

thm_2EDecode_2Eencode__then__decode__prod (TMUqZCPPzaUhcEuz13Vu9USWaSZxydJDsSi)

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Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 3 We define c_2Ebool_2E2T to be $(ap (ap (c_2Emin_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{3}$$

Definition 6 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS_sum$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \tag{4}$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \tag{5}$$

Definition 7 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap (c_2Eoption_2Eoption_ABS A_27a) (ty_2Elist_2Elist A_27a))$.
Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (6)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (7)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod A_27a A_27b) (x, y))$.

Definition 9 We define $c_2Ebool_2E_21$ to be $(ap (c_2Ebool_2E_21) (\lambda V0t \in 2.V0t))$.

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21))$.

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EAPPEND A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{(ty_2Elist_2Elist A_27a)}) \quad (9)$$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x) \text{ of type } \iota \Rightarrow \iota)$.

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40) P)))$.

Definition 13 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Emin_2E_40) t1 t2))))$.

Definition 14 We define $c_2EDecode_2Ewf_decoder$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1d \in ((ty_2Eoption_2Eoption_ABS A_27a) p)$.

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (10)$$

Definition 15 We define $c_2EDecode_2Edec2enc$ to be $\lambda A_27a : \iota.\lambda V0d \in ((ty_2Eoption_2Eoption_ABS (ty_2Epair_2Epair_ABS A_27a) (ty_2Elist_2Elist A_27a)) d)$.

Definition 16 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40) (ty_2Eone_2Eone)) (\lambda V0x \in ty_2Eone_2Eone.V0x)$.

Definition 17 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS_prod A_27a A_27b) (e, 0))$.

Definition 18 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap (c_2Eoption_2Eoption_ABS A_27a) (0))$.

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (11)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (12)$$

Definition 19 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})$

Definition 20 We define $c_2EDecode_2Enc2dec$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1e \in ((ty_2Elist_2Elist$

Let $c_2EEncode_2Eencode_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2EEncode_2Eencode_prod \\ A_27a\ A_27b \in (((ty_2Elist_2Elist\ 2)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})^{(ty_2Elist_2Elist\ 2)^{A_27b}})^{(ty_2Elist_2Elist} \end{aligned} \quad (13)$$

Definition 21 We define $c_2EDecode_2Edecode_prod$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (2^{(ty_2Epair_2Eprod$

Definition 22 We define $c_2EEncode_2Elift_prod$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p1 \in (2^{A_27a}).\lambda V1p2 \in (2$

Let $c_2Elist_2EisPREFIX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EisPREFIX\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (14)$$

Definition 23 We define $c_2EEncode_2Ewf_encoder$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1e \in ((ty_2Elist$

Definition 24 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Definition 25 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 26 We define $c_2Ecombin_2El$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a})\ A$

Definition 27 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0p \in (2^{A_27a}).(\forall V1e \in \\ ((ty_2Elist_2Elist\ 2)^{A_27a}).(\forall V2l \in (ty_2Elist_2Elist \\ 2).(\forall V3x \in A_27a.(\forall V4t \in (ty_2Elist_2Elist\ 2). \\ ((p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder\ A_27a)\ V0p)\ V1e)) \Rightarrow (((ap \\ (ap\ (ap\ (c_2EDecode_2Enc2dec\ A_27a)\ V0p)\ V1e)\ V2l) = (ap\ (c_2Eoption_2ESOME \\ (ty_2Epair_2Eprod\ A_27a\ (ty_2Elist_2Elist\ 2)))\ (ap\ (ap\ (c_2Epair_2E_2C \\ A_27a\ (ty_2Elist_2Elist\ 2))\ V3x)\ V4t))) \Leftrightarrow ((p\ (ap\ V0p\ V3x)) \wedge (V2l = \\ (ap\ (ap\ (c_2Elist_2EAPPEND\ 2)\ (ap\ V1e\ V3x))\ V4t))))))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0p \in (2^{A_27a}). (\forall V1e \in \\ & ((ty_2Elist_2Elist\ 2)^{A_27a}). ((p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder \\ & A_27a)\ V0p)\ V1e)) \Rightarrow (p\ (ap\ (ap\ (c_2EDecode_2Ewf_decoder\ A_27a) \\ & V0p)\ (ap\ (ap\ (c_2EDecode_2Eenc2dec\ A_27a)\ V0p)\ V1e)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0p \in (2^{A_27a}). (\forall V1d \in \\ & ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a\ (ty_2Elist_2Elist \\ & 2)))^{(ty_2Elist_2Elist\ 2)}). ((p\ (ap\ (ap\ (c_2EDecode_2Ewf_decoder \\ & A_27a)\ V0p)\ V1d)) \Rightarrow (p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder\ A_27a) \\ & V0p)\ (ap\ (c_2EDecode_2Edec2enc\ A_27a)\ V1d)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0p \in (2^{A_27a}). (\forall V1e \in \\ & ((ty_2Elist_2Elist\ 2)^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap \\ & (c_2EEncode_2Ewf_encoder\ A_27a)\ V0p)\ V1e)) \wedge (p\ (ap\ V0p\ V2x))) \Rightarrow \\ & ((ap\ (ap\ (c_2EDecode_2Edec2enc\ A_27a)\ (ap\ (ap\ (c_2EDecode_2Eenc2dec \\ & A_27a)\ V0p)\ V1e))\ V2x) = (ap\ V1e\ V2x)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0xb \in ((ty_2Elist_2Elist\ 2)^{A_27a}). (\forall V1yb \in ((\\ & ty_2Elist_2Elist\ 2)^{A_27b}). (\forall V2x \in A_27a. (\forall V3y \in \\ & A_27b. ((ap\ (ap\ (ap\ (c_2EEncode_2Eencode_prod\ A_27a\ A_27b)\ V0xb) \\ & V1yb)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2x)\ V3y)) = (ap\ (ap\ (c_2Elist_2EAPPEND \\ & 2)\ (ap\ V0xb\ V2x))\ (ap\ V1yb\ V3y)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0p1 \in (2^{A_27a}). (\forall V1p2 \in (2^{A_27b}). (\forall V2e1 \in \\ & ((ty_2Elist_2Elist\ 2)^{A_27a}). (\forall V3e2 \in ((ty_2Elist_2Elist \\ & 2)^{A_27b}). ((p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder\ A_27a)\ V0p1) \\ & V2e1)) \wedge (p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder\ A_27b)\ V1p2)\ V3e2))) \Rightarrow \\ & (p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder\ (ty_2Epair_2Eprod\ A_27a \\ & A_27b))\ (ap\ (ap\ (c_2EEncode_2Elift_prod\ A_27a\ A_27b)\ V0p1)\ V1p2)) \\ & (ap\ (ap\ (c_2EEncode_2Eencode_prod\ A_27a\ A_27b)\ V2e1)\ V3e2)))))) \end{aligned} \quad (20)$$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True)))) \end{aligned} \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee \neg(p V1B)))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge \neg(p V1B)))))) \quad (37)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (38)$$

