

thm\_2EOmega\_\_Automata\_2EOMEGA\_\_CONJ\_\_CLOSURE  
 (TMdCn-  
 pDbEbL6HqYEC7Xq33d7C5mBFPuLhkW)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow Q)$  of type  $\iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{3}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{4}$$

**Definition 6** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p$   
of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

**Definition 9** We define  $c\_2Ebool\_2E\_EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 10** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E$

Assume the following.

$$True \tag{5}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{7}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \tag{9}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b1.nonempty\ A\_27b1 \Rightarrow \\
& \quad \forall A\_27b2.nonempty\ A\_27b2 \Rightarrow (\forall V0Phi\_I1 \in (2^{A\_27b1}). \\
& (\forall V1t0 \in ty\_2Enum\_2Enum. (\forall V2Phi\_R1 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b1)}). \\
& (\forall V3i \in (A\_27a^{ty\_2Enum\_2Enum}). (\forall V4Psi1 \in (2^{(ty\_2Epair\_2Eprod\ (A\_27a^{ty\_2Enum\_2Enum})\ (A\_27b1^{ty\_2Enum\_2Enum})}). \\
& (\forall V5Phi\_I2 \in (2^{A\_27b2}). (\forall V6Phi\_R2 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b2)}). \\
& (\forall V7Psi2 \in (2^{(ty\_2Epair\_2Eprod\ (A\_27a^{ty\_2Enum\_2Enum})\ (A\_27b2^{ty\_2Enum\_2Enum})}). \\
& \quad (((\exists V8q1 \in (A\_27b1^{ty\_2Enum\_2Enum}). ((p\ (ap\ V0Phi\_I1\ (ap \\
& \quad V8q1\ V1t0))) \wedge ((\forall V9t \in ty\_2Enum\_2Enum. (p\ (ap\ V2Phi\_R1\ ( \\
& \quad ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b1)\ (ap\ V3i\ (ap\ c\_2Earithmetic\_2E\_2B \\
& \quad V9t)\ V1t0)))\ (ap\ V8q1\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V9t)\ V1t0)))))) \wedge \\
& (p\ (ap\ V4Psi1\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (A\_27a^{ty\_2Enum\_2Enum})\ (A\_27b1^{ty\_2Enum\_2Enum})) \\
& \quad V3i)\ V8q1)))))) \wedge (\exists V10q2 \in (A\_27b2^{ty\_2Enum\_2Enum}). ((p \\
& \quad (ap\ V5Phi\_I2\ (ap\ V10q2\ V1t0))) \wedge ((\forall V11t \in ty\_2Enum\_2Enum. \\
& \quad (p\ (ap\ V6Phi\_R2\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b2)\ (ap\ V3i\ (ap \\
& \quad (ap\ c\_2Earithmetic\_2E\_2B\ V11t)\ V1t0)))\ (ap\ V10q2\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& \quad V11t)\ V1t0)))))) \wedge (p\ (ap\ V7Psi2\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (A\_27a^{ty\_2Enum\_2Enum}) \\
& \quad (A\_27b2^{ty\_2Enum\_2Enum})\ V3i)\ V10q2)))))) \Leftrightarrow (\exists V12q1 \in ( \\
& \quad A\_27b1^{ty\_2Enum\_2Enum}). (\exists V13q2 \in (A\_27b2^{ty\_2Enum\_2Enum}). \\
& \quad (((p\ (ap\ V0Phi\_I1\ (ap\ V12q1\ V1t0))) \wedge (p\ (ap\ V5Phi\_I2\ (ap\ V13q2\ V1t0)))) \wedge \\
& \quad ((\forall V14t \in ty\_2Enum\_2Enum. ((p\ (ap\ V2Phi\_R1\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& \quad A\_27a\ A\_27b1)\ (ap\ V3i\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V14t)\ V1t0))) \\
& \quad (ap\ V12q1\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V14t)\ V1t0)))))) \wedge (p\ (ap\ V6Phi\_R2 \\
& \quad (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b2)\ (ap\ V3i\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& \quad V14t)\ V1t0)))\ (ap\ V13q2\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V14t)\ V1t0)))))) \wedge \\
& \quad ((p\ (ap\ V4Psi1\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ (A\_27a^{ty\_2Enum\_2Enum})\ ( \\
& \quad A\_27b1^{ty\_2Enum\_2Enum})\ V3i)\ V12q1))) \wedge (p\ (ap\ V7Psi2\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\
& \quad (A\_27a^{ty\_2Enum\_2Enum})\ (A\_27b2^{ty\_2Enum\_2Enum})\ V3i)\ V13q2)))))))))
\end{aligned}$$