

thm\_2EEncode\_2Eencode\_\_blist\_\_def\_\_compute  
(TMc1h3miNG8C7ryDwBJqWMwSvPEbuEB6LfE)

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Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (1)$$

Let  $c\_2Elist\_2ETL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ETL\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (2)$$

Let  $c\_2Elist\_2EHD : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EHD\ A\_27a \in (A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)} \quad (3)$$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (4)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (5)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (6)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (7)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \quad (8)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2EEncode\_2Eencode\_blist : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2EEncode\_2Eencode\_blist\ A\_27a\ A\_27b \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27b)})^{(ty\_2Elist\_2Elist\ A\_27a)^{A\_27b}})^{ty\_2Enum\_2E0} \quad (9)$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (10)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (11)$$

**Definition 3** We define  $c\_2Ebool\_2E2$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a})))$

**Definition 5** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (ap\ c\_2Enum\_2ESUC\_REP\ m))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (12)$$

**Definition 6** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ n))$

**Definition 7** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (13)$$

**Definition 8** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2D\ n))$

**Definition 9** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 10** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ t1)\ t2))$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad (\forall V0e \in ((ty\_2Elist\_2Elist\ A\_27a)^{A\_27b}). (\forall V1l \in \\
& \quad (ty\_2Elist\_2Elist\ A\_27b). ((ap\ (ap\ (ap\ (c\_2EEncode\_2Eencode\_blist \\
& \quad A\_27a\ A\_27b)\ c\_2Enum\_2E0)\ V0e)\ V1l) = (c\_2Elist\_2ENIL\ A\_27a)))) \wedge \\
& \quad (\forall V2m \in ty\_2Enum\_2Enum. (\forall V3e \in ((ty\_2Elist\_2Elist \\
& \quad A\_27a)^{A\_27b}). (\forall V4l \in (ty\_2Elist\_2Elist\ A\_27b). ((ap\ (ap \\
& \quad (ap\ (c\_2EEncode\_2Eencode\_blist\ A\_27a\ A\_27b)\ (ap\ c\_2Enum\_2ESUC \\
& \quad V2m))\ V3e)\ V4l) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ V3e)\ (ap\ (c\_2Elist\_2EHD \\
& \quad A\_27b)\ V4l)))\ (ap\ (ap\ (ap\ (c\_2EEncode\_2Eencode\_blist\ A\_27a\ A\_27b) \\
& \quad V2m)\ V3e)\ (ap\ (c\_2Elist\_2ETL\ A\_27b)\ V4l))))))))) \\
& \hspace{15em} (14)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in ((A\_27a^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}). \\
& \quad (\forall V1g \in (A\_27a^{ty\_2Enum\_2Enum}). ((\forall V2n \in ty\_2Enum\_2Enum. \\
& \quad ((ap\ V1g\ (ap\ c\_2Enum\_2ESUC\ V2n)) = (ap\ (ap\ V0f\ V2n)\ (ap\ c\_2Enum\_2ESUC \\
& \quad V2n)))) \Leftrightarrow ((\forall V3n \in ty\_2Enum\_2Enum. ((ap\ V1g\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap\ c\_2Earithmetic\_2EBIT1\ V3n))) = (ap\ (ap\ V0f\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D \\
& \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ V3n))) \\
& \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))))) \\
& \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ V3n)))))) \wedge \\
& \quad (\forall V4n \in ty\_2Enum\_2Enum. ((ap\ V1g\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap\ c\_2Earithmetic\_2EBIT2\ V4n))) = (ap\ (ap\ V0f\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap\ c\_2Earithmetic\_2EBIT1\ V4n)))\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap\ c\_2Earithmetic\_2EBIT2\ V4n))))))))) \\
& \hspace{15em} (15)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow ((p \\
& \quad V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \\
& \hspace{15em} (16)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad (\forall V0e \in ((ty\_2Elist\_2Elist\ A\_27a)^{A\_27b}).(\forall V1l \in \\
& \quad (ty\_2Elist\_2Elist\ A\_27b).((ap\ (ap\ (ap\ (c\_2EEncode\_2Eencode\_blist \\
& \quad A\_27a\ A\_27b)\ c\_2Enum\_2E0)\ V0e)\ V1l) = (c\_2Elist\_2ENIL\ A\_27a)))) \wedge \\
& \quad ((\forall V2m \in ty\_2Enum\_2Enum.(\forall V3e \in ((ty\_2Elist\_2Elist \\
& \quad A\_27a)^{A\_27b}).(\forall V4l \in (ty\_2Elist\_2Elist\ A\_27b).((ap\ (ap \\
& \quad (ap\ (c\_2EEncode\_2Eencode\_blist\ A\_27a\ A\_27b)\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap\ c\_2Earithmetic\_2EBIT1\ V2m)))\ V3e)\ V4l) = (ap\ (ap\ (c\_2Elist\_2EAPPEND \\
& \quad A\_27a)\ (ap\ V3e\ (ap\ (c\_2Elist\_2EHD\ A\_27b)\ V4l)))\ (ap\ (ap\ (ap\ (c\_2EEncode\_2Eencode\_blist \\
& \quad A\_27a\ A\_27b)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap\ c\_2Earithmetic\_2EBIT1\ V2m)))\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))\ V3e)\ (ap \\
& \quad (c\_2Elist\_2ETL\ A\_27b)\ V4l)))))) \wedge (\forall V5m \in ty\_2Enum\_2Enum. \\
& \quad (\forall V6e \in ((ty\_2Elist\_2Elist\ A\_27a)^{A\_27b}).(\forall V7l \in \\
& \quad (ty\_2Elist\_2Elist\ A\_27b).((ap\ (ap\ (ap\ (c\_2EEncode\_2Eencode\_blist \\
& \quad A\_27a\ A\_27b)\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& \quad V5m)))\ V6e)\ V7l) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ V6e\ (ap\ ( \\
& \quad c\_2Elist\_2EHD\ A\_27b)\ V7l)))\ (ap\ (ap\ (ap\ (c\_2EEncode\_2Eencode\_blist \\
& \quad A\_27a\ A\_27b)\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& \quad V5m)))\ V6e)\ (ap\ (c\_2Elist\_2ETL\ A\_27b)\ V7l)))))))))
\end{aligned}$$