

thm\_2EHolSmt\_2Et023  
 (TMUz6gnWPYrLrcvxEHmM2ErD8ozYfjUeuqJ)

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Let  $c\_2Enum\_2ZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ZERO\_REP \in \omega \quad (1)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let  $c\_2Enum\_2ABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2ABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (3)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be ( $ap\ c\_2Enum\_2ABS\_num\ c\_2Enum\_2ZERO\_REP$ ).

**Definition 3** We define  $c\_2Earithmetic\_2ZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (5)$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be ( $ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x)$ ).

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x)))$ .

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2ABS\_num\ m)$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EBIT2 n) V0)$

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 9** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(c\_2EEXP (c\_2EMOD n) x)$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

**Definition 10** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(c\_2Ebit\_2EMOD\_2EXP (c\_2EDIV n) x)$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

**Definition 11** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V2m \in ty\_2Enum\_2Enum.(c\_2Ebit\_2EMOD\_2EXP (c\_2Ebit\_2EDIV\_2EXP h) l)$

**Definition 12** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 13** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 14** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF))$

**Definition 15** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.(c\_2Ebool\_2E\_7E t1) t2))))$

**Definition 16** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge P x)) \text{ else } (\lambda x.x \in A \wedge \neg P x)$  of type  $\iota \Rightarrow \iota$ .

**Definition 17** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a) P)))$

**Definition 18** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(c\_2Eprim\_rec (c\_2Emin\_2E\_40 V0m) V1n)$

**Definition 19** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(c\_2Earithmetic\_2EBIT2 (c\_2Eprim\_rec\_2E\_3C V0m) V1n)$

**Definition 20** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.(c\_2Ebool\_2E\_7E t1) t2))))$

**Definition 21** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(c\_2Earithmetic\_2E\_3E (c\_2Ebool\_2E\_5C\_2F V0m) V1n)$

**Definition 22** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(c\_2Earithmetic\_2E\_3C\_3D (c\_2Earithmetic\_2E\_3E\_3D V0m) V1n)$

**Definition 23** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 24** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ebool\_2ECOND$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 25** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x.$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (12)$$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (13)$$

Let  $c\_2Enumeral\_bit\_2EFDUB : \iota$  be given. Assume the following.

$$c\_2Enumeral\_bit\_2EFDUB \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})}) \quad (14)$$

**Definition 26** We define  $c\_2Earithmetic\_2EDIV2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap (ap c\_2Earithmetic\_2EODD$

**Definition 27** We define  $c\_2Enumeral\_bit\_2EiDIV2$  to be  $c\_2Earithmetic\_2EDIV2.$

Let  $c\_2Enumeral\_bit\_2ESFUNPOW : \iota$  be given. Assume the following.

$$c\_2Enumeral\_bit\_2ESFUNPOW \in (((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{(ty\_2Enum\_2Enum)}) \quad (15)$$

Let  $ty\_2Efcp\_2Efinit\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Efcp\_2Efinit\_image A0) \quad (16)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Ebool\_2Eitself A0) \quad (17)$$

Let  $c\_2Ebool\_2Eth\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Ebool\_2Eth\_value A\_27a \in (ty\_2Ebool\_2Eitself A\_27a) \quad (18)$$

Let  $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Efcp\_2Edimindex A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (19)$$

**Definition 28** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap c\_2Ebool\_2E\_2F\_5C$

**Definition 29** We define  $c\_2Efcp\_2Efinite\_index$  to be  $\lambda A\_27a : \iota. (ap (c\_2Emin\_2E\_40 (A\_27a^{ty\_2Enum\_2Enum}))$

Let  $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A0.nonempty A0 \Rightarrow & \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Efcp\_2Ecart \\ & A0 A1) \end{aligned} \quad (20)$$

Let  $c\_2Efcp\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow & \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Efcp\_2Edest\_cart \\ & A\_27a A\_27b \in ((A\_27a^{ty\_2Efcp\_2Efinite\_image A\_27b})^{(ty\_2Efcp\_2Ecart A\_27a A\_27b)}) \end{aligned} \quad (21)$$

**Definition 30** We define  $c\_2Efcp\_2Efcp\_index$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in (ty\_2Efcp\_2Ecart A\_27a A\_27b)$

