thm_2EDeepSyntax_2Ein__aset (TMWkhLkh1nvzNL5sgkVS8btcQJCZvhdDsTo)

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Let $ty\_2EDeepSyntax\_2Edeep\_\_form : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EDeepSyntax\_2Edeep\_\_form \tag{1}$$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{2}$$

Let $c\_2EDeepSyntax\_2ExDivided : \iota$ be given. Assume the following.

$$c\_2EDeepSyntax\_2ExDivided \in ((ty\_2EDeepSyntax\_2Edeep\_\_form^{ty\_2Einteger\_2Eint})^{ty\_2Einteger\_2Eint}) \tag{3}$$

Let $c\_2EDeepSyntax\_2ExEQ : \iota$ be given. Assume the following.

$$c\_2EDeepSyntax\_2ExEQ \in (ty\_2EDeepSyntax\_2Edeep\_\_form^{ty\_2Einteger\_2Eint}) \tag{4}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{5}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{6}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{7}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (8)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (9)$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{2}))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

Definition 7 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (11)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (12)$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \quad (13)$$

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (t$

Let $c\_2Einteger\_2Etint\_\_add : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ t} \quad (14)$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \quad (15)$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}} \quad (16)$$

Definition 11 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_$

Definition 12 *We define* c_2Einteger_2Eint__add *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Intege$

Let $c\_2EDeepSyntax\_2ELTx : \iota$ be given. Assume the following.

$$c\_2EDeepSyntax\_2ELTx \in (ty\_2EDeepSyntax\_2Edeep\_\_form^{ty\_2Einteger\_2Eint}) \quad (17)$$

Let $c\_2EDeepSyntax\_2ExLT : \iota$ be given. Assume the following.

$$c\_2EDeepSyntax\_2ExLT \in (ty\_2EDeepSyntax\_2Edeep\_\_form^{ty\_2Einteger\_2Eint}) \quad (18)$$

Let $c\_2EDeepSyntax\_2EUnrelatedBool : \iota$ be given. Assume the following.

$$c\_2EDeepSyntax\_2EUnrelatedBool \in (ty\_2EDeepSyntax\_2Edeep\_\_form^{2}) \quad (19)$$

Let $c\_2EDeepSyntax\_2ENegn : \iota$ be given. Assume the following.

$$c\_2EDeepSyntax\_2ENegn \in (ty\_2EDeepSyntax\_2Edeep\_\_form^{ty\_2EDeepSyntax\_2Edeep\_\_form}) \quad (20)$$

Let $c\_2EDeepSyntax\_2EDisjn : \iota$ be given. Assume the following.

$$c\_2EDeepSyntax\_2EDisjn \in ((ty\_2EDeepSyntax\_2Edeep\_\_form^{ty\_2EDeepSyntax\_2Edeep\_\_form})^{ty\_2EDeepS} \quad (21)$$

Let $c\_2EDeepSyntax\_2EConjn : \iota$ be given. Assume the following.

$$c\_2EDeepSyntax\_2EConjn \in ((ty\_2EDeepSyntax\_2Edeep\_\_form^{ty\_2EDeepSyntax\_2Edeep\_\_form})^{ty\_2EDeep} \quad (22)$$

Let $c\_2EDeepSyntax\_2EAset : \iota$ be given. Assume the following.

$$c\_2EDeepSyntax\_2EAset \in (((2^{ty\_2Einteger\_2Eint})^{ty\_2EDeepSyntax\_2Edeep\_\_form})^{2}) \quad (23)$$

Definition 13 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 14 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 15 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 16 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 17 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 18 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 19 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{24}$$

Definition 20 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{25}$$

Definition 21 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

Definition 22 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 23 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 24 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Assume the following.

