

thm_2EEncodeVar_2Efixed__width__univ
(TMUty56z9HPRqvwGvXgy6823LTuT6Ap2jyK)

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Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (1)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (2)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (3)$$

Let $c_2ECoder_2Edecoder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2ECoder_2Edecoder\ A_27a \in ((A_27a^{(ty_2Elist_2Elist\ 2)})^{(ty_2Epair_2Eprod\ (2^{A_27a}))})^{(ty_2Epair_2Eprod\ (2^{A_27a}))}) \quad (4)$$

Let $c_2ECoder_2Ewf_coder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2ECoder_2Ewf_coder\ A_27a \in (2^{(ty_2Epair_2Eprod\ (2^{A_27a}))})^{(ty_2Epair_2Eprod\ ((ty_2Elist_2Elist\ 2)^{A_27a}))} ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a\ A_27a))^{(ty_2Elist_2Elist\ 2)^{A_27a}})} \quad (5)$$

Let $c_2ECoder_2Eencoder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2ECoder_2Eencoder\ A_27a \in (2^{(ty_2Elist_2Elist\ 2)^{A_27a}})^{(ty_2Epair_2Eprod\ (2^{A_27a}))} ((ty_2Epair_2Eprod\ ((ty_2Elist_2Elist\ 2)^{A_27a}))^{(ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a\ A_27a))}) \quad (6)$$

Let $c_2ECoder_2Edomain : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2ECoder_2Edomain\ A_27a \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ (2^{A_27a}))})^{(ty_2Epair_2Eprod\ (2^{A_27a}))} ((ty_2Epair_2Eprod\ ((ty_2Elist_2Elist\ 2)^{A_27a}))^{(ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a\ A_27a))}) \quad (7)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (8)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (9)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 3 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))))$

Definition 5 We define $c_2EEncodeVar_2Efixed_width$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.\lambda V1c \in 2$

Let $c_2EEncodeVar_2Eof_length : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2EEncodeVar_2Eof_length\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 6 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 7 We define $c_2Ebool_2ERES_FORALL$ to be $\lambda A_27a : \iota.(\lambda V0p \in (2^{A_27a}).(\lambda V1m \in (2^{A_27a}).(ap\ V1m\ V0p)))$

Definition 8 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Definition 10 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Definition 11 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21))$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0c \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ ((ty_2Elist_2Elist\ 2)^{A_27a})\ ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a\ (ty_2Elist_2Elist\ 2))\ (ty_2Elist_2Elist\ 2))))).((p\ (ap\ (c_2ECoder_2Ewf_coder\ A_27a)\ V0c)) \Rightarrow (\forall V1x \in A_27a.((p\ (ap\ (ap\ (c_2ECoder_2Edecoder\ A_27a)\ V0c)\ (ap\ (ap\ (c_2ECoder_2Edecoder\ A_27a)\ V0c)\ V1x)) \Rightarrow ((ap\ (ap\ (c_2ECoder_2Edecoder\ A_27a)\ V0c)\ (ap\ (ap\ (c_2ECoder_2Eencoder\ A_27a)\ V0c)\ V1x)) = V1x)))))) \quad (11) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0c \in (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ ((ty_2Elist_2Elist\ 2)^{A_27a})\ ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a\ (ty_2Elist_2Elist\ 2))\ (ty_2Elist_2Elist\ 2))))).((p\ (ap\ (c_2ECoder_2Ewf_coder\ A_27a)\ V0c)) \Rightarrow (\forall V1l \in (ty_2Elist_2Elist\ 2).(p\ (ap\ (ap\ (c_2ECoder_2Edecoder\ A_27a)\ V0c)\ (ap\ (ap\ (c_2ECoder_2Edecoder\ A_27a)\ V0c)\ V1l)))))) \quad (12) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ & A_27a).(\forall V1n \in ty_2Enum_2Enum.((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & (ty_2Elist_2Elist\ A_27a))\ V0l)\ (ap\ (c_2EENcodeVar_2Eof_length \\ & A_27a)\ V1n))) \Leftrightarrow ((ap\ (c_2Elist_2ELENGTH\ A_27a)\ V0l) = V1n)))) \end{aligned} \quad (13)$$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ & A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p\ V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ & 2^{A_27a}).((\forall V2x \in A_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p \\ & V0P) \vee (\forall V3x \in A_27a.(p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (22)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B))))))) \quad (25)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (26)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (27)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}).(\forall V1f \in (2^{A_{27a}}).((p (ap (ap (c.2Ebool_2ERES_FORALL A_{27a}) V0P) V1f)) \Leftrightarrow (\forall V2x \in A_{27a}.((p (ap (ap (c.2Ebool_2EIN A_{27a}) V2x) V0P)) \Rightarrow (p (ap V1f V2x))))))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (37)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (42)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0phi \in (2^{A_27a}). (\forall V1c \in \\ & (ty_2Epair_2Eprod\ (2^{A_27a})\ (ty_2Epair_2Eprod\ ((ty_2Elist_2Elist \\ & 2)^{A_27a})\ ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_27a\ (ty_2Elist_2Elist \\ & 2))^{(ty_2Elist_2Elist\ 2)}))))). (\forall V2n \in ty_2Enum_2Enum. \\ & (((p\ (ap\ (c_2ECoder_2Ewf_coder\ A_27a)\ V1c)) \wedge (p\ (ap\ (ap\ (c_2EEncodeVar_2Efixed_width \\ & A_27a)\ V2n)\ V1c))) \Rightarrow ((\forall V3x \in A_27a. ((p\ (ap\ (ap\ (c_2ECoder_2Edomain \\ & A_27a)\ V1c)\ V3x)) \Rightarrow (p\ (ap\ V0phi\ V3x)))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\ & (ty_2Elist_2Elist\ 2))\ (ap\ (c_2EEncodeVar_2Eof_length\ 2)\ V2n)) \\ & (\lambda V4w \in (ty_2Elist_2Elist\ 2). (ap\ V0phi\ (ap\ (ap\ (c_2ECoder_2Edecoder \\ & A_27a)\ V1c)\ V4w)))))))))) \end{aligned}$$