

thm\_2EHolSmt\_2Et035  
 (TMKn3pkRXnAFS5xVyXeUqmuYyV6gj9WQRKb)

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Let  $c\_2Enum\_2ZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ZERO\_REP \in \omega \quad (1)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let  $c\_2Enum\_2ABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2ABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (3)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be ( $ap\ c\_2Enum\_2ABS\_num\ c\_2Enum\_2ZERO\_REP$ ).

**Definition 3** We define  $c\_2Earithmetic\_2ZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (5)$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be ( $ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x)$ ).

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x)))$ .

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2ABS\_num\ m)$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2EBIT2\ n)\ V)$

**Definition 8** We define  $c_2Earthmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x.$

Let  $c_2\text{Earithmetic\_2EXP} : \iota$  be given. Assume the following.

$$c_2Earithmetic_2EXP \in ((ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)ty\_2Enum\_2Enum) \quad (7)$$

Let  $c_2 \in \text{Arithmetc\_EMOD} : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 9** We define  $\text{c\_EBit\_EMOD\_EXP}$  to be  $\lambda V0x \in \text{ty\_Enum\_Enum}. \lambda V1n \in \text{ty\_Enum\_Enum}.$

Let  $c_{\text{Arithmetric\_EDIV}} : \iota$  be given. Assume the following.

$$c_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

**Definition 10** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

Let  $c_2Earithmetic_2E_2D : \iota$  be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enumty_2Enum_2Enum_2Enum)ty_2Enum_2Enum) \quad (10)$$

**Definition 11** We define `c_2EBit_2EBITS` to be  $\lambda V0h \in ty\_2Enum\_2Enum. \lambda V1l \in ty\_2Enum\_2Enum. \lambda V2m \in ty\_2Enum\_2Enum. \lambda V3n \in ty\_2Enum\_2Enum.$

**Definition 12** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 13** We define  $c_{\text{min\_3D\_3D\_3E}}$  to be  $\lambda P \in 2.\lambda Q \in 2.\text{inj\_o} (p \ P \Rightarrow p \ Q)$  of type  $\iota$ .

**Definition 14** We define  $c_{\text{Ebool\_2E\_7E}}$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c_{\text{Emin\_2E\_3D\_3D\_3E}}\ V0t)\ c_{\text{Ebool\_2E}}))$

**Definition 15** We define  $c_{\text{Ebool\_2E\_2F\_5C}}$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_{\text{Ebool\_2E\_21}}) 2)) (\lambda V2t \in$

**Definition 16** We define  $c_{\text{2Emin\_2E\_40}}$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \ (ap \ P \ x)) \ \text{then} \ (\lambda x. x \in A \wedge x \neq x) \ \text{else} \ (\lambda x. x \in A \wedge x = x)$  of type  $\iota \Rightarrow \iota$ .

**Definition 17** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.\_27a : \iota.(\lambda V0P \in (2^A\_{-27}a)).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 18** We define  $c_2Eprim\_rec_2E_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 19** We define  $c\_2Earthmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 20** We define  $c_{\text{CBool}} \_ 2E \_ 5C \_ 2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_{\text{CBool}} \_ 2E \_ 21 \_ 2) (\lambda V2t \in$

**Definition 21** We define  $c\_2Earthmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 22** We define  $c_{\text{Ebool\_ECOND}}$  to be  $\lambda A. \lambda 27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A. 27a. (\lambda V2t2 \in A. 27a. ($

**Definition 23** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (ap (c\_2Ebool\_2Bool$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (11)$$

**Definition 24** We define  $c\_2Eenumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (12)$$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (13)$$

Let  $c\_2Eenumeral\_bit\_2EFDUB : \iota$  be given. Assume the following.

$$c\_2Eenumeral\_bit\_2EFDUB \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum} \quad (14)$$

**Definition 25** We define  $c\_2Earithmetic\_2EDIV2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EODD$

**Definition 26** We define  $c\_2Eenumeral\_bit\_2EiDIV2$  to be  $c\_2Earithmetic\_2EDIV2$ .

Let  $c\_2Eenumeral\_bit\_2ESFUNPOW : \iota$  be given. Assume the following.