$$((\forall V0pos \in 2.(\forall V1f1 \in ty\_2EDeepSyntax\_2Edeep\_\_form.(\forall V2f2 \in ty\_2EDeepSyntax\_2Edeep\_\_form.((ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V0pos)\ (ap\ (ap\ c\_2EDeepSyntax\_2EConjn\ V1f1)\ V2f2)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ ty\_2Einteger\_2Eint)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V0pos)\ V1f1))\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V0pos)\ V2f2))))))\wedge((\forall V3pos \in 2.(\forall V4f1 \in ty\_2EDeepSyntax\_2Edeep\_\_form.(\forall V5f2 \in ty\_2EDeepSyntax\_2Edeep\_\_form.((ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V3pos)\ (ap\ (ap\ c\_2EDeepSyntax\_2EDisjn\ V4f1)\ V5f2)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ ty\_2Einteger\_2Eint)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V3pos)\ V4f1))\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V3pos)\ V5f2))))))\wedge((\forall V6pos \in 2.(\forall V7f \in ty\_2EDeepSyntax\_2Edeep\_\_form.((ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V6pos)\ (ap\ c\_2EDeepSyntax\_2ENegn\ V7f)) = (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ (ap\ \mathsf{c\_2Ebool\_2E\_7E}\ V6pos))\ V7f))))\wedge((\forall V8pos \in 2.(\forall V9b \in 2.((ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V8pos)\ (ap\ c\_2EDeepSyntax\_2EUnrelatedBool\ V9b)) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Einteger\_2Eint))))\wedge((\forall V10pos \in 2.(\forall V11i \in ty\_2Einteger\_2Eint.((ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V10pos)\ (ap\ c\_2EDeepSyntax\_2ExLT\ V11i)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (2^{ty\_2Einteger\_2Eint}))\ V10pos)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Einteger\_2Eint)\ V11i)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Einteger\_2Eint)))\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Einteger\_2Eint)))))\wedge((\forall V12pos \in 2.(\forall V13i \in ty\_2Einteger\_2Eint.((ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V12pos)\ (ap\ c\_2EDeepSyntax\_2ELTx\ V13i)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (2^{ty\_2Einteger\_2Eint}))\ V12pos)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Einteger\_2Eint))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Einteger\_2Eint)\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V13i)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Einteger\_2Eint))))))\wedge((\forall V14pos \in 2.(\forall V15i \in ty\_2Einteger\_2Eint.((ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V14pos)\ (ap\ c\_2EDeepSyntax\_2ExEQ\ V15i)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (2^{ty\_2Einteger\_2Eint}))\ V14pos)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Einteger\_2Eint)\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V15i)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Einteger\_2Eint)))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Einteger\_2Eint)\ V15i)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Einteger\_2Eint))))))\wedge(\forall V16pos \in 2.(\forall V17i1 \in ty\_2Einteger\_2Eint.(\forall V18i2 \in ty\_2Einteger\_2Eint.((ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V16pos)\ (ap\ (ap\ c\_2EDeepSyntax\_2ExDivided\ V17i1)\ V18i2)) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Einteger\_2Eint)))))))))))) \tag{26}$$

Assume the following.

$$True \tag{27}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg (p\ V0t)))) \tag{31}$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{32}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{34}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{35}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg (\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg (p\ (ap\ V0P\ V2x)))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\exists V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \vee (\exists V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.((\exists V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \wedge (p\ V1Q))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\exists V2x \in A\_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \wedge (\exists V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{44}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg ((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg (p\ V0A)) \vee (\neg (p\ V1B)))) \wedge ((\neg ((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg (p\ V0A)) \wedge (\neg (p\ V1B))))))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a))))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1y)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))) \Leftrightarrow (V0x = V1y)))) \tag{49}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{50}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{51}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{52}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{53}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{54}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \tag{55}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{56}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p))))))))))) \tag{57}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p))))))))))) \tag{58}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{59}$$

