thm_2EHolSmt_2Et034
(TMGD52UkCvWXJmX7NaWLcivCgkNRjkZy6Jj)

October 26, 2020

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty \; ty\_2Enum\_2Enum \tag{1}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o \; (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap \; (ap \; (\text{c\_2Emin\_2E\_3D} \; (2^2)) \; (\lambda V0x \in 2.V0x)) \; (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap \; (ap \; (\text{c\_2Emin\_2E\_3D} \; (2^{A\_27})$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap \; (\text{c\_2Ebool\_2E\_21} \; 2) \; (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o \; (p \; P \Rightarrow p \; Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap \; (ap \; \text{c\_2Emin\_2E\_3D\_3D\_3E} \; V0t) \; \text{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap \; (\text{c\_2Ebool\_2E\_21} \; 2) \; (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{3}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap \; c\_2Enum\_2EABS\_\_num \;$ (

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p \; (ap \; P \; x))$ **then** $(the \; (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

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Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\text{c\_2Emin\_2E\_40}$

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 13 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 14 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 15 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (5)$$

Definition 16 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

Definition 17 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 18 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\text{c\_2Ebool\_2E}$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(6)$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(7)$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(8)$$

Definition 19 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(9)$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \qquad (10)$$

Definition 20 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 21 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 22 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 23 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$

$$(11)$$

Definition 24 *We define* c_2Earithmetic_2EDIV2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \qquad (12)$$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \qquad (13)$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (\\ ty\_2Ebool\_2Eitself\ A\_27a) \qquad (14)$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)})$$

$$(15)$$

Definition 25 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ $ c_2Ebool_2E_2F_5C

Definition 26 *We define* c_2Efcp_2Efinite_index *to be* $\lambda A\_27a : \iota.(ap\ ($ c_2Emin_2E_40 $(A\_27a^{ty\_2Enum\_2Enu}$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart \\ A0\ A1)$$

$$(16)$$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart \\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)})$$

$$(17)$$

Definition 27 *We define* c_2Efcp_2Efcp_index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Definition 28 *We define* c_2Efcp_2EFCP *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap\ $

Definition 29 *We define* c_2Ebit_2EDIV_2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$

$$(18)$$

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Definition 30 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 31 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Definition 32 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Definition 33 *We define* c_2Ewords_2En2w *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap$ (c_2Efcp_2EF

Definition 34 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod$$
$$A0\ A1)$$

$$(19)$$

Let $c\_2Epair\_2EABS\__prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\__prod$$
$$A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})})$$

$$(20)$$

Definition 35 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\__set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\__set\_2EGSPEC$$
$$A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})})$$

$$(21)$$

Definition 36 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_2$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum})$$

$$(22)$$

Definition 37 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c_2Ebool_2EF)$.

Let $c\_2Ewords\_2EBIT\__SET : \iota$ be given. Assume the following.

$$c\_2Ewords\_2EBIT\__SET \in (((2^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$

$$(23)$$

Definition 38 *We define* c_2Ebit_2ESBIT *to be* $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (c\_2Eboo$

Let $c\_2Esum\__num\_2ESUM : \iota$ be given. Assume the following.

$$c\_2Esum\__num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})}))^{ty\_2Enum\_2Enum})$$

$$(24)$$

Definition 39 *We define* c_2Ewords_2Ew2n *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ c$

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Definition 40 *We define* c_2Ewords_2Eword__add *to be* $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1$

Assume the following.

$$((ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ \text{c\_2Earithmetic\_2EZERO}) = \text{c\_2Enum\_2E0}) \tag{25}$$

Assume the following.

$$True \tag{26}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1){\Rightarrow}(p\ V1t2)){\Rightarrow}(((p\ \\ V1t2){\Rightarrow}(p\ V0t1)){\Rightarrow}((p\ V0t1){\Leftrightarrow}(p\ V1t2)))))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(False{\Rightarrow}(p\ V0t))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\vee}(p\ V0t)){\Leftrightarrow}True){\wedge}((((p\ V0t){\vee}True){\Leftrightarrow}True){\wedge} \\ (((False{\vee}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\vee}False){\Leftrightarrow}(p\ V0t)){\wedge}(((p\ V0t){\vee} \\ (p\ V0t)){\Leftrightarrow}(p\ V0t))))))) \tag{29}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Rightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\Rightarrow}True){\Leftrightarrow} \\ True){\wedge}(((False{\Rightarrow}(p\ V0t)){\Leftrightarrow}True){\wedge}((((p\ V0t){\Rightarrow}(p\ V0t)){\Leftrightarrow}True){\wedge}((\\ (p\ V0t){\Rightarrow}False){\Leftrightarrow}({\neg}(p\ V0t)))))))) \tag{30}$$

Assume the following.

$$((\forall V0t \in 2.(({\neg}({\neg}(p\ V0t))){\Leftrightarrow}(p\ V0t))){\wedge}((({\neg}True){\Leftrightarrow}False){\wedge} \\ (({\neg}False){\Leftrightarrow}True))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.((V0x = V0x){\Leftrightarrow} \\ True)) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y){\Leftrightarrow}(V1y = V0x)))) \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\Leftrightarrow}True){\Leftrightarrow} \\ (p\ V0t)){\wedge}(((False{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}({\neg}(p\ V0t))){\wedge}(((p\ V0t){\Leftrightarrow}False){\Leftrightarrow}({\neg}(\\ p\ V0t)))))))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V0t1)\ V1t2) = V1t2))))$$

