thm_2EOmega_2Eeval__step__extra4 (TMcvhq4dG1kABSsio1gJQw21KZLwg1s9ajZ)

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Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{3}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{4}$$

Let $c\_2EOmega\_2Eevallower : \iota$ be given. Assume the following.

$$c\_2EOmega\_2Eevallower \in ((2^{(ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint))})^{ty\_2Ein} \tag{5}$$

Let $c\_2EOmega\_2Eevalupper : \iota$ be given. Assume the following.

$$c\_2EOmega\_2Eevalupper \in ((2^{(ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint))})^{ty\_2Ein} \tag{6}$$

Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \quad (8)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (9)$$

Theorem 1

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1ups \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint)).(\forall V2lows \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Einteger\_2Eint)).(\forall V3ex \in 2.(\forall V4ex\_27 \in 2.(\forall V5p \in 2.(((((p\ (ap\ (ap\ c\_2EOmega\_2Eevalupper\ V0x)\ V1ups)) \wedge (p\ (ap\ (ap\ c\_2EOmega\_2Eevallower\ V0x)\ V2lows))) \wedge (p\ V3ex)) \wedge ((p\ V4ex\_27) \wedge (p\ V5p))) \Leftrightarrow ((((p\ (ap\ (ap\ c\_2EOmega\_2Eevalupper\ V0x)\ V1ups)) \wedge (p\ (ap\ (ap\ c\_2EOmega\_2Eevallower\ V0x)\ V2lows))) \wedge ((p\ V3ex) \wedge (p\ V4ex\_27))) \wedge (p\ V5p)))))))))$$