

thm_2Ealignment_2Ealigned_add_sub
 (TMJUWsYpqAi3xibf2CSEr23n7sFHbE7WXYm)

October 26, 2020

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (3)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be ($ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP$).

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Definition 4 We define c_2Ebool_2ET to be ($ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x)$)

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x)))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1 n) V0)$

Definition 8 We define `c_2Earthmetic_2ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. \text{nonempty } A \Rightarrow \text{nonempty } (\text{ty_}2\text{Ebool_}2\text{Eitself } A) \quad (7)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (\text{ty_2Ebool_2Eitself } A_27a) \quad (8)$$

Let $c_2Efcp_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Efcp_2Edimindex A_27a \in (\text{ty_2Enum_2Enum}^{(\text{ty_2Ebool_2Eitself } A_27a)}) \quad (9)$$

Let $c \in \mathbb{R}$ and $E, D : \iota$ be given. Assume the following.

Let $ty_2Efc_{finite_image} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. \text{nonempty } A \Rightarrow \text{nonempty } (\text{ty_}2Efc\text{p_}2Efinite_image \ A) \quad (11)$$

Definition 9 We define $c_{\text{2Ebool_2EF}}$ to be $(ap(c_{\text{2Ebool_2E_21}}) \ (\lambda V0t \in 2.V0t))$.

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_7E$ to be $(\lambda Vt \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E))$

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 13 We define $c_{\text{Emin}} \cdot 2E_{\text{A40}}$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \text{ } x)) \text{ then } (\lambda x. x \in A \wedge_P$ of type $\iota \Rightarrow \iota$.

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : i.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 15 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 17 We define $c_2Efcp_2Efinite_Index$ to be $\lambda A_\mathcal{Z}a : \iota.(ap(c_2Emin_2E40(A_\mathcal{Z}a)^{c_2Elen_2Ename_2E40}))$

$$\forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty}(\text{ty_}Efc\text{p_}Ec\text{art } A0 \text{ } A1) \quad (12)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_{_27a}.nonempty\ A_{_27a} \Rightarrow \forall A_{_27b}.nonempty\ A_{_27b} \Rightarrow c_2Efcpc_2Edest_cart \\ & A_{_27a}\ A_{_27b} \in ((A_{_27a}^{(ty_2Efcpc_2Efinite_image\ A_{_27b})})^{(ty_2Efcpc_2Ecart\ A_{_27a}\ A_{_27b})}) \end{aligned} \quad (13)$$

Definition 18 We define $c_2Efcpc_2Efcpc_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcpc_2Ecarr\ A_27a)$

Definition 19 We define $c_{\text{Ebool}} \text{ECOND}$ to be $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

Definition 20 We define c_2 Earthmet_2EMIN to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 21 We define $c_{\text{C_Ebool_2E_5C_2F}}$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_{\text{C_Ebool_2E_21}}\ 2)\ (\lambda V2t \in$

Definition 22 We define $c_{_2Earthmet_2E_{3C_3D}}$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 23 We define $c_{\text{2EFcp_2EFCP}}$ to be $\lambda A_{\text{27a}} : \iota. \lambda A_{\text{27b}} : \iota. (\lambda V0g \in (A_{\text{27a}})^{\text{ty_2Enum_2Enum}}). (ap$

Definition 24 We define $c_2Ewords_2Eword_slice$ to be $\lambda A_27a : \iota. \lambda V0h \in ty_2Enum_2Enum. \lambda V1l \in ty_2Enum$

Definition 27 We define $c_Ebool_EBOUNDED$ to be $(\lambda V0v \in 2.c_Ebool_ET)$.

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum})$$

metric_2EODD: ι be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum})$$

We define \mathcal{C} 2F_{arithmeti}c 2F_3F to be $\lambda V0m \in tu$ 2F_{enum} 2F_{enum}

Definition 29 We define \mathfrak{c} 2Earthmetic 2E 3E 3D to be $\lambda V0m \in tu\ 2Enum\ 2Enum\ \lambda V1n \in tu\ 2Enum\ 2Enum$.