Theorem 1

$$(\forall V0pos \in 2.(\forall V1f1 \in ty\_2EDeepSyntax\_2Edeep\_\_form.(\forall V2f2 \in ty\_2EDeepSyntax\_2Edeep\_\_form.(\forall V3P \in (2^{ty\_2Einteger\_2Eint}).(\forall V4f \in ty\_2EDeepSyntax\_2Edeep\_\_form.(\forall V5a0 \in 2.(\forall V6i \in ty\_2Einteger\_2Eint.(\forall V7i1 \in ty\_2Einteger\_2Eint.(\forall V8i2 \in ty\_2Einteger\_2Eint.(((\exists V9a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V9a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V0pos)\ (ap\ (ap\ c\_2EDeepSyntax\_2EConjn\ V1f1)\ V2f2))))\wedge(p\ (ap\ V3P\ V9a))))\Leftrightarrow((\exists V10a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V10a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V0pos)\ V1f1)))\wedge(p\ (ap\ V3P\ V10a))))\vee(\exists V11a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V11a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V0pos)\ V2f2)))\wedge(p\ (ap\ V3P\ V11a))))))\wedge(((\exists V12a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V12a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V0pos)\ (ap\ (ap\ c\_2EDeepSyntax\_2EDisjn\ V1f1)\ V2f2))))\wedge(p\ (ap\ V3P\ V12a))))\Leftrightarrow((\exists V13a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V13a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V0pos)\ V1f1)))\wedge(p\ (ap\ V3P\ V13a))))\vee(\exists V14a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V14a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V0pos)\ V2f2)))\wedge(p\ (ap\ V3P\ V14a))))))\wedge(((\exists V15a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V15a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ \mathsf{c\_2Ebool\_2ET})\ (ap\ c\_2EDeepSyntax\_2ENegn\ V4f))))\wedge(p\ (ap\ V3P\ V15a))))\Leftrightarrow(\exists V16a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V16a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ \mathsf{c\_2Ebool\_2EF})\ V4f)))\wedge(p\ (ap\ V3P\ V16a)))))\wedge(((\exists V17a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V17a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ \mathsf{c\_2Ebool\_2EF})\ (ap\ c\_2EDeepSyntax\_2ENegn\ V4f))))\wedge(p\ (ap\ V3P\ V17a))))\Leftrightarrow(\exists V18a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V18a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ \mathsf{c\_2Ebool\_2ET})\ V4f)))\wedge(p\ (ap\ V3P\ V18a)))))\wedge(((\exists V19a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V19a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V0pos)\ (ap\ c\_2EDeepSyntax\_2EUnrelatedBool\ V5a0))))\wedge(p\ (ap\ V3P\ V19a))))\Leftrightarrow False)\wedge(((\exists V20a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V20a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ \mathsf{c\_2Ebool\_2ET})\ (ap\ c\_2EDeepSyntax\_2ExLT\ V6i))))\wedge(p\ (ap\ V3P\ V20a))))\Leftrightarrow(p\ (ap\ V3P\ V6i)))\wedge(((\exists V21a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V21a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ \mathsf{c\_2Ebool\_2EF})\ (ap\ c\_2EDeepSyntax\_2ExLT\ V6i))))\wedge(p\ (ap\ V3P\ V21a))))\Leftrightarrow False)\wedge(((\exists V22a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V22a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ \mathsf{c\_2Ebool\_2ET})\ (ap\ c\_2EDeepSyntax\_2ELTx\ V6i))))\wedge(p\ (ap\ V3P\ V22a))))\Leftrightarrow False)\wedge(((\exists V23a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V23a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ \mathsf{c\_2Ebool\_2EF})\ (ap\ c\_2EDeepSyntax\_2ELTx\ V6i))))\wedge(p\ (ap\ V3P\ V23a))))\Leftrightarrow(p\ (ap\ V3P\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V6i)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))))\wedge(((\exists V24a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V24a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ \mathsf{c\_2Ebool\_2ET})\ (ap\ c\_2EDeepSyntax\_2ExEQ\ V6i))))\wedge(p\ (ap\ V3P\ V24a))))\Leftrightarrow(p\ (ap\ V3P\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V6i)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))))\wedge(((\exists V25a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V25a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ \mathsf{c\_2Ebool\_2EF})\ (ap\ c\_2EDeepSyntax\_2ExEQ\ V6i))))\wedge(p\ (ap\ V3P\ V25a))))\Leftrightarrow(p\ (ap\ V3P\ V6i)))\wedge((\exists V26a \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Einteger\_2Eint)\ V26a)\ (ap\ (ap\ c\_2EDeepSyntax\_2EAset\ V0pos)\ (ap\ (ap\ c\_2EDeepSyntax\_2ExDivided\ V7i1)\ V8i2))))\wedge(p\ (ap\ V3P\ V26a))))\Leftrightarrow False))))))))))))))))))))))$$