

thm_2EDecode_2Edec_bnum_def_compute
 (TM-
 RhRuOK31EXB5hKmWZVRuD87NxCtVnfVqk)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj_o (t \ t))))$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 8 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap (c_2Ebool_2E_21 2) (\lambda V3t3 \in 2. inj_o (t1 \ t2))))))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty \ ty_2Enum_2Enum \quad (1)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (2)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (3)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty}(\text{ty_2Epair_2Eprod } A0\ A1) \quad (4)$$

Let $c\text{-}2Epair\text{-}2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2\text{Epair_2ESND}_A_27a\ A_27b \in (A_27b^{(ty_2\text{Epair_2Eprod}\ A_27a\ A_27b)}) \quad (5)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2\text{Epair_2EFST } A_27a \ A_27b \in (A_27a^{(ty_2\text{Epair_2Eprod } A_27a \ A_27b)}) \quad (6)$$

Definition 9 We define $c_2Epair_2Epair_CASE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_2Epair_2Epair_CASE)$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Eoption_2Eoption } A0) \quad (7)$$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Eoption_2Eoption_CASE
A_27a\ A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption\ A_27a)})$$

(8)

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

nonempty *ty_2Eone_2Eone* (9)

Definition 10 We define c_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2E. \dots))$

Definition 11 We define $c_2Eb0o_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Eb0o_2E_7E))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty} (\text{ty_2Esum_2Esum } A0 \ A1) \quad (10)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.\text{nonempty } A.27a \Rightarrow \forall A.27b.\text{nonempty } A.27b \Rightarrow c.2Esum.2EABS_sum A.27a A.27b \in ((ty.2Esum.2Esum A.27a A.27b)^{(((2^{A-27b})^A-2^{27a})^2)}) \quad (11)$$

Definition 12 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS\ (A_27a\ V0)\ A_27b))$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow c_2Eoption_2Eoption_ABS\ A_{27a} \in ((ty_2Eoption_2Eoption\ A_{27a})(ty_2Esum_2Esum\ A_{27a}\ ty_2Eone_2Eone)) \quad (12)$$

Definition 13 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a) (c_2Eoption_2ENONE))$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (13)$$

Let $c_2Elist_2Elist_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2Elist_CASE \\ & A_27a A_27b \in (((A_27b^{(ty_2Elist_2Elist A_27a)^{A_27a})})^{A_27b})^{(ty_2Elist_2Elist A_27a)}) \end{aligned} \quad (14)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ & A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (15)$$

Definition 14 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2Epair_2C A_27a A_27b) (c_2Epair_2Epair_2C A_27a A_27b)))$

Definition 15 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_2Esum_2EABS A_27a A_27b) (c_2Esum_2EINL A_27a)))$

Definition 16 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_2ESOME A_27a) (c_2Eoption_2Eoption_2ESOME A_27a)))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (16)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (17)$$

Definition 17 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2EDecode_2Edec_bnum : \iota$ be given. Assume the following.

$$c_2EDecode_2Edec_bnum \in (((ty_2Eoption_2Eoption (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2))))^{(ty_2Elist_2Elist 2)})^{ty_2Enum_2Enum}) \quad (18)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \quad (19)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \quad (20)$$

Definition 18 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num m)$

Definition 19 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2EBIT2 n) (c_2Earithmetic_2EBIT2 n)))$

Definition 20 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (21)$$

Definition 21 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2D n))$

Definition 22 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Assume the following.