**Definition 31** We define  $c\_2Efcp\_2EFCP$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}). (ap$

**Definition 32** We define  $c\_2Ewords\_2Eword\_1comp$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Efcp\_2Ecart 2 A\_27a).$

**Definition 33** We define  $c\_2Ecombin\_2EFAIL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V0x = V1y))$

**Definition 34** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap (ap c\_2Earithmetic\_2EBIT1 V0n))$

**Definition 35** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap$

**Definition 36** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota. \lambda V0n \in ty\_2Enum\_2Enum. (ap (c\_2Efcp\_2EFC$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty ty\_2Eone\_2Eone \quad (22)$$

Assume the following.

$$(\forall V0c \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2D V0c) V0c) = c\_2Enum\_2E0)) \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow & (\forall V0f \in ((A\_27a^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}). \\ & (\forall V1g \in (A\_27a^{ty\_2Enum\_2Enum}). ((\forall V2n \in ty\_2Enum\_2Enum. \\ & ((ap V1g (ap c\_2Enum\_2ESUC V2n)) = (ap (ap V0f V2n) (ap c\_2Enum\_2ESUC \\ & V2n)))) \Leftrightarrow (\forall V3n \in ty\_2Enum\_2Enum. ((ap V1g (ap c\_2Earithmetic\_2ENUMERAL \\ & (ap c\_2Earithmetic\_2EBIT1 V3n))) = (ap (ap V0f (ap (ap c\_2Earithmetic\_2E\_2D \\ & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V3n)))) \\ & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))) \\ & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Earithmetic\_2ENUMERAL \\ & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V3n))))))) \wedge \\ & (\forall V4n \in ty\_2Enum\_2Enum. ((ap V1g (ap c\_2Earithmetic\_2ENUMERAL \\ & (ap c\_2Earithmetic\_2EBIT2 V4n))) = (ap (ap V0f (ap c\_2Earithmetic\_2ENUMERAL \\ & (ap c\_2Earithmetic\_2EBIT1 V4n))) (ap c\_2Earithmetic\_2ENUMERAL \\ & (ap c\_2Earithmetic\_2EBIT2 V4n))))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum.(((ap (ap c\_2Earithmetic\_2EXP \\ 
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \\ 
& V0n) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\ 
& c\_2Earithmetic\_2EZERO))) \wedge ((ap (ap c\_2Earithmetic\_2EXP V0n) \\ 
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = \\ 
& V0n))) \\
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2D \\ 
& (ap c\_2Enum\_2ESUC V0a)) V0a) = (ap c\_2Earithmetic\_2ENUMERAL (ap \\ 
& c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \\
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty\_2Enum\_2Enum.(\forall V1l \in ty\_2Enum\_2Enum.( \\ 
& (ap (ap c\_2Ebit\_2EBITS V0h) V1l) c\_2Enum\_2E0) = c\_2Enum\_2E0))) \\
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum.((p (ap c\_2Earithmetic\_2EODD V0n)) \Leftrightarrow \\ 
& ((ap (ap c\_2Earithmetic\_2EMOD V0n) (ap c\_2Earithmetic\_2ENUMERAL \\ 
& (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) = (ap c\_2Earithmetic\_2ENUMERAL \\ 
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \\
\end{aligned} \tag{28}$$

Assume the following.

$$True \tag{29}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ 
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ 
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \\
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ 
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ 
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \\
\end{aligned} \tag{33}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (34)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.((ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) \\ & V0t1) V1t2) = V1t2))) \end{aligned} \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (38)$$

Assume the following.

