

thm\_2EHolSmt\_2Ed018  
(TMPujqxxtw6a2TFQt3fLM9paKs9b3XTUPXq)

October 26, 2020

**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2E_2ET` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 3** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A-27a}))))$

**Definition 4** We define `c_2Ebool_2E_2EF` to be  $(\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V0t \in 2. V0t))$ .

**Definition 5** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow Q)$  of type  $\iota$ .

**Definition 6** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V2t \in 2. V2t))))$

**Definition 7** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define `c_2Ebool_2ECOND` to be  $\lambda A. 27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. 27a. (\lambda V2t2 \in A. 27a. (\text{ap } (\text{c\_2Emin\_2E\_40 } (2^A))))))$

**Definition 9** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V2t \in 2. V2t))))$

**Definition 10** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D\_3D\_3E } V0t) (\text{c\_2Ebool\_2E\_21 } 2))))$

Assume the following.

$$\text{True} \tag{1}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p \Rightarrow V0t1) \Rightarrow (p \Rightarrow V1t2)) \Rightarrow (((p \Rightarrow V1t2) \Rightarrow (p \Rightarrow V0t1)) \Rightarrow ((p \Rightarrow V0t1) \Leftrightarrow (p \Rightarrow V1t2)))))) \tag{2}$$

Assume the following.

$$(\forall V0t \in 2. (\text{False} \Rightarrow (p \Rightarrow V0t))) \tag{3}$$

Assume the following.

$$(\forall V0t \in 2. ((p \Rightarrow V0t) \vee \neg (p \Rightarrow V0t))) \tag{4}$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t))) \quad (5)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (6)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (10)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p V0t)))))) \end{aligned} \quad (12)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ & \forall V0f \in (A\_27b^{A\_27a}).(\forall V1b \in 2.(\forall V2x \in A\_27a. \\ & (\forall V3y \in A\_27a.((ap V0f (ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) \\ & V1b) V2x) V3y)) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27b) V1b) (ap V0f \\ & V2x)) (ap V0f V3y)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in 2. (\forall V1t1 \in 2. (\forall V2t2 \in 2. ((p \text{ (ap (ap} \\
& (\text{ap (c\_2Ebool\_2ECOND 2) V0b) V1t1) V2t2})) \Leftrightarrow (((\neg(p \text{ V0b})) \vee (p \text{ V1t1})) \wedge \\
& ((p \text{ V0b}) \vee (p \text{ V2t2})))))))))
\end{aligned} \tag{15}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p \text{ V0t}))) \Leftrightarrow (p \text{ V0t}))) \tag{16}$$

Assume the following.

$$(\forall V0A \in 2. ((p \text{ V0A}) \Rightarrow ((\neg(p \text{ V0A})) \Rightarrow \text{False}))) \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p \text{ V0A}) \vee (p \text{ V1B}))) \Rightarrow \text{False}) \Leftrightarrow \\
& ((p \text{ V0A}) \Rightarrow \text{False}) \Rightarrow ((\neg(p \text{ V1B})) \Rightarrow \text{False}))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p \text{ V0A})) \vee (p \text{ V1B}))) \Rightarrow \text{False}) \Leftrightarrow \\
& ((p \text{ V0A}) \Rightarrow ((\neg(p \text{ V1B})) \Rightarrow \text{False}))))))
\end{aligned} \tag{19}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p \text{ V0A})) \Rightarrow \text{False}) \Rightarrow (((p \text{ V0A}) \Rightarrow \text{False}) \Rightarrow \text{False}))) \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \text{ V0p}) \Leftrightarrow ( \\
& (p \text{ V1q}) \Leftrightarrow (p \text{ V2r}))) \Leftrightarrow (((p \text{ V0p}) \vee ((p \text{ V1q}) \vee (p \text{ V2r}))) \wedge (((p \text{ V0p}) \vee (\neg( \\
& p \text{ V2r})) \vee (\neg(p \text{ V1q})))) \wedge (((p \text{ V1q}) \vee ((\neg(p \text{ V2r})) \vee (\neg(p \text{ V0p})))) \wedge ((p \text{ V2r}) \vee \\
& ((\neg(p \text{ V1q})) \vee (\neg(p \text{ V0p})))))))))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \text{ V0p}) \Leftrightarrow ( \\
& (p \text{ V1q}) \wedge (p \text{ V2r}))) \Leftrightarrow (((p \text{ V0p}) \vee (\neg(p \text{ V1q})) \vee (\neg(p \text{ V2r}))) \wedge (((p \text{ V1q}) \vee \\
& (\neg(p \text{ V0p})) \wedge ((p \text{ V2r}) \vee (\neg(p \text{ V0p}))))))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \text{ V0p}) \Leftrightarrow ( \\
& (p \text{ V1q}) \vee (p \text{ V2r}))) \Leftrightarrow (((p \text{ V0p}) \vee (\neg(p \text{ V1q})) \wedge (((p \text{ V0p}) \vee (\neg(p \text{ V2r}))) \wedge \\
& ((p \text{ V1q}) \vee ((p \text{ V2r}) \vee (\neg(p \text{ V0p})))))))))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \text{ V0p}) \Leftrightarrow ( \\
& (p \text{ V1q}) \Rightarrow (p \text{ V2r}))) \Leftrightarrow (((p \text{ V0p}) \vee (p \text{ V1q})) \wedge (((p \text{ V0p}) \vee (\neg(p \text{ V2r}))) \wedge (( \\
& \neg(p \text{ V1q})) \vee ((p \text{ V2r}) \vee (\neg(p \text{ V0p}))))))))))
\end{aligned} \tag{24}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))) \quad (25)$$

**Theorem 1**

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0p \in 2. (\forall V1x \in A\_27a. (\forall V2y \in A\_27a. ((\neg(p \ V0p)) \vee (V1x = (ap \ (ap \ (ap \ (c.2Ebool\_2ECOND \ A\_27a) \ V0p) \ V1x) \ V2y)))))))$$