

thm\_2EHolSmt\_2Et002 (TM-  
GYW9QghhqnNxNLH92twjqB8RgN3Jgu4EQ)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2ET` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

**Definition 3** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow q)$  of type  $\iota$ .

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A-27a}))))$

**Definition 5** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2)) (\lambda V2t \in 2. V2t)))$

**Definition 6** We define `c_2Ebool_2EF` to be  $(\text{ap } (\text{c\_2Ebool\_2E\_21 } 2)) (\lambda V0t \in 2. V0t)$ .

**Definition 7** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define `c_2Ebool_2ECOND` to be  $\lambda A. \lambda a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. \lambda V2t2 \in A. \lambda V3t3 \in A. \text{c\_2Ebool\_2E\_21 } (V0t1 \wedge V2t2 \wedge V3t3)))$

**Definition 9** We define `c_2Ecombin_2EUPDATE` to be  $\lambda A. \lambda a : \iota. \lambda A. \lambda b : \iota. \lambda V0a \in A. \lambda V1b \in A. \lambda V2c \in A. \text{c\_2Ebool\_2E\_21 } (V0a \wedge V1b \wedge V2c)$

**Definition 10** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2)) (\lambda V2t \in 2. V2t)))$

**Definition 11** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D\_3D\_3E } V0t)) (\text{c\_2Ebool\_2E\_21 } 2)))$

Assume the following.

$$\text{True} \tag{1}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((\text{True} \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge \text{True}) \Leftrightarrow \\ & (p V0t)) \wedge (((\text{False} \wedge (p V0t)) \Leftrightarrow \text{False}) \wedge (((p V0t) \wedge \text{False}) \Leftrightarrow \text{False}) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{3}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{5}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t))))))
\end{aligned} \tag{6}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \tag{7}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow ( \\
& \forall V0f \in (A\_27b^{A\_27a}).(\forall V1b \in 2.(\forall V2x \in A\_27a. \\
& (\forall V3y \in A\_27a.((ap \ V0f \ (ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ V1b) \ V2x) \ V3y)) = (ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27b) \ V1b) \ (ap \ V0f \ V2x)) \ (ap \ V0f \ V3y))))))
\end{aligned} \tag{8}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in 2.(\forall V1t1 \in 2.(\forall V2t2 \in 2.((p \ (ap \ (ap \\
& (ap \ (c\_2Ebool\_2ECOND \ 2) \ V0b) \ V1t1) \ V2t2)) \Leftrightarrow (((\neg(p \ V0b)) \vee (p \ V1t1)) \wedge \\
& ((p \ V0b) \vee (p \ V2t2))))))
\end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\
& (\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a. \\
& (\forall V5y\_27 \in A\_27a.(((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge (((p \ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\
& ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ V1Q) \ V3x\_27) \ V5y\_27))))))
\end{aligned} \tag{10}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (11)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (12)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (13)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p))))) \quad (14)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (15)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (16)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow ( \\ & \quad \forall V0x \in A\_27a. (\forall V1y \in A\_27a. (\forall V2f \in (A\_27b^{A\_27a}). \\ & (\forall V3z \in A\_27b. ((V0x = V1y) \vee ((ap \ V2f \ V1y) = (ap \ (ap \ (ap \ (ap \ (c\_2Ecombin\_2EUPDATE \\ & \quad \quad \quad A\_27a \ A\_27b) \ V0x) \ V3z) \ V2f) \ V1y)))))) \end{aligned}$$