thm_2EHolSmt_2Er102 (TMWZ-ABGvWqmmCY1y5NivVaHjrhxgR8xXwDL)

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Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (1)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (2)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (3)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (4)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (5)$$

Definition 10 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

Definition 11 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 12 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\textsf{c\_2Emin\_2E\_40}$

Definition 13 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 14 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\textsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 15 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$

(6)

Definition 16 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 17 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod$$
$$A0\ A1)$$

(7)

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \qquad (8)$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2}$$

(9)

Definition 18 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap\ (\textsf{c\_2Emin\_2E\_40}\ (t)$

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum$$
$$ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})$$

(10)

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum}$$

(11)

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}$$

(12)

2

Definition 19 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_$

Definition 20 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint_

Definition 21 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (13)$$

Let $c\_2Einteger\_2Etint\_\_lt : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_}$$
$$(14)$$

Definition 22 *We define* c_2Einteger_2Eint__lt *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_$

Definition 23 *We define* c_2Einteger_2Eint__le *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2E$

Assume the following.
$$True \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow$$
$$(p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\quad (16)$$
$$p\ V0t))))))))$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$\mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_of\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow$$
$$True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$\mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V0n))))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$\mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))))))\Leftrightarrow$$
$$False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$\mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))))))\Leftrightarrow$$
$$False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))))$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow False)\wedge(((p$$
$$(ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))))$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow False)\wedge(((p$$
$$(ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap$$
$$c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap$$
$$\mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))))\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$\mathsf{c\_2Enum\_2E0})))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap$$
$$c\_2Einteger\_2Eint\_neg\ (ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))))\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$\mathsf{c\_2Enum\_2E0})))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap$$
$$c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V1m))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))\wedge(((p\ ($$
$$ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))))))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V0n)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))))))\Leftrightarrow$$
$$False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_neg$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V0n))))\ (ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V1m))))\Leftrightarrow True)\wedge((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_neg$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V0n))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_of\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1m)))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V1m)\ V0n)))))))))))))))))$$

$$(17)$$

Theorem 1

$$((p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_\_le} \ (ap \ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})) \ (ap \ c\_2Einteger\_2Eint\_\_of\_\_num \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ \mathsf{c\_2Earithmetic\_2EZERO}))))) \Leftrightarrow True)$$