

thm_2EPast_Temporal_Logic_2EPWHEN_EXPRESSIVE
(TMY5cJBnc2ER3JChbw3Q6HwxEKHuKuMgdNy)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

nonempty ty_2Enum_2Enum (1)

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (2)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^\omega) \quad (3)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (4)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap \ (ap \ (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_{\text{2Ebool_2E}_21}$ to be $\lambda A_{\text{27a}} : \iota.(\lambda V0P \in (2^{A_{\text{27a}}}).(ap\ (ap\ (c_{\text{2Emin_2E}_3D}\ (2^{A_{\text{27}}}))\ V)\ P))$

Definition 4 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ ($

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap P x))$ then $(the (\lambda x.x \in A \wedge p$ of type $\iota \rightarrow \iota$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (5)$$

Definition 6 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EAABS_num\ c_2Enum_2EZERO_REP)$.

Definition 7 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 10 We define c_2Ebool_2ECOND to be $\lambda A._27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A._27a.(\lambda V2t2 \in A._27a.($

Definition 11 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (ap (c_2Ebool_2B$

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E))$

Definition 13 We define $c_2Ebool_2E_3F$ to be $\lambda A._27a : \iota.(\lambda V0P \in (2^A_{27}a)).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 14 We define $c_2Eprim_rec_2E\lambda_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 15 We define $c_2EPast_Temporal_Logic_2EPSNEXT$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1t0 \in$

Definition 16 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 17 We define $c_2EPast_Temporal_Logic_2EPNEXT$ to be $\lambda V0a \in (2ty_2Enum_2Enum). \lambda V1t0 \in$

Definition 18 We define c_2 Earthmetic_2E_3C_3D to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 19 We define $c_2EPast_Temporal_Logic_2EPALWAYS$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1t0$

Definition 20 We define $c_2EPast_Temporal_Logic_2EPEVENTUAL$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V$

Definition 21 We define $c_2EPast_Temporal_Logic_2EPSWHEN$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in$

Definition 22 We define $c_2EPast_Temporal_Logic_2EPSUNTIL$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in$

Definition 23 We define $c_2EPast_Temporal_Logic_2EPSBEFORE$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}), \lambda V1b$

Definition 24 We define c-2EPast-Temporal-Logic-2EPWHEN to be $\lambda V0a \in (2^{ty_2Enum_2Enum}), \lambda V1b \in (2^{ty_2Enum_2Enum})$

Definition 25 We define $\mathcal{L}_{\text{2EPast-Temporal-Logic-2EPIUNTH}}$ to be $\lambda V0a \in (2^{\text{ty_2Enum_2Enum}}) \lambda V1b \in (2^{\text{ty_2Enum_2Enum}})$

Definition 26 We define \mathcal{L} 2EPast-Temporal-Logic 2EPBEFORE to be $\lambda V0a \in (2^{ty_2Enum_2Enum}) \lambda V1b \in$

Let $c, 2E, 2B : t$ be given. Assume the following

$c : 2\text{Earithmetic} \cdot 2E \cdot 2B \in ((tu : 2E\text{enum} \cdot 2E\text{enum}^{\text{ty}} \cdot 2E\text{num} \cdot 2E\text{num})^{\text{ty}} \cdot 2E\text{num} \cdot 2E\text{num})$

(6)

Definition 27 We define c₂E temporal logic to be $\lambda V \lambda P \in (2^{(g-1)kam} \rightarrow L_{\text{temp}}). \lambda V \lambda P \in ty_2$

Definition 28 We define $c_{\mathcal{E}}\text{-Temporal-Logic}_{\mathcal{E}}$ EVENTUAL to be $\lambda V \circ P \in (2^{c_{\mathcal{E}}\text{-Temporal-Logic}_{\mathcal{E}}})^P$. $\lambda V \exists t \in t_0$

Definition 29 We define $c_2E\ temporal_Logic_2Ewatch$ to be $\lambda V\, Uq \in (2^{e_9-2Ename-2Ename}).\lambda V\, 1b \in (2^{e_9-2Ename})$.

