

thm_2ETemporal_Logic_2EALWAYS_AS_UNTIL
 (TMNZyCAHJvYJc6LkRvvSmx5nYmJLgci8jtD)

October 26, 2020

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (2)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1t \in 2.V1t)) P)))$

Definition 4 We define $c_2ETemporal_Logic_2EALWAYS$ to be $\lambda V0P \in (2^{ty_2Enum_2Enum}). \lambda V1t0 \in ty_2E$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \Rightarrow p \ Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj_o (V0t1 = V1t2))))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (3)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (4)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (5)$$

Definition 7 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num (c_2Enum_2E0 V0m))$

Definition 8 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Definition 10 We define $c_2ETemporal_Logic_2EWATCH$ to be $\lambda V0q \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum})$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge p(x))) \text{ else } \perp$

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 (c_2Ebool_2E_3F A_27a)))))$

Definition 13 We define $c_2ETemporal_Logic_2EWHEN$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum})$

Definition 14 We define $c_2ETemporal_Logic_2EUNTIL$ to be $\lambda V0a \in (2^{ty_2Enum_2Enum}).\lambda V1b \in (2^{ty_2Enum_2Enum})$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Definition 15 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$

Definition 16 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_3F A_27a)))$

Definition 17 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Assume the following.

$$\begin{aligned} & ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0n) = V0n)) \wedge (\forall V1m \in ty_2Enum_2Enum.(\forall V2n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V1m) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V1m) V2n))))))) \\ & \quad (8) \end{aligned}$$

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V1n) = V1n) \wedge ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V1n) = V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))) \\ & \quad (9) \end{aligned}$$

Assume the following.

$$(\forall V0a \in ty_2Enum_2Enum. (\forall V1c \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2D (ap (ap c_2Earithmetic_2E_2B V0a) V1c)) V1c) = V0a))) \quad (10)$$

Assume the following.

$$(\forall V0c \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D V0c) V0c) = c_2Enum_2E0)) \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27b. ((ap (\lambda V2x \in A_27b. V0t1) V1t2) = V0t1))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t) \Leftrightarrow (p V0t)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (20)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (21)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (23)$$

Assume the following.

$$(\forall V0P \in (2^{ty_2Enum_2Enum}).(((p (ap V0P c_2Enum_2E0)) \wedge (\forall V1n \in ty_2Enum_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c_2Enum_2ESUC V1n))))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p (ap V0P V2n)))))) \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0e \in A_27a.(\forall V1f \in \\ & ((A_27a^{ty_2Enum_2Enum})^{A_27a}).(p (ap (c_2Ebool_2E_3F_21 (A_27a^{ty_2Enum_2Enum})) \\ & (\lambda V2fn1 \in (A_27a^{ty_2Enum_2Enum}).(ap (ap c_2Ebool_2E_2F_5C \\ & (ap (ap (c_2Emin_2E_3D A_27a) (ap V2fn1 c_2Enum_2E0)) V0e)) (ap \\ & (c_2Ebool_2E_21 ty_2Enum_2Enum) (\lambda V3n \in ty_2Enum_2Enum.(\\ & ap (ap (c_2Emin_2E_3D A_27a) (ap V2fn1 (ap c_2Enum_2ESUC V3n))) \\ & (ap (ap V1f (ap V2fn1 V3n)) V3n))))))))))) \end{aligned} \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\ (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V1q) \vee (p V2r)) \vee ((\neg(p V1q) \vee (p V2r)) \vee ((\neg(p V1q) \vee (p V0p))))))) \wedge (((p V1q) \vee ((\neg(p V2r) \vee ((\neg(p V0p) \vee ((\neg(p V1q) \vee (p V0p))))))))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (((p V1q) \vee ((p V2r) \vee ((\neg(p V0p))))))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (((\neg(p V1q) \vee ((p V2r) \vee ((\neg(p V0p))))))))))) \end{aligned} \quad (33)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (38)$$

Theorem 1

$$\begin{aligned} & (\forall V0a \in (2^{ty_2Enum_2Enum}).((ap c_2ETemporal_Logic_2EALWAYS \\ V0a) = (ap (ap c_2ETemporal_Logic_2EUNTIL V0a) (\lambda V1t \in ty_2Enum_2Enum. \\ c_2Ebool_2EF)))) \end{aligned}$$