

# thm\_2EPast\_\_Temporal\_\_Logic\_2ESEPARATE\_\_PSUNTIL\_\_THM (TMLdmT3rxxSn9XuFroArK8X71y7pyisjZYe)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.^{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num ($

**Definition 5** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{5}$$

**Definition 6** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 7** We define  $c\_Ebool\_2EF$  to be  $(ap (c\_Ebool\_2E\_21) 2) (\lambda V0t \in 2.V0t)$ .

**Definition 8** We define  $c\_Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 9** We define  $c\_Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21) 2) (\lambda V2t \in 2.V2t)))$

**Definition 10** We define  $c\_Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap (c\_Ebool\_2E\_21) 2) (\lambda V3t3 \in 2.V3t3))))$

**Definition 11** We define  $c\_Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_Ebool\_2E\_21) 2) (\lambda V1t1 \in 2.V1t1)) (\lambda V2t2 \in 2.V2t2)))$

**Definition 12** We define  $c\_Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_2E\_3D\_3D\_3E V0t) c\_Ebool\_2E\_21))$

**Definition 13** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_Emin\_2E\_3D\_3D\_3E) V0P)))$

**Definition 14** We define  $c\_Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_Ebool\_2E\_21) 2) (\lambda V2t2 \in 2.V2t2)) (\lambda V3t3 \in 2.V3t3)))$

**Definition 15** We define  $c\_EPast\_Temporal\_Logic\_2EPSNEXT$  to be  $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1t0 \in 2.V1t0$

**Definition 16** We define  $c\_Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21) 2) (\lambda V2t \in 2.V2t)))$

**Definition 17** We define  $c\_EPast\_Temporal\_Logic\_2EPNEXT$  to be  $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1t0 \in 2.V1t0$

**Definition 18** We define  $c\_Earithmic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_Ebool\_2E\_21) 2) (\lambda V2t2 \in 2.V2t2)) (\lambda V3t3 \in 2.V3t3)))$

**Definition 19** We define  $c\_EPast\_Temporal\_Logic\_2EPALWAYS$  to be  $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1t0 \in 2.V1t0$

**Definition 20** We define  $c\_EPast\_Temporal\_Logic\_2EPEVENTUAL$  to be  $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1t0 \in 2.V1t0$

**Definition 21** We define  $c\_EPast\_Temporal\_Logic\_2EPSWHEN$  to be  $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in 2.V1b$

**Definition 22** We define  $c\_EPast\_Temporal\_Logic\_2EPSUNTIL$  to be  $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in 2.V1b$

**Definition 23** We define  $c\_EPast\_Temporal\_Logic\_2EPSBEFORE$  to be  $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in 2.V1b$

**Definition 24** We define  $c\_EPast\_Temporal\_Logic\_2EPWHEN$  to be  $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in 2.V1b$

**Definition 25** We define  $c\_EPast\_Temporal\_Logic\_2EPUNTIL$  to be  $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in 2.V1b$

**Definition 26** We define  $c\_EPast\_Temporal\_Logic\_2EPBEFORE$  to be  $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in 2.V1b$

**Definition 27** We define  $c\_ETemporal\_Logic\_2ENEXT$  to be  $\lambda V0P \in (2^{ty\_2Enum\_2Enum}).(\lambda V1t \in ty\_2Enum\_2Enum.V1t)$

Let  $c\_Earithmic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_Earithmic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \tag{6}$$

**Definition 28** We define  $c\_ETemporal\_Logic\_2EALWAYS$  to be  $\lambda V0P \in (2^{ty\_2Enum\_2Enum}).\lambda V1t0 \in ty\_2Enum\_2Enum.V1t0$

**Definition 29** We define  $c\_ETemporal\_Logic\_2EEVENTUAL$  to be  $\lambda V0P \in (2^{ty\_2Enum\_2Enum}).\lambda V1t0 \in ty\_2Enum\_2Enum.V1t0$