Definition 30 We define $c_2ETemporal_Logic_2EWHEN$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum})$

Definition 31 We define $c_2ETemporal_Logic_2EUNTIL$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum})$

Definition 32 We define $c_2ETemporal_Logic_2EBEFORE$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum})$

Definition 33 We define $c_2ETemporal_Logic_2ESWHEN$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum})$

Definition 34 We define $c_2ETemporal_Logic_2ESUNTIL$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum})$

Definition 35 We define $c_2ETemporal_Logic_2ENEXT$ to be $\lambda V0P \in (2^{ty_2Enum_2Enum}).(\lambda V1t \in ty_2Enum_2Enum)$

Definition 36 We define $c_2ETemporal_Logic_2ESBEFORE$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum})$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (8)$$

Definition 37 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 38 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2EXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 39 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 40 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap c_2Enum_2ESUC (ap$

Definition 41 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 42 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EXP$

Definition 43 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EXP$

Definition 44 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Assume the following.

$$\begin{aligned}
 & (\forall V0P \in (2^{ty_2Enum_2Enum}).(\forall V1t0 \in ty_2Enum_2Enum. \\
 & ((p (ap (ap c_2ETemporal_Logic_2EALWAYS V0P) V1t0)) \Leftrightarrow ((p (ap V0P \\
 & V1t0)) \wedge (p (ap (ap c_2ETemporal_Logic_2ENEXT (ap c_2ETemporal_Logic_2EALWAYS \\
 & V0P)) V1t0)))))) \\
 \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0P \in (2^{ty_2Enum_2Enum}).(\forall V1t0 \in ty_2Enum_2Enum. \\
 & ((p (ap (ap c_2ETemporal_Logic_2EEVENTUAL V0P) V1t0)) \Leftrightarrow ((p (ap \\
 & V0P V1t0)) \vee (p (ap (ap c_2ETemporal_Logic_2ENEXT (ap c_2ETemporal_Logic_2EEVENTUAL \\
 & V0P)) V1t0)))))) \\
 \end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in (2^{ty_2Enum_2Enum}).(\forall V1b \in (2^{ty_2Enum_2Enum}. \\
 & (\forall V2t0 \in ty_2Enum_2Enum.((p (ap (ap c_2ETemporal_Logic_2EWHEN \\
 & V0a) V1b) V2t0)) \Leftrightarrow (p (ap (ap (c_2Ebool_2ECOND 2) (ap V1b V2t0)) \\
 & (ap V0a V2t0)) (ap (ap c_2ETemporal_Logic_2ENEXT (ap (ap c_2ETemporal_Logic_2EWHEN \\
 & V0a) V1b) V2t0))))))) \\
 \end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in (2^{ty_2Enum_2Enum}).(\forall V1b \in (2^{ty_2Enum_2Enum}. \\
 & (\forall V2t0 \in ty_2Enum_2Enum.((p (ap (ap c_2ETemporal_Logic_2EUNTIL \\
 & V0a) V1b) V2t0)) \Leftrightarrow ((\neg(p (ap V1b V2t0))) \Rightarrow ((p (ap V0a V2t0)) \wedge (p (ap \\
 & (ap c_2ETemporal_Logic_2ENEXT (ap (ap c_2ETemporal_Logic_2EUNTIL \\
 & V0a) V1b) V2t0))))))) \\
 \end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in (2^{ty_2Enum_2Enum}).(\forall V1b \in (2^{ty_2Enum_2Enum}. \\
 & (\forall V2t0 \in ty_2Enum_2Enum.((p (ap (ap c_2ETemporal_Logic_2EBEFORE \\
 & V0a) V1b) V2t0)) \Leftrightarrow ((\neg(p (ap V1b V2t0))) \wedge ((p (ap V0a V2t0)) \vee (p (ap \\
 & (ap c_2ETemporal_Logic_2ENEXT (ap (ap c_2ETemporal_Logic_2EBEFORE \\
 & V0a) V1b) V2t0))))))) \\
 \end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in (2^{ty_2Enum_2Enum}).(\forall V1b \in (2^{ty_2Enum_2Enum}. \\
 & (\forall V2t0 \in ty_2Enum_2Enum.((p (ap (ap c_2ETemporal_Logic_2ESWHEN \\
 & V0a) V1b) V2t0)) \Leftrightarrow (p (ap (ap (c_2Ebool_2ECOND 2) (ap V1b V2t0)) \\
 & (ap V0a V2t0)) (ap (ap c_2ETemporal_Logic_2ENEXT (ap (ap c_2ETemporal_Logic_2ESWHEN \\
 & V0a) V1b) V2t0))))))) \\
 \end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in (2^{ty_2Enum_2Enum}).(\forall V1b \in (2^{ty_2Enum_2Enum}). \\
 & (\forall V2t0 \in ty_2Enum_2Enum.((p (ap (ap (ap c_2ETemporal_Logic_2ESUNTIL \\
 & V0a) V1b) V2t0)) \Leftrightarrow ((\neg(p (ap V1b V2t0))) \Rightarrow ((p (ap V0a V2t0)) \wedge (p (ap \\
 & (ap c_2ETemporal_Logic_2ENEXT (ap (ap c_2ETemporal_Logic_2ESUNTIL \\
 & V0a) V1b)) V2t0)))))))) \\
 \end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in (2^{ty_2Enum_2Enum}).(\forall V1b \in (2^{ty_2Enum_2Enum}). \\
 & (\forall V2t0 \in ty_2Enum_2Enum.((p (ap (ap (ap c_2ETemporal_Logic_2ESBEFORE \\
 & V0a) V1b) V2t0)) \Leftrightarrow ((\neg(p (ap V1b V2t0))) \wedge ((p (ap V0a V2t0)) \vee (p (ap \\
 & (ap c_2ETemporal_Logic_2ENEXT (ap (ap c_2ETemporal_Logic_2ESBEFORE \\
 & V0a) V1b)) V2t0)))))))) \\
 \end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\
 & ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\
 & ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\
 & (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\
 & V0m) V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\
 & V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))) \\
 \end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\
 & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\
 & V1n) V0m))) \\
 \end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\
 & \forall V2p \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) \\
 & (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\
 & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) V2p)))))) \\
 \end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\
 & (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & (ap c_2Enum_2ESUC V0m)) V1n)))) \\
 \end{aligned} \tag{23}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D \\
 c_2Enum_2E0) V0n))) \tag{24}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earthmetic_2E_3C_3D V1n) V0m))))) \quad (25)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge \\
& ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
& (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
& (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
& V0m) V1n))))))))))) \\
\end{aligned} \tag{26}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earthmetic_2E_3C_3D V0m) V1n)) \wedge (p (ap (ap c_2Earthmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (ap (ap c_2Earthmetic_2E_3C_3D V0m) V2p))))))) \quad (27)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(p(ap(ap(c_2Earthmetic_2E_3C_3D
V0m) V0m))) \quad (28)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m))))))) \quad (29)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))) \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\ & (ap c_2Enum_2ESUC V1n)) V0m)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\ & V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\ & V1n)) V0m))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\ & c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)) V0n))) \end{aligned} \quad (33)$$