Definition 30 We define $c : \text{2Eprim_rec_2EPRE}$ to be $\lambda V0m \in tu : \text{2Enum_2Enum} (ap (ap (ap (ap (c : \text{2Ebool_2Ebool})))))))$

Let c 2E arithmetic 2EXP; t be given. Assume the following

$\exists \exists E\text{arithm} \exists \exists E\text{FXYR} \in ((t_1 \exists E\text{num} \exists E\text{num})^{\text{ty}} \exists E\text{num} \exists E\text{num})$

(16)

Let c_2 be given. Assume the following.

$$\partial E_1 \cap \{t\} = \partial E_2 \wedge \{t\} \cup \partial E_3 \cap \{t\} \cup \partial E_4 \cap \{t\}$$

D. Section 21 Work 6 - 25. LOEÜSÜHG to b. NO. 5 to 2E. 2E. (17)

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum^{ty_2Enum_2Enum}})})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 35 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Efcp_2Ecart\ 2\ A_27a). (ap\ (ap\ c_2Esum_num_2ESUM\ A_27a)\ V0w)$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (19)$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Definition 36 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (c_2Ebit_2EDIV\ V0x\ V1n)$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (21)$$

Definition 37 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (c_2Ebit_2EMOD\ V0x\ V1n)$

Definition 38 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum. \lambda V1l \in ty_2Enum_2Enum. \lambda V2m \in ty_2Enum_2Enum. (c_2Ebit_2EBITS\ V0h\ V1l\ V2m)$

Definition 39 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (c_2Ebit_2EBIT\ V0b\ V1n)$

Definition 40 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota. \lambda V0n \in ty_2Enum_2Enum. (ap\ (c_2Efcp_2EFC\ A_27a)\ V0n)$

Definition 41 We define $c_2Ewords_2Eword_2comp$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Efcp_2Ecart\ 2\ A_27a). (ap\ (c_2Ebit_2EBITS\ A_27a)\ V0w)$

Definition 42 We define $c_2Ewords_2Eword_add$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_2Efcp_2Ecart\ 2\ A_27a). (\lambda V1n \in ty_2Enum_2Enum. (c_2Ebit_2EBIT\ V1n\ V0v))$

Definition 43 We define $c_2Ewords_2Eword_sub$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_2Efcp_2Ecart\ 2\ A_27a). (\lambda V1n \in ty_2Enum_2Enum. (c_2Ebit_2EBIT\ V1n\ V0v))$

Definition 44 We define $c_2Ewords_2Eword_bits$ to be $\lambda A_27a : \iota. \lambda V0h \in ty_2Enum_2Enum. \lambda V1l \in ty_2Enum_2Enum. (c_2Ebit_2EBITS\ V0h\ V1l)$

Definition 45 We define $c_2Ewords_2Ew2w$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w \in (ty_2Efcp_2Ecart\ 2\ A_27a). (\lambda V1n \in ty_2Enum_2Enum. (c_2Ebit_2EBIT\ V1n\ V0w))$

Definition 46 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). (\lambda V1n \in ty_2Enum_2Enum. (c_2Ebit_2EBIT\ V1n\ V0f))$

Definition 47 We define $c_2Ewords_2Eword_extract$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0h \in ty_2Enum_2Enum. (c_2Ebit_2EBITS\ V0h\ V0h)$

