thm_2EPast__Temporal__Logic_2ESWHEN__EXPRESSIVE (TMRGfGrbdAD-KUWuz7CPk5Ggg5BSux22KwFt)

October 26, 2020

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{3}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{4}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{5}$$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$ (

Definition 7 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in 2$

Definition 9 *We define* c_2ETemporal__Logic_2EWATCH *to be* $\lambda V0q \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in (2^{ty\_2En}$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A$.**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 12 *We define* c_2ETemporal__Logic_2EUNTIL *to be* $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in (2^{ty\_2En}$

Definition 13 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 14 *We define* c_2ETemporal__Logic_2EBEFORE *to be* $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in (2^{ty\_2}$

Definition 15 *We define* c_2ETemporal__Logic_2EALWAYS *to be* $\lambda V0P \in (2^{ty\_2Enum\_2Enum}).\lambda V1t0 \in ty\_2$

Definition 16 *We define* c_2ETemporal__Logic_2EEVENTUAL *to be* $\lambda V0P \in (2^{ty\_2Enum\_2Enum}).\lambda V1t0 \in t$

Definition 17 *We define* c_2ETemporal__Logic_2ESUNTIL *to be* $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in (2^{ty\_2E}$

Definition 18 *We define* c_2ETemporal__Logic_2ESBEFORE *to be* $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in (2^{ty\_}$

Definition 19 *We define* c_2ETemporal__Logic_2EWHEN *to be* $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in (2^{ty\_2En}$

Definition 20 *We define* c_2ETemporal__Logic_2ESWHEN *to be* $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in (2^{ty\_2E}$

Assume the following.

$$(\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}).\\ ((ap\ (ap\ \text{c\_2ETemporal\_\_Logic\_2EUNTIL}\ V0a)\ V1b) = (ap\ (ap\ \text{c\_2ETemporal\_\_Logic\_2EWHEN}\\ V1b)\ (\lambda V2t \in ty\_2Enum\_2Enum.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ (ap\\ V0a\ V2t))\ (ap\ V1b\ V2t)))))))) \tag{6}$$

Assume the following.

$$(\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}).\\ ((ap\ (ap\ \text{c\_2ETemporal\_\_Logic\_2EBEFORE}\ V0a)\ V1b) = (ap\ (ap\ \text{c\_2ETemporal\_\_Logic\_2EWHEN}\\ (\lambda V2t \in ty\_2Enum\_2Enum.(ap\ \text{c\_2Ebool\_2E\_7E}\ (ap\ V1b\ V2t)))\ (\\ \lambda V3t \in ty\_2Enum\_2Enum.(ap\ (ap\ \text{c\_2Ebool\_2E\_5C\_2F}\ (ap\ V0a\ V3t))\\ (ap\ V1b\ V3t))))))) \tag{7}$$

Assume the following.

$$(\forall V0a \in (2^{ty\_2Enum\_2Enum}).((ap\ \text{c\_2ETemporal\_\_Logic\_2EALWAYS}\\ V0a) = (\lambda V1t \in ty\_2Enum\_2Enum.(ap\ \text{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ (ap\\ \text{c\_2ETemporal\_\_Logic\_2ESWHEN}\ (\lambda V2t \in ty\_2Enum\_2Enum.\text{c\_2Ebool\_2ET}))\\ (\lambda V3t \in ty\_2Enum\_2Enum.(ap\ \text{c\_2Ebool\_2E\_7E}\ (ap\ V0a\ V3t))))\ V1t))))) \tag{8}$$

Assume the following.

$$(\forall V0a \in (2^{ty\_2Enum\_2Enum}).((ap\ \mathsf{c\_2ETemporal\_\_Logic\_2EEVENTUAL}\ V0a) = (ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2ESWHEN}\ (\lambda V1t \in ty\_2Enum\_2Enum.\ \mathsf{c\_2Ebool\_2ET}))\ V0a))) \tag{9}$$

Assume the following.