$$c\_2Eenumeral\_bit\_2ESFUNPOW \in (((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{(ty\_2Enum\_2Enum)})^{ty\_2Enum\_2Enum} \quad (15)$$

**Definition 27** We define  $c\_2Earithmetic\_2EMIN$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 28** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

Let  $ty\_2Efcp\_2Efinit\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Efcp\_2Efinit\_image A0) \quad (16)$$

Let  $ty\_2Ebool\_2Eitsel : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ebool\_2Eitsel A0) \quad (17)$$

Let  $c\_2Ebool\_2Ethet\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ebool\_2Ethet\_value A\_27a \in (ty\_2Ebool\_2Eitsel A\_27a) \quad (18)$$

Let  $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Efcp\_2Edimindex A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitsel A\_27a)}) \quad (19)$$

**Definition 29** We define  $c\_Ebool\_E_3F\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}).(ap (ap c\_Ebool\_E\_2F\_5C))$

**Definition 30** We define  $c\_Efcp\_Efinite\_index$  to be  $\lambda A\_27a : \iota. (ap (c\_Emin\_E\_40 (A\_27a^{ty\_2Enum\_2Enum}))$

Let  $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Efcp\_2Ecart \\ & \quad A0 A1) \end{aligned} \tag{20}$$

Let  $c\_2Efcp\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Efcp\_2Edest\_cart \\ & \quad A\_27a A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_image A\_27b)})^{(ty\_2Efcp\_2Ecart A\_27a A\_27b)}) \end{aligned} \tag{21}$$

**Definition 31** We define  $c\_Efcp\_2Efcp\_index$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in (ty\_2Efcp\_2Ecart A\_27a A\_27b)$

**Definition 32** We define  $c\_Efcp\_2EFCP$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap (ap c\_Efcp\_2EFCP)))$

**Definition 33** We define  $c\_Earithmetic\_EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmetic\_EBIT1))$

**Definition 34** We define  $c\_Ewords\_2Eword\_bits$  to be  $\lambda A\_27a : \iota. \lambda V0h \in ty\_2Enum\_2Enum. \lambda V1l \in ty\_2Enum\_2Enum$

**Definition 35** We define  $c\_Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in (A\_27b^{A\_27c}). \lambda V1g \in ty\_2Enum\_2Enum$

**Definition 36** We define  $c\_Ebit\_2ESBIT$  to be  $\lambda V0b \in 2. \lambda V1n \in ty\_2Enum\_2Enum.(ap (ap (c\_Ebool\_2Ebit\_2ESBIT)))$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \tag{22}$$

**Definition 37** We define  $c\_Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Efcp\_2Ecart 2 A\_27a).(ap (ap c\_Ewords\_2Ew2n))$

**Definition 38** We define  $c\_Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.(ap (c\_Ebit\_2EBIT))$

**Definition 39** We define  $c\_Ewords\_2En2w$  to be  $\lambda A\_27a : \iota. \lambda V0n \in ty\_2Enum\_2Enum.(ap (c\_Efcp\_2EFCP))$

**Definition 40** We define  $c\_Ewords\_2Ew2w$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0w \in (ty\_2Efcp\_2Ecart 2 A\_27a)$

**Definition 41** We define  $c\_Ewords\_2Eword\_extract$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0h \in ty\_2Enum\_2Enum$