Definition 48 We define $c_2Ewords_2Eword_mul$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_2Efcp_2Ecart\ 2\ A_27a). (\lambda V1n \in ty_2Enum_2Enum. (c_2Ebit_2EBIT\ V1n\ V0v))$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0p \in ty_2Enum_2Enum. (& \\ \forall V1w \in (ty_2Efcop_2Ecart 2 A_27a).((p (ap (ap (c_2Ealignment_2Ealigned & \\ A_27a) V0p) V1w)) \Leftrightarrow ((V0p = c_2Enum_2E0) \vee ((ap (ap (ap (c_2Ewords_2Eword_extract & \\ A_27a A_27a) (ap (ap c_2Earithmetic_2E_2D V0p) (ap c_2Earithmetic_2ENUMERAL & \\ (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) c_2Enum_2E0) & \\ V1w) = (ap (c_2Ewords_2En2w A_27a) c_2Enum_2E0))))))) & \\ (22) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0p \in ty_2Enum_2Enum. (& \\ \forall V1w \in (ty_2Efcop_2Ecart 2 A_27a).((p (ap (ap c_2Earithmetic_2E_3C_3D & \\ (ap (c_2Efcop_2Edimindex A_27a) (c_2Ebool_2Ethe_value A_27a))) & \\ V0p)) \Rightarrow ((p (ap (ap (c_2Ealignment_2Ealigned A_27a) V0p) V1w)) \Leftrightarrow & \\ (V1w = (ap (c_2Ewords_2En2w A_27a) c_2Enum_2E0))))))) & \\ (23) \end{aligned}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) & \\ c_2Enum_2E0) = V0m)) \quad (24)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (& \\ ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (& \\ ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B & \\ (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B & \\ V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC & \\ V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))) & \\ (25) \end{aligned}$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (& \\ (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B & \\ V1n) V0m)))) & \\ (26) \end{aligned}$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (& \\ \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) & \\ (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B & \\ (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p)))))) & \\ (27) \end{aligned}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((V0m = c_2Enum_2E0) \vee (\exists V1n \in & \\ ty_2Enum_2Enum. (V0m = (ap c_2Enum_2ESUC V1n))))) \quad (28)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0m)) V1n)))))) \quad (29)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D c_2Enum_2E0) V0n))) \quad (30)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m)))))) \quad (31)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2E_2D V0m) c_2Enum_2E0) = V0m))) \quad (32)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D (ap c_2Enum_2ESUC V0m)) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m))) \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge \\ & ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\ & V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\ & (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A V0m) V1n))))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge \\ & (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p))))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p \\
 & \quad ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m)))))) \\
 \end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & \quad ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
 & \quad V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)))))) \\
 \end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & \quad (ap c_2Enum_2ESUC V1n)) V0m)))))) \\
 \end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
 & \quad V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
 & \quad V1n)) V0m)))))) \\
 \end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\
 & c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
 & \quad c_2Earithmetic_2EZERO)) V0n))) \\
 \end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C V0m) \\
 & \quad (ap (ap c_2Earithmetic_2E_2D V1n) V2p))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
 & \quad (ap (ap c_2Earithmetic_2E_2B V0m) V2p)) V1n)))))) \\
 \end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
 & \quad ap (ap c_2Earithmetic_2E_2D V0m) V1n)) V2p)) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C \\
 & \quad V0m) (ap (ap c_2Earithmetic_2E_2B V1n) V2p))) \wedge (p (ap (ap c_2Eprim_rec_2E_3C \\
 & \quad c_2Enum_2E0) V2p)))))) \\
 \end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0P \in (2^{ty_2Enum_2Enum}).(\forall V1a \in ty_2Enum_2Enum. \\
 & (\forall V2b \in ty_2Enum_2Enum.((p (ap V0P (ap (ap c_2Earithmetic_2E_2D \\
 & V1a) V2b))) \Leftrightarrow (\forall V3d \in ty_2Enum_2Enum.((V2b = (ap (ap c_2Earithmetic_2E_2B \\
 & V1a) V3d)) \Rightarrow (p (ap V0P c_2Enum_2E0))) \wedge ((V1a = (ap (ap c_2Earithmetic_2E_2B \\
 & V2b) V3d)) \Rightarrow (p (ap V0P V3d))))))) \\
 \end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum. \\
 & (\forall V2p \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
 & (ap (ap c_2Earithmetic_2EMIN V1m) V0n) V2p)) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C \\
 & V1m) V2p)) \vee (p (ap (ap c_2Eprim_rec_2E_3C V0n) V2p)))) \wedge ((p (ap \\
 & (ap c_2Eprim_rec_2E_3C V2p) (ap (ap c_2Earithmetic_2EMIN V1m) \\
 & V0n))) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C V2p) V1m)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C \\
 & V2p) V0n))))))) \\
 \end{aligned} \tag{44}$$

Assume the following.

$$True \tag{45}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Leftrightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \tag{46}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \tag{47}$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \tag{48}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{49}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge \\
 ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))) \tag{50}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
 & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
 & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \\
 \end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \end{aligned} \quad (53)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \quad (54))$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\ True)) \quad (55)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (56)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & (p V0t))))))) \end{aligned} \quad (57)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (\\ (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C))))))) \quad (58)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p \\ V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \end{aligned} \quad (59)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee \\ (p V1B)))))) \quad (60)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (61)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (62)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1) \wedge (\neg(p V1t2))))))) \quad (63)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27))))))) \Rightarrow \\ & 2. (((((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\ & V5y_27)))))))))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. \\ & ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ & (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V2t1) V3t2) = V3t2)))))) \end{aligned} \quad (66)$$

Assume the following.

$$(\forall V0v \in 2. ((p (ap c_2Ebool_2EBOUNDED V0v)) \Leftrightarrow \text{True})) \quad (67)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg((ap c_2Enum_2ESUC V0n) = c_2Enum_2E0))) \quad (68)$$

Assume the following.

