# thm_2EHolSmt_2Er250
# (TMWRWgJwYJuH7c2pfnRiNjqr9rm1VdyriDT)

October 26, 2020

**Definition 1** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
*of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

**Definition 3** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

**Definition 4** *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

**Definition 5** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
*of type* $\iota$.

**Definition 6** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{3}$$

**Definition 7** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{6}$$

**Definition 8** *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap \ c\_2Enum\_2EABS\_num \ ($

**Definition 9** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p \ (ap \ P \ x))$ **then** $(the \ (\lambda x.x \in A \wedge p$
*of type* $\iota \Rightarrow \iota$.

**Definition 10** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap \ V0P \ (ap \ (c\_2Emin\_2E\_40$

**Definition 11** *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

**Definition 12** *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

**Definition 13** *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap \ (c\_2Ebool\_2E\_21 \ 2) \ (\lambda V2t \in$

**Definition 14** *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (7)$$

**Definition 15** *We define* c_2Enum_2E0 *to be* $(ap \ c\_2Enum\_2EABS\_\_num \ c\_2Enum\_2EZERO\_\_REP).$

**Definition 16** *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 17** *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap \ (ap \ (ap \ (c\_2Ebool\_2E$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(8)$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(9)$$

**Definition 18** *We define* c_2Enumeral_2EiiSUC *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap \ c\_2Enum\_2ESUC \ (ap$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(10)$$

**Definition 19** *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow nonempty \ (ty\_2Efcp\_2Efinite\_\_image \ A0) \qquad (11)$$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow nonempty \ (ty\_2Ebool\_2Eitself \ A0) \qquad (12)$$

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Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in ( \\ ty\_2Ebool\_2Eitself\ A\_27a) \tag{13}$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{14}$$

**Definition 20** *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ $c_2Ebool_2E_2F_5C

**Definition 21** *We define* c_2Efcp_2Efinite__index *to be* $\lambda A\_27a : \iota.(ap\ ($c_2Emin_2E_40 $(A\_27a^{ty\_2Enum\_2Enu}}$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart \\ A0\ A1) \tag{15}$$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart \\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \tag{16}$$

**Definition 22** *We define* c_2Efcp_2Efcp__index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

**Definition 23** *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

**Definition 24** *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 25** *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{17}$$

**Definition 26** *We define* c_2Earithmetic_2EMIN *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

**Definition 27** *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

**Definition 28** *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap\ $

**Definition 29** *We define* c_2Ewords_2Eword__bits *to be* $\lambda A\_27a : \iota.\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2$

**Definition 30** *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 31** *We define* c_2Ebit_2ESBIT *to be* $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ ($c_2Eboo

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Let $c\_2Esum\_\_num\_2ESUM : \iota$ be given. Assume the following.

$$c\_2Esum\_\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})ty\_2Enum\_2Enum)$$

$$(18)$$

**Definition 32** *We define* c_2Ewords_2Ew2n *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ c$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})ty\_2Enum\_2Enum)$$

$$(19)$$

**Definition 33** *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})ty\_2Enum\_2Enum)$$

$$(20)$$

**Definition 34** *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

**Definition 35** *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

**Definition 36** *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

**Definition 37** *We define* c_2Ewords_2En2w *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap$ (c_2Efcp_2EFC

**Definition 38** *We define* c_2Ewords_2Ew2w *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27$

**Definition 39** *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

**Definition 40** *We define* c_2Ewords_2Eword__extract *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0h \in ty\_2Enum\_2Enum$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \qquad (21)$$

Let $ty\_2Efcp\_2Ebit0 : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Efcp\_2Ebit0\ A0) \qquad (22)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D} \atop V0m)\ V0m))) \qquad (23)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ V1n))\ V2p)){\Leftrightarrow}(p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)))))))$$

$$(24)$$

Assume the following.

$$True \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \land (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \land (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{27}$$

Assume the following.