Assume the following.

$$True \quad (34)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow (\\ & \forall V0t1 \in A_27a. (\forall V1t2 \in A_27b. ((ap (\lambda V2x \in A_27b. \\ & V0t1) V1t2) = V0t1))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ & A_27a. (p V0t)) \Leftrightarrow (p V0t))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge \\ & ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))) \end{aligned} \quad (40)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (41)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (43)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (44)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (45)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee ((p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (47)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))) \wedge (((\neg(p V0A) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B)))))))))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (49)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (50)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (51)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1) \wedge (\neg(p V1t2))))))) \quad (52)$$

Assume the following.

$$(\forall V0P \in (2^{ty_2Enum_2Enum}). (((p (ap V0P c_2Enum_2E0)) \wedge (\forall V1n \in ty_2Enum_2Enum. ((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c_2Enum_2ESUC V1n))))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. (p (ap V0P V2n)))))) \quad (53)$$

Assume the following.

$((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A V6n) V7m))))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V10n) V11m))))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP V14n) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))) \wedge ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL V15n)) (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2EEEXP V15n) V16m))))))) \wedge (((ap c_2Enum_2ESUC c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum.((ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2ESUC V17n))))))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Eprim_rec_2EPRE V18n))))))) \wedge ((\forall V19n \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum.((\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m))))))) \wedge ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C V23n) c_2Enum_2E0)) \Leftrightarrow False))) \wedge ((\forall V24n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) V24n))))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.((\forall V26m \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C V25n) c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V26m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V25n) V26m))))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V27n)) \Leftrightarrow False))) \wedge ((\forall V28n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E V28n) c_2Enum_2E0) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V28n))))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.((\forall V30m \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C V29n) c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V30m)))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C V30m) V29n))))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C c_2Enum_2E0) V31n)) \Leftrightarrow True))) \wedge ((\forall V32n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C c_2Enum_2E0) V32n)) \Leftrightarrow False))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C c_2Enum_2E0) V32n)) \Leftrightarrow True)))$

Assume the following.

