

thm_2EPast__Temporal__Logic_2ESEPARATE__PNEXT__THM (TMaps5GQaX8bZLUoLJCLKBGodojfeQhR6ti)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2EPast_Temporal_Logic_2ElnitPoint$ to be $(\lambda V0t \in ty_2Enum_2Enum.(ap\ (ap\ ($

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ ($

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ ($

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define c_Ebool_2EF to be $(ap (c_Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 9 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 10 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21) 2) (\lambda V2t \in 2.V2t)))$

Definition 11 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_Ebool_2E_21) 2) (\lambda V3t3 \in 2.V3t3))))$

Definition 12 We define $c_Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_Ebool_2E_21) 2) (\lambda V1t1 \in 2.V1t1)) (\lambda V2t2 \in 2.V2t2)))$

Definition 13 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21) 2) (\lambda V2t \in 2.V2t)))$

Definition 14 We define $c_EPast_Temporal_Logic_2EPNEXT$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1t0 \in 2.V1t0$

Let $c_Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (6)$$

Definition 15 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E_21))$

Definition 16 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_Emin_2E_40) 2) (\lambda V1P \in 2.V1P))))$

Definition 17 We define $c_ETemporal_Logic_2EWATCH$ to be $\lambda V0q \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum})$

Definition 18 We define $c_ETemporal_Logic_2EBEFORE$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum})$

Definition 19 We define $c_ETemporal_Logic_2ENEXT$ to be $\lambda V0P \in (2^{ty_2Enum_2Enum}).(\lambda V1t \in ty_2Enum_2Enum)$

Definition 20 We define $c_ETemporal_Logic_2ESUNTIL$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum})$

Let $c_Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (8)$$

Definition 21 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 22 We define $c_Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 23 We define $c_Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 24 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 25 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 26 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1$

Definition 27 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT2$

Definition 28 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 29 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Assume the following.

$$\begin{aligned} & (\forall V0a \in (2^{ty_2Enum_2Enum}).(\forall V1b \in (2^{ty_2Enum_2Enum}). \\ & (\forall V2t0 \in ty_2Enum_2Enum.((p (ap (ap (ap c_2ETemporal_Logic_2EBEFORE \\ & V0a) V1b) V2t0)) \Leftrightarrow ((\neg(p (ap V1b V2t0))) \wedge ((p (ap V0a V2t0)) \vee (p (ap \\ & (ap c_2ETemporal_Logic_2ENEXT (ap (ap c_2ETemporal_Logic_2EBEFORE \\ & V0a) V1b)) V2t0)))))))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in (2^{ty_2Enum_2Enum}).(\forall V1b \in (2^{ty_2Enum_2Enum}). \\ & (\forall V2t0 \in ty_2Enum_2Enum.((p (ap (ap (ap c_2ETemporal_Logic_2ESUNTIL \\ & V0a) V1b) V2t0)) \Leftrightarrow ((\neg(p (ap V1b V2t0))) \Rightarrow ((p (ap V0a V2t0)) \wedge (p (ap \\ & (ap c_2ETemporal_Logic_2ENEXT (ap (ap c_2ETemporal_Logic_2ESUNTIL \\ & V0a) V1b)) V2t0)))))))))) \end{aligned} \quad (13)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D c_2Enum_2E0) V0n))) \quad (14)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge (\\
& ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
& (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
& (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
& V0m) V1n))))))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\
& c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))) V0n))
\end{aligned} \tag{19}$$

Assume the following.

