

thm_2EHolSmt_2Er103 (TM- MxT9TD6fzDGZpcZvAGV6y8EAfEt5H7eTw)

October 26, 2020

Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x) \text{ of type } \iota \Rightarrow \iota.$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota.$

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40 } A$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty ty_2Enum_2Enum} \tag{1}$$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty (ty_2Epair_2Eprod } A0 \ A1) \tag{2}$$

Let `ty_2Einteger_2Eint` : ι be given. Assume the following.

$$\text{nonempty ty_2Einteger_2Eint} \tag{3}$$

Let `c_2Einteger_2Eint_REP_CLASS` : ι be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}) \text{ ty_2Einteger_2Eint}})) \tag{4}$$

Definition 4 We define `c_2Ebool_2ET` to be $(\text{ap (ap (c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap (ap (c_2Emin_2E_3D } (2^{A-27a})$

Definition 6 We define `c_2Einteger_2Eint_REP` to be $\lambda V0a \in \text{ty_2Einteger_2Eint}. (\text{ap (c_2Emin_2E_40 (ty_2Emin_2E_40$

Let `c_2Einteger_2Etint_lt` : ι be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}) \text{ (ty_2Epair_2Eprod ty_2Enum_2Enum)}})) \tag{5}$$

Definition 7 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$.

Definition 8 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21))$.

Definition 11 We define $c_2Einteger_2Eint_le$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint$.

Definition 12 We define $c_2Einteger_2Eint_ge$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{6}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{7}$$

Definition 13 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 14 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)_{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})_{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})_{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \tag{8}$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})_{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})_{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \tag{9}$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})_{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}} \tag{10}$$

Definition 15 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)$.

Definition 16 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})_{ty_2Enum_2Enum})_{ty_2Enum_2Enum} \tag{11}$$

Definition 17 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t)))$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (12)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (13)$$

Definition 18 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 19 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 21 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 22 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Definition 23 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Einteger_2Etint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (15)$$

Definition 24 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (16)$$

Definition 25 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint$

Definition 26 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 27 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 28 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (17)$$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee (\neg(p\ V0t)))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (25)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (29)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\neg(\exists V1x \in A.27a. (p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a. (\neg(p (ap V0P V2x)))))) \quad (30)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_lt V0x) V1y)) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le (ap (ap c_2Einteger_2Eint_add V0x) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) V1y)))))) \quad (31)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_le V0x) V1y)) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_add V1y) (ap c_2Einteger_2Eint_neg V0x))))))))) \quad (32)$$

Assume the following.

$$(\forall V0c \in ty_2Einteger_2Eint. (\forall V1x \in ty_2Einteger_2Eint. (\forall V2y \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_add V0c) V1x))) \Rightarrow ((p (ap (ap c_2Einteger_2Eint_lt V2y) (ap c_2Einteger_2Eint_neg V1x))) \Rightarrow ((p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg V0c) V2y))) \Leftrightarrow False))))))))) \quad (33)$$

Assume the following.

$$(\forall V0y \in ty_2Einteger_2Eint. (\forall V1x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_add V1x) V0y) = (ap (ap c_2Einteger_2Eint_add V0y) V1x)))) \quad (34)$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_add \\
V2x) (ap (ap c_2Einteger_2Eint_add V1y) V0z)) = (ap (ap c_2Einteger_2Eint_add \\
& (ap (ap c_2Einteger_2Eint_add V2x) V1y)) V0z))))))
\end{aligned} \tag{35}$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_add V0x) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) = V0x)) \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_add V0x) \\
V1y)) = (ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\
& V0x)) (ap c_2Einteger_2Eint_neg V1y))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_mul V0x) \\
V1y)) = (ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_neg \\
& V0x)) V1y))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_mul V0x) \\
V1y)) = (ap (ap c_2Einteger_2Eint_mul V0x) (ap c_2Einteger_2Eint_neg \\
& V1y))))))
\end{aligned} \tag{40}$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint. ((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_neg V0x)) = V0x)) \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((\neg (p (ap (ap c_2Einteger_2Eint_le V0x) V1y))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_lt \\
& V1y) V0x))))))
\end{aligned} \tag{42}$$