$$\begin{aligned} & ((\forall V0l \in (ty_2Elist_2Elist 2).((ap (ap c_2EDecode_2Edec_bnum \\ c_2Enum_2E0) V0l) = (ap (c_2Eoption_2ESOME (ty_2Epair_2Eprod \\ ty_2Enum_2Enum (ty_2Elist_2Elist 2))) (ap (ap (c_2Epair_2E_2C \\ ty_2Enum_2Enum (ty_2Elist_2Elist 2)) c_2Enum_2E0) V0l)))) \wedge \\ & (\forall V1m \in ty_2Enum_2Enum.(\forall V2l \in (ty_2Elist_2Elist \\ 2).((ap (ap c_2EDecode_2Edec_bnum (ap c_2Enum_2ESUC V1m)) V2l) = \\ & (ap (ap (ap (c_2Elist_2Elist_CASE 2 (ty_2Eoption_2Eoption (\\ ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2)))) V2l) \\ & (c_2Eoption_2ENONE (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist \\ 2)))) (\lambda V3h \in 2.(\lambda V4t \in (ty_2Elist_2Elist 2).(ap (ap (\\ ap (c_2Eoption_2Eoption_CASE (ty_2Epair_2Eprod ty_2Enum_2Enum \\ (ty_2Elist_2Elist 2)) (ty_2Eoption_2Eoption (ty_2Epair_2Eprod \\ ty_2Enum_2Enum (ty_2Elist_2Elist 2)))) (ap (ap c_2EDecode_2Edec_bnum \\ V1m) V4t)) (c_2Eoption_2ENONE (ty_2Epair_2Eprod ty_2Enum_2Enum \\ (ty_2Elist_2Elist 2)))) (\lambda V5v \in (ty_2Epair_2Eprod ty_2Enum_2Enum \\ (ty_2Elist_2Elist 2)).(ap (ap (c_2Epair_2Epair_CASE (ty_2Eoption_2Eoption \\ (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2))) ty_2Enum_2Enum \\ (ty_2Elist_2Elist 2)) V5v) (\lambda V6n \in ty_2Enum_2Enum.(\lambda V7t_27 \in \\ (ty_2Elist_2Elist 2).(ap (c_2Eoption_2ESOME (ty_2Epair_2Eprod \\ ty_2Enum_2Enum (ty_2Elist_2Elist 2))) (ap (ap (c_2Epair_2E_2C \\ ty_2Enum_2Enum (ty_2Elist_2Elist 2)) (ap (ap c_2Earithmetic_2E_2B \\ (ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap \\ c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V6n)) (ap (ap \\ (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) V3h) (ap c_2Earithmetic_2ENUMERAL \\ (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) c_2Enum_2E0))) \\ V7t_27))))))))))))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0f \in ((A_27a^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}). \\
& \quad (\forall V1g \in (A_27a^{ty_2Enum_2Enum}).((\forall V2n \in ty_2Enum_2Enum. \\
& \quad ((ap V1g (ap c_2Enum_2ESUC V2n)) = (ap (ap V0f V2n) (ap c_2Enum_2ESUC \\
& \quad V2n)))) \Leftrightarrow ((\forall V3n \in ty_2Enum_2Enum.((ap V1g (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT1 V3n))) = (ap (ap V0f (ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V3n))) \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2ZERO)))) \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V3n)))))) \wedge \\
& \quad (\forall V4n \in ty_2Enum_2Enum.((ap V1g (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V4n))) = (ap (ap V0f (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT1 V4n))) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V4n)))))))) \\
\end{aligned} \tag{23}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{24}$$

Theorem 1

$((\forall V0l \in (ty_2Elist_2Elist 2)).((ap (ap c_2EDecode_2Edec_bnum c_2Enum_2E0) V0l) = (ap (c_2Eoption_2ESOME (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2))) (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum (ty_2Elist_2Elist 2)) c_2Enum_2E0) V0l)))) \wedge$
 $((\forall V1m \in ty_2Enum_2Enum.(\forall V2l \in (ty_2Elist_2Elist 2).((ap (ap c_2EDecode_2Edec_bnum (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V1m))) V2l) = (ap (ap (ap (c_2Elist_2Elist_CASE 2 (ty_2Eoption_2Eoption (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2))) V2l) (c_2Eoption_2ENONE (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2)))) (\lambda V3h \in 2.(\lambda V4t \in (ty_2Elist_2Elist 2).(ap (ap (c_2Eoption_2Eoption_CASE (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2)) (ty_2Eoption_2Eoption (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2)))) (\lambda V4t \in (ap (ap c_2EDecode_2Edec_bnum (ap (ap c_2Earithmetic_2E_2D (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V1m))) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) (c_2Eoption_2ENONE (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2)))) (\lambda V5v \in (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2)).(ap (ap (c_2Epair_2Epair_CASE (ty_2Eoption_2Eoption (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2))) ty_2Enum_2Enum (ty_2Elist_2Elist 2)) V5v) (\lambda V6n \in ty_2Enum_2Enum.(\lambda V7t_27 \in (ty_2Elist_2Elist 2).(ap (c_2Eoption_2ESOME (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2))) (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum (ty_2Elist_2Elist 2)) (ap (ap c_2Earithmetic_2E_2B (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V6n)) (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) V3h) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) c_2Enum_2E0)))) V7t_27))))))))))) \wedge (\forall V8m \in ty_2Enum_2Enum.(\forall V9l \in (ty_2Elist_2Elist 2).((ap (ap c_2EDecode_2Edec_bnum (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 V8m))) V9l) = (ap (ap (ap (c_2Elist_2Elist_CASE 2 (ty_2Eoption_2Eoption (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2))) V9l) (c_2Eoption_2ENONE (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2)))) (\lambda V10h \in 2.(\lambda V11t \in (ty_2Elist_2Elist 2).(ap (ap (ap (c_2Eoption_2Eoption_CASE (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2)) (ty_2Eoption_2Eoption (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2)))) (\lambda V11t \in (ap (ap c_2EDecode_2Edec_bnum (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V8m))) V11t) (c_2Eoption_2ENONE (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2)))) (\lambda V12v \in (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2)).(ap (ap (c_2Epair_2Epair_CASE (ty_2Eoption_2Eoption (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2))) ty_2Enum_2Enum (ty_2Elist_2Elist 2)) V12v) (\lambda V13n \in ty_2Enum_2Enum.(\lambda V14t_27 \in (ty_2Elist_2Elist 2).(ap (c_2Eoption_2ESOME (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Elist_2Elist 2))) (ap (c_2Epair_2E_2C ty_2Enum_2Enum (ty_2Elist_2Elist 2)) (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V13n)) (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) V10h) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) c_2Enum_2E0)))) V14t_27)))))))))))$