$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$\textsf{c\_2Enum\_2E0})\ V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \textsf{c\_2Enum\_2E0}) = V1n)) \wedge ((\forall V2n \in$
$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V3m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Enumeral\_2EiZ}\ (ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \textsf{c\_2Enum\_2E0})\ V4n) = \textsf{c\_2Enum\_2E0})) \wedge$
$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V5n)\ \textsf{c\_2Enum\_2E0}) = \textsf{c\_2Enum\_2E0})) \wedge ((\forall V6n \in ty\_2Enum\_2Enum.$
$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($
$ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V7m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V6n)\ V7m))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$\textsf{c\_2Enum\_2E0})\ V8n) = \textsf{c\_2Enum\_2E0})) \wedge ((\forall V9n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \textsf{c\_2Enum\_2E0}) = V9n)) \wedge ((\forall V10n \in$
$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V11m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$V10n)\ V11m))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$
$\textsf{c\_2Enum\_2E0})\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$
$V12n))) = \textsf{c\_2Enum\_2E0})) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2EEXP\ \textsf{c\_2Enum\_2E0})\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \textsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \textsf{c\_2Enum\_2E0})) \wedge ((\forall V14n \in$
$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \textsf{c\_2Enum\_2E0}) =$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO})))) \wedge$
$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V15n))$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m))))) \wedge (((ap\ \textsf{c\_2Enum\_2ESUC}$
$\textsf{c\_2Enum\_2E0}) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$
$\textsf{c\_2Earithmetic\_2EZERO}))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum.($
$(ap\ \textsf{c\_2Enum\_2ESUC}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \textsf{c\_2Enum\_2ESUC}\ V17n)))) \wedge (((ap\ \textsf{c\_2Eprim\_\_rec\_2EPRE}\ \textsf{c\_2Enum\_2E0}) =$
$\textsf{c\_2Enum\_2E0}) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \textsf{c\_2Eprim\_\_rec\_2EPRE}$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \textsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum.$
$(((ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \textsf{c\_2Enum\_2E0}) \Leftrightarrow (V19n = \textsf{c\_2Earithmetic\_2EZERO}))) \wedge$
$((\forall V20n \in ty\_2Enum\_2Enum.((\textsf{c\_2Enum\_2E0} = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V20n)) \Leftrightarrow (V20n = \textsf{c\_2Earithmetic\_2EZERO}))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum.$
$(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V21n) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V22m)) \Leftrightarrow (V21n = V22m)))) \wedge$
$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V23n)\ \textsf{c\_2Enum\_2E0})) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ \textsf{c\_2Enum\_2E0})\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V24n))) \Leftrightarrow (p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ \textsf{c\_2Earithmetic\_2EZERO})$
$V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V25n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V26m))) \Leftrightarrow (p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V25n)\ V26m))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3E}$
$\textsf{c\_2Enum\_2E0})\ V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V28n))\ \textsf{c\_2Enum\_2E0})) \Leftrightarrow (p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ \textsf{c\_2Earithmetic\_2EZERO})$
$V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V29n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V30m))) \Leftrightarrow (p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V30m)\ V29n))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$
$\textsf{c\_2Enum\_2E0})\ V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum.$

Assume the following.

$$
\begin{aligned}
&(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.( \\
&((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
&V0n)) = V0n) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
&V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = V0n) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ ( \\
&ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ ( \\
&ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ ( \\
&ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
&(((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
&V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
&(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
&((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
&V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
&(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
&((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
&V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
&(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
&((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
&V0n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
&V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)) \wedge (((ap \\
&\mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
&V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
&(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
&((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
&V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
&(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
&((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
&V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
&(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
&((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
&V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
&(ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
&(((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
&V0n)) = (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC} \\
&(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = ( \\
&ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ ( \\
&ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
&V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
&V0n)\ V1m)))) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
&(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = \\
&(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
&V0n)\ V1m)))) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
&(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = \\
&(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
&V0n)\ V1m)))) \wedge ((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
&(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = \\
&(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
&V0n)\ V1m))))))))))))))))))))))))
\end{aligned}
$$

$$(29)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h \in ty\_2Enum\_2Enum.($$
$$\forall V1l \in ty\_2Enum\_2Enum.(\forall V2w \in (ty\_2Efcp\_2Ecart\ 2$$
$$A\_27a).((ap\ (ap\ (ap\ \mathsf{c\_2Ewords\_2Eword\_\_bits}\ A\_27a)\ V0h)\ V1l)\ V2w) =$$
$$(ap\ (ap\ (ap\ \mathsf{c\_2Ewords\_2Eword\_\_extract}\ A\_27a\ A\_27a)\ V0h)\ V1l)\ V2w)))))$$

$$(30)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Efcp\_2Ecart$$
$$2\ A\_27a).(\forall V1h \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$(ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_2D}\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ ($$
$$c\_2Ebool\_2Ethe\_\_value\ A\_27a)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V1h)) \Rightarrow$$
$$((ap\ (ap\ (ap\ \mathsf{c\_2Ewords\_2Eword\_\_bits}\ A\_27a)\ V1h)\ \mathsf{c\_2Enum\_2E0})$$
$$V0w) = V0w))))$$

$$(31)$$

Assume the following.

$$((ap\ (c\_2Efcp\_2Edimindex\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0$$
$$(ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone))))\ (c\_2Ebool\_2Ethe\_\_value$$
$$(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))))) =$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))$$

$$(32)$$

**Theorem 1**

$$(\forall V0x \in (ty\_2Efcp\_2Ecart\ 2\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0$$
$$(ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))).((ap\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_extract}$$
$$(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone)))$$
$$(ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ (ty\_2Efcp\_2Ebit0\ ty\_2Eone\_2Eone))))$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ \mathsf{c\_2Enum\_2E0})$$
$$V0x) = V0x))$$