(35)

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3)))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3))))))$$

(36)

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27))))))))$$

(37)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V1i \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Eprim\_\_rec\_2E\_3C\ V1i)\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27b)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27b)))) \Rightarrow ((ap\ (ap\ (c\_2Efcp\_2Efcp\_\_index\ A\_27a\ A\_27b)\ (ap\ (c\_2Efcp\_2EFCP\ A\_27a\ A\_27b)\ V0g))\ V1i) = (ap\ V0g\ V1i)))))$$

(38)

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Assume the following.

$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$\mathsf{c\_2Enum\_2E0})\ V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \mathsf{c\_2Enum\_2E0}) = V1n))\wedge((\forall V2n \in$
$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V3m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V4n) = \mathsf{c\_2Enum\_2E0}))\wedge$
$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V5n)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V6n \in ty\_2Enum\_2Enum.$
$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($
$ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V7m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$\mathsf{c\_2Enum\_2E0})\ V8n) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V9n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \mathsf{c\_2Enum\_2E0}) = V9n))\wedge((\forall V10n \in$
$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V11m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$
$\mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$
$V12n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2EEXP\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V14n \in$
$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \mathsf{c\_2Enum\_2E0}) =$
$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\wedge$
$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V15n))$
$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap\ \mathsf{c\_2Enum\_2ESUC}$
$\mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$
$\mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V17n \in ty\_2Enum\_2Enum.($
$(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \mathsf{c\_2Enum\_2ESUC}\ V17n))))\wedge(((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Enum\_2E0}) =$
$\mathsf{c\_2Enum\_2E0})\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}$
$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum.$
$(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow(V19n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge$
$((\forall V20n \in ty\_2Enum\_2Enum.((\mathsf{c\_2Enum\_2E0} = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V20n))\Leftrightarrow(V20n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V21n \in ty\_2Enum\_2Enum.$
$(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V21n) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge$
$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V23n)\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V24n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})$
$V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V25n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}$
$\mathsf{c\_2Enum\_2E0})\ V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V28n))\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})$
$V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$
$V29n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$
$\mathsf{c\_2Enum\_2E0})\ V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum.$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((\text{c\_2Earithmetic\_2EZERO} = (ap\ \text{c\_2Earithmetic\_2EBIT1}\ V0n)) \Leftrightarrow False) \wedge$$
$$((((ap\ \text{c\_2Earithmetic\_2EBIT1}\ V0n) = \text{c\_2Earithmetic\_2EZERO}) \Leftrightarrow$$
$$False) \wedge (((\text{c\_2Earithmetic\_2EZERO} = (ap\ \text{c\_2Earithmetic\_2EBIT2}$$
$$V0n)) \Leftrightarrow False) \wedge ((((ap\ \text{c\_2Earithmetic\_2EBIT2}\ V0n) = \text{c\_2Earithmetic\_2EZERO}) \Leftrightarrow$$
$$False) \wedge ((((ap\ \text{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \text{c\_2Earithmetic\_2EBIT2}$$
$$V1m)) \Leftrightarrow False) \wedge ((((ap\ \text{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \text{c\_2Earithmetic\_2EBIT1}$$
$$V1m)) \Leftrightarrow False) \wedge ((((ap\ \text{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \text{c\_2Earithmetic\_2EBIT1}$$
$$V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ \text{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \text{c\_2Earithmetic\_2EBIT2}$$
$$V1m)) \Leftrightarrow (V0n = V1m)))))))))))$$

(40)

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ c\_2Earithmetic\_2EEVEN\ \text{c\_2Earithmetic\_2EZERO})) \wedge$$
$$((p\ (ap\ c\_2Earithmetic\_2EEVEN\ (ap\ \text{c\_2Earithmetic\_2EBIT2}\ V0n))) \wedge$$
$$((\neg(p\ (ap\ c\_2Earithmetic\_2EEVEN\ (ap\ \text{c\_2Earithmetic\_2EBIT1}\ V0n)))) \wedge$$
$$((\neg(p\ (ap\ c\_2Earithmetic\_2EODD\ \text{c\_2Earithmetic\_2EZERO}))) \wedge ((\neg(p\ (ap\ c\_2Earithmetic\_2EODD\ (ap\ \text{c\_2Earithmetic\_2EBIT2}\ V0n)))) \wedge$$
$$(p\ (ap\ c\_2Earithmetic\_2EODD\ (ap\ \text{c\_2Earithmetic\_2EBIT1}\ V0n)))))))))$$

(41)

Assume the following.

