

thm_2EOmega__Automata_2EOMEGA__DISJ__CLOSURE
 (TME_n-
 rpS98yzPpMxsb7NDPMYBMyfJwXkYxz4)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1t1 \in 2. (\lambda V2t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. t2)))))))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. t2))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod \\ & \quad A0 A1) \end{aligned} \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ & \quad A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \tag{2}$$

Definition 6 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2Eprod A_27a A_27b) (inj_o (x, y)))$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40 A_27a) (c_2Epair_2E_2C A_27a A_27b))))$

Let $c_2Enum_2ZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (3)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

nonempty *ty_2Enum_2Enum* (4)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (5)$$

Definition 9 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 10 We define $\text{c_2Earithmetic_2EZERO}$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^\omega) \quad (7)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Definition 12 We define `c_2Earthmetic_2EBIT1` to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earthmetic\ 0\ n)\ V)$

Definition 13 We define `c_2Earithmetic_2ENUMERAL` to be $\lambda V0x \in ty_Enum_2Enum. V0x$.

Definition 14 We define $c_{\text{Ebool_2E_5C_2F}}$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_{\text{Ebool_2E_21}}\ 2)\ (\lambda V2t \in$

Definition 15 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t\ t \in 2.V0t))$.

Definition 16 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3D\ t\ 0)\ t))$

Assume the following.

(All)

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap c_2Enum_2ESUC V0m) = (ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \quad (10)$$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (14)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow & \forall A_27b.\text{nonempty } A_27b \Rightarrow \\ \forall V0t1 \in A_27a.(\forall V1t2 \in A_27b.(& (ap (\lambda V2x \in A_27b. \\ V0t1) V1t2) = V0t1))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow & (\forall V0t \in 2.((\forall V1x \in \\ A_27a.(p V0t) \Leftrightarrow (p V0t))) \end{aligned} \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (17)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (19)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (21)$$

Assume the following.

$$(\forall V0P \in (2^{ty_2Enum_2Enum}).(((p (ap V0P c_2Enum_2E0)) \wedge (\forall V1n \in ty_2Enum_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c_2Enum_2ESUC V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p (ap V0P V2n))))) \quad (22)$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b1.\text{nonempty } A_27b1 \Rightarrow \\
& \quad \forall A_27b2.\text{nonempty } A_27b2 \Rightarrow (\forall V0Phi_I1 \in (2^{A_27b1}). \\
& \quad (\forall V1t0 \in ty_2Enum_2Enum.(\forall V2Phi_R1 \in (2^{(ty_2Epair_2Eprod \ A_27a \ A_27b1)}). \\
& \quad (\forall V3i \in (A_27a^{ty_2Enum_2Enum}).(\forall V4Psi1 \in (2^{(ty_2Epair_2Eprod \ (A_27a^{ty_2Enum_2Enum}) \ (A_27b1^{ty_2Enum_2Enum}))}. \\
& \quad (\forall V5Phi_I2 \in (2^{A_27b2}).(\forall V6Phi_R2 \in (2^{(ty_2Epair_2Eprod \ A_27a \ A_27b2)}). \\
& \quad (\forall V7Psi2 \in (2^{(ty_2Epair_2Eprod \ (A_27a^{ty_2Enum_2Enum}) \ (A_27b2^{ty_2Enum_2Enum}))}. \\
& \quad (((\exists V8q1 \in (A_27b1^{ty_2Enum_2Enum})).((p (ap V0Phi_I1 (ap \\
& \quad V8q1 V1t0))) \wedge ((\forall V9t \in ty_2Enum_2Enum.(p (ap V2Phi_R1 (\\
& \quad ap (ap (c_2Epair_2E_2C A_27a A_27b1) (ap V3i (ap (ap c_2Earithmetic_2E_2B \\
& \quad V9t) V1t0))) (ap V8q1 (ap (ap c_2Earithmetic_2E_2B V9t) V1t0))))))) \wedge \\
& \quad (p (ap V4Psi1 (ap (ap (c_2Epair_2E_2C (A_27a^{ty_2Enum_2Enum}) (A_27b1^{ty_2Enum_2Enum})) \\
& \quad V3i) V8q1)))))) \vee (\exists V10q2 \in (A_27b2^{ty_2Enum_2Enum})).((p \\
& \quad (ap V5Phi_I2 (ap V10q2 V1t0))) \wedge ((\forall V11t \in ty_2Enum_2Enum. \\
& \quad (p (ap V6Phi_R2 (ap (ap (c_2Epair_2E_2C A_27a A_27b2) (ap V3i (ap \\
& \quad (ap c_2Earithmetic_2E_2B V11t) V1t0))) (ap V10q2 (ap (ap c_2Earithmetic_2E_2B \\
& \quad V11t) V1t0)))))) \wedge (p (ap V7Psi2 (ap (ap (c_2Epair_2E_2C (A_27a^{ty_2Enum_2Enum}) \\
& \quad (A_27b2^{ty_2Enum_2Enum})) V3i) V10q2)))))) \Leftrightarrow (\exists V12p \in (2^{ty_2Enum_2Enum}). \\
& \quad (\exists V13q1 \in (A_27b1^{ty_2Enum_2Enum})).(\exists V14q2 \in (A_27b2^{ty_2Enum_2Enum}). \\
& \quad (((((\neg(p (ap V12p V1t0))) \wedge (p (ap V0Phi_I1 (ap V13q1 V1t0)))) \vee ((\\
& \quad p (ap V12p V1t0)) \wedge (p (ap V5Phi_I2 (ap V14q2 V1t0)))))) \wedge ((\forall V15t \in \\
& \quad ty_2Enum_2Enum.(((\neg(p (ap V12p (ap (ap c_2Earithmetic_2E_2B V15t) \\
& \quad V1t0)))) \wedge ((p (ap V2Phi_R1 (ap (ap (c_2Epair_2E_2C A_27a A_27b1) \\
& \quad (ap V3i (ap (ap c_2Earithmetic_2E_2B V15t) V1t0))) (ap V13q1 (ap \\
& \quad (ap c_2Earithmetic_2E_2B V15t) V1t0)))))) \wedge (\neg(p (ap V12p (ap (ap \\
& \quad c_2Earithmetic_2E_2B V15t) (ap (ap c_2Earithmetic_2E_2B V1t0) \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))))))) \vee \\
& \quad ((p (ap V12p (ap (ap c_2Earithmetic_2E_2B V15t) V1t0))) \wedge ((p (ap \\
& \quad V6Phi_R2 (ap (ap (c_2Epair_2E_2C A_27a A_27b2) (ap V3i (ap (ap c_2Earithmetic_2E_2B \\
& \quad V15t) V1t0))) (ap V14q2 (ap (ap c_2Earithmetic_2E_2B V15t) V1t0)))))) \wedge \\
& \quad (p (ap V12p (ap (ap c_2Earithmetic_2E_2B V15t) (ap (ap c_2Earithmetic_2E_2B \\
& \quad V1t0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))))))) \wedge (((\neg(p (ap V12p V1t0))) \wedge (p \\
& \quad (ap V4Psi1 (ap (ap (c_2Epair_2E_2C (A_27a^{ty_2Enum_2Enum}) (A_27b1^{ty_2Enum_2Enum})) \\
& \quad V3i) V13q1)))) \vee ((p (ap V12p V1t0)) \wedge (p (ap V7Psi2 (ap (ap (c_2Epair_2E_2C \\
& \quad (A_27a^{ty_2Enum_2Enum}) (A_27b2^{ty_2Enum_2Enum})) V3i) V14q2)))))))))))))))))))
\end{aligned}$$