$$True \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))
\end{aligned} \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p (ap V0P\ c_2Enum_2E0)) \wedge \\ & (\forall V1n \in ty_2Enum_2Enum.((p (ap V0P\ V1n)) \Rightarrow (p (ap V0P\ (ap\ c_2Enum_2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p (ap V0P\ V2n)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m)) \Leftrightarrow (\neg (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m)))))))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = c_2Enum_2E0) \wedge (\forall V0m \in \\
& ty_2Enum_2Enum. ((ap c_2Eprim_rec_2EPRE (ap c_2Enum_2ESUC V0m)) = \\
& V0m)))
\end{aligned} \tag{34}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{35}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{38}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \wedge (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (\neg(p \vee 1q)) \vee (\neg(p \vee 2r)))) \wedge (((p \vee 1q) \vee \\
& (\neg(p \vee 0p))) \wedge ((p \vee 2r) \vee (\neg(p \vee 0p))))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \vee (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (\neg(p \vee 1q))) \wedge ((p \vee 0p) \vee (\neg(p \vee 2r)))) \wedge \\
& ((p \vee 1q) \vee ((p \vee 2r) \vee (\neg(p \vee 0p))))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow (\\
& (p \vee 1q) \Rightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge ((p \vee 0p) \vee (\neg(p \vee 2r))) \wedge (\\
& \neg(p \vee 1q)) \vee ((p \vee 2r) \vee (\neg(p \vee 0p))))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee 0p) \Leftrightarrow (\neg(p \vee 1q))) \Leftrightarrow (((p \vee 0p) \vee \\
& (p \vee 1q)) \wedge ((\neg(p \vee 1q)) \vee (\neg(p \vee 0p))))))
\end{aligned} \tag{44}$$

