

thm\_2EHolSmt\_2Et030  
(TMPmAo2ciPh691DUiwnVevK2y9nhBVQVsync)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 7** We define  $c\_Earithmetic\_EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmetic\_2E21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 8** We define  $c\_Earithmetic\_ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_Earithmetic\_EEXP : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (7)$$

Let  $c\_Earithmetic\_EMOD : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (8)$$

**Definition 9** We define  $c\_Ebit\_EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_Earithmetic\_EDIV : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (9)$$

**Definition 10** We define  $c\_Ebit\_EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_Earithmetic\_E\_2D : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (10)$$

**Definition 11** We define  $c\_Ebit\_EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.V0h$ .

**Definition 12** We define  $c\_Ebool\_EF$  to be  $(ap (c\_Ebool\_E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 13** We define  $c\_Emin\_E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 14** We define  $c\_Ebool\_E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_E\_3D\_3D\_3E V0t) c\_Ebool\_E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 15** We define  $c\_Ebool\_E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_E\_21 2) (\lambda V2t \in 2.V2t) (\lambda V0t1 \in 2.V0t1))))$ .

**Definition 16** We define  $c\_Emin\_E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then  $(the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 17** We define  $c\_Ebool\_E\_3F$  to be  $\lambda A.^{27a} : \iota.(\lambda V0P \in (2^A)^{27a}).(ap V0P (ap (c\_Emin\_E\_40 A) P))$ .

**Definition 18** We define  $c\_Eprim\_rec\_E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V0m$ .

**Definition 19** We define  $c\_Earithmetic\_E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V0m$ .

**Definition 20** We define  $c\_Ebool\_E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_E\_21 2) (\lambda V2t \in 2.V2t) (\lambda V0t1 \in 2.V0t1))))$ .

**Definition 21** We define  $c\_Earithmetic\_E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V0m$ .

**Definition 22** We define  $c\_Earithmetic\_E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V0m$ .

**Definition 23** We define  $c\_Ebool\_ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 24** We define  $c\_Eprim\_rec\_EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_Ebool\_2E$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 25** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (12)$$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (13)$$

Let  $c\_2Enumeral\_bit\_2EFDUB : \iota$  be given. Assume the following.

$$c\_2Enumeral\_bit\_2EFDUB \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{(ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum}}) \quad (14)$$

**Definition 26** We define  $c\_2Earithmetic\_2EDIV2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap (ap c\_2Earithmetic$

**Definition 27** We define  $c\_2Enumeral\_bit\_2EiDIV2$  to be  $c\_2Earithmetic\_2EDIV2$ .

Let  $c\_2Enumeral\_bit\_2ESFUNPOW : \iota$  be given. Assume the following.

$$c\_2Enumeral\_bit\_2ESFUNPOW \in (((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{(ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum}}) \quad (15)$$

Let  $ty\_2Efc\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty\_2Efc\_2Ecart A0 A1) \quad (16)$$

**Definition 28** We define  $c\_2Ecombin\_2EFAIL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V$

**Definition 29** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap (ap c\_2Earithmetic$

**Definition 30** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap$

Let  $ty\_2Efc\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Efc\_2Efinite\_image A0) \quad (17)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Ebool\_2Eitself A0) \quad (18)$$

Let  $c\_2Ebool\_2Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \quad (19)$$

Let  $c\_2Efcf\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcf\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (20)$$

**Definition 31** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ (ap\ c\_2Ebool\_2E\_2F\_5C$

**Definition 32** We define  $c\_2Efcf\_2Efinite\_index$  to be  $\lambda A\_27a : \iota. (ap\ (c\_2Emin\_2E\_40\ (A\_27a^{ty\_2Enum\_2Enum}$

Let  $c\_2Efcf\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcf\_2Edest\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcf\_2Efinite\_image\ A\_27b)})^{(ty\_2Efcf\_2Ecart\ A\_27a\ A\_27b)}) \quad (21)$$

**Definition 33** We define  $c\_2Efcf\_2Efcf\_index$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in (ty\_2Efcf\_2Ecart\ A\_27a$

