

thm_2EDeCode_2Edecode__unit (TMGKQU- STKP9SfxwvJMJsRuq6NbgcLi8rN9)

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Definition 1 We define `c_2Emin_2E3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E7E` to be $(ap (ap (c_2Emin_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let `ty_2Eone_2Eone` : ι be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 3 We define `c_2Emin_2E40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge p))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define `c_2Eone_2Eone` to be $(ap (c_2Emin_2E40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone))$

Definition 5 We define `c_2Ebool_2E21` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E3D (2^{A-27a}))))$

Definition 6 We define `c_2Ebool_2E2F` to be $(ap (c_2Ebool_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define `c_2Emin_2E3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define `c_2Ebool_2E7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E3D_3D_3E V0t) c_2Ebool_2E2F))$

Definition 9 We define `c_2Ebool_2E2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21 2) (\lambda V2t \in 2.V2t))))$

Let `ty_2Esum_2Esum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let `c_2Esum_2EABS__sum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A-27b})^{A-27a})^2}) \tag{3}$$

Definition 10 We define c_Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_Esum_2EABS$
Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (5)$$

Definition 11 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a) (c_2Eoption_2Eoption A_27a))$
Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (6)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (7)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2EABS_prod A_27a A_27b) (V0x V1y))$

Definition 13 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_2Esum_2EABS A_27a A_27b) (V0e))$

Definition 14 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_ABS A_27a) (V0x))$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (8)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2EAPPEND A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(ty_2Elist_2Elist A_27a)}) \quad (9)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (10)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (11)$$

Definition 15 We define `c_2Epair_2EUNCURRY` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a}$

Definition 16 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 17 We define `c_2Ebool_2ECOND` to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 18 We define `c_2EDecode_2Eenc2dec` to be $\lambda A_27a : \iota. \lambda V0p \in (2^{A_27a}). \lambda V1e \in ((ty_2Elist_2El$

Let `c_2Elist_2ENIL` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (12)$$

Definition 19 We define `c_2EEncode_2Eencode_unit` to be $\lambda V0v0 \in ty_2Eone_2Eone. (c_2Elist_2ENIL\ 2)$

Definition 20 We define `c_2EDecode_2Edecode_unit` to be $\lambda V0p \in (2^{ty_2Eone_2Eone}). (ap\ (ap\ (c_2EDecode_2E$

Let `c_2Elist_2EisPREFIX` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EisPREFIX\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (13)$$

Definition 21 We define `c_2EEncode_2Ewf_encoder` to be $\lambda A_27a : \iota. \lambda V0p \in (2^{A_27a}). \lambda V1e \in ((ty_2Elist_2E$

Definition 22 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Let `c_2Elist_2ECONS` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0p \in (2^{A_27a}). (\forall V1e \in \\ & ((ty_2Elist_2Elist\ 2)^{A_27a}). (\forall V2l \in (ty_2Elist_2Elist\ 2). \\ & (((ap\ (ap\ (ap\ (c_2EDecode_2Eenc2dec\ A_27a)\ V0p)\ V1e)\ V2l) = \\ & (c_2Eoption_2ENONE\ (ty_2Epair_2Eprod\ A_27a\ (ty_2Elist_2Elist\ 2)))) \\ & \Leftrightarrow (\forall V3x \in A_27a. (\forall V4t \in (ty_2Elist_2Elist\ 2). \\ & ((p\ (ap\ V0p\ V3x)) \Rightarrow (\neg(V2l = (ap\ (ap\ (c_2Elist_2EAPPEND\ 2)\ (ap\ V1e\ V3x))\ V4t)))))))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0p \in (2^{A_27a}). (\forall V1e \in \\ & ((ty_2Elist_2Elist\ 2)^{A_27a}). (\forall V2l \in (ty_2Elist_2Elist\ 2). \\ & (\forall V3x \in A_27a. (\forall V4t \in (ty_2Elist_2Elist\ 2). \\ & ((p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder\ A_27a)\ V0p)\ V1e)) \Rightarrow (((ap \\ & (ap\ (ap\ (c_2EDecode_2Eenc2dec\ A_27a)\ V0p)\ V1e)\ V2l) = (ap\ (c_2Eoption_2ESOME \\ & (ty_2Epair_2Eprod\ A_27a\ (ty_2Elist_2Elist\ 2)))\ (ap\ (ap\ (c_2Epair_2E_2C \\ & A_27a\ (ty_2Elist_2Elist\ 2))\ V3x)\ V4t))) \Leftrightarrow ((p\ (ap\ V0p\ V3x)) \wedge (V2l = \\ & (ap\ (ap\ (c_2Elist_2EAPPEND\ 2)\ (ap\ V1e\ V3x))\ V4t)))))))))) \end{aligned} \quad (16)$$