Assume the following.

$$2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow 2.(((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((ap (c_2Ecombin_2El A_27a) V0x) = V0x)) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow ((\forall V0l1 \in (ty_2Elist_2Elist A_27a).(\forall V1l2 \in (ty_2Elist_2Elist A_27a).(\forall V2l3 \in (ty_2Elist_2Elist A_27a).(((ap (ap (c_2Elist_2EAPPEND A_27a) V0l1) V1l2) = (ap (ap (c_2Elist_2EAPPEND A_27a) V0l1) V2l3))) \Leftrightarrow (V1l2 = V2l3)))))) \wedge (\forall V3l1 \in (ty_2Elist_2Elist A_27a).(\forall V4l2 \in (ty_2Elist_2Elist A_27a).(((ap (ap (c_2Elist_2EAPPEND A_27a) V4l2) V3l1) = (ap (ap (c_2Elist_2EAPPEND A_27a) V5l3) V3l1)) \Leftrightarrow (V4l2 = V5l3)))))) \quad (41)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b). (\exists V1q \in A_27a. \\ & (\exists V2r \in A_27b. (V0x = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b) \\ & V1q)\ V2r)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (c_2Epair_2EFST\ A_27a \\ & A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (c_2Epair_2ESND\ A_27a \\ & A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (44)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (48)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (49)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & p\ V2r)) \vee (\neg(p\ V1q)))))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{54}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0p1 \in (2^{A_27a}). (\forall V1p2 \in (2^{A_27b}). (\forall V2e1 \in \\
& ((ty_2Elist_2Elist\ 2)^{A_27a}). (\forall V3e2 \in ((ty_2Elist_2Elist \\
& 2)^{A_27b}). (\forall V4l \in (ty_2Epair_2Eprod\ A_27a\ A_27b). (\forall V5t \in \\
& (ty_2Elist_2Elist\ 2). (((p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder \\
& A_27a)\ V0p1)\ V2e1)) \wedge ((p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder\ A_27b) \\
& V1p2)\ V3e2)) \wedge (p\ (ap\ (ap\ (ap\ (c_2EEncode_2Elift_prod\ A_27a\ A_27b) \\
& V0p1)\ V1p2)\ V4l)))))) \Rightarrow ((ap\ (ap\ (ap\ (ap\ (c_2EDecode_2Edecode_prod \\
& A_27a\ A_27b)\ (ap\ (ap\ (c_2EEncode_2Elift_prod\ A_27a\ A_27b)\ V0p1) \\
& V1p2))\ (ap\ (ap\ (c_2EDecode_2Eenc2dec\ A_27a)\ V0p1)\ V2e1))\ (ap\ (ap \\
& (c_2EDecode_2Eenc2dec\ A_27b)\ V1p2)\ V3e2))\ (ap\ (ap\ (c_2Elist_2EAPPEND \\
& 2)\ (ap\ (ap\ (ap\ (c_2EEncode_2Eencode_prod\ A_27a\ A_27b)\ V2e1)\ V3e2) \\
& V4l))\ V5t)) = (ap\ (c_2Eoption_2ESOME\ (ty_2Epair_2Eprod\ (ty_2Epair_2Eprod \\
& A_27a\ A_27b)\ (ty_2Elist_2Elist\ 2)))\ (ap\ (ap\ (c_2Epair_2E_2C\ (\\
& ty_2Epair_2Eprod\ A_27a\ A_27b)\ (ty_2Elist_2Elist\ 2))\ V4l)\ V5t))))))
\end{aligned}$$