**Definition 42** We define  $c\_Ecombin\_2EFAIL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V1y))$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty ty\_2Eone\_2Eone \tag{23}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((ap (ap c\_Earithmetic\_2E\_2B V0m) \\ c\_2Enum\_2E0) = V0m)) \tag{24}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ & (ap c\_2Enum\_2ESUC V0m)) V1n)))))) \end{aligned} \quad (25)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ c\_2Enum\_2E0) V0n))) \quad (26)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ V0n) c\_2Enum\_2E0)) \Leftrightarrow (V0n = c\_2Enum\_2E0))) \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\ & (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\ & (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\ (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\ & (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\ (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge \\ & ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\ & (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\ & V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\ & (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\ V0m) V1n)))))))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & ((ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = c\_2Enum\_2E0) \Leftrightarrow ((V0m = \\ & c\_2Enum\_2E0) \wedge (V1n = c\_2Enum\_2E0)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\ & (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p))))))) \end{aligned} \quad (30)$$

Assume the following.

$$(\forall V0c \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2D V0c) \\ V0c) = c\_2Enum\_2E0))) \quad (31)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\
 & V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))))))
 \end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
 & V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
 & V1n)) V0m))))))
 \end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap \\
 & c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
 & c\_2Earithmetic\_2EZERO)) V0n)))
 \end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
 & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0f \in ((A\_27a^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}). \\
 & (\forall V1g \in (A\_27a^{ty\_2Enum\_2Enum}). ((\forall V2n \in ty\_2Enum\_2Enum. \\
 & ((ap V1g (ap c\_2Enum\_2ESUC V2n)) = (ap (ap V0f V2n) (ap c\_2Enum\_2ESUC \\
 & V2n))) \Leftrightarrow ((\forall V3n \in ty\_2Enum\_2Enum. ((ap V1g (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT1 V3n)) = (ap (ap V0f (ap (ap c\_2Earithmetic\_2E\_2D \\
 & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V3n))) \\
 & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \\
 & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V3n))))))) \wedge \\
 & (\forall V4n \in ty\_2Enum\_2Enum. ((ap V1g (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT2 V4n)) = (ap (ap V0f (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT1 V4n))) (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT2 V4n)))))))))))
 \end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2EEEXP \\
 & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \\
 & V0n) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
 & c\_2Earithmetic\_2EZERO)))) \wedge ((ap (ap c\_2Earithmetic\_2EEEXP V0n) \\
 & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = \\
 & V0n)))
 \end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & (ap (ap c\_2Earithmetic\_2EMIN V1m) V0n)) V2p)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & V1m) V2p)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) V2p)))) \wedge (( \\
 & p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V2p) (ap (ap c\_2Earithmetic\_2EMIN \\
 & V1m) V0n))) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V2p) V1m)) \wedge (p \\
 & (ap (ap c\_2Earithmetic\_2E\_3C\_3D V2p) V0n)))))))
 \end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2D \\
 & (ap c\_2Enum\_2ESUC V0a)) V0a) = (ap c\_2Earithmetic\_2ENUMERAL (ap \\
 & c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \\
 \end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1l \in ty\_2Enum\_2Enum. (\forall V0h \in ty\_2Enum\_2Enum. \\
 & (ap (ap (ap c\_2EBIT\_2EBITS V0h) V1l) c\_2Enum\_2E0) = c\_2Enum\_2E0)))
 \end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. ((p (ap c\_2Earithmetic\_2EODD V0n)) \Leftrightarrow \\
 & ((ap (ap c\_2Earithmetic\_2EMOD V0n) (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) = (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))
 \end{aligned} \tag{40}$$

Assume the following.

$$True \tag{41}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
 V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \tag{42}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{43}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \tag{44}$$

Assume the following.

$$\begin{aligned}
 & \forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\
 & A\_27a. (p V0t)) \Leftrightarrow (p V0t)))
 \end{aligned} \tag{45}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (48)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))))) \quad (49)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (50)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (51)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (52)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.((ap(ap(ap(c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap(ap(ap(c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) V0t1) V1t2) = V1t2)))))) \end{aligned} \quad (53)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B))))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B))))))) \quad (54)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (55)$$

Assume the following.