**Definition 30** We define  $c\_2ETemporal\_Logic\_2EWATCH$  to be  $\lambda V0q \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in (2^{ty\_2E$

**Definition 31** We define  $c\_2ETemporal\_Logic\_2EWHEN$  to be  $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in (2^{ty\_2E$

**Definition 32** We define  $c\_2ETemporal\_Logic\_2ESWHEN$  to be  $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in (2^{ty\_2E$

**Definition 33** We define  $c\_2ETemporal\_Logic\_2EBEFORE$  to be  $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in (2^{ty\_2E$

**Definition 34** We define  $c\_2ETemporal\_Logic\_2ESUNTIL$  to be  $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in (2^{ty\_2E$

**Definition 35** We define  $c\_2ETemporal\_Logic\_2EUNTIL$  to be  $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in (2^{ty\_2E$

**Definition 36** We define  $c\_2ETemporal\_Logic\_2ESBEFORE$  to be  $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in (2^{ty\_2E$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 37** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 38** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 39** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 40** We define  $c\_2Enumeral\_2EiSUC$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2ESUC\ (ap$

**Definition 41** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 42** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 43** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 44** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Assume the following.

$$\begin{aligned}
& (\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}). \\
& (\forall V2t0 \in ty\_2Enum\_2Enum.((p (ap (ap (ap (ap c\_2ETemporal\_Logic\_2ESUNTIL \\
& V0a) V1b) V2t0)) \Leftrightarrow (\exists V3delta \in ty\_2Enum\_2Enum.((\forall V4t \in \\
& ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C V4t) V3delta)) \Rightarrow \\
& ((p (ap V0a (ap (ap c\_2Earithmetic\_2E\_2B V4t) V2t0))) \wedge (\neg (p (ap V1b \\
& (ap (ap c\_2Earithmetic\_2E\_2B V4t) V2t0)))))) \wedge (p (ap V1b (ap (ap \\
& c\_2Earithmetic\_2E\_2B V3delta) V2t0))))))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(\forall V1t0 \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2ETemporal\_Logic\_2EALWAYS V0P) V1t0)) \Leftrightarrow ((p (ap V0P \\
& V1t0)) \wedge (p (ap (ap c\_2ETemporal\_Logic\_2ENEXT (ap c\_2ETemporal\_Logic\_2EALWAYS \\
& V0P)) V1t0))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(\forall V1t0 \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2ETemporal\_Logic\_2EEVENTUAL V0P) V1t0)) \Leftrightarrow ((p (ap \\
& V0P V1t0)) \vee (p (ap (ap c\_2ETemporal\_Logic\_2ENEXT (ap c\_2ETemporal\_Logic\_2EEVENTUAL \\
& V0P)) V1t0))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}). \\
& (\forall V2t0 \in ty\_2Enum\_2Enum.((p (ap (ap (ap c\_2ETemporal\_Logic\_2EWHEN \\
& V0a) V1b) V2t0)) \Leftrightarrow (p (ap (ap (ap (c\_2Ebool\_2ECOND 2) (ap V1b V2t0)) \\
