thm_2EOmega_2Ealternative__equivalence (TMb5U8sn8Q8gofBXDu5m7ShtR9wSATkW4NL)

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Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \quad (1)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (3)$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (4)$$

Let $c\_2EOmega\_2Enightmare : \iota$ be given. Assume the following.

$$c\_2EOmega\_2Enightmare \in (((((2^{(ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint))})^{(ty} \quad (5)$$

Let $c\_2EOmega\_2Eevallower : \iota$ be given. Assume the following.

$$c\_2EOmega\_2Eevallower \in ((2^{(ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint))})^{ty\_2Ein} \quad (6)$$

Let $c\_2EOmega\_2Eevalupper : \iota$ be given. Assume the following.

$$c\_2EOmega\_2Eevalupper \in ((2^{(ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint))})^{ty\_2Ein} \quad (7)$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (8)$$

Definition 1 *We define* $\mathsf{c\_2Emin\_2E\_3D}$ *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota{\Rightarrow}\iota$.

Definition 2 *We define* $\mathsf{c\_2Ebool\_2ET}$ *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* $\mathsf{c\_2Ebool\_2E\_21}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* $\mathsf{c\_2Ebool\_2EF}$ *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* $\mathsf{c\_2Emin\_2E\_3D\_3D\_3E}$ *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P{\Rightarrow}p\ Q)$ *of type* ι.

Definition 6 *We define* $\mathsf{c\_2Ebool\_2E\_7E}$ *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* $\mathsf{c\_2Ebool\_2E\_2F\_5C}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{9}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{10}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{11}$$

Definition 8 *We define* $\mathsf{c\_2Enum\_2ESUC}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* $\mathsf{c\_2Emin\_2E\_40}$ *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A{\wedge}p$ *of type* $\iota{\Rightarrow}\iota$.

Definition 10 *We define* $\mathsf{c\_2Ebool\_2E\_3F}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 11 *We define* $\mathsf{c\_2Eprim\_\_rec\_2E\_3C}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* $\mathsf{c\_2Ebool\_2E\_5C\_2F}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 13 *We define* $\mathsf{c\_2Earithmetic\_2E\_3C\_3D}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2EOmega\_2Ereal\_\_shadow : \iota$ be given. Assume the following.

$$c\_2EOmega\_2Ereal\_\_shadow \in ((2^{(ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint))})^{(ty} \tag{12}$$

Let $c\_2EOmega\_2Edark\_\_shadow : \iota$ be given. Assume the following.

$$c\_2EOmega\_2Edark\_\_shadow \in ((2^{(ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint))})^{(ty} \tag{13}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{14}$$

Definition 14 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 15 *We define* c_2EOmega_2Efst__nzero *to be* $\lambda A\_27a : \iota.\lambda V0x \in (ty\_2Epair\_2Eprod\ ty\_2Enum.$

Let $c\_2Elist\_2EEVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEVERY\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \tag{15}$$

Assume the following.

$$(\forall V0rs \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint)).(\forall V1x \in ty\_2Einteger\_2Eint.(\forall V2uppers \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint)).(\forall V3lowers \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint)).(\forall V4c \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ (ap\ (ap\ (ap\ c\_2EOmega\_2Enightmare\ V1x)\ V4c)\ V2uppers)\ V3lowers)\ V0rs)) \Rightarrow ((p\ (ap\ (ap\ c\_2EOmega\_2Eevalupper\ V1x)\ V2uppers)) \wedge (p\ (ap\ (ap\ c\_2EOmega\_2Eevallower\ V1x)\ V3lowers)))))))) \tag{16}$$

Assume the following.

$$(\forall V0uppers \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint)).(\forall V1lowers \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint)).(\forall V2m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint))\ (\mathsf{c\_2EOmega\_2Efst\_\_nzero}\ ty\_2Einteger\_2Eint))\ V0uppers)) \wedge ((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint))\ (\mathsf{c\_2EOmega\_2Efst\_\_nzero}\ ty\_2Einteger\_2Eint))\ V1lowers)) \wedge (p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint))\ (\lambda V3p \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint).(ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (c\_2Epair\_2EFST\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint)\ V3p))\ V2m)))\ V0uppers)))) \Rightarrow ((\exists V4x \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ c\_2EOmega\_2Eevalupper\ V4x)\ V0uppers)) \wedge (p\ (ap\ (ap\ c\_2EOmega\_2Eevallower\ V4x)\ V1lowers)))) \Leftrightarrow ((p\ (ap\ (ap\ c\_2EOmega\_2Ereal\_\_shadow\ V0uppers)\ V1lowers)) \wedge ((p\ (ap\ (ap\ c\_2EOmega\_2Edark\_\_shadow\ V0uppers)\ V1lowers)) \vee (\exists V5x \in ty\_2Einteger\_2Eint.(p\ (ap\ (ap\ (ap\ (ap\ c\_2EOmega\_2Enightmare\ V5x)\ V2m)\ V0uppers)\ V1lowers)\ V1lowers)))))))))) \tag{17}$$

