

thm_2EHolSmt_2Er070 (TM- FKHe1LyEiPpzvKnmvViEWEq39GStd1zEV)

October 26, 2020

Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x) \text{ of type } \iota \Rightarrow \iota).$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A. 2^A. \lambda VOP \in (2^{A \rightarrow 2^A}). \text{(ap } VOP \text{ (ap } (c_2Emin_2E_40 \ A \ _)))$

Let `c_2Enum_2EZERO__REP` : ι be given. Assume the following.

$$c_2Enum_2EZERO__REP \in \text{omega} \quad (1)$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \quad (2)$$

Let `c_2Enum_2EABS__num` : ι be given. Assume the following.

$$c_2Enum_2EABS__num \in (ty_2Enum_2Enum^{\text{omega}}) \quad (3)$$

Definition 4 We define `c_2Enum_2E0` to be $(\text{ap } c_2Enum_2EABS__num \ c_2Enum_2EZERO__REP).$

Definition 5 We define `c_2Earithmic_2EZERO` to be `c_2Enum_2E0`.

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (ty_2Epair_2Eprod \ A0 \ A1) \quad (4)$$

Let `ty_2Einteger_2Eint` : ι be given. Assume the following.

$$\text{nonempty } ty_2Einteger_2Eint \quad (5)$$

Let `c_2Einteger_2Eint__REP__CLASS` : ι be given. Assume the following.

$$c_2Einteger_2Eint__REP__CLASS \in ((2^{(ty_2Epair_2Eprod \ ty_2Enum_2Enum \ ty_2Enum_2Enum)}) \ ty_2Einteger_2Eint) \quad (6)$$

Definition 6 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 8 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 (ty_2Eint$

Let $c_2Einteger_2Eint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)_{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}))_{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (7)$$

Let $c_2Einteger_2Eint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}))_{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (8)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}))_{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (9)$$

Definition 9 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)$

Definition 10 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Let $c_2Einteger_2Eint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_lt \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}))_{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (10)$$

Definition 11 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})_{ty_2Enum_2Enum} \quad (11)$$

Definition 12 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 13 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 14 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Ebool_2E_21 2) (\lambda V3t3 \in 2.V3t3))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})_{ty_2Enum_2Enum} \quad (12)$$

Definition 16 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Einteger_2Etint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum) (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum) (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)))$$
(13)

Definition 17 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Definition 18 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum})$$
(14)

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega})$$
(15)

Definition 19 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 20 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 21 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2$

Definition 22 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 23 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 24 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum) (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum) (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)))$$
(16)

Definition 25 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint$

Definition 26 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum})$$
(17)

Definition 27 We define $c_2Einteger_2Eint_le$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2E$

Assume the following.

$$True$$
(18)

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2))))))$$
(19)

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (22)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \wedge (p V1t2)) \Leftrightarrow ((p V1t2) \wedge (p V0t1)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (26)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\neg(\exists V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge (((\neg(p V0A) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))) \quad (32)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint.((p (ap (ap c_2Einteger_2Eint_lt V0x) V1y)) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le (ap (ap c_2Einteger_2Eint_add V0x) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) V1y)))))) \quad (33)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint.((p (ap (ap c_2Einteger_2Eint_le V0x) V1y)) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_add V1y) (ap c_2Einteger_2Eint_neg V0x)))))) \quad (34)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint.((V0x = V1y) \Leftrightarrow ((ap c_2Einteger_2Eint_of_num c_2Enum_2E0) = (ap (ap c_2Einteger_2Eint_add V1y) (ap c_2Einteger_2Eint_neg V0x)))))) \quad (35)$$

Assume the following.

$$(\forall V0c \in ty_2Einteger_2Eint.(\forall V1x \in ty_2Einteger_2Eint.(\forall V2y \in ty_2Einteger_2Eint.(((ap c_2Einteger_2Eint_of_num c_2Enum_2E0) = (ap (ap c_2Einteger_2Eint_add V0c) V1x)) \Rightarrow ((p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_add V0c) V2y))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le V1x) V2y)))))) \quad (36)$$

Assume the following.

