

thm_2EEncode_2Ewf_encode_sum (TMRpCee26cBVZxtQyQrJcAPssCEjYsAycUo)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (1)$$

Let $c_2Elist_2EisPREFIX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EisPREFIX\ A_27a \in (((ty_2Elist_2Elist\ A_27a))(ty_2Elist_2Elist\ A_27a)) \quad (2)$$

Definition 2 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 3 We define c_2Ebool_2E2 to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Definition 6 We define $c_2EEncode_2Ewf_encoder$ to be $\lambda A_27a : \iota.\lambda V0p \in (2^{A_27a}).\lambda V1e \in ((ty_2Elist_2Elist\ 2)^{A_27a})$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (3)$$

Let $c_2EEncode_2Eencode_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2EEncode_2Eencode_sum\ A_27a\ A_27b \in (((ty_2Elist_2Elist\ 2)^{(ty_2Esum_2Esum\ A_27a\ A_27b)})(ty_2Elist_2Elist\ 2)^{A_27b})(ty_2Elist_2Elist\ 2)^{A_27a} \quad (4)$$

Let $c_2Esum_2Esum_CASE : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.nonempty\ A_27c \Rightarrow c_2Esum_2Esum_CASE\ A_27a\ A_27b\ A_27c \in (((A_27c^{(A_27c^{A_27b})})^{(A_27c^{A_27a})})(ty_2Elist_2Elist\ 2)^{A_27c}) \quad (5)$$

Definition 7 We define $c_2EEncode_2Elift_sum$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p1 \in (2^{A_27a}).\lambda V1p2 \in (2^A$

Definition 8 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E\ V0t) c_2Ebool_2EF$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)(ty_2Elist_2Elist\ A_27a))^{A_27a}) \quad (6)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (7)$$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap\ P\ x))$ **then** *(the* $(\lambda x.x \in A \wedge$