$$(\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}).\ (\forall V2t0 \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2EWHEN}\ V0a)\ V1b)\ V2t0)) \Leftrightarrow (\neg(p\ (ap\ (ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2ESWHEN}\ (\lambda V3t \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ V0a\ V3t))))\ V1b)\ V2t0))))))) \tag{10}$$

Assume the following.

$$(\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}).\ ((ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2ESUNTIL}\ V0a)\ V1b) = (ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2ESWHEN}\ V1b)\ (\lambda V2t \in ty\_2Enum\_2Enum.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ (ap\ V0a\ V2t))\ (ap\ V1b\ V2t))))))) \tag{11}$$

Assume the following.

$$(\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}).\ ((ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2ESBEFORE}\ V0a)\ V1b) = (ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2ESWHEN}\ (\lambda V2t \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ V1b\ V2t))))\ (\lambda V3t \in ty\_2Enum\_2Enum.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_5C\_2F}\ (ap\ V0a\ V3t))\ (ap\ V1b\ V3t))))))) \tag{12}$$

Assume the following.

$$(\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}).\ (\forall V2t0 \in ty\_2Enum\_2Enum.((\neg(p\ (ap\ (ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2ESWHEN}\ V0a)\ V1b)\ V2t0))) \Leftrightarrow (p\ (ap\ (ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2EWHEN}\ (\lambda V3t \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ V0a\ V3t))))\ V1b)\ V2t0)))))) \tag{13}$$

Assume the following.

$$True \tag{14}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \quad (17)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (18)$$

Theorem 1

$$(\forall V0a \in (2^{ty\_2Enum\_2Enum}).(\forall V1b \in (2^{ty\_2Enum\_2Enum}).(((ap\ \mathsf{c\_2ETemporal\_\_Logic\_2EALWAYS}\ V0a) = (\lambda V2t \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2ESWHEN}\ (\lambda V3t \in ty\_2Enum\_2Enum.\mathsf{c\_2Ebool\_2ET}))\ (\lambda V4t \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ V0a\ V4t))))\ V2t)))) \wedge (((ap\ \mathsf{c\_2ETemporal\_\_Logic\_2EEVENTUAL}\ V0a) = (\lambda V5t \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2ESWHEN}\ (\lambda V6t \in ty\_2Enum\_2Enum.\mathsf{c\_2Ebool\_2ET})\ V0a)\ V5t))) \wedge (((ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2ESUNTIL}\ V0a)\ V1b) = (\lambda V7t \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2ESWHEN}\ V1b)\ (\lambda V8t \in ty\_2Enum\_2Enum.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ (ap\ V0a\ V8t))\ (ap\ V1b\ V8t))))\ V7t))) \wedge (((ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2EUNTIL}\ V0a)\ V1b) = (\lambda V9t \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2ESWHEN}\ (\lambda V10t \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ V1b\ V10t))))\ (\lambda V11t \in ty\_2Enum\_2Enum.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ (ap\ V0a\ V11t))\ (ap\ V1b\ V11t))))\ V9t)))) \wedge (((ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2EWHEN}\ V0a)\ V1b) = (\lambda V12t \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2ESWHEN}\ (\lambda V13t \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ V0a\ V13t))))\ V1b)\ V12t)))) \wedge (((ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2EBEFORE}\ V0a)\ V1b) = (\lambda V14t \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2ESWHEN}\ V1b)\ (\lambda V15t \in ty\_2Enum\_2Enum.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_5C\_2F}\ (ap\ V0a\ V15t))\ (ap\ V1b\ V15t))))\ V14t)))) \wedge ((ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2ESBEFORE}\ V0a)\ V1b) = (\lambda V16t \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ \mathsf{c\_2ETemporal\_\_Logic\_2ESWHEN}\ (\lambda V17t \in ty\_2Enum\_2Enum.(ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ V1b\ V17t))))\ (\lambda V18t \in ty\_2Enum\_2Enum.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_5C\_2F}\ (ap\ V0a\ V18t))\ (ap\ V1b\ V18t))))\ V16t)))))))))))$$