

thm_2EDeepSyntax_2Eneginf_exoriginal_eq_rhs (TMYEJrNVrtAqq4cLzTjcNiHo5YAZuPacMk7)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \tag{3}$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})_{ty_2Einteger_2Eint}) \tag{4}$$

Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p\ (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p\ x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_27$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a})))$

Definition 5 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E_40\ (ty_2Einteger_2Eint_REP_CLASS\ a)))$

Let $c_2Einteger_2Eint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)_{ty_2Einteger_2Eint_add})_{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})_{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)} \tag{5}$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (6)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})} \quad (7)$$

Definition 6 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 7 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Let $ty_2EDeepSyntax_2Edeep_form : \iota$ be given. Assume the following.

$$nonempty\ ty_2EDeepSyntax_2Edeep_form \quad (8)$$

Let $c_2EDeepSyntax_2EBset : \iota$ be given. Assume the following.

$$c_2EDeepSyntax_2EBset \in (((2^{ty_2Einteger_2Eint})^{ty_2EDeepSyntax_2Edeep_form})^2) \quad (9)$$

Definition 8 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Let $c_2EDeepSyntax_2Eneginf : \iota$ be given. Assume the following.

$$c_2EDeepSyntax_2Eneginf \in (ty_2EDeepSyntax_2Edeep_form)^{ty_2EDeepSyntax_2Edeep_form} \quad (10)$$

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (11)$$

Definition 9 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Definition 10 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 11 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 12 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E7E))$

Definition 13 We define $c_2Einteger_2Eint_le$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint$

Let $c_2EDeepSyntax_2EEval_form : \iota$ be given. Assume the following.

$$c_2EDeepSyntax_2EEval_form \in ((2^{ty_2Einteger_2Eint})^{ty_2EDeepSyntax_2Edeep_form}) \quad (12)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (13)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (14)$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (15)$$

Let $c_2EDeepSyntax_2Ealldivide : \iota$ be given. Assume the following.

$$c_2EDeepSyntax_2Ealldivide \in ((2^{ty_2Einteger_2Eint})^{ty_2EDeepSyntax_2Edeep_form}) \quad (16)$$

Definition 15 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 16 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x)$

Definition 17 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 18 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Assume the following.

$$\begin{aligned} & (\forall V0f \in ty_2EDeepSyntax_2Edeep_form. (\forall V1d \in ty_2Einteger_2Eint. \\ & (\forall V2i \in ty_2Einteger_2Eint. ((p\ (ap\ (ap\ c_2EDeepSyntax_2Ealldivide \\ & V0f)\ V1d)) \Rightarrow ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_of_num \\ & c_2Enum_2E0))\ V2i)) \wedge ((p\ (ap\ (ap\ c_2Einteger_2Eint_le\ V2i)\ V1d)) \wedge \\ & (p\ (ap\ (ap\ c_2EDeepSyntax_2Eeval_form\ (ap\ c_2EDeepSyntax_2Eneginf \\ & V0f))\ V2i)))) \Rightarrow (\exists V3x \in ty_2Einteger_2Eint. (p\ (ap\ (ap\ c_2EDeepSyntax_2Eeval_form \\ & V0f)\ V3x)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0f \in ty_2EDeepSyntax_2Edeep_form. (\forall V1d \in ty_2Einteger_2Eint. \\ & (\forall V2x \in ty_2Einteger_2Eint. ((p\ (ap\ (ap\ c_2EDeepSyntax_2Ealldivide \\ & V0f)\ V1d)) \wedge (p\ (ap\ (ap\ c_2Einteger_2Eint_lt\ (ap\ c_2Einteger_2Eint_of_num \\ & c_2Enum_2E0))\ V1d)) \Rightarrow ((p\ (ap\ (ap\ c_2EDeepSyntax_2Eeval_form \\ & V0f)\ V2x)) \Rightarrow ((\exists V3i \in ty_2Einteger_2Eint. ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt \\ & (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))\ V3i)) \wedge ((p\ (ap\ (\\ & ap\ c_2Einteger_2Eint_le\ V3i)\ V1d)) \wedge (p\ (ap\ (ap\ c_2EDeepSyntax_2Eeval_form \\ & (ap\ c_2EDeepSyntax_2Eneginf\ V0f))\ V3i)))))) \vee (\exists V4j \in ty_2Einteger_2Eint. \\ & (\exists V5b \in ty_2Einteger_2Eint. ((p\ (ap\ (ap\ c_2Einteger_2Eint_lt \\ & (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))\ V4j)) \wedge ((p\ (ap\ (\\ & ap\ c_2Einteger_2Eint_le\ V4j)\ V1d)) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & ty_2Einteger_2Eint)\ V5b)\ (ap\ (ap\ c_2EDeepSyntax_2EBset\ c_2Ebool_2ET) \\ & V0f)))) \wedge (p\ (ap\ (ap\ c_2EDeepSyntax_2Eeval_form\ V0f)\ (ap\ (ap\ c_2Einteger_2Eint_add \\ & V5b)\ V4j)))))))))) \end{aligned} \quad (18)$$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (23)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (24)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (26)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (27)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p V0P) \vee (\exists V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A.27a.((p V0P) \vee (p (ap V1Q V3x)))))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (31)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0P \in ((2^{A.27b})^{A.27a}).((\forall V1x \in A.27a.(\exists V2y \in A.27b.(p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}).(\forall V4x \in A.27a.(p (ap (ap V0P V4x) (ap V3f V4x))))))) \quad (32)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27b.((ap (ap (c.2Ecombin_2EK A.27a A.27b) V0x) V1y) = V0x))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r))) \vee (\neg(p V1q)))))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p))))))))) \quad (39)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{43}$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in ty_2EDeepSyntax_2Edeep_form. (\forall V1d \in ty_2Einteger_2Eint. \\
& (((p (ap (ap c_2EDeepSyntax_2Ealldivide V0f) V1d)) \wedge (p (ap (ap c_2Einteger_2Eint_lt \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V1d))) \Rightarrow ((\exists V2x \in \\
& ty_2Einteger_2Eint. (p (ap (ap c_2EDeepSyntax_2EEval_form V0f) \\
& V2x))) \Leftrightarrow ((\exists V3i \in ty_2Einteger_2Eint. ((p (ap (ap (c_2Ecombin_2EK \\
& 2 ty_2Einteger_2Eint) (ap (ap c_2Ebool_2E_2F_5C (ap (ap c_2Einteger_2Eint_lt \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V3i)) (ap (ap c_2Einteger_2Eint_le \\
& V3i) V1d))) V3i)) \wedge (p (ap (ap c_2EDeepSyntax_2EEval_form (ap c_2EDeepSyntax_2Eneginf \\
& V0f)) V3i)))) \vee (\exists V4b \in ty_2Einteger_2Eint. (\exists V5j \in \\
& ty_2Einteger_2Eint. (((p (ap (ap (c_2Ebool_2EIN ty_2Einteger_2Eint) \\
& V4b) (ap (ap c_2EDeepSyntax_2EBset c_2Ebool_2ET) V0f))) \wedge (p (ap \\
& (ap (c_2Ecombin_2EK 2 ty_2Einteger_2Eint) (ap (ap c_2Ebool_2E_2F_5C \\
& (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) V5j)) (ap (ap c_2Einteger_2Eint_le V5j) V1d))) \\
& V5j))) \wedge (p (ap (ap c_2EDeepSyntax_2EEval_form V0f) (ap (ap c_2Einteger_2Eint_add \\
& V4b) V5j))))))))))
\end{aligned}$$