Assume the following.

$$(\forall V0uppers \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint)).(\forall V1lowers \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint)).(((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint))\ (\mathsf{c\_2EOmega\_2Efst\_\_nzero}\ ty\_2Einteger\_2Eint))\ V0uppers))\wedge((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint))\ (\mathsf{c\_2EOmega\_2Efst\_\_nzero}\ ty\_2Einteger\_2Eint))\ V1lowers))\wedge(p\ (ap\ (ap\ c\_2EOmega\_2Edark\_\_shadow\ V0uppers)\ V1lowers))))\Rightarrow(p\ (ap\ (ap\ c\_2EOmega\_2Ereal\_\_shadow\ V0uppers)\ V1lowers))))) \tag{18}$$

Assume the following.

$$True \tag{19}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t)\Rightarrow False)\Rightarrow(\neg(p\ V0t)))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t))\Rightarrow((p\ V0t)\Rightarrow False))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow(p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge(((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True\vee(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\vee True)\Leftrightarrow True)\wedge(((False\vee(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\vee False)\Leftrightarrow(p\ V0t))\wedge(((p\ V0t)\vee(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge(((p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{25}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge((\neg False)\Leftrightarrow True))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x))))\Leftrightarrow(\forall V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.(((\exists V2x \in A\_27a.(p\ (ap\ V0P\ V2x)))\vee(p\ V1Q))\Leftrightarrow(\exists V3x \in A\_27a.((p\ (ap\ V0P\ V3x))\vee(p\ V1Q)))))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P)\vee(\exists V2x \in A\_27a.(p\ (ap\ V1Q\ V2x))))\Leftrightarrow(\exists V3x \in A\_27a.((p\ V0P)\vee(p\ (ap\ V1Q\ V3x))))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.((\exists V2x \in A\_27a.((p\ (ap\ V0P\ V2x))\wedge(p\ V1Q)))\Leftrightarrow((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x)))\wedge(p\ V1Q))))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\exists V2x \in A\_27a.((p\ V0P)\wedge(p\ (ap\ V1Q\ V2x))))\Leftrightarrow((p\ V0P)\wedge(\exists V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{33}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A)\vee((p\ V1B)\vee(p\ V2C)))\Leftrightarrow(((p\ V0A)\vee(p\ V1B))\vee(p\ V2C)))))) \tag{34}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A)\vee(p\ V1B))\Leftrightarrow((p\ V1B)\vee(p\ V0A))))) \tag{35}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\wedge(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\vee(\neg(p\ V1B))))\wedge((\neg((p\ V0A)\vee(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\wedge(\neg(p\ V1B))))))) \tag{36}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{37}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{38}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow(((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{39}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{40}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{41}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{42}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{43}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{44}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{45}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))) \tag{46}$$

Theorem 1

$$(\forall V0uppers \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint)).(\forall V1lowers \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint)).(\forall V2m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint))\ (\mathsf{c\_2EOmega\_2Efst\_\_nzero}\ ty\_2Einteger\_2Eint))\ V0uppers)) \wedge ((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint))\ (\mathsf{c\_2EOmega\_2Efst\_\_nzero}\ ty\_2Einteger\_2Eint))\ V1lowers)) \wedge (p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint))\ (\lambda V3p \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint).(ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (c\_2Epair\_2EFST\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint)\ V3p))\ V2m)))\ V0uppers)))) \Rightarrow ((\exists V4x \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ c\_2EOmega\_2Eevalupper\ V4x)\ V0uppers)) \wedge (p\ (ap\ (ap\ c\_2EOmega\_2Eevallower\ V4x)\ V1lowers)))) \Leftrightarrow ((p\ (ap\ (ap\ c\_2EOmega\_2Edark\_\_shadow\ V0uppers)\ V1lowers)) \vee (\exists V5x \in ty\_2Einteger\_2Eint.(p\ (ap\ (ap\ (ap\ (ap\ (ap\ c\_2EOmega\_2Enightmare\ V5x)\ V2m)\ V0uppers)\ V1lowers)\ V1lowers)))))))))$$