

thm\_2ETemporal\_\_Logic\_2ESWHEN\_\_IMP  
 (TMJu-  
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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{3}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (omega^{omega}) \tag{4}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{5}$$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num ($

**Definition 7** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

**Definition 9** We define  $c\_2ETemporal\_Logic\_2EWATCH$  to be  $\lambda V0q \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in (2^{ty\_2Enum\_2Enum})$

**Definition 10** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E))$

**Definition 11** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 12** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A) P)))$

**Definition 13** We define  $c\_2ETemporal\_Logic\_2ESWHEN$  to be  $\lambda V0a \in (2^{ty\_2Enum\_2Enum}).\lambda V1b \in (2^{ty\_2Enum\_2Enum})$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (6)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (7)$$

**Definition 14** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 15** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap c\_2Ebool\_2E\_2F\_5C A) P))$

Assume the following.

$$\begin{aligned} & ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\ & c\_2Enum\_2E0) V0n) = V0n)) \wedge (\forall V1m \in ty\_2Enum\_2Enum.(\forall V2n \in \\ & ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Enum\_2ESUC \\ & V1m)) V2n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V1m) \\ & V2n))))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & ((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap ( \\ & ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B \\ & (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\ & V0m) V1n))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\ & V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)))))))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty\_2Enum\_2Enum.(\forall V1c \in ty\_2Enum\_2Enum.( \\ & (ap (ap c\_2Earithmetic\_2E\_2D (ap (ap c\_2Earithmetic\_2E\_2B V0a) \\ & V1c)) V1c) = V0a))) \end{aligned} \quad (10)$$

Assume the following.

$$(\forall V0c \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmic\_2E\_2D V0c) V0c) = c\_2Enum\_2E0)) \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (15)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (17)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (20)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap V0P c\_2Enum\_2E0)) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c\_2Enum\_2ESUC V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p (ap V0P V2n)))))) \quad (21)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0e \in A_{27a}. (\forall V1f \in \\
& ((A_{27a}^{ty\_2Enum\_2Enum})^{A_{27a}}). (p (ap (c\_2Ebool\_2E\_3F\_21 (A_{27a}^{ty\_2Enum\_2Enum})) \\
& (\lambda V2fn1 \in (A_{27a}^{ty\_2Enum\_2Enum}). (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& (ap (ap (c\_2Emin\_2E\_3D A_{27a}) (ap V2fn1 c\_2Enum\_2E0)) V0e)) (ap \\
& (c\_2Ebool\_2E\_21 ty\_2Enum\_2Enum) (\lambda V3n \in ty\_2Enum\_2Enum. ( \\
& ap (ap (c\_2Emin\_2E\_3D A_{27a}) (ap V2fn1 (ap c\_2Enum\_2ESUC V3n))) \\
& (ap (ap V1f (ap V2fn1 V3n)) V3n))))))))) \\
& \tag{22}
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0a \in (2^{ty\_2Enum\_2Enum}). (\forall V1b \in (2^{ty\_2Enum\_2Enum}). \\
& (\forall V2t0 \in ty\_2Enum\_2Enum. ((p (ap (ap (ap c\_2ETemporal\_Logic\_2ESWHEN \\
& V0a) V1b) V2t0)) \Leftrightarrow (\forall V3q \in (2^{ty\_2Enum\_2Enum}). ((p (ap (ap \\
& (ap c\_2ETemporal\_Logic\_2EWATCH V3q) V1b) V2t0)) \Rightarrow (\exists V4t \in \\
& ty\_2Enum\_2Enum. ((\neg (p (ap V3q (ap (ap c\_2Earithmetic\_2E\_2B V4t) \\
& V2t0)))) \wedge ((p (ap V1b (ap (ap c\_2Earithmetic\_2E\_2B V4t) V2t0))) \wedge \\
& (p (ap V0a (ap (ap c\_2Earithmetic\_2E\_2B V4t) V2t0)))))))))))))
\end{aligned}$$