& (ap V0a V2t0)) (ap (ap c\_2ETemporal\_Logic\_2ENEXT (ap (ap c\_2ETemporal\_Logic\_2EWHEN \\
& V0a) V1b)) V2t0))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}). \\
& (\forall V2t0 \in ty\_2Enum\_2Enum.((p (ap (ap (ap c\_2ETemporal\_Logic\_2EUNTIL \\
& V0a) V1b) V2t0)) \Leftrightarrow ((\neg (p (ap V1b V2t0))) \Rightarrow ((p (ap V0a V2t0)) \wedge (p (ap \\
& (ap c\_2ETemporal\_Logic\_2ENEXT (ap (ap c\_2ETemporal\_Logic\_2EUNTIL \\
& V0a) V1b)) V2t0))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}). \\
& (\forall V2t0 \in ty\_2Enum\_2Enum.((p (ap (ap (ap c\_2ETemporal\_Logic\_2EBEFORE \\
& V0a) V1b) V2t0)) \Leftrightarrow ((\neg (p (ap V1b V2t0))) \wedge ((p (ap V0a V2t0)) \vee (p (ap \\
& (ap c\_2ETemporal\_Logic\_2ENEXT (ap (ap c\_2ETemporal\_Logic\_2EBEFORE \\
& V0a) V1b)) V2t0))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}). \\
& (\forall V2t0 \in ty\_2Enum\_2Enum.((p (ap (ap (ap (ap c\_2ETemporal\_Logic\_2ESWHEN \\
& V0a) V1b) V2t0)) \Leftrightarrow (p (ap (ap (ap (c\_2Ebool\_2ECOND 2) (ap V1b V2t0)) \\
& (ap V0a V2t0)) (ap (ap c\_2ETemporal\_Logic\_2ENEXT (ap (ap c\_2ETemporal\_Logic\_2ESWHEN \\
& V0a) V1b)) V2t0)))))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}). \\
& (\forall V2t0 \in ty\_2Enum\_2Enum.((p (ap (ap (ap (ap c\_2ETemporal\_Logic\_2ESUNTIL \\
& V0a) V1b) V2t0)) \Leftrightarrow ((\neg(p (ap V1b V2t0))) \Rightarrow ((p (ap V0a V2t0)) \wedge (p (ap \\
& (ap c\_2ETemporal\_Logic\_2ENEXT (ap (ap c\_2ETemporal\_Logic\_2ESUNTIL \\
& V0a) V1b)) V2t0)))))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}). \\
& (\forall V2t0 \in ty\_2Enum\_2Enum.((p (ap (ap (ap (ap c\_2ETemporal\_Logic\_2ESBEFORE \\
& V0a) V1b) V2t0)) \Leftrightarrow ((\neg(p (ap V1b V2t0))) \wedge ((p (ap V0a V2t0)) \vee (p (ap \\
& (ap c\_2ETemporal\_Logic\_2ENEXT (ap (ap c\_2ETemporal\_Logic\_2ESBEFORE \\
& V0a) V1b)) V2t0)))))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1t0 \in ty\_2Enum\_2Enum. \\
& ((\neg(p (ap (ap c\_2ETemporal\_Logic\_2EALWAYS V0a) V1t0))) \Leftrightarrow (p (ap \\
& (ap c\_2ETemporal\_Logic\_2EEVENTUAL (\lambda V2t \in ty\_2Enum\_2Enum. \\
& (ap c\_2Ebool\_2E\_7E (ap V0a V2t)))) V1t0))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1t0 \in ty\_2Enum\_2Enum. \\
& ((\neg(p (ap (ap c\_2ETemporal\_Logic\_2EEVENTUAL V0a) V1t0))) \Leftrightarrow (p \\
& (ap (ap c\_2ETemporal\_Logic\_2EALWAYS (\lambda V2t \in ty\_2Enum\_2Enum. \\
& (ap c\_2Ebool\_2E\_7E (ap V0a V2t)))) V1t0))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}). \\
& (\forall V2t0 \in ty\_2Enum\_2Enum.((\neg(p (ap (ap (ap (ap c\_2ETemporal\_Logic\_2EWHEN \\
& V0a) V1b) V2t0)) \Leftrightarrow (p (ap (ap (ap (ap c\_2ETemporal\_Logic\_2ESWHEN ( \\
& \lambda V3t \in ty\_2Enum\_2Enum.(ap c\_2Ebool\_2E\_7E (ap V0a V3t)))) V1b) \\
& V2t0)))))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}). \\
& (\forall V2t0 \in ty\_2Enum\_2Enum.((\neg(p (ap (ap (ap (ap c\_2ETemporal\_Logic\_2EUNTIL \\
& V0a) V1b) V2t0))) \Leftrightarrow (p (ap (ap (ap (ap c\_2ETemporal\_Logic\_2ESBEFORE \\