$$(\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2R \in 2. (((((p V0P) \vee (p V1Q)) \Rightarrow (p V2R)) \Leftrightarrow (((p V0P) \Rightarrow (p V2R)) \wedge ((p V1Q) \Rightarrow (p V2R))))))) \quad (56)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (57)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (58)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27))))))) \quad (59)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) V1Q) V3x\_27) \\ & V5y\_27))))))))))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}). (\forall V1v \in \\ & A\_27a. ((\forall V2x \in A\_27a. ((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p ( \\ & ap V0f V1v)))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow \forall A\_27c. \\ & nonempty A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27a^{A\_27c}). \\ & (\forall V2x \in A\_27c. ((ap (ap (ap (c\_2Ecombin\_2Eo A\_27c A\_27b A\_27a) \\ & V0f) V1g) V2x) = (ap V0f (ap V1g V2x))))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned}
 & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow \\
 & \forall V0x \in (ty\_2Efcp\_2Ecart\ A_{27a}\ A_{27b}).(\forall V1y \in (ty\_2Efcp\_2Ecart\ \\
 & A_{27a}\ A_{27b}).((V0x = V1y) \Leftrightarrow (\forall V2i \in ty\_2Enum\_2Enum.((p\ (ap\ \\
 & (ap\ c\_2Eprim\_rec\_2E\_3C\ V2i)\ (ap\ (c\_2Efcp\_2Edimindex\ A_{27b})\ (\\
 & c\_2Ebool\_2Ethe\_value\ A_{27b})))) \Rightarrow ((ap\ (ap\ (c\_2Efcp\_2Efcp\_index\ \\
 & A_{27a}\ A_{27b})\ V0x)\ V2i) = (ap\ (ap\ (c\_2Efcp\_2Efcp\_index\ A_{27a}\ A_{27b})\ \\
 & V1y)\ V2i))))))) \\
 \end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
 & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow \\
 & \forall V0g \in (A_{27a}^{ty\_2Enum\_2Enum}).(\forall V1i \in ty\_2Enum\_2Enum. \\
 & ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V1i)\ (ap\ (c\_2Efcp\_2Edimindex\ A_{27b})\ (\\
 & (c\_2Ebool\_2Ethe\_value\ A_{27b})))) \Rightarrow ((ap\ (ap\ (c\_2Efcp\_2Efcp\_index\ \\
 & A_{27a}\ A_{27b})\ (ap\ (c\_2Efcp\_2EFCP\ A_{27a}\ A_{27b})\ V0g))\ V1i) = (ap\ V0g\ \\
 & V1i)))))) \\
 \end{aligned} \tag{64}$$

Assume the following.

$((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enum\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum.((\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A V6n) V7m))))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D V10n) V11m))))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEEXP V14n) c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))) \wedge ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2EEEXP V15n) V16m))))))) \wedge (((ap c\_2Enum\_2ESUC c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum.((ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enum\_2ESUC V17n))))))) \wedge (((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Eprim\_rec\_2EPRE V18n))))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum.((\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m))))))) \wedge ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL V23n)) \Leftrightarrow False))) \wedge ((\forall V24n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL V24n)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) V24n))))))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL V25n) V26m))))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E c\_2Enum\_2E0) V27n)) \Leftrightarrow False))) \wedge ((\forall V28n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL V28n)) c\_2Enum\_2E0) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) V28n))))))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.((\forall V30m \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E c\_2Enum\_2E0) V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) V29n))))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3D c\_2Enum\_2E0) V31n)) \Leftrightarrow True))) \wedge ((\forall V32n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C c\_2Enum\_2E0) V32n)) \Leftrightarrow False))) \wedge ((\forall V33n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3D c\_2Enum\_2E0) V33n)) \Leftrightarrow True))) \wedge ((\forall V34n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3D c\_2Enum\_2E0) V34n)) \Leftrightarrow False)))$

Assume the following.

