definition 7 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (8)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (9)$$

Definition 8 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge p\ x)) \text{ of type } \iota \Rightarrow \iota.$

Definition 10 We define $c_2EDecode_2Edec2enc$ to be $\lambda A_27a : \iota.\lambda V0d \in ((ty_2Eoption_2Eoption\ (ty_2Eone_2Eone\ A_27a))\ V0d)$

Definition 11 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone.\ V0x))$

Definition 12 We define $c_2Ebool_2E_21$ to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21))$

Definition 14 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 15 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (ty_2Eone_2Eone\ A_27a))$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)\ (ty_2Elist_2Elist\ A_27a))\ (ty_2Elist_2Elist\ A_27a)) \quad (10)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (11)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (12)$$

Definition 16 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a})$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 18 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 19 We define $c_2EDecode_2Eenc2dec$ to be $\lambda A_27a : \iota. \lambda V0p \in (2^{A_27a}). \lambda V1e \in ((ty_2Elist_2El$

Definition 20 We define $c_2EEncode_2Eencode_unit$ to be $\lambda V0v0 \in ty_2Eone_2Eone. (c_2Elist_2ENIL\ 2)$

Definition 21 We define $c_2EDecode_2Edecode_unit$ to be $\lambda V0p \in (2^{ty_2Eone_2Eone}). (ap\ (ap\ (c_2EDecode$

Let $c_2Elist_2EisPREFIX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EisPREFIX\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (13)$$

Definition 22 We define $c_2EEncode_2Ewf_encoder$ to be $\lambda A_27a : \iota. \lambda V0p \in (2^{A_27a}). \lambda V1e \in ((ty_2Elist$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0p \in (2^{A_27a}). (\forall V1e \in ((ty_2Elist_2Elist\ 2)^{A_27a}). (\forall V2x \in A_27a. (((p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder\ A_27a)\ V0p)\ V1e)) \wedge (p\ (ap\ V0p\ V2x))) \Rightarrow ((ap\ (ap\ (c_2EDecode_2Edec2enc\ A_27a)\ (ap\ (ap\ (c_2EDecode_2Eenc2dec\ A_27a)\ V0p)\ V1e))\ V2x) = (ap\ V1e\ V2x)))))) \quad (14)$$

Assume the following.

$$(\forall V0p \in (2^{ty_2Eone_2Eone}). (p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder\ ty_2Eone_2Eone)\ V0p)\ c_2EEncode_2Eencode_unit))) \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (17)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{19}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p V0t))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\
& 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow \\
& (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))))
\end{aligned} \tag{23}$$

Theorem 1

$$\begin{aligned}
& (\forall V0p \in (2^{ty_2Eone_2Eone}).(\forall V1x \in ty_2Eone_2Eone. \\
& ((p (ap V0p V1x)) \Rightarrow ((ap (ap (c_2EDecode_2Edec2enc ty_2Eone_2Eone) \\
& (ap c_2EDecode_2Edecode_unit V0p)) V1x) = (ap c_2EEncode_2Encode_unit \\
& V1x))))))
\end{aligned}$$