$$\begin{aligned}
& (\forall V0c \in ty_2Einteger_2Eint. (\forall V1x \in ty_2Einteger_2Eint. \\
& (\forall V2y \in ty_2Einteger_2Eint. (((ap\ c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0) = (ap\ (ap\ c_2Einteger_2Eint_add\ V0c)\ V1x)) \Rightarrow ((p\ (\\
& ap\ (ap\ c_2Einteger_2Eint_le\ (ap\ c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0))\ (ap\ (ap\ c_2Einteger_2Eint_add\ (ap\ c_2Einteger_2Eint_neg \\
& V0c))\ V2y))) \Leftrightarrow (p\ (ap\ (ap\ c_2Einteger_2Eint_le\ (ap\ c_2Einteger_2Eint_neg \\
& V1x))\ V2y))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty_2Einteger_2Eint. (\forall V1x \in ty_2Einteger_2Eint. \\
& ((ap\ (ap\ c_2Einteger_2Eint_add\ V1x)\ V0y) = (ap\ (ap\ c_2Einteger_2Eint_add \\
& V0y)\ V1x))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2x \in ty_2Einteger_2Eint. ((ap\ (ap\ c_2Einteger_2Eint_add \\
& V2x)\ (ap\ (ap\ c_2Einteger_2Eint_add\ V1y)\ V0z)) = (ap\ (ap\ c_2Einteger_2Eint_add \\
& (ap\ (ap\ c_2Einteger_2Eint_add\ V2x)\ V1y))\ V0z))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap\ (ap\ c_2Einteger_2Eint_add \\
& V0x)\ (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)) = V0x))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap\ (ap\ c_2Einteger_2Eint_mul \\
& (ap\ c_2Einteger_2Eint_of_num\ (ap\ c_2Earithmic_2ENUMERAL \\
& (ap\ c_2Earithmic_2EBIT1\ c_2Earithmic_2EZERO))))\ V0x) = V0x))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap\ c_2Einteger_2Eint_neg\ (ap\ (ap\ c_2Einteger_2Eint_add\ V0x) \\
& V1y)) = (ap\ (ap\ c_2Einteger_2Eint_add\ (ap\ c_2Einteger_2Eint_neg \\
& V0x))\ (ap\ c_2Einteger_2Eint_neg\ V1y))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap\ c_2Einteger_2Eint_neg\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x) \\
& V1y)) = (ap\ (ap\ c_2Einteger_2Eint_mul\ (ap\ c_2Einteger_2Eint_neg \\
& V0x))\ V1y))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap\ c_2Einteger_2Eint_neg\ (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x) \\
& V1y)) = (ap\ (ap\ c_2Einteger_2Eint_mul\ V0x)\ (ap\ c_2Einteger_2Eint_neg \\
& V1y))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap\ c_2Einteger_2Eint_neg \\
& (ap\ c_2Einteger_2Eint_neg\ V0x)) = V0x))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((\neg(p\ (ap\ (ap\ c_2Einteger_2Eint_le\ V0x)\ V1y))) \Leftrightarrow (p\ (ap\ (ap\ c_2Einteger_2Eint_lt \\
& V1y)\ V0x))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (((p\ (ap\ (ap\ c_2Einteger_2Eint_le\ V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c_2Einteger_2Eint_le \\
& V1y)\ V0x))) \Leftrightarrow (V0x = V1y))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& ((ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (((ap\ c_2Einteger_2Eint_neg\ V0x) = (ap\ c_2Einteger_2Eint_neg \\
& V1y)) \Leftrightarrow (V0x = V1y))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Einteger_2Eint. (\forall V1n \in ty_2Enum_2Enum. \\
& (\forall V2m \in ty_2Enum_2Enum. (((ap (ap c_2Einteger_2Eint_add \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0p) = V0p) \wedge (((\\
& ap (ap c_2Einteger_2Eint_add V0p) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) = V0p) \wedge (((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) \wedge \\
& (((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_neg V0p)) = \\
& V0p) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V1n))) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V2m))) = (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B \\
& V1n) V2m)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V1n))) (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V2m)))) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Einteger_2Eint) (ap \\
& (ap c_2Earithmetic_2E_3C_3D V2m) V1n)) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V1n) \\
& V2m)))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V2m) \\
& V1n)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V1n))) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V2m))) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Einteger_2Eint) (ap (\\
& ap c_2Earithmetic_2E_3C_3D V1n) V2m)) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V2m) \\
& V1n)))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V1n) \\
& V2m)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V1n))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V2m)))) = (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B V1n) V2m))))))))))))))
\end{aligned}$$

(50)

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& \quad ((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow \\
& \quad True) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V0n)))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT1 V0n)))))) \Leftrightarrow \\
& \quad False) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT2 V0n)))))) \Leftrightarrow \\
& \quad False) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT1 V0n)))) \\
& \quad \quad (ap c_2Integer_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow False) \wedge (((p \\
& \quad \quad (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT2 V0n)))) \\
& \quad \quad (ap c_2Integer_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow False) \wedge (((p \\
& \quad \quad (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_neg (ap \\
& \quad \quad c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL (ap \\
& \quad \quad c_2Arithmetic_2EBIT1 V0n)))) (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap \\
& \quad \quad c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad (ap c_2Arithmetic_2EBIT2 V0n)))) (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap \\
& \quad \quad c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL V0n))) \\
& \quad \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V1m)))) \Leftrightarrow (p (ap (ap c_2Arithmetic_2E_3C_3D V0n) V1m))) \wedge (((p (\\
& \quad \quad ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL V0n))) (ap c_2Integer_2Eint_neg \\
& \quad \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad (ap c_2Arithmetic_2EBIT1 V1m)))))) \Leftrightarrow False) \wedge (((p (ap (ap c_2Integer_2Eint_le \\
& \quad \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V0n))) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT2 V1m)))))) \Leftrightarrow \\
& \quad False) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_neg \\
& \quad \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V0n)))) (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V1m)))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_neg \\
& \quad \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V0n)))) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL V1m)))))) \Leftrightarrow (p (ap (ap c_2Arithmetic_2E_3C_3D \\
& \quad \quad V1m) V0n)))))))))
\end{aligned}$$

(51)

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (53)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (54)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (60)$$

Theorem 1

$$(\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint.(\forall V2z \in ty_2Einteger_2Eint.((V0x = (ap (ap c_2Einteger_2Eint_add V1y) V2z)) \Leftrightarrow ((ap (ap c_2Einteger_2Eint_add V1y) (ap (ap c_2Einteger_2Eint_add V2z) (ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) V0x))) = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))))))$$