$$(\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2R \in 2.(((p V0P) \vee (p V1Q)) \Rightarrow (p V2R)) \Leftrightarrow (((p V0P) \Rightarrow (p V2R)) \wedge ((p V1Q) \Rightarrow (p V2R))))))) \quad (39)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))) \quad (40)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27))))))) \Rightarrow & ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a.(((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap (ap (c\_2Ebool\_2ECOND A\_27a) \\ & V0P) V2x) V4y) = (ap (ap (c\_2Ebool\_2ECOND A\_27a) V1Q) V3x\_27) \\ & V5y\_27)))))))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A_{\_27a}.nonempty\ A_{\_27a} \Rightarrow & (\forall V0f \in (2^{A_{\_27a}}).(\forall V1v \in \\ A_{\_27a}.((\forall V2x \in A_{\_27a}.((V2x = V1v) \Rightarrow (p (ap\ V0f\ V2x)))) \Leftrightarrow (p ( \\ ap\ V0f\ V1v)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A_{\_27a}.nonempty\ A_{\_27a} \Rightarrow & \forall A_{\_27b}.nonempty\ A_{\_27b} \Rightarrow \\ \forall V0x \in (ty\_2Efcp\_2Ecart\ A_{\_27a}\ A_{\_27b}).(\forall V1y \in (ty\_2Efcp\_2Ecart\ \\ A_{\_27a}\ A_{\_27b}).((V0x = V1y) \Leftrightarrow (\forall V2i \in ty\_2Enum\_2Enum.((p (ap \\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V2i) (ap\ (c\_2Efcp\_2Edimindex\ A_{\_27b}) ( \\ c\_2Ebool\_2Ethe\_value\ A_{\_27b})))) \Rightarrow ((ap\ (ap\ (c\_2Efcp\_2Efcp\_index \\ A_{\_27a}\ A_{\_27b})\ V0x)\ V2i) = (ap\ (ap\ (c\_2Efcp\_2Efcp\_index\ A_{\_27a}\ A_{\_27b}) \\ V1y)\ V2i))))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} \forall A_{\_27a}.nonempty\ A_{\_27a} \Rightarrow & \forall A_{\_27b}.nonempty\ A_{\_27b} \Rightarrow \\ \forall V0g \in (A_{\_27a}^{ty\_2Enum\_2Enum}).(\forall V1i \in ty\_2Enum\_2Enum. \\ ((p (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V1i) (ap\ (c\_2Efcp\_2Edimindex\ A_{\_27b}) \\ (c\_2Ebool\_2Ethe\_value\ A_{\_27b})))) \Rightarrow ((ap\ (ap\ (c\_2Efcp\_2Efcp\_index \\ A_{\_27a}\ A_{\_27b})\ (ap\ (c\_2Efcp\_2EFCP\ A_{\_27a}\ A_{\_27b})\ V0g))\ V1i) = (ap\ V0g \\ V1i)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0n) = V0n)) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge (\forall V2n \in ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2B V3m) = ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2E0) V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2E0) V3m))) \wedge (\forall V4n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& (\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2A V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge (\forall V6n \in ty\_2Enum\_2Enum.((\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL V6n) V7m)))) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2E0) V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A V6n) V7m)))) \wedge (\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge (\forall V9n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge (\forall V10n \in ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2E_2D (ap c_2Earithmetic_2ENUMERAL V10n) (ap c_2Earithmetic_2ENUMERAL V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V10n) V11m)))) \wedge (\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V12n)))) = c_2Enum_2E0)) \wedge (\forall V13n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 V13n)))) = c_2Enum_2E0)) \wedge (\forall V14n \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2EEEXP V14n) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \wedge \\
& (\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.((ap (ap c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL V15n)) (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2EEEXP V15n) V16m)))) \wedge (((ap c_2Enum_2ESUC c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \wedge (\forall V17n \in ty\_2Enum\_2Enum.((ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2ESUC V17n)))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = c_2Enum_2E0) \wedge (\forall V18n \in ty\_2Enum\_2Enum.((ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Eprim_rec_2EPRE V18n)))) \wedge (\forall V19n \in ty\_2Enum\_2Enum.((((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge (\forall V20n \in ty\_2Enum\_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge (\forall V21n \in ty\_2Enum\_2Enum.((\forall V22m \in ty\_2Enum\_2Enum.(((ap c_2Earithmetic_2ENUMERAL V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge (\forall V24n \in ty\_2Enum\_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V24n)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) V24n)))) \wedge (\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL V25n)) (ap c_2Earithmetic_2ENUMERAL V26m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V25n) V26m)))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge (\forall V28n \in ty\_2Enum\_2Enum.((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V28n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V29n)) \Leftrightarrow True)) \wedge (\forall V30m \in ty\_2Enum\_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V30m)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V30m)))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge (\forall V32n \in ty\_2Enum\_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V32n)) \Leftrightarrow True)))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((c\_2Earithmetic\_2EZERO = (ap c\_2Earithmetic\_2EBIT1 V0n)) \Leftrightarrow False) \wedge \\
& (((ap c\_2Earithmetic\_2EBIT1 V0n) = c\_2Earithmetic\_2EZERO) \Leftrightarrow \\
& False) \wedge ((c\_2Earithmetic\_2EZERO = (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap c\_2Earithmetic\_2EBIT2 V0n) = c\_2Earithmetic\_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap c\_2Earithmetic\_2EBIT1 V0n) = (ap c\_2Earithmetic\_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap c\_2Earithmetic\_2EBIT2 V0n) = (ap c\_2Earithmetic\_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap c\_2Earithmetic\_2EBIT1 V0n) = (ap c\_2Earithmetic\_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap c\_2Earithmetic\_2EBIT2 V0n) = (ap c\_2Earithmetic\_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))))))) \\
\end{aligned} \tag{47}$$

Assume the following.