& (\lambda V3t \in ty\_2Enum\_2Enum.(ap c\_2Ebool\_2E\_7E (ap V0a V3t)))) V1b) \\
& V2t0))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}). \\
& (\forall V2t0 \in ty\_2Enum\_2Enum.((\neg(p (ap (ap (ap (ap c\_2ETemporal\_Logic\_2EBEFORE \\
& V0a) V1b) V2t0))) \Leftrightarrow (p (ap (ap (ap (ap c\_2ETemporal\_Logic\_2ESUNTIL \\
& (\lambda V3t \in ty\_2Enum\_2Enum.(ap c\_2Ebool\_2E\_7E (ap V0a V3t)))) V1b) \\
& V2t0))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}). \\
& (\forall V2t0 \in ty\_2Enum\_2Enum.((\neg(p (ap (ap (ap (ap c\_2ETemporal\_Logic\_2ESWHEN \\
& V0a) V1b) V2t0))) \Leftrightarrow (p (ap (ap (ap (ap c\_2ETemporal\_Logic\_2EWHEN (\lambda V3t \in \\
& ty\_2Enum\_2Enum.(ap c\_2Ebool\_2E\_7E (ap V0a V3t)))) V1b) V2t0))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}). \\
& (\forall V2t0 \in ty\_2Enum\_2Enum.((\neg(p (ap (ap (ap (ap c\_2ETemporal\_Logic\_2ESUNTIL \\
& V0a) V1b) V2t0))) \Leftrightarrow (p (ap (ap (ap (ap c\_2ETemporal\_Logic\_2EBEFORE \\
& (\lambda V3t \in ty\_2Enum\_2Enum.(ap c\_2Ebool\_2E\_7E (ap V0a V3t)))) V1b) \\
& V2t0))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}). \\
& (\forall V2t0 \in ty\_2Enum\_2Enum.((\neg(p (ap (ap (ap (ap c\_2ETemporal\_Logic\_2ESBEFORE \\
& V0a) V1b) V2t0))) \Leftrightarrow (p (ap (ap (ap (ap c\_2ETemporal\_Logic\_2EUNTIL ( \\
& \lambda V3t \in ty\_2Enum\_2Enum.(ap c\_2Ebool\_2E\_7E (ap V0a V3t)))) V1b) \\
& V2t0))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\
& ((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap ( \\
& ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0m) V1n))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\
& V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n))))))
\end{aligned} \tag{29}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B V1n) V0m)))) \quad (30)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \forall V2p \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p)))))) \quad (31)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0m)) V1n)))))) \quad (32)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Enum\_2E0) V0n))) \quad (33)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( (\neg (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m)))))) \quad (34)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)))))))))) \quad (35)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \forall V2p \in ty\_2Enum\_2Enum.(((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p)))))) \quad (36)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V0m))) \quad (37)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m))))) \quad (38)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))))) \quad (39)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V1n) V0m))))) \quad (40)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0m) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V1n) V0m))))) \quad (41)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap c\_2Enum\_2ESUC V0n) = (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0n))) \quad (42)$$