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = c_2Enum_2E0) \wedge (\forall V0m \in \\
& ty_2Enum_2Enum. ((ap c_2Eprim_rec_2EPRE (ap c_2Enum_2ESUC V0m)) = \\
& V0m))) \\
\end{aligned} \tag{57}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Eprim_rec_2E_3C \\
V0n) V0n)))) \tag{58}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{59}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{62}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{63}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
 & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
 & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
 & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
 & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
 & ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{67}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
 (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \tag{68}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \tag{69}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \tag{70}$$

Theorem 1

$$\begin{aligned}
& (\forall V0a \in (2^{ty_2Enum_2Enum}).(\forall V1b \in (2^{ty_2Enum_2Enum}). \\
& (((ap c_2EPast_Temporal_Logic_2EPALWAYS V0a) = (\lambda V2t \in ty_2Enum_2Enum. \\
& (ap (ap (ap c_2EPast_Temporal_Logic_2EPWHEN (\lambda V3t \in ty_2Enum_2Enum. \\
& c_2Ebool_2EF)) (\lambda V4t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E \\
& (ap V0a V4t)))) V2t))) \wedge (((ap c_2EPast_Temporal_Logic_2EPEVENTUAL \\
& V0a) = (\lambda V5t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E (ap (ap (ap \\
& c_2EPast_Temporal_Logic_2EPWHEN (\lambda V6t \in ty_2Enum_2Enum. \\
& c_2Ebool_2EF)) V0a) V5t)))) \wedge (((ap (ap c_2EPast_Temporal_Logic_2EPSUNTIL \\
& V0a) V1b) = (\lambda V7t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E (ap (ap \\
& (ap c_2EPast_Temporal_Logic_2EPWHEN (\lambda V8t \in ty_2Enum_2Enum. \\
& (ap c_2Ebool_2E_7E (ap V1b V8t)))) (\lambda V9t \in ty_2Enum_2Enum.(\\
& ap (ap c_2Emin_2E_3D_3D_3E (ap V0a V9t)) (ap V1b V9t)))) V7t)))) \wedge \\
& (((ap (ap c_2EPast_Temporal_Logic_2EPUNTIL V0a) V1b) = (\lambda V10t \in \\
& ty_2Enum_2Enum.(ap (ap (ap c_2EPast_Temporal_Logic_2EPWHEN \\
& V1b) (\lambda V11t \in ty_2Enum_2Enum.(ap (ap c_2Emin_2E_3D_3D_3E (\\
& ap V0a V11t)) (ap V1b V11t)))) V10t))) \wedge (((ap (ap c_2EPast_Temporal_Logic_2EPSWHEN \\
& V0a) V1b) = (\lambda V12t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E (ap (\\
& ap (ap c_2EPast_Temporal_Logic_2EPWHEN (\lambda V13t \in ty_2Enum_2Enum. \\
& (ap c_2Ebool_2E_7E (ap V0a V13t)))) V12t))) \wedge (((ap (ap c_2EPast_Temporal_Logic_2EPBEFORE \\
& V0a) V1b) = (\lambda V14t \in ty_2Enum_2Enum.(ap (ap (ap c_2EPast_Temporal_Logic_2EPWHEN \\
& (\lambda V15t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E (ap V1b V15t)))) \\
& (\lambda V16t \in ty_2Enum_2Enum.(ap (ap c_2Ebool_2E_5C_2F (ap V0a V16t)) \\
& (ap V1b V16t)))) V14t))) \wedge ((ap (ap c_2EPast_Temporal_Logic_2EPSBEFORE \\
& V0a) V1b) = (\lambda V17t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E (ap (\\
& ap (ap c_2EPast_Temporal_Logic_2EPWHEN V1b) (\lambda V18t \in ty_2Enum_2Enum. \\
& (ap (ap c_2Ebool_2E_5C_2F (ap V0a V18t)) (ap V1b V18t)))) V17t)))))))))))
\end{aligned}$$