**Definition 34** We define  $c\_2Efcf\_2EFCF$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}). (ap$

**Definition 35** We define  $c\_2Ewords\_2Een2w$  to be  $\lambda A\_27a : \iota. \lambda V0n \in ty\_2Enum\_2Enum. (ap\ (c\_2Efcf\_2EFCF$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (22)$$

Assume the following.

$$(\forall V0c \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0c)\ V0c) = c\_2Enum\_2E0)) \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in ((A\_27a^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}). \\ & \quad (\forall V1g \in (A\_27a^{ty\_2Enum\_2Enum}). ((\forall V2n \in ty\_2Enum\_2Enum. \\ & \quad ((ap\ V1g\ (ap\ c\_2Enum\_2ESUC\ V2n)) = (ap\ (ap\ V0f\ V2n)\ (ap\ c\_2Enum\_2ESUC\ V2n)))) \Leftrightarrow ((\forall V3n \in ty\_2Enum\_2Enum. ((ap\ V1g\ (ap\ c\_2Earithmetic\_2ENUMERAL \\ & \quad (ap\ c\_2Earithmetic\_2EBIT1\ V3n))) = (ap\ (ap\ V0f\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D \\ & \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ V3n))) \\ & \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))))) \\ & \quad (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ V3n)))))) \wedge \\ & \quad (\forall V4n \in ty\_2Enum\_2Enum. ((ap\ V1g\ (ap\ c\_2Earithmetic\_2ENUMERAL \\ & \quad (ap\ c\_2Earithmetic\_2EBIT2\ V4n))) = (ap\ (ap\ V0f\ (ap\ c\_2Earithmetic\_2ENUMERAL \\ & \quad (ap\ c\_2Earithmetic\_2EBIT1\ V4n)))\ (ap\ c\_2Earithmetic\_2ENUMERAL \\ & \quad (ap\ c\_2Earithmetic\_2EBIT2\ V4n))))))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2EEXP \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \\
& V0n) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO))) \wedge ((ap (ap c\_2Earithmetic\_2EEXP V0n) \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = \\
& V0n)))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2D ( \\
& ap c\_2Enum\_2ESUC V0a)) V0a) = (ap c\_2Earithmetic\_2ENUMERAL (ap \\
& c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1l \in ty\_2Enum\_2Enum. ( \\
& (ap (ap (ap c\_2Ebit\_2EBITS V0h) V1l) c\_2Enum\_2E0) = c\_2Enum\_2E0)))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((p (ap c\_2Earithmetic\_2EODD V0n)) \Leftrightarrow \\
& ((ap (ap c\_2Earithmetic\_2EMOD V0n) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) = (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{28}$$

Assume the following.

$$True \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))
\end{aligned} \tag{30}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{31}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
& (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p V0t))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in \\
& A\_27a.(((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\
& V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF) \\
& V0t1)\ V1t2) = V1t2))))))
\end{aligned} \tag{40}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \tag{41}$$

Assume the following.

$$(\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2R \in 2.(((p V0P) \vee (p V1Q)) \Rightarrow (p V2R)) \Leftrightarrow (((p V0P) \Rightarrow (p V2R)) \wedge ((p V1Q) \Rightarrow (p V2R)))))) \tag{42}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0f \in (A\_27b^{A\_27a}).(\forall V1b \in 2.(\forall V2x \in A\_27a. \\ (\forall V3y \in A\_27a.((ap\ V0f\ (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\ V1b)\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27b)\ V1b)\ (ap\ V0f \\ V2x))\ (ap\ V0f\ V3y))))))) \end{aligned} \quad (44)$$

Assume the following.