$$(((ap\ \text{c\_2Earithmetic\_2EDIV2}\ \text{c\_2Enum\_2E0}) = \text{c\_2Enum\_2E0}) \wedge ((\forall V0n \in$$
$$ty\_2Enum\_2Enum.((ap\ \text{c\_2Earithmetic\_2EDIV2}\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \text{c\_2Earithmetic\_2EBIT1}\ V0n))) = (ap\ \text{c\_2Earithmetic\_2ENUMERAL}$$
$$V0n))) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((ap\ \text{c\_2Earithmetic\_2EDIV2}$$
$$(ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Earithmetic\_2EBIT2}\ V1n))) =$$
$$(ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Enum\_2ESUC}\ V1n))))))$$

(42)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg(p\ (ap\ (ap$$
$$(\text{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (\text{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))))$$

(43)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in$$
$$A\_27a.(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\text{c\_2Ebool\_2EIN}\ A\_27a)$$
$$V0x)\ (ap\ (ap\ (\text{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x =$$
$$V1y) \vee (p\ (ap\ (ap\ (\text{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V2s)))))))$$

(44)

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)))$$

(45)

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False)))$$

(46)

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Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow \\ (((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False)))))) \tag{47}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow \\ ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False)))))) \tag{48}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{49}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow(\\ (p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(\\ p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee \\ ((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))))))) \tag{50}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow(\\ (p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge \\ ((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{51}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow(\\ (p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\\ \neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{52}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee \\ (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{53}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{54}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{55}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{56}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \qquad (57)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \qquad (58)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1i \in ty\_2Enum\_2Enum.(\\
(ap\ (ap\ c\_2Ewords\_2EBIT\_\_SET\ V1i)\ V0n) = (ap\ (ap\ (ap\ (\textsf{c\_2Ebool\_2ECOND}\\
(2^{ty\_2Enum\_2Enum}))\ (ap\ (ap\ (\textsf{c\_2Emin\_2E\_3D}\ ty\_2Enum\_2Enum)\ V0n)\\
\textsf{c\_2Enum\_2E0}))\ (\textsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Enum\_2Enum))\ (ap\ (\\
ap\ (ap\ (\textsf{c\_2Ebool\_2ECOND}\ (2^{ty\_2Enum\_2Enum}))\ (ap\ c\_2Earithmetic\_2EODD\\
V0n))\ (ap\ (ap\ (\textsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Enum\_2Enum)\ V1i)\ (ap\\
(ap\ c\_2Ewords\_2EBIT\_\_SET\ (ap\ \textsf{c\_2Enum\_2ESUC}\ V1i))\ (ap\ (ap\ c\_2Earithmetic\_2EDIV\\
V0n)\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT2}\\
\textsf{c\_2Earithmetic\_2EZERO})))))))\ (ap\ (ap\ c\_2Ewords\_2EBIT\_\_SET\ (ap\\
\textsf{c\_2Enum\_2ESUC}\ V1i))\ (ap\ (ap\ c\_2Earithmetic\_2EDIV\ V0n)\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\\
(ap\ \textsf{c\_2Earithmetic\_2EBIT2}\ \textsf{c\_2Earithmetic\_2EZERO})))))))))) \qquad (59)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ \textsf{c\_2Enum\_2E0})\\
(ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))) \qquad (60)$$

Assume the following.

$$(\forall V0i \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
(p\ (ap\ (ap\ \textsf{c\_2Ebit\_2EBIT}\ V0i)\ V1n))\Leftrightarrow(p\ (ap\ (ap\ (\textsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\\
V0i)\ (ap\ (ap\ c\_2Ewords\_2EBIT\_\_SET\ \textsf{c\_2Enum\_2E0})\ V1n)))))) \qquad (61)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0a \in (ty\_2Efcp\_2Ecart\\
2\ A\_27a).(\forall V1b \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).((p\ (ap\ (ap\\
(\textsf{c\_2Efcp\_2Efcp\_\_index}\ 2\ A\_27a)\ (ap\ (ap\ (\textsf{c\_2Ewords\_2Eword\_\_add}\\
A\_27a)\ V0a)\ V1b))\ \textsf{c\_2Enum\_2E0}))\Leftrightarrow(\neg((p\ (ap\ (ap\ (\textsf{c\_2Efcp\_2Efcp\_\_index}\\
2\ A\_27a)\ V0a)\ \textsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (ap\ (ap\ (\textsf{c\_2Efcp\_2Efcp\_\_index}\\
2\ A\_27a)\ V1b)\ \textsf{c\_2Enum\_2E0}))))))) \qquad (62)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in (ty\_2Efcp\_2Ecart\\
2\ A\_27a).(\forall V1y \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(((ap\ (ap\ (\\
\textsf{c\_2Ewords\_2Eword\_\_add}\ A\_27a)\ (ap\ (\textsf{c\_2Ewords\_2En2w}\ A\_27a)\ (ap\\
\textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO}))))\\
V0x) = V1y)\Rightarrow((p\ (ap\ (ap\ (\textsf{c\_2Efcp\_2Efcp\_\_index}\ 2\ A\_27a)\ V0x)\ \textsf{c\_2Enum\_2E0}))\Rightarrow\\
(\neg(p\ (ap\ (ap\ (\textsf{c\_2Efcp\_2Efcp\_\_index}\ 2\ A\_27a)\ V1y)\ \textsf{c\_2Enum\_2E0}))))))))$$

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