thm_2EOmega__Automata_2ECO__BUECHI__DISJ__CLOSURE (TMcwcSCe-DUsnZHAMjvN7eqshRnWwKB2mo5R)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 6 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A$**.if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{3}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{4}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{5}$$

Definition 9 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 10 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{6}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{7}$$

Definition 11 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{8}$$

Definition 12 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 13 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 14 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 15 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 16 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2E}$

Assume the following.

$$\begin{gathered}(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \text{c\_2Enum\_2E0})\ V0m) = V0m) \wedge (((ap\ (\\ ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \text{c\_2Enum\_2E0}) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ (ap\ \text{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \text{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\ V0m)\ V1n))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \text{c\_2Enum\_2ESUC} \\ V1n)) = (ap\ \text{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))))\end{gathered} \tag{9}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))) \tag{10}$$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27b.((ap\ (\lambda V2x \in A\_27b.\ V0t1)\ V1t2) = V0t1))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{18}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \tag{20}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0})) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n))))) \tag{21}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b1.nonempty\ A\_27b1 \Rightarrow \forall A\_27b2.nonempty\ A\_27b2 \Rightarrow (\forall V0Phi\_\_I1 \in (2^{A\_27b1}).$$
$$(\forall V1t0 \in ty\_2Enum\_2Enum.(\forall V2Phi\_\_R1 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b1)}).$$
$$(\forall V3i \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V4Psi1 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b1)}).$$
$$(\forall V5Phi\_\_I2 \in (2^{A\_27b2}).(\forall V6Phi\_\_R2 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b2)}).$$
$$(\forall V7Psi2 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b2)}).(\forall V8t \in$$
$$ty\_2Enum\_2Enum.(((\exists V9q1 \in (A\_27b1^{ty\_2Enum\_2Enum}).((p\ (ap\ V0Phi\_\_I1\ (ap\ V9q1\ V1t0)))\wedge((\forall V10t \in ty\_2Enum\_2Enum.$$
$$(p\ (ap\ V2Phi\_\_R1\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b1)\ (ap\ V3i\ (ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ V10t)\ V1t0)))\ (ap\ V9q1\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V10t)\ V1t0))))))\wedge(\exists V11t1 \in ty\_2Enum\_2Enum.(\forall V12t2 \in$$
$$ty\_2Enum\_2Enum.(p\ (ap\ V4Psi1\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b1)$$
$$(ap\ V3i\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V11t1)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V12t2)\ V1t0))))\ (ap\ V9q1\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V11t1)\ (ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ V12t2)\ V1t0)))))))))))\vee(\exists V13q2 \in$$
$$(A\_27b2^{ty\_2Enum\_2Enum}).((p\ (ap\ V5Phi\_\_I2\ (ap\ V13q2\ V1t0)))\wedge($$
$$(\forall V14t \in ty\_2Enum\_2Enum.(p\ (ap\ V6Phi\_\_R2\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}$$
$$A\_27a\ A\_27b2)\ (ap\ V3i\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V14t)\ V1t0)))$$
$$(ap\ V13q2\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V14t)\ V1t0))))))\wedge(\exists V15t1 \in$$
$$ty\_2Enum\_2Enum.(\forall V16t2 \in ty\_2Enum\_2Enum.(p\ (ap\ V7Psi2$$
$$(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b2)\ (ap\ V3i\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V15t1)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V16t2)\ V1t0))))\ (ap\ V13q2\ ($$
$$ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V15t1)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V16t2)\ V1t0)))))))))))\Leftrightarrow(\exists V17p \in (2^{ty\_2Enum\_2Enum}).$$
$$(\exists V18q1 \in (A\_27b1^{ty\_2Enum\_2Enum}).(\exists V19q2 \in (A\_27b2^{ty\_2Enum\_2Enum}).$$
$$((((\neg(p\ (ap\ V17p\ V1t0)))\wedge(p\ (ap\ V0Phi\_\_I1\ (ap\ V18q1\ V1t0))))\vee(($$
$$p\ (ap\ V17p\ V1t0))\wedge(p\ (ap\ V5Phi\_\_I2\ (ap\ V19q2\ V1t0)))))\wedge((\forall V20t \in$$
$$ty\_2Enum\_2Enum.(((\neg(p\ (ap\ V17p\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V20t)$$
$$V1t0))))\wedge((p\ (ap\ V2Phi\_\_R1\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b1)$$
$$(ap\ V3i\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V20t)\ V1t0)))\ (ap\ V18q1\ (ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ V20t)\ V1t0)))))\wedge(\neg(p\ (ap\ V17p\ (ap\ (ap$$
$$c\_2Earithmetic\_2E\_2B\ V20t)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1t0)$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))))))\vee$$
$$((p\ (ap\ V17p\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V20t)\ V1t0)))\wedge((p\ (ap$$
$$V6Phi\_\_R2\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b2)\ (ap\ V3i\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V20t)\ V1t0)))\ (ap\ V19q2\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V20t)\ V1t0)))))\wedge$$
$$(p\ (ap\ V17p\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V20t)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V1t0)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO}))))))))))\wedge(\exists V21t1 \in ty\_2Enum\_2Enum.$$
$$(\forall V22t2 \in ty\_2Enum\_2Enum.(((\neg(p\ (ap\ V17p\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V8t)\ V1t0))))\wedge(p\ (ap\ V4Psi1\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b1)$$
$$(ap\ V3i\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V21t1)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V22t2)\ V1t0))))\ (ap\ V18q1\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V21t1)\ ($$
$$ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V22t2)\ V1t0)))))))\vee((p\ (ap\ V17p\ (ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ V8t)\ V1t0)))\wedge(p\ (ap\ V7Psi2\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}$$
$$A\_27a\ A\_27b2)\ (ap\ V3i\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V21t1)\ (ap\ (ap$$
$$c\_2Earithmetic\_2E\_2B\ V22t2)\ V1t0))))\ (ap\ V19q2\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V21t1)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V22t2)\ V1t0))))))))))))))))))))))))$$