Assume the following.

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty\_2Enum\_2Enum.((ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW \\
& c\_2Enumeral\_bit\_2EiDIV2) c\_2Enum\_2E0) V0x) = V0x)) \wedge ((\forall V1y \in \\
& ty\_2Enum\_2Enum.((ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW c\_2Enumeral\_bit\_2EiDIV2) \\
& V1y) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V2n \in ty\_2Enum\_2Enum. \\
& (\forall V3x \in ty\_2Enum\_2Enum.((ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW \\
& c\_2Enumeral\_bit\_2EiDIV2) (ap c\_2Earithmetic\_2ENUMERAL (ap \\
& c\_2Earithmetic\_2EBIT1 V2n))) V3x) = (ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW \\
& (ap c\_2Enumeral\_bit\_2EFDUB c\_2Enumeral\_bit\_2EiDIV2)) (ap \\
& c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Enumeral\_bit\_2EiDIV2 \\
& V3x)))) \wedge (\forall V4n \in ty\_2Enum\_2Enum.(\forall V5x \in ty\_2Enum\_2Enum. \\
& ((ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW c\_2Enumeral\_bit\_2EiDIV2) \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 V4n))) \\
& V5x) = (ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW (ap c\_2Enumeral\_bit\_2EFDUB \\
& c\_2Enumeral\_bit\_2EiDIV2)) (ap c\_2Earithmetic\_2ENUMERAL V4n)) \\
& (ap c\_2Enumeral\_bit\_2EiDIV2 (ap c\_2Enumeral\_bit\_2EiDIV2 V5x))))))) \\
& (50)
\end{aligned}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V0n) c\_2Enum\_2E0)))) \quad (51)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) (ap c\_2Enum\_2ESUC V1n))) \Leftrightarrow ( \\
& (V0m = V1n) \vee (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n))))))
\end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) \\
& (ap (c\_2Efcp\_2Edimindex A\_27a) (c\_2Ebool\_2Ethet\_value A\_27a)))) \\
& (53)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0index\_20too\_20large \in \\
& 2.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2i \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap (c\_2Efcp\_2Efcp\_index 2 A\_27a) (ap (c\_2Ewords\_2En2w \\
& A\_27a) V1n)) \Leftrightarrow (p (ap (ap (c\_2Ebool\_2ECOND 2) (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V2i) (ap (c\_2Efcp\_2Edimindex A\_27a) (c\_2Ebool\_2Ethet\_value A\_27a))) \\
& (ap (ap c\_2Ebit\_2EBIT V2i) V1n)) (ap (ap (ap (c\_2Ecombin\_2EFAIL \\
& ((2^{ty\_2Enum\_2Enum})^{(ty\_2Efcp\_2Ecart 2 A\_27a)}) 2) (c\_2Efcp\_2Efcp\_index \\
& 2 A\_27a)) V0index\_20too\_20large) (ap (c\_2Ewords\_2En2w A\_27a) \\
& V1n)) V2i)))))))
\end{aligned} \quad (54)$$

Assume the following.

$$((ap (c_2Efcp_2Edimindex ty_2Eone_2Eone) (c_2Ebool_2Ethethe_value ty_2Eone_2Eone)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \Rightarrow \\ (55)$$

**Theorem 1**

$$(\forall V0x \in (ty_2Efcp_2Ecart 2 ty_2Eone_2Eone). ((\neg(p (ap (ap (c_2Efcp_2Efcp_index 2 ty_2Eone_2Eone) V0x) c_2Enum_2E0))) \Rightarrow \\ ((ap (c_2Ewords_2En2w ty_2Eone_2Eone) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = (ap (c_2Ewords_2Eword_1comp ty_2Eone_2Eone) V0x))))$$