thm_2EDecode_2Ewf__decode__bnum (TMKQ5QBNQTDgPkyQ9bV4DfTrov7uawD2S5V)

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Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{1}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj__o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj__o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^{A_27a}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V2t \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2EABS__prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS__prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})}) \tag{3}$$

Definition 6 *We define* c_2Epair_2E_2C *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{4}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{5}$$

Let $c_2Esum_2EABS__sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS__sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{(((2^{A_27b})^{A_27a})^{2})}) \tag{6}$$

Definition 7 *We define* c_2Esum_2EINL *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \tag{7}$$

Let $c_2Eoption_2Eoption__ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption__ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \tag{8}$$

Definition 8 *We define* c_2Eoption_2ESOME *to be* $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption__A$

Definition 9 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c_2Emin_2E_3D_3D_3E}\ V0t)\ \mathsf{c_2Ebool_2E}$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \tag{9}$$

Definition 11 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 12 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (\mathsf{c_2Emin_2E_40}$

Definition 13 *We define* c_2Ebool_2ECOND *to be* $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 14 *We define* c_2EDecode_2Ewf__decoder *to be* $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1d \in ((ty_2Eopti$

Definition 15 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c_2Emin_2E_40}\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2$

Definition 16 *We define* c_2Esum_2EINR *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS$

Definition 17 *We define* c_2Eoption_2ENONE *to be* $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption__ABS\ A_27a)\ ($

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \tag{10}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \tag{11}$$

Definition 18 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27}$

Definition 19 *We define* c_2EDecode_2Eenc2dec *to be* $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1e \in ((ty_2Elist_2El$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{12}$$

Let $c_2EEncode_2Eencode__bnum : \iota$ be given. Assume the following.

$$c_2EEncode_2Eencode__bnum \in (((ty_2Elist_2Elist\ 2)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{13}$$

Definition 20 *We define* c_2EDecode_2Edecode__bnum *to be* $\lambda V0m \in ty_2Enum_2Enum.\lambda V1p \in (2^{ty_2En}$

Let $c_2Elist_2EisPREFIX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EisPREFIX\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27} \tag{14}$$

Definition 21 *We define* c_2EEncode_2Ewf__encoder *to be* $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1e \in ((ty_2Elist_$

Let $c_2Enum_2EZERO__REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO__REP \in omega \tag{15}$$

Let $c_2Enum_2EABS__num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS__num \in (ty_2Enum_2Enum^{omega}) \tag{16}$$

Definition 22 *We define* c_2Enum_2E0 *to be* $(ap\ c_2Enum_2EABS__num\ c_2Enum_2EZERO__REP)$.

Definition 23 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c_2Enum_2EREP__num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP__num \in (omega^{ty_2Enum_2Enum}) \tag{17}$$

Let $c_2Enum_2ESUC__REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC__REP \in (omega^{omega}) \tag{18}$$

Definition 24 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS__num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{19}$$

Definition 25 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 26 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{20}$$

Definition 27 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enu$

Definition 28 *We define* c_2EEncode_2Ewf__pred *to be* $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).(ap\ (\mathsf{c_2Ebool_2E_3F}\ A_$

Definition 29 *We define* c_2EEncode_2Ewf__pred__bnum *to be* $\lambda V0m \in ty_2Enum_2Enum.\lambda V1p \in (2^{ty_2E}$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0p \in (2^{A_27a}).(\forall V1e \in ((ty_2Elist_2Elist\ 2)^{A_27a}).((p\ (ap\ (ap\ (\mathsf{c_2EEncode_2Ewf__encoder}\ A_27a)\ V0p)\ V1e)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c_2EDecode_2Ewf__decoder}\ A_27a)\ V0p)\ (ap\ (ap\ (\mathsf{c_2EDecode_2Eenc2dec}\ A_27a)\ V0p)\ V1e)))))) \tag{21}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1p \in (2^{ty_2Enum_2Enum}).((p\ (ap\ (ap\ \mathsf{c_2EEncode_2Ewf__pred__bnum}\ V0m)\ V1p)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c_2EEncode_2Ewf__encoder}\ ty_2Enum_2Enum)\ V1p)\ (ap\ c_2EEncode_2Eencode__bnum\ V0m)))))) \tag{22}$$

Assume the following.

$$True \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{24}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{26}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{27}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27))))))) \tag{28}$$

Theorem 1

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1p \in (2^{ty_2Enum_2Enum}).$$
$$((p\ (ap\ (ap\ \mathsf{c_2EEncode_2Ewf__pred__bnum}\ V0m)\ V1p))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c_2EDecode_2Ewf__decoder}\ ty_2Enum_2Enum)\ V1p)\ (ap\ (ap\ \mathsf{c_2EDecode_2Edecode__bnum}\ V0m)\ V1p))))))$$