

thm_2EPast_Temporal_Logic_2EWHEN_EXPRESSIVE (TMHa5HEWv6ggjCn7qF3QUrcJrphinFTFegK)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (2)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1t0 \in ty_2Enum_2Enum)))$

Definition 4 We define $c_2ETemporal_Logic_2EALWAYS$ to be $\lambda V0P \in (2^{ty_2Enum_2Enum}). \lambda V1t0 \in ty_2Enum_2Enum. inj_o (V0P = V1t0)$

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x) \text{ of type } \iota \Rightarrow \iota)$

Definition 6 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 (A_27a)) (\lambda V1t0 \in ty_2Enum_2Enum)))$

Definition 7 We define $c_2ETemporal_Logic_2EEVENTUAL$ to be $\lambda V0P \in (2^{ty_2Enum_2Enum}). \lambda V1t0 \in ty_2Enum_2Enum. inj_o (V0P \neq V1t0)$

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj_o (V1t2 = V2t))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (3)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (4)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (5)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Definition 13 We define $c_2ETemporal_Logic_2EWATCH$ to be $\lambda V0q \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum}.\lambda V0a \in (2^{ty_2Enum_2Enum}))$

Definition 14 We define $c_2ETemporal_Logic_2EUNTIL$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum}.\lambda V2t \in 2.V2t))$

Definition 15 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 16 We define $c_2ETemporal_Logic_2EBEFORE$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum}.\lambda V2t \in 2.V2t))$

Definition 17 We define $c_2ETemporal_Logic_2ESUNTIL$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum}.\lambda V2t \in 2.V2t))$

Definition 18 We define $c_2ETemporal_Logic_2ESBEFORE$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum}.\lambda V2t \in 2.V2t))$

Definition 19 We define $c_2ETemporal_Logic_2ESWHEN$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum}.\lambda V2t \in 2.V2t))$

Definition 20 We define $c_2ETemporal_Logic_2EWHEN$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum}.\lambda V2t \in 2.V2t))$

Assume the following.

$$\begin{aligned} & (\forall V0a \in (2^{ty_2Enum_2Enum}).((ap c_2ETemporal_Logic_2EALWAYS \\ & V0a) = (ap (ap c_2ETemporal_Logic_2EWHEN (\lambda V1t \in ty_2Enum_2Enum. \\ & c_2Ebool_2EF)) (\lambda V2t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E \\ & (ap V0a V2t))))))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in (2^{ty_2Enum_2Enum}).((ap c_2ETemporal_Logic_2EEVENTUAL \\ & V0a) = (\lambda V1t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E (ap (ap \\ & c_2ETemporal_Logic_2EWHEN (\lambda V2t \in ty_2Enum_2Enum.c_2Ebool_2EF)) \\ & V0a) V1t)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in (2^{ty_2Enum_2Enum}).(\forall V1b \in (2^{ty_2Enum_2Enum}). \\ & ((ap (ap c_2ETemporal_Logic_2EUNTIL V0a) V1b) = (ap (ap c_2ETemporal_Logic_2EWHEN \\ & V1b) (\lambda V2t \in ty_2Enum_2Enum.(ap (ap c_2Emin_2E_3D_3D_3E (ap \\ & V0a V2t)) (ap V1b V2t))))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in (2^{ty_2Enum_2Enum}).(\forall V1b \in (2^{ty_2Enum_2Enum}). \\
 & ((ap (ap c_2ETemporal_Logic_2EBEFORE V0a) V1b) = (ap (ap c_2ETemporal_Logic_2EWHEN \\
 & (\lambda V2t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E (ap V1b V2t)))) \\
 & \lambda V3t \in ty_2Enum_2Enum.(ap (ap c_2Ebool_2E_5C_2F (ap V0a V3t) \\
 & (ap V1b V3t))))))) \\
 & (9)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in (2^{ty_2Enum_2Enum}).(\forall V1b \in (2^{ty_2Enum_2Enum}. \\
 & (\forall V2t0 \in ty_2Enum_2Enum.((p (ap (ap (ap c_2ETemporal_Logic_2ESWHEN \\
 & V0a) V1b) V2t0)) \Leftrightarrow (\neg(p (ap (ap (ap c_2ETemporal_Logic_2EWHEN \\
 & \lambda V3t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E (ap V0a V3t))) V1b) \\
 & V2t0))))))) \\
 & (10)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in (2^{ty_2Enum_2Enum}).(\forall V1b \in (2^{ty_2Enum_2Enum}. \\
 & ((ap (ap c_2ETemporal_Logic_2ESUNTIL V0a) V1b) = (ap (ap c_2ETemporal_Logic_2ESWHEN \\
 & V1b) (\lambda V2t \in ty_2Enum_2Enum.(ap (ap c_2Emin_2E_3D_3D_3E (ap \\
 & V0a V2t)) (ap V1b V2t))))))) \\
 & (11)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in (2^{ty_2Enum_2Enum}).(\forall V1b \in (2^{ty_2Enum_2Enum}. \\
 & ((ap (ap c_2ETemporal_Logic_2ESBEFORE V0a) V1b) = (ap (ap c_2ETemporal_Logic_2ESWHEN \\
 & (\lambda V2t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E (ap V1b V2t)))) \\
 & \lambda V3t \in ty_2Enum_2Enum.(ap (ap c_2Ebool_2E_5C_2F (ap V0a V3t) \\
 & (ap V1b V3t))))))) \\
 & (12)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in (2^{ty_2Enum_2Enum}).(\forall V1b \in (2^{ty_2Enum_2Enum}. \\
 & (\forall V2t0 \in ty_2Enum_2Enum.((\neg(p (ap (ap (ap c_2ETemporal_Logic_2EWHEN \\
 & V0a) V1b) V2t0)) \Leftrightarrow (p (ap (ap (ap c_2ETemporal_Logic_2ESWHEN \\
 & \lambda V3t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E (ap V0a V3t))) V1b) \\
 & V2t0))))))) \\
 & (13)
 \end{aligned}$$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (15)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (17)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (18)$$