Theorem 1

$$\begin{aligned}
& (\forall V0a \in (2^{ty_2Enum_2Enum}).(\forall V1b \in (2^{ty_2Enum_2Enum}). \\
& (\forall V2c \in (2^{ty_2Enum_2Enum}).(((ap\ c_2EPast_Temporal_Logic_2EPNEXT \\
& (\lambda V3t \in ty_2Enum_2Enum.(ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ V0a\ V3t)) \\
& (ap\ (ap\ c_2ETemporal_Logic_2ENEXT\ V1b)\ V3t)))) = (\lambda V4t \in ty_2Enum_2Enum. \\
& (ap\ (ap\ c_2Ebool_2E_5C_2F\ (ap\ c_2EPast_Temporal_Logic_2EInitPoint \\
& V4t))\ (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ V1b\ V4t))\ (ap\ (ap\ c_2EPast_Temporal_Logic_2EPNEXT \\
& V0a)\ V4t)))))) \wedge (((ap\ c_2EPast_Temporal_Logic_2EPNEXT\ (\lambda V5t \in \\
& ty_2Enum_2Enum.(ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ V0a\ V5t))\ (ap\ (ap \\
& (ap\ c_2ETemporal_Logic_2ESUNTIL\ V1b)\ V2c)\ V5t)))) = (\lambda V6t \in \\
& ty_2Enum_2Enum.(ap\ (ap\ c_2Ebool_2E_5C_2F\ (ap\ (ap\ c_2EPast_Temporal_Logic_2EPNEXT \\
& (\lambda V7t \in ty_2Enum_2Enum.(ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ V0a\ V7t)) \\
& (ap\ V2c\ V7t))))\ V6t))\ (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ (ap\ (ap\ c_2ETemporal_Logic_2ESUNTIL \\
& V1b)\ V2c)\ V6t))\ (ap\ (ap\ c_2EPast_Temporal_Logic_2EPNEXT\ (\lambda V8t \in \\
& ty_2Enum_2Enum.(ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ V0a\ V8t))\ (ap\ V1b \\
& V8t))))\ V6t)))))) \wedge (((ap\ c_2EPast_Temporal_Logic_2EPNEXT\ (\\
& \lambda V9t \in ty_2Enum_2Enum.(ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ V0a\ V9t)) \\
& (ap\ (ap\ (ap\ c_2ETemporal_Logic_2EBEFORE\ V1b)\ V2c)\ V9t)))) = (\lambda V10t \in \\
& ty_2Enum_2Enum.(ap\ (ap\ c_2Ebool_2E_5C_2F\ (ap\ (ap\ c_2EPast_Temporal_Logic_2EPNEXT \\
& (\lambda V11t \in ty_2Enum_2Enum.(ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ V0a\ V11t)) \\
& (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ V1b\ V11t))\ (ap\ c_2Ebool_2E_7E\ (ap \\
& V2c\ V11t))))))\ V10t))\ (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ (ap\ (ap\ c_2ETemporal_Logic_2EBEFORE \\
& V1b)\ V2c)\ V10t))\ (ap\ (ap\ c_2EPast_Temporal_Logic_2EPNEXT\ (\lambda V12t \in \\
& ty_2Enum_2Enum.(ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ V0a\ V12t))\ (ap\ c_2Ebool_2E_7E \\
& (ap\ V2c\ V12t))))))\ V10t)))))) \wedge (((ap\ c_2EPast_Temporal_Logic_2EPNEXT \\
& (\lambda V13t \in ty_2Enum_2Enum.(ap\ (ap\ c_2Ebool_2E_5C_2F\ (ap\ V0a\ V13t)) \\
& (ap\ (ap\ c_2ETemporal_Logic_2ENEXT\ V1b)\ V13t)))) = (\lambda V14t \in \\
& ty_2Enum_2Enum.(ap\ (ap\ c_2Ebool_2E_5C_2F\ (ap\ V1b\ V14t))\ (ap\ (ap \\
& c_2EPast_Temporal_Logic_2EPNEXT\ V0a)\ V14t)))))) \wedge (((ap\ c_2EPast_Temporal_Logic_2EPNEXT \\
& (\lambda V15t \in ty_2Enum_2Enum.(ap\ (ap\ c_2Ebool_2E_5C_2F\ (ap\ V0a\ V15t)) \\
& (ap\ (ap\ (ap\ c_2ETemporal_Logic_2ESUNTIL\ V1b)\ V2c)\ V15t)))) = (\\
& \lambda V16t \in ty_2Enum_2Enum.(ap\ (ap\ c_2Ebool_2E_5C_2F\ (ap\ (ap\ c_2EPast_Temporal_Logic_2EPNEXT \\
& (\lambda V17t \in ty_2Enum_2Enum.(ap\ (ap\ c_2Ebool_2E_5C_2F\ (ap\ V0a\ V17t)) \\
& (ap\ V2c\ V17t))))\ V16t))\ (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ (ap\ (ap\ c_2ETemporal_Logic_2ESUNTIL \\
& V1b)\ V2c)\ V16t))\ (ap\ (ap\ c_2EPast_Temporal_Logic_2EPNEXT\ V1b) \\
& V16t)))))) \wedge (((ap\ c_2EPast_Temporal_Logic_2EPNEXT\ (\lambda V18t \in \\
& ty_2Enum_2Enum.(ap\ (ap\ c_2Ebool_2E_5C_2F\ (ap\ V0a\ V18t))\ (ap\ (ap \\
& (ap\ c_2ETemporal_Logic_2EBEFORE\ V1b)\ V2c)\ V18t)))) = (\lambda V19t \in \\
& ty_2Enum_2Enum.(ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ (ap\ c_2EPast_Temporal_Logic_2EPNEXT \\
& (\lambda V20t \in ty_2Enum_2Enum.(ap\ (ap\ c_2Ebool_2E_5C_2F\ (ap\ V0a\ V20t)) \\
& (ap\ c_2Ebool_2E_7E\ (ap\ V2c\ V20t))))))\ V19t))\ (ap\ (ap\ c_2Ebool_2E_5C_2F \\
& (ap\ (ap\ (ap\ c_2ETemporal_Logic_2EBEFORE\ V1b)\ V2c)\ V19t))\ (ap\ (\\
& ap\ c_2EPast_Temporal_Logic_2EPNEXT\ (\lambda V21t \in ty_2Enum_2Enum. \\
& (ap\ (ap\ c_2Ebool_2E_5C_2F\ (ap\ V0a\ V21t))\ (ap\ V1b\ V21t))))\ V19t)))))))))
\end{aligned}$$