$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ((c\_2Earthmetic\_2EZERO = (ap c\_2Earthmetic\_2EBIT1 V0n)) \Leftrightarrow False) \wedge (((ap c\_2Earthmetic\_2EBIT1 V0n) = c\_2Earthmetic\_2EZERO) \Leftrightarrow False) \wedge (((c\_2Earthmetic\_2EZERO = (ap c\_2Earthmetic\_2EBIT2 V0n)) \Leftrightarrow False) \wedge (((ap c\_2Earthmetic\_2EBIT2 V0n) = c\_2Earthmetic\_2EZERO) \Leftrightarrow False) \wedge (((ap c\_2Earthmetic\_2EBIT1 V0n) = (ap c\_2Earthmetic\_2EBIT2 V1m)) \Leftrightarrow False) \wedge (((ap c\_2Earthmetic\_2EBIT2 V0n) = (ap c\_2Earthmetic\_2EBIT1 V1m)) \Leftrightarrow False) \wedge (((ap c\_2Earthmetic\_2EBIT1 V0n) = (ap c\_2Earthmetic\_2EBIT1 V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap c\_2Earthmetic\_2EBIT2 V0n) = (ap c\_2Earthmetic\_2EBIT2 V1m)) \Leftrightarrow (V0n = V1m)))))))$

Assume the following.

$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Earithmetic\_2EZERO) V0n)) \leftrightarrow$   
 $True) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 V0n)) c\_2Earithmetic\_2EZERO)) \leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 V0n)) c\_2Earithmetic\_2EZERO)) \leftrightarrow False) \wedge$   
 $((((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \leftrightarrow (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1m) V0n)))) \wedge ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) V1m)))))))))))))))$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((p (ap c\_2Earithmetic\_2EEVEN c\_2Earithmetic\_2EZERO)) \wedge \\
& \quad ((p (ap c\_2Earithmetic\_2EEVEN (ap c\_2Earithmetic\_2EBIT2 V0n))) \wedge \\
& \quad ((\neg(p (ap c\_2Earithmetic\_2EEVEN (ap c\_2Earithmetic\_2EBIT1 V0n)))) \wedge \\
& \quad ((\neg(p (ap c\_2Earithmetic\_2EODD c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad \neg(p (ap c\_2Earithmetic\_2EODD (ap c\_2Earithmetic\_2EBIT2 V0n))) \wedge \\
& \quad (p (ap c\_2Earithmetic\_2EODD (ap c\_2Earithmetic\_2EBIT1 V0n)))))))))) \\
& \hspace{10em} (68)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Ebit\_2EDIV\_2EXP V0n) \\
& \quad c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2x \in ty\_2Enum\_2Enum. ((ap (ap c\_2Ebit\_2EDIV\_2EXP V1n) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2x)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW c\_2Enumeral\_bit\_2EiDIV2) \\
& \quad V1n) V2x)))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty\_2Enum\_2Enum.((ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW \\
& c\_2Enumeral\_bit\_2EiDIV2) c\_2Enum\_2E0) V0x) = V0x)) \wedge ((\forall V1y \in \\
& ty\_2Enum\_2Enum.((ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW c\_2Enumeral\_bit\_2EiDIV2) \\
& V1y) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V2n \in ty\_2Enum\_2Enum. \\
& (\forall V3x \in ty\_2Enum\_2Enum.((ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW \\
& c\_2Enumeral\_bit\_2EiDIV2) (ap c\_2Earithmetic\_2ENUMERAL (ap \\
& c\_2Earithmetic\_2EBIT1 V2n))) V3x) = (ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW \\
& (ap c\_2Enumeral\_bit\_2EFDUB c\_2Enumeral\_bit\_2EiDIV2)) (ap \\
& c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Enumeral\_bit\_2EiDIV2 \\
& V3x)))) \wedge (\forall V4n \in ty\_2Enum\_2Enum.(\forall V5x \in ty\_2Enum\_2Enum. \\
& ((ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW c\_2Enumeral\_bit\_2EiDIV2) \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 V4n))) \\
& V5x) = (ap (ap (ap c\_2Enumeral\_bit\_2ESFUNPOW (ap c\_2Enumeral\_bit\_2EFDUB \\
& c\_2Enumeral\_bit\_2EiDIV2)) (ap c\_2Earithmetic\_2ENUMERAL V4n)) \\
& (ap c\_2Enumeral\_bit\_2EiDIV2 (ap c\_2Enumeral\_bit\_2EiDIV2 V5x))))))) \\
& (70)
\end{aligned}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V0n) c\_2Enum\_2E0)))) \quad (71)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) (ap c\_2Enum\_2ESUC V1n))) \Leftrightarrow ( \\
& (V0m = V1n) \vee (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n))))))
\end{aligned} \quad (72)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (73)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (74)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \quad (75)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \quad (76)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (77)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
 & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
 & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{78}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
 & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{79}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
 & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
 & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \\
 \end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned}
 & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) \\
 & (ap (c\_2Efcp\_2Edimindex A\_27a) (c\_2Ebool\_2Ethe\_value A\_27a))) \\
 \end{aligned} \tag{82}$$