Assume the following.

$$True \quad (43)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (44)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (45)$$



Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (46)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (47)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge (p V1t2) \wedge (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (48)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (49)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (50)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (51)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (52)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (53)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (54)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (56)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))) \quad (57)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A) \vee (p V1B)))))) \quad (58)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (59)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (60)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2)))))) \quad (61)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}). (((p (ap V0P c\_2Enum\_2E0)) \wedge (\forall V1n \in ty\_2Enum\_2Enum. ((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c\_2Enum\_2ESUC V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum. (p (ap V0P V2n)))))) \quad (62)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ ( \\
& ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ ( \\
& ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ ( \\
& ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap \\
& c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC \\
& (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ c\_2Earithmetic\_2EZERO)) = ( \\
& ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ ( \\
& ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Earithmetic\_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow \neg (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m)))))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge (\forall V0m \in \\
& ty\_2Enum\_2Enum. ((ap c\_2Eprim\_rec\_2EPRE (ap c\_2Enum\_2ESUC V0m)) = \\
& V0m)))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\neg (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0n) V0n))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False)))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False)))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg( \\
& p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee \\
& ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (( \\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{76}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{77}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{78}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{79}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}). \\
& (\forall V2c \in (2^{ty\_2Enum\_2Enum}).(\forall V3d \in (2^{ty\_2Enum\_2Enum}). \\
& (((ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL V0a) (\lambda V4t \in \\
& ty\_2Enum\_2Enum.(ap (ap c\_2Ebool\_2E\_5C\_2F (ap V1b V4t)) (ap V2c \\
& V4t)))) = (\lambda V5t \in ty\_2Enum\_2Enum.(ap (ap c\_2Ebool\_2E\_5C\_2F \\
& (ap (ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL V0a) V1b) V5t)) \\
& (ap (ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL V0a) V2c) V5t)))) \wedge \\
& (((ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL V0a) (\lambda V6t \in \\
& ty\_2Enum\_2Enum.(ap (ap c\_2Ebool\_2E\_2F\_5C (ap V1b V6t)) (ap (ap \\
& c\_2ETemporal\_Logic\_2ENEXT V2c) V6t)))) = (\lambda V7t \in ty\_2Enum\_2Enum. \\
& (ap (ap c\_2Ebool\_2E\_5C\_2F (ap (ap c\_2Ebool\_2E\_2F\_5C (ap V1b V7t)) \\
& (ap (ap c\_2ETemporal\_Logic\_2ENEXT V2c) V7t))) (ap (ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL \\
& V0a) (\lambda V8t \in ty\_2Enum\_2Enum.(ap (ap c\_2Ebool\_2E\_2F\_5C (ap V0a \\
