

thm_2EHolSmt_2Ep002
(TMJf2dSazdFj9tG6FsCR3RzstnQnJWZHu87)

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Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (1)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (2)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (3)$$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (4)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (6)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (8)$$

Definition 4 We define $c_Ebool_2E_2T$ to be $(ap (ap (c_Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_Emin_2E_3D (2^{A-27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x))$

Definition 6 We define $c_Eenum_2E_2SUC$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_Eenum_2E_2ABS_num (ap (ap (c_Emin_2E_3D (2^{A-27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (9)$$

Definition 7 We define $c_2Earithmetic_2E_2BIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B (ap (ap (c_Emin_2E_3D (2^{A-27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x))$

Definition 8 We define $c_2Earithmetic_2E_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 9 We define $c_Ebool_2E_2F$ to be $(ap (c_Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$.

Definition 10 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E_2F))$

Definition 12 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2)) (\lambda V2t \in 2.V2t))$

Definition 13 We define $c_Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) then (the (\lambda x.x \in A \wedge p (ap P x))) of type $\iota \Rightarrow \iota$.$

Definition 14 We define $c_Ebool_2E_23F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_Emin_2E_40 (ap (ap (c_Emin_2E_3D (2^{A-27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x))$

Definition 15 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V1n$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (p (ap (ap c_Eprim_rec_2E_3C (ap c_2Earithmetic_2E_2ENUMERAL (ap c_2Earithmetic_2E_2BIT1 c_2Earithmetic_2E_2ZERO))) (ap (c_2Ewords_2E_2dimword A_27a) (c_2Ebool_2E_2the_value A_27a)))) \quad (10)$$

Theorem 1

$$\forall A_27a.nonempty A_27a \Rightarrow (p (ap (ap c_Eprim_rec_2E_3C (ap c_2Earithmetic_2E_2ENUMERAL (ap c_2Earithmetic_2E_2BIT1 c_2Earithmetic_2E_2ZERO))) (ap (c_2Ewords_2E_2dimword A_27a) (c_2Ebool_2E_2the_value A_27a))))$$