$((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2EiZ (ap (ap c_2Earithmetic_2E_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum.((\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A V6n) V7m))))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V10n) V11m))))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP V14n) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))) \wedge ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL V15n)) (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2EEEXP V15n) V16m))))))) \wedge (((ap c_2Enum_2ESUC c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \wedge ((\forall V17n \in ty_2Enum_2Enum.((ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2ESUC V17n))))))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Eprim_rec_2EPRE V18n))))))) \wedge ((\forall V19n \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum.((\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m))))))) \wedge ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V23n)) \Leftrightarrow False))) \wedge ((\forall V24n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V24n)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) V24n))))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V25n)) (ap c_2Earithmetic_2ENUMERAL V26m)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V25n) V26m))))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V27n)) \Leftrightarrow False))) \wedge ((\forall V28n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL V28n)) c_2Enum_2E0) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) V28n))))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.((\forall V30m \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V29n)) (ap c_2Earithmetic_2ENUMERAL V30m)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V29n))))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V31n)) \Leftrightarrow True))) \wedge ((\forall V32n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C c_2Enum_2E0) V32n)) \Leftrightarrow False))) \wedge ((\forall V33n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V33n)) \Leftrightarrow True))) \wedge ((\forall V34n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V34n)) \Leftrightarrow False)))$

Assume the following.

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{71}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Eprim_rec_2E_3C \\
V0n) V0n)))) \tag{72}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Eprim_rec_2E_3C \\
V0n) c_2Enum_2E0)))) \tag{73}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{74}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{75}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \tag{76}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \tag{77}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{78}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
 & ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \\
 \end{aligned} \tag{79}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
 & ((\neg(p V0p))) \wedge ((p V2r) \vee ((\neg(p V0p))))))))))) \\
 \end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \wedge ((p V0p) \vee ((\neg(p V2r)))) \wedge \\
 & ((p V1q) \vee ((p V2r) \vee ((\neg(p V0p))))))))))) \\
 \end{aligned} \tag{81}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
 (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \tag{82}$$

Assume the following.