Assume the following.

$$((ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)) = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)) \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in ty_2Einteger_2Eint. (\forall V1n \in ty_2Enum_2Enum. \\ & (\forall V2m \in ty_2Enum_2Enum. (((ap\ (ap\ c_2Einteger_2Eint_add \\ & (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0))\ V0p) = V0p) \wedge (((\\ & ap\ (ap\ c_2Einteger_2Eint_add\ V0p)\ (ap\ c_2Einteger_2Eint_of_num \\ & c_2Enum_2E0)) = V0p) \wedge (((ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num \\ & c_2Enum_2E0)) = (ap\ c_2Einteger_2Eint_of_num\ c_2Enum_2E0)) \wedge \\ & (((ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_neg\ V0p)) = \\ & V0p) \wedge (((ap\ (ap\ c_2Einteger_2Eint_add\ (ap\ c_2Einteger_2Eint_of_num \\ & (ap\ c_2Earithmetic_2ENUMERAL\ V1n)))\ (ap\ c_2Einteger_2Eint_of_num \\ & (ap\ c_2Earithmetic_2ENUMERAL\ V2m))) = (ap\ c_2Einteger_2Eint_of_num \\ & (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\ & V1n)\ V2m)))))) \wedge (((ap\ (ap\ c_2Einteger_2Eint_add\ (ap\ c_2Einteger_2Eint_of_num \\ & (ap\ c_2Earithmetic_2ENUMERAL\ V1n)))\ (ap\ c_2Einteger_2Eint_neg \\ & (ap\ c_2Einteger_2Eint_of_num\ (ap\ c_2Earithmetic_2ENUMERAL \\ & V2m)))) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Einteger_2Eint)\ (ap \\ & (ap\ c_2Earithmetic_2E_3C_3D\ V2m)\ V1n))\ (ap\ c_2Einteger_2Eint_of_num \\ & (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ (ap\ c_2Earithmetic_2E_2D\ V1n) \\ & V2m))))\ (ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num \\ & (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ (ap\ c_2Earithmetic_2E_2D\ V2m) \\ & V1n)))))) \wedge (((ap\ (ap\ c_2Einteger_2Eint_add\ (ap\ c_2Einteger_2Eint_neg \\ & (ap\ c_2Einteger_2Eint_of_num\ (ap\ c_2Earithmetic_2ENUMERAL \\ & V1n)))\ (ap\ c_2Einteger_2Eint_of_num\ (ap\ c_2Earithmetic_2ENUMERAL \\ & V2m))) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Einteger_2Eint)\ (ap\ (\\ & ap\ c_2Earithmetic_2E_3C_3D\ V1n)\ V2m))\ (ap\ c_2Einteger_2Eint_of_num \\ & (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ (ap\ c_2Earithmetic_2E_2D\ V2m) \\ & V1n))))\ (ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num \\ & (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ (ap\ c_2Earithmetic_2E_2D\ V1n) \\ & V2m)))))) \wedge (((ap\ (ap\ c_2Einteger_2Eint_add\ (ap\ c_2Einteger_2Eint_neg \\ & (ap\ c_2Einteger_2Eint_of_num\ (ap\ c_2Earithmetic_2ENUMERAL \\ & V1n)))\ (ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num \\ & (ap\ c_2Earithmetic_2ENUMERAL\ V2m)))) = (ap\ c_2Einteger_2Eint_neg \\ & (ap\ c_2Einteger_2Eint_of_num\ (ap\ c_2Earithmetic_2ENUMERAL \\ & (ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V2m))))))))))))) \end{aligned} \quad (44)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (54)$$

Theorem 1

$$(\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_le V0x) V1y)) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_ge V1y) V0x))))))$$