$$(\forall V0b \in 2.(\forall V1t1 \in 2.(\forall V2t2 \in 2.((p\ (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ 2)\ V0b)\ V1t1)\ V2t2)) \Leftrightarrow (((\neg(p\ V0b)) \vee (p\ V1t1)) \wedge ((p\ V0b) \vee (p\ V2t2)))))) \quad (45)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\ 2.(((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))))) \Rightarrow \\ (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ (\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a. \\ (\forall V5y\_27 \in A\_27a.(((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27 \\ V5y\_27))))))))) \end{aligned} \quad (47)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}).(\forall V1v \in A\_27a.((\forall V2x \in A\_27a.((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (ap\ V0f\ V1v)))))) \quad (48)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0x \in (ty\_2Efc\_2Ecart\ A\_27a\ A\_27b).(\forall V1y \in (ty\_2Efc\_2Ecart\ A\_27a\ A\_27b).((V0x = V1y) \Leftrightarrow (\forall V2i \in ty\_2Enum\_2Enum.((ap\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V2i)\ (ap\ (c\_2Efc\_2Edimindex\ A\_27b)\ (c\_2Ebool\_2Ethe\_value\ A\_27b)))) \Rightarrow ((ap\ (ap\ (c\_2Efc\_2Efc\_index\ A\_27a\ A\_27b)\ V0x)\ V2i) = (ap\ (ap\ (c\_2Efc\_2Efc\_index\ A\_27a\ A\_27b)\ V1y)\ V2i)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$



Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((c\_2Earithmic\_2EZERO = (ap\ c\_2Earithmic\_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
& (((ap\ c\_2Earithmic\_2EBIT1\ V0n) = c\_2Earithmic\_2EZERO) \Leftrightarrow \\
& False) \wedge (((c\_2Earithmic\_2EZERO = (ap\ c\_2Earithmic\_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmic\_2EBIT2\ V0n) = c\_2Earithmic\_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap\ c\_2Earithmic\_2EBIT1\ V0n) = (ap\ c\_2Earithmic\_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmic\_2EBIT2\ V0n) = (ap\ c\_2Earithmic\_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmic\_2EBIT1\ V0n) = (ap\ c\_2Earithmic\_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c\_2Earithmic\_2EBIT2\ V0n) = (ap\ c\_2Earithmic\_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))))) \\
& \tag{51}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((p\ (ap\ c\_2Earithmic\_2EVEN\ c\_2Earithmic\_2EZERO)) \wedge \\
& ((p\ (ap\ c\_2Earithmic\_2EVEN\ (ap\ c\_2Earithmic\_2EBIT2\ V0n))) \wedge \\
& ((\neg(p\ (ap\ c\_2Earithmic\_2EVEN\ (ap\ c\_2Earithmic\_2EBIT1\ V0n)))) \wedge \\
& ((\neg(p\ (ap\ c\_2Earithmic\_2EODD\ c\_2Earithmic\_2EZERO))) \wedge (( \\
& \neg(p\ (ap\ c\_2Earithmic\_2EODD\ (ap\ c\_2Earithmic\_2EBIT2\ V0n)))) \wedge \\
& (p\ (ap\ c\_2Earithmic\_2EODD\ (ap\ c\_2Earithmic\_2EBIT1\ V0n))))))))) \\
& \tag{52}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_2Ebit\_2EDIV\_2EXP\ V0n) \\
& c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2x \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_2Ebit\_2EDIV\_2EXP\ V1n) \\
& (ap\ c\_2Earithmic\_2ENUMERAL\ V2x)) = (ap\ c\_2Earithmic\_2ENUMERAL \\
& (ap\ (ap\ (ap\ c\_2Enumeral\_bit\_2ESFUNPOW\ c\_2Enumeral\_bit\_2EiDIV2) \\
& V1n)\ V2x)))))) \\
& \tag{53}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty\_2Enum\_2Enum.((ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW \\
& \quad c\_2Enumeral\_bit\_2EiDIV2) c\_2Enum\_2E0) V0x) = V0x)) \wedge ((\forall V1y \in \\
& ty\_2Enum\_2Enum.((ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW c\_2Enumeral\_bit\_2EiDIV2) \\
& \quad V1y) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V2n \in ty\_2Enum\_2Enum. \\
& (\forall V3x \in ty\_2Enum\_2Enum.((ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW \\
& \quad c\_2Enumeral\_bit\_2EiDIV2) (ap c\_2Earithmetic\_2ENUMERAL (ap \\
& c\_2Earithmetic\_2EBIT1 V2n))) V3x) = (ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW \\
& \quad (ap c\_2Enumeral\_bit\_2EFDUB c\_2Enumeral\_bit\_2EiDIV2)) (ap \\
& \quad c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Enumeral\_bit\_2EiDIV2 \\
& \quad V3x)))))) \wedge (\forall V4n \in ty\_2Enum\_2Enum.(\forall V5x \in ty\_2Enum\_2Enum. \\
& ((ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW c\_2Enumeral\_bit\_2EiDIV2) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 V4n))) \\
& V5x) = (ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW (ap c\_2Enumeral\_bit\_2EFDUB \\
& \quad c\_2Enumeral\_bit\_2EiDIV2)) (ap c\_2Earithmetic\_2ENUMERAL V4n)) \\
& \quad (ap c\_2Enumeral\_bit\_2EiDIV2 (ap c\_2Enumeral\_bit\_2EiDIV2 V5x))))))))))
\end{aligned} \tag{54}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V0n) c\_2Enum\_2E0)))) \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) (ap c\_2Enum\_2ESUC V1n)))) \Leftrightarrow ( \\
& (V0m = V1n) \vee (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) \\
(ap (c\_2Efc\_2Edimindex A\_27a) (c\_2Ebool\_2Ethe\_value A\_27a)))) \tag{57}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0i \in ty\_2Enum\_2Enum.( \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0i) (ap (c\_2Efc\_2Edimindex A\_27a) \\
& (c\_2Ebool\_2Ethe\_value A\_27a)))) \Rightarrow (\neg(p (ap (ap (c\_2Efc\_2Efc\_index \\
& \quad 2 A\_27a) (ap (c\_2Ewords\_2En2w A\_27a) c\_2Enum\_2E0)) V0i))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0index\_20too\_20large \in \\
& \quad 2.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2i \in ty\_2Enum\_2Enum. \\
& \quad ((p\ (ap\ (ap\ (c\_2Efc\_2Efc\_index\ 2\ A_{.27a})\ (ap\ (c\_2Ewords\_2En2w \\
& \quad A_{.27a})\ V1n))\ V2i)) \Leftrightarrow (p\ (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ 2)\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\
& \quad V2i)\ (ap\ (c\_2Efc\_2Edimindex\ A_{.27a})\ (c\_2Ebool\_2Ethe\_value\ A_{.27a)))) \\
& \quad (ap\ (ap\ c\_2Ebit\_2EBIT\ V2i)\ V1n))\ (ap\ (ap\ (ap\ (ap\ (c\_2Ecombin\_2EFAIL \\
& \quad ((2^{ty\_2Enum\_2Enum})^{ty\_2Efc\_2Ecart\ 2\ A_{.27a})\ 2)\ (c\_2Efc\_2Efc\_index \\
& \quad 2\ A_{.27a}))\ V0index\_20too\_20large)\ (ap\ (c\_2Ewords\_2En2w\ A_{.27a}) \\
& \quad V1n))\ V2i))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& ((ap\ (c\_2Efc\_2Edimindex\ ty\_2Eone\_2Eone)\ (c\_2Ebool\_2Ethe\_value \\
& \quad ty\_2Eone\_2Eone)) = (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO)))
\end{aligned} \tag{60}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1x \in (ty\_2Efc\_2Ecart\ 2\ ty\_2Eone\_2Eone). \\
& (((p\ V0p) \Leftrightarrow ((ap\ (c\_2Ewords\_2En2w\ ty\_2Eone\_2Eone)\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))) = V1x)) \Leftrightarrow \\
& (V1x = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ (ty\_2Efc\_2Ecart\ 2\ ty\_2Eone\_2Eone)) \\
& \quad V0p)\ (ap\ (c\_2Ewords\_2En2w\ ty\_2Eone\_2Eone)\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))))\ (ap\ (c\_2Ewords\_2En2w \\
& \quad ty\_2Eone\_2Eone)\ c\_2Enum\_2E0))))))
\end{aligned}$$