Assume the following.

$$(\forall V0p \in (2^{ty_2Eone_2Eone}).(p (ap (ap (c_2EEncode_2Ewf_encoder ty_2Eone_2Eone) V0p) c_2EEncode_2Eencode_unit))) \quad (17)$$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (27)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. ((\text{ap } (\text{ap } (\text{ap } (\text{c_2Ebool_2ECOND } A_27a) \text{ c_2Ebool_2ET}) V0t1) \\ V1t2) = V0t1) \wedge ((\text{ap } (\text{ap } (\text{ap } (\text{c_2Ebool_2ECOND } A_27a) \text{ c_2Ebool_2EF}) \\ V0t1) V1t2) = V1t2)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow \\ (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow ((\forall V0l \in (\text{ty_2Elist_2Elist} \\ A_27a). ((\text{ap } (\text{ap } (\text{c_2Elist_2EAPPEND } A_27a) (\text{c_2Elist_2ENIL } A_27a)) \\ V0l) = V0l) \wedge (\forall V1l1 \in (\text{ty_2Elist_2Elist } A_27a). (\forall V2l2 \in \\ (\text{ty_2Elist_2Elist } A_27a). (\forall V3h \in A_27a. ((\text{ap } (\text{ap } (\text{c_2Elist_2EAPPEND} \\ A_27a) (\text{ap } (\text{ap } (\text{c_2Elist_2ECONS } A_27a) V3h) V1l1)) V2l2) = (\text{ap } (\text{ap} \\ (\text{c_2Elist_2ECONS } A_27a) V3h) (\text{ap } (\text{ap } (\text{c_2Elist_2EAPPEND } A_27a) \\ V1l1) V2l2)))))))))) \end{aligned} \quad (31)$$

Assume the following.

$$(\forall V0v \in \text{ty_2Eone_2Eone}. (V0v = \text{c_2Eone_2Eone})) \quad (32)$$

Theorem 1

$$\begin{aligned} (\forall V0p \in (\text{2ty_2Eone_2Eone}). (\forall V1l \in (\text{ty_2Elist_2Elist} \\ 2). ((\text{ap } (\text{ap } (\text{c_2EDecode_2Edecode_unit } V0p) V1l) = (\text{ap } (\text{ap } (\text{ap } (\\ \text{c_2Ebool_2ECOND } (\text{ty_2Eoption_2Eoption } (\text{ty_2Epair_2Eprod } \text{ty_2Eone_2Eone} \\ (\text{ty_2Elist_2Elist } 2)))) (\text{ap } V0p \text{ c_2Eone_2Eone})) (\text{ap } (\text{c_2Eoption_2ESOME} \\ (\text{ty_2Epair_2Eprod } \text{ty_2Eone_2Eone} (\text{ty_2Elist_2Elist } 2)))) (\text{ap} \\ (\text{ap } (\text{c_2Epair_2E_2C } \text{ty_2Eone_2Eone} (\text{ty_2Elist_2Elist } 2)) \text{ c_2Eone_2Eone}) \\ V1l))) (\text{c_2Eoption_2ENONE } (\text{ty_2Epair_2Eprod } \text{ty_2Eone_2Eone} \\ (\text{ty_2Elist_2Elist } 2)))))))))) \end{aligned}$$