Assume the following.

$$\begin{aligned}
 & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \\
 & \forall V0w \in (ty\_2Efcp\_2Ecart 2 A\_27a). (\forall V1i \in ty\_2Enum\_2Enum. \\
 & ((p (ap (ap c\_2Eprim\_rec\_2E\_3C V1i) (ap (c\_2Efcp\_2Edimindex A\_27b) \\
 & (c\_2Ebool\_2Ethe\_value A\_27b)))) \Rightarrow ((p (ap (ap (c\_2Efcp\_2Efcp\_index \\
 & 2 A\_27b) (ap (c\_2Ewords\_2Ew2w A\_27a A\_27b) V0w)) V1i)) \Leftrightarrow ((p (ap \\
 & (ap c\_2Eprim\_rec\_2E\_3C V1i) (ap (c\_2Efcp\_2Edimindex A\_27a) ( \\
 & c\_2Ebool\_2Ethe\_value A\_27a))) \wedge (p (ap (ap (c\_2Efcp\_2Efcp\_index \\
 & 2 A\_27a) V0w) V1i))))))) \\
 \end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. nonempty\ A_{27a} \Rightarrow (\forall V0index\_20too\_20large \in \\
& 2. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2i \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap (c\_2Efcp\_2Efcp\_index 2 A_{27a}) (ap (c\_2Ewords\_2En2w \\
& A_{27a}) V1n)) V2i)) \Leftrightarrow (p (ap (ap (ap (c\_2Ebool\_2ECOND 2) (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V2i) (ap (c\_2Efcp\_2Edimindex A_{27a}) (c\_2Ebool\_2Eth\_value A_{27a}))) \\
& (ap (ap c\_2Ebit\_2EBIT V2i) V1n)) (ap (ap (ap (c\_2Ecombin\_2EFAIL \\
& ((2^{ty\_2Enum\_2Enum}(ty\_2Efcp\_2Ecart 2 A_{27a})) 2) (c\_2Efcp\_2Efcp\_index \\
& 2 A_{27a})) V0index\_20too\_20large) (ap (c\_2Ewords\_2En2w A_{27a}) \\
& V1n)) V2i)))))))
\end{aligned} \tag{84}$$

Assume the following.

$$((ap (c\_2Efcp\_2Edimindex ty\_2Eone\_2Eone) (c\_2Ebool\_2Eth\_value \\
ty\_2Eone\_2Eone)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
c\_2Earithmetic\_2EZERO))) \tag{85}$$

### Theorem 1

$$\begin{aligned}
& (\forall V0x \in (ty\_2Efcp\_2Ecart 2 ty\_2Eone\_2Eone). (((ap (c\_2Ewords\_2En2w \\
& ty\_2Eone\_2Eone) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO))) = V0x) \vee (\neg((ap (ap (ap (c\_2Ewords\_2Eword\_extract \\
& ty\_2Eone\_2Eone ty\_2Eone\_2Eone) c\_2Enum\_2E0) c\_2Enum\_2E0) V0x) = \\
& (ap (c\_2Ewords\_2En2w ty\_2Eone\_2Eone) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))))
\end{aligned}$$