& V8t)) (ap (ap c\_2Ebool\_2E\_2F\_5C (ap V2c V8t)) (ap (ap c\_2EPast\_Temporal\_Logic\_2EPSNEXT \\
& V1b) V8t)))))) \wedge (((ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL \\
& V0a) (\lambda V9t \in ty\_2Enum\_2Enum.(ap (ap c\_2Ebool\_2E\_2F\_5C (ap V1b \\
& V9t)) (ap (ap (ap c\_2ETemporal\_Logic\_2ESUNTIL V2c) V3d) V9t)))) = \\
& (\lambda V10t \in ty\_2Enum\_2Enum.(ap (ap c\_2Ebool\_2E\_5C\_2F (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& (ap (ap (ap c\_2ETemporal\_Logic\_2ESUNTIL V2c) V3d) V10t)) (ap ( \\
& ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL (\lambda V11t \in ty\_2Enum\_2Enum. \\
& (ap (ap c\_2Ebool\_2E\_2F\_5C (ap V0a V11t)) (ap (ap c\_2EPast\_Temporal\_Logic\_2EPNEXT \\
& V2c) V11t)))) V1b) V10t))) (ap (ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL \\
& V0a) (\lambda V12t \in ty\_2Enum\_2Enum.(ap (ap c\_2Ebool\_2E\_2F\_5C (ap \\
& V3d V12t)) (ap (ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL (\lambda V13t \in \\
& ty\_2Enum\_2Enum.(ap (ap c\_2Ebool\_2E\_2F\_5C (ap V0a V13t)) (ap (ap \\
& c\_2EPast\_Temporal\_Logic\_2EPNEXT V2c) V13t)))) V1b) V12t)))) \\
& V10t)))) \wedge (((ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL V0a) \\
& (\lambda V14t \in ty\_2Enum\_2Enum.(ap (ap c\_2Ebool\_2E\_2F\_5C (ap V1b V14t)) \\
& (ap (ap (ap c\_2ETemporal\_Logic\_2EBEFORE V2c) V3d) V14t)))) = ( \\
& \lambda V15t \in ty\_2Enum\_2Enum.(ap (ap c\_2Ebool\_2E\_5C\_2F (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& (ap (ap (ap c\_2ETemporal\_Logic\_2EBEFORE V2c) V3d) V15t)) (ap ( \\
& ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL (\lambda V16t \in ty\_2Enum\_2Enum. \\
& (ap (ap c\_2Ebool\_2E\_2F\_5C (ap V0a V16t)) (ap c\_2Ebool\_2E\_7E (ap \\
& (ap c\_2EPast\_Temporal\_Logic\_2EPNEXT V3d) V16t)))))) V1b) V15t))) \\
& (ap (ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL V0a) (\lambda V17t \in \\
& ty\_2Enum\_2Enum.(ap (ap c\_2Ebool\_2E\_2F\_5C (ap V2c V17t)) (ap (ap \\
& c\_2Ebool\_2E\_2F\_5C (ap c\_2Ebool\_2E\_7E (ap V3d V17t))) (ap (ap (ap \\
& c\_2EPast\_Temporal\_Logic\_2EPSUNTIL (\lambda V18t \in ty\_2Enum\_2Enum. \\
& (ap (ap c\_2Ebool\_2E\_2F\_5C (ap V0a V18t)) (ap c\_2Ebool\_2E\_7E (ap \\
& (ap c\_2EPast\_Temporal\_Logic\_2EPNEXT V3d) V18t)))))) V1b) V17t)))) \\
& V15t)))) \wedge (((ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL (\lambda V19t \in \\
& ty\_2Enum\_2Enum.(ap (ap c\_2Ebool\_2E\_2F\_5C (ap V0a V19t)) (ap V1b \\
& V19t)))) V2c) = (\lambda V20t \in ty\_2Enum\_2Enum.(ap (ap c\_2Ebool\_2E\_2F\_5C \\
& (ap (ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL V0a) V2c) V20t)) \\
& (ap (ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL V1b) V2c) V20t)))) \wedge \\
& (((ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL (\lambda V21t \in ty\_2Enum\_2Enum. \\
& (ap (ap c\_2Ebool\_2E\_5C\_2F (ap V0a V21t)) (ap (ap c\_2ETemporal\_Logic\_2ENEXT \\
& V1b) V21t)))) V2c) = (\lambda V22t \in ty\_2Enum\_2Enum.(ap (ap c\_2Ebool\_2E\_5C\_2F \\
& (ap V2c V22t)) (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap c\_2Ebool\_2E\_5C\_2F \\
& (ap V0a V22t)) (ap (ap c\_2ETemporal\_Logic\_2ENEXT V1b) V22t)))) \\
& (ap (ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL (\lambda V23t \in ty\_2Enum\_2Enum. \\
& (ap (ap c\_2Ebool\_2E\_5C\_2F (ap V1b V23t)) (ap (ap c\_2EPast\_Temporal\_Logic\_2EPNEXT \\
& V0a) V23t)))) (ap c\_2EPast\_Temporal\_Logic\_2EPSNEXT V2c) V22t)))) \wedge \\
& (((ap (ap c\_2EPast\_Temporal\_Logic\_2EPSUNTIL (\lambda V24t \in ty\_2Enum\_2Enum. \\
& (ap (ap c\_2Ebool\_2E\_5C\_2F (ap V0a V24t)) (ap (ap (ap c\_2ETemporal\_Logic\_2ESUNTIL \\
& V1b) V2c) V24t)))) V3d) = (\lambda V25t \in ty\_2Enum\_2Enum.(ap (ap c\_2Ebool\_2E\_2F\_5C \\
& (ap (ap c\_2Ebool\_2E\_5C\_2F (ap (ap (ap c\_2ETemporal\_Logic\_2ESUNTIL \\
& V1b) V2c) V25t)) (ap (ap (ap c\_2EPast\_Temporal\_Logic\_2EPBEFORE \\
& (\lambda V26t \in ty\_2Enum\_2Enum.(ap (ap c\_2Ebool\_2E\_5C\_2F (ap V3d V26t))
\end{aligned}$$