of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (8)$$

Definition 13 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS$

Definition 14 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad (\forall V0xb \in ((ty_2Elist_2Elist\ 2)^{A_27a}).(\forall V1yb \in (\\ & \quad (ty_2Elist_2Elist\ 2)^{A_27b}).(\forall V2x \in A_27a.((ap (ap (ap \\ & (c_2EEncode_2Encode_sum\ A_27a\ A_27b)\ V0xb)\ V1yb) (ap (c_2Esum_2EINL \\ & \quad A_27a\ A_27b)\ V2x)) = (ap (ap (c_2Elist_2ECONS\ 2)\ c_2Ebool_2ET) \\ & \quad (ap\ V0xb\ V2x)))))) \wedge (\forall V3xb \in ((ty_2Elist_2Elist\ 2)^{A_27a}). \\ & \quad (\forall V4yb \in ((ty_2Elist_2Elist\ 2)^{A_27b}).(\forall V5y \in A_27b. \\ & \quad ((ap (ap (ap (c_2EEncode_2Encode_sum\ A_27a\ A_27b)\ V3xb)\ V4yb) \\ & \quad (ap (c_2Esum_2EINR\ A_27a\ A_27b)\ V5y)) = (ap (ap (c_2Elist_2ECONS \\ & \quad 2)\ c_2Ebool_2EF) (ap\ V4yb\ V5y))))))))) \end{aligned} \quad (9)$$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(\text{False} \Rightarrow (p \ V0t))) \quad (11)$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((\text{True} \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge \text{True}) \Leftrightarrow \\ &(p \ V0t)) \wedge (((\text{False} \wedge (p \ V0t)) \Leftrightarrow \text{False}) \wedge (((p \ V0t) \wedge \text{False}) \Leftrightarrow \text{False}) \wedge \\ &(((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} &((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge ((\neg \text{True}) \Leftrightarrow \text{False}) \wedge \\ &((\neg \text{False}) \Leftrightarrow \text{True}))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} &\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ &A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} &(\forall V0t \in 2.(((\text{True} \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow \text{True}) \Leftrightarrow \\ &(p \ V0t)) \wedge (((\text{False} \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow \text{False}) \Leftrightarrow (\neg(\\ &p \ V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} &(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\ &((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} &(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ &2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))) \Rightarrow \\ &(((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} &\forall A_27a.\text{nonempty } A_27a \Rightarrow ((\forall V0l \in (\text{ty_2Elist_2Elist } \\ &A_27a).((p \ (\text{ap} \ (\text{ap} \ (\text{c_2Elist_2EisPREFIX } A_27a) \ (\text{c_2Elist_2ENIL } \\ &A_27a)) \ V0l)) \Leftrightarrow \text{True})) \wedge ((\forall V1x \in A_27a.(\forall V2l \in (\text{ty_2Elist_2Elist } \\ &A_27a).((p \ (\text{ap} \ (\text{ap} \ (\text{c_2Elist_2EisPREFIX } A_27a) \ (\text{ap} \ (\text{ap} \ (\text{c_2Elist_2ECONS } \\ &A_27a) \ V1x) \ V2l)) \ (\text{c_2Elist_2ENIL } A_27a))) \Leftrightarrow \text{False}))) \wedge (\forall V3x1 \in \\ &A_27a.(\forall V4l1 \in (\text{ty_2Elist_2Elist } A_27a).(\forall V5x2 \in \\ &A_27a.(\forall V6l2 \in (\text{ty_2Elist_2Elist } A_27a).((p \ (\text{ap} \ (\text{ap} \ (\text{c_2Elist_2EisPREFIX } \\ &A_27a) \ (\text{ap} \ (\text{ap} \ (\text{c_2Elist_2ECONS } A_27a) \ V5x2) \ V6l2)) \ (\text{ap} \ (\text{ap} \ (\text{c_2Elist_2ECONS } \\ &A_27a) \ V3x1) \ V4l1))) \Leftrightarrow ((V3x1 = V5x2) \wedge (p \ (\text{ap} \ (\text{ap} \ (\text{c_2Elist_2EisPREFIX } \\ &A_27a) \ V6l2) \ V4l1))))))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0y \in A_27a. (\forall V1x \in A_27a. (((ap\ (c_2Esum_2EINL \\
& A_27a\ A_27b)\ V1x) = (ap\ (c_2Esum_2EINL\ A_27a\ A_27b)\ V0y)) \Leftrightarrow (V1x = \\
& V0y)))) \wedge (\forall V2y \in A_27b. (\forall V3x \in A_27b. (((ap\ (c_2Esum_2EINR \\
& A_27a\ A_27b)\ V3x) = (ap\ (c_2Esum_2EINR\ A_27a\ A_27b)\ V2y)) \Leftrightarrow (V3x = \\
& V2y))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0ss \in (ty_2Esum_2Esum\ A_27a\ A_27b). ((\exists V1x \in A_27a. \\
& (V0ss = (ap\ (c_2Esum_2EINL\ A_27a\ A_27b)\ V1x))) \vee (\exists V2y \in A_27b. \\
& (V0ss = (ap\ (c_2Esum_2EINR\ A_27a\ A_27b)\ V2y))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow ((\forall V0x \in A_27a. (\forall V1f \in (A_27c^{A_27a}). \\
& (\forall V2f1 \in (A_27c^{A_27b}). ((ap\ (ap\ (ap\ (c_2Esum_2Esum_CASE \\
& A_27a\ A_27b\ A_27c)\ (ap\ (c_2Esum_2EINL\ A_27a\ A_27b)\ V0x))\ V1f)\ V2f1) = \\
& (ap\ V1f\ V0x)))))) \wedge (\forall V3y \in A_27b. (\forall V4f \in (A_27c^{A_27a}). \\
& (\forall V5f1 \in (A_27c^{A_27b}). ((ap\ (ap\ (ap\ (c_2Esum_2Esum_CASE \\
& A_27a\ A_27b\ A_27c)\ (ap\ (c_2Esum_2EINR\ A_27a\ A_27b)\ V3y))\ V4f)\ V5f1) = \\
& (ap\ V5f1\ V3y))))))
\end{aligned} \tag{21}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0p1 \in (2^{A_27a}). (\forall V1p2 \in (2^{A_27b}). (\forall V2e1 \in \\
& ((ty_2Elist_2Elist\ 2)^{A_27a}). (\forall V3e2 \in ((ty_2Elist_2Elist \\
& 2)^{A_27b}). (((p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder\ A_27a)\ V0p1) \\
& V2e1)) \wedge (p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder\ A_27b)\ V1p2)\ V3e2))) \Rightarrow \\
& (p\ (ap\ (ap\ (c_2EEncode_2Ewf_encoder\ (ty_2Esum_2Esum\ A_27a\ A_27b)) \\
& (ap\ (ap\ (c_2EEncode_2Elift_sum\ A_27a\ A_27b)\ V0p1)\ V1p2))\ (ap\ (\\
& ap\ (c_2EEncode_2Encode_sum\ A_27a\ A_27b)\ V2e1)\ V3e2))))))
\end{aligned}$$