Theorem 1

$$\begin{aligned} & (\forall V0a \in (2^{ty_2Enum_2Enum}).(\forall V1b \in (2^{ty_2Enum_2Enum}). \\ & (((ap c_2ETemporal_Logic_2EALWAYS V0a) = (\lambda V2t \in ty_2Enum_2Enum. \\ & (ap (ap (ap c_2ETemporal_Logic_2EWHEN (\lambda V3t \in ty_2Enum_2Enum. \\ & c_2Ebool_2EF)) (\lambda V4t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E \\ & (ap V0a V4t)))) V2t))) \wedge (((ap c_2ETemporal_Logic_2EEVENTUAL \\ & V0a) = (\lambda V5t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E (ap (ap (ap \\ & c_2ETemporal_Logic_2EWHEN (\lambda V6t \in ty_2Enum_2Enum.c_2Ebool_2EF)) \\ & V0a) V5t)))) \wedge (((ap (ap c_2ETemporal_Logic_2ESUNTIL V0a) V1b) = \\ & (\lambda V7t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E (ap (ap (ap c_2ETemporal_Logic_2EWHEN \\ & (\lambda V8t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E (ap V1b V8t)))) (\\ & \lambda V9t \in ty_2Enum_2Enum.(ap (ap c_2Emin_2E_3D_3D_3E (ap V0a V9t)) \\ & (ap V1b V9t)))) V7t)))) \wedge (((ap (ap c_2ETemporal_Logic_2EUNTIL \\ & V0a) V1b) = (\lambda V10t \in ty_2Enum_2Enum.(ap (ap (ap c_2ETemporal_Logic_2EWHEN \\ & V1b) (\lambda V11t \in ty_2Enum_2Enum.(ap (ap c_2Emin_2E_3D_3D_3E (\\ & ap V0a V11t)) (ap V1b V11t)))) V10t)))) \wedge (((ap (ap c_2ETemporal_Logic_2ESWHEN \\ & V0a) V1b) = (\lambda V12t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E (ap (\\ & ap (ap c_2ETemporal_Logic_2EWHEN (\lambda V13t \in ty_2Enum_2Enum. \\ & (ap c_2Ebool_2E_7E (ap V0a V13t)))) V1b) V12t)))) \wedge (((ap (ap c_2ETemporal_Logic_2EBEFORE \\ & V0a) V1b) = (\lambda V14t \in ty_2Enum_2Enum.(ap (ap (ap c_2ETemporal_Logic_2EWHEN \\ & (\lambda V15t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E (ap V1b V15t)))) \\ & (\lambda V16t \in ty_2Enum_2Enum.(ap (ap c_2Ebool_2E_5C_2F (ap V0a V16t)) \\ & (ap V1b V16t)))) V14t)))) \wedge (((ap (ap c_2ETemporal_Logic_2ESBEFORE \\ & V0a) V1b) = (\lambda V17t \in ty_2Enum_2Enum.(ap c_2Ebool_2E_7E (ap (\\ & ap (ap c_2ETemporal_Logic_2EWHEN V1b) (\lambda V18t \in ty_2Enum_2Enum. \\ & (ap (ap c_2Ebool_2E_5C_2F (ap V0a V18t)) (ap V1b V18t)))) V17t))))))))))) \end{aligned}$$