

# thm\_2EOmega\_\_Automata\_2ECO\_\_BUECHI\_\_CONJ\_\_CLOSURE (TMWL6TkTqrd9k1gJpDyMpCZ3xswpKy5VAsR)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow P \Rightarrow Q)$  of type  $\iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (2)$$

**Definition 6** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \quad (3)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (4)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (5)$$

**Definition 7** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 8** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (6)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (7)$$

**Definition 9** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num\ (ap\ c\_2Enum\_2ESUC\_REP\ V0m))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (8)$$

**Definition 10** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V0n)$

**Definition 11** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

**Definition 12** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2. V0t))$ .

**Definition 13** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_21\ 2)\ V0t))$

**Definition 14** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P\ x) \text{ then } (\lambda x. x \in A \wedge P\ x) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 15** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A. 2^A : \iota. (\lambda V0P \in (2^A)^{2^A}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ V0P)\ V0P))$

**Definition 16** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m\ V1n)\ V0m)$

**Definition 17** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2. (ap\ (ap\ c\_2Emin\_2E\_40\ V2t)\ V2t))\ V1t2)\ V0t1))$

**Definition 18** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & \quad \forall V2p \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)\ V2p) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ & \quad \quad (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)\ V2p)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & \quad (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V1n)) \vee (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V1n)\ V0m)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1n) V0m)) \Rightarrow (\exists V2p \in ty\_2Enum\_2Enum. \\
& (V0m = (ap (ap c\_2Earithmetic\_2E\_2B V1n) (ap (ap c\_2Earithmetic\_2E\_2B \\
& V2p) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO))))))))))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \Rightarrow (\exists V2p \in ty\_2Enum\_2Enum. \\
& (V1n = (ap (ap c\_2Earithmetic\_2E\_2B V0m) V2p))))))
\end{aligned} \tag{12}$$

Assume the following.

$$True \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))
\end{aligned} \tag{14}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \tag{16}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\
& A\_27a. (p V0t)) \Leftrightarrow (p V0t)))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p V0t))))))
\end{aligned} \tag{20}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b1.nonempty\ A\_27b1 \Rightarrow \\
& \quad \forall A\_27b2.nonempty\ A\_27b2 \Rightarrow (\forall V0Phi\_I1 \in (2^{A\_27b1}). \\
& (\forall V1t0 \in ty\_2Enum\_2Enum. (\forall V2Phi\_R1 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b1)}). \\
& (\forall V3i \in (A\_27a^{ty\_2Enum\_2Enum}). (\forall V4Psi1 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b1)}). \\
& (\forall V5Phi\_I2 \in (2^{A\_27b2}). (\forall V6Phi\_R2 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b2)}). \\
& (\forall V7Psi2 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b2)}). (((\exists V8q1 \in \\
& (A\_27b1^{ty\_2Enum\_2Enum}). ((p\ (ap\ V0Phi\_I1\ (ap\ V8q1\ V1t0))) \wedge (( \\
& \forall V9t \in ty\_2Enum\_2Enum. (p\ (ap\ V2Phi\_R1\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& A\_27a\ A\_27b1)\ (ap\ V3i\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V9t)\ V1t0))) \\
& (ap\ V8q1\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V9t)\ V1t0)))))) \wedge (\exists V10t1 \in \\
& ty\_2Enum\_2Enum. (\forall V11t2 \in ty\_2Enum\_2Enum. (p\ (ap\ V4Psi1 \\
& (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b1)\ (ap\ V3i\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V10t1)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V11t2)\ V1t0))))\ (ap\ V8q1\ (ap \\
& (ap\ c\_2Earithmetic\_2E\_2B\ V10t1)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V11t2)\ V1t0)))))))))) \wedge (\exists V12q2 \in (A\_27b2^{ty\_2Enum\_2Enum}). \\
& ((p\ (ap\ V5Phi\_I2\ (ap\ V12q2\ V1t0))) \wedge ((\forall V13t \in ty\_2Enum\_2Enum. \\
& (p\ (ap\ V6Phi\_R2\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b2)\ (ap\ V3i\ (ap \\
& (ap\ c\_2Earithmetic\_2E\_2B\ V13t)\ V1t0)))\ (ap\ V12q2\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V13t)\ V1t0)))))) \wedge (\exists V14t1 \in ty\_2Enum\_2Enum. (\forall V15t2 \in \\
& ty\_2Enum\_2Enum. (p\ (ap\ V7Psi2\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b2) \\
& (ap\ V3i\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V14t1)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V15t2)\ V1t0))))\ (ap\ V12q2\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V14t1)\ ( \\
& ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V15t2)\ V1t0)))))))))) \Leftrightarrow (\exists V16q1 \in \\
& (A\_27b1^{ty\_2Enum\_2Enum}). (\exists V17q2 \in (A\_27b2^{ty\_2Enum\_2Enum}). \\
& (((p\ (ap\ V0Phi\_I1\ (ap\ V16q1\ V1t0))) \wedge (p\ (ap\ V5Phi\_I2\ (ap\ V17q2\ V1t0)))) \wedge \\
& ((\forall V18t \in ty\_2Enum\_2Enum. ((p\ (ap\ V2Phi\_R1\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& A\_27a\ A\_27b1)\ (ap\ V3i\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V18t)\ V1t0))) \\
& (ap\ V16q1\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V18t)\ V1t0)))) \wedge (p\ (ap\ V6Phi\_R2 \\
& (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b2)\ (ap\ V3i\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V18t)\ V1t0)))\ (ap\ V17q2\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V18t)\ V1t0)))))) \wedge \\
& (\exists V19t1 \in ty\_2Enum\_2Enum. (\forall V20t2 \in ty\_2Enum\_2Enum. \\
& ((p\ (ap\ V4Psi1\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b1)\ (ap\ V3i\ (ap\ ( \\
& ap\ c\_2Earithmetic\_2E\_2B\ V19t1)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V20t2) \\
& V1t0))))\ (ap\ V16q1\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V19t1)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V20t2)\ V1t0)))))) \wedge (p\ (ap\ V7Psi2\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b2) \\
& (ap\ V3i\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V19t1)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V20t2)\ V1t0))))\ (ap\ V17q2\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V19t1)\ ( \\
& ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V20t2)\ V1t0))))))))))))))
\end{aligned}$$