$$\begin{aligned}
 & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \\
 & \forall V0h \in \text{ty_2Enum_2Enum}. (\forall V1l \in \text{ty_2Enum_2Enum}. ((\\
 & \text{ap } (\text{ap } (\text{ap } (\text{c_2Ewords_2Eword_extract } A_27b A_27a) V0h) V1l) (\text{ap } \\
 & (\text{c_2Ewords_2En2w } A_27b) \text{ c_2Enum_2E0})) = (\text{ap } (\text{c_2Ewords_2En2w } \\
 & A_27a) \text{ c_2Enum_2E0)))) \\
 \end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
 & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\
 & \text{nonempty } A_27c \Rightarrow (\forall V0w \in (\text{ty_2Efcp_2Ecart } 2 A_27c). (\forall V1h \in \\
 & \text{ty_2Enum_2Enum}. (\forall V2l \in \text{ty_2Enum_2Enum}. (\forall V3m \in \text{ty_2Enum_2Enum}. \\
 & (\forall V4n \in \text{ty_2Enum_2Enum}. ((\text{ap } (\text{ap } (\text{ap } (\text{c_2Ewords_2Eword_extract } \\
 & A_27b A_27a) V1h) V2l) (\text{ap } (\text{ap } (\text{ap } (\text{c_2Ewords_2Eword_extract } A_27c \\
 & A_27b) V3m) V4n) V0w)) = (\text{ap } (\text{ap } (\text{ap } (\text{c_2Ewords_2Eword_extract } \\
 & A_27c A_27a) (\text{ap } (\text{ap } \text{c_2Earithmetic_2EMIN } V3m) (\text{ap } (\text{ap } \text{c_2Earithmetic_2EMIN } \\
 & (\text{ap } (\text{ap } \text{c_2Earithmetic_2E_2B } V1h) V4n)) (\text{ap } (\text{ap } \text{c_2Earithmetic_2EMIN } \\
 & (\text{ap } (\text{ap } \text{c_2Earithmetic_2E_2D } (\text{ap } (\text{c_2Efcp_2Edimindex } A_27c) (\\
 & \text{c_2Ebool_2Eth_value } A_27c))) (\text{ap } \text{c_2Earithmetic_2ENUMERAL } \\
 & (\text{ap } \text{c_2Earithmetic_2EBIT1 } \text{c_2Earithmetic_2EZERO}))))))) (\text{ap } (\text{ap } \\
 & \text{c_2Earithmetic_2E_2D } (\text{ap } (\text{ap } \text{c_2Earithmetic_2E_2B } (\text{ap } (\text{c_2Efcp_2Edimindex } \\
 & A_27b) (\text{c_2Ebool_2Eth_value } A_27b))) V4n)) (\text{ap } \text{c_2Earithmetic_2ENUMERAL } \\
 & (\text{ap } \text{c_2Earithmetic_2EBIT1 } \text{c_2Earithmetic_2EZERO}))))))) (\text{ap } \\
 & (\text{ap } \text{c_2Earithmetic_2E_2B } V2l) V4n) V0w))))))) \\
 \end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. nonempty A_{27a} \Rightarrow \forall A_{27b}. nonempty A_{27b} \Rightarrow \\
& \forall V0a \in (ty_2Efc_2Ecart 2 A_{27a}). (\forall V1b \in (ty_2Efc_2Ecart \\
& 2 A_{27a}). (\forall V2h \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& V2h) (ap (c_2Efc_2Edimindex A_{27a}) (c_2Ebool_2Ethet_value A_{27a})))) \Rightarrow \\
& ((ap (ap (ap (c_2Ewords_2Eword_extract A_{27b} A_{27b}) V2h) c_2Enum_2E0) \\
& (ap (ap (c_2Ewords_2Eword_add A_{27b}) (ap (ap (c_2Ewords_2Eword_extract \\
& A_{27a} A_{27b}) V2h) c_2Enum_2E0) V0a)) (ap (ap (ap (c_2Ewords_2Eword_extract \\
& A_{27a} A_{27b}) V2h) c_2Enum_2E0) V1b))) = (ap (ap (ap (c_2Ewords_2Eword_extract \\
& A_{27a} A_{27b}) V2h) c_2Enum_2E0) (ap (ap (c_2Ewords_2Eword_add \\
& A_{27a}) V0a) V1b)))))) \\
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. nonempty A_{27a} \Rightarrow \forall A_{27b}. nonempty A_{27b} \Rightarrow \\
& \forall V0a \in (ty_2Efc_2Ecart 2 A_{27a}). (\forall V1b \in (ty_2Efc_2Ecart \\
& 2 A_{27a}). (\forall V2h \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& V2h) (ap (c_2Efc_2Edimindex A_{27a}) (c_2Ebool_2Ethet_value A_{27a})))) \Rightarrow \\
& ((ap (ap (ap (c_2Ewords_2Eword_extract A_{27b} A_{27b}) V2h) c_2Enum_2E0) \\
& (ap (ap (c_2Ewords_2Eword_mul A_{27b}) (ap (ap (c_2Ewords_2Eword_extract \\
& A_{27a} A_{27b}) V2h) c_2Enum_2E0) V0a)) (ap (ap (ap (c_2Ewords_2Eword_extract \\
& A_{27a} A_{27b}) V2h) c_2Enum_2E0) V1b))) = (ap (ap (ap (c_2Ewords_2Eword_extract \\
& A_{27a} A_{27b}) V2h) c_2Enum_2E0) (ap (ap (c_2Ewords_2Eword_mul \\
& A_{27a}) V0a) V1b)))))) \\
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. nonempty A_{27a} \Rightarrow ((\forall V0w \in (ty_2Efc_2Ecart \\
& 2 A_{27a}). ((ap (ap (c_2Ewords_2Eword_add A_{27a}) V0w) (ap (c_2Ewords_2En2w \\
& A_{27a}) c_2Enum_2E0)) = V0w)) \wedge (\forall V1w \in (ty_2Efc_2Ecart 2 \\
& A_{27a}). ((ap (ap (c_2Ewords_2Eword_add A_{27a}) (ap (c_2Ewords_2En2w \\
& A_{27a}) c_2Enum_2E0)) V1w) = V1w))) \\
\end{aligned} \tag{87}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0v \in (ty_2Efc_2Ecart \\
& 2 A_{27a}). (\forall V1w \in (ty_2Efc_2Ecart 2 A_{27a}). ((ap (ap (c_2Ewords_2Eword_mul \\
& A_{27a}) V0v) V1w) = (ap (ap (c_2Ewords_2Eword_mul A_{27a}) V1w) V0v)))) \\
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0v \in (ty_{.2E}fc_{.2E}cart \\
& 2 A_{.27a}).(\forall V1w \in (ty_{.2E}fc_{.2E}cart 2 A_{.27a}).(((ap (ap (\\
& c_{.2E}words_{.2E}word_{.mul} A_{.27a}) (ap (c_{.2E}words_{.2E}En2w A_{.27a}) c_{.2E}num_{.2E}0) \\
& V0v) = (ap (c_{.2E}words_{.2E}En2w A_{.27a}) c_{.2E}num_{.2E}0)) \wedge (((ap (ap (c_{.2E}words_{.2E}word_{.mul} \\
& A_{.27a}) V0v) (ap (c_{.2E}words_{.2E}En2w A_{.27a}) c_{.2E}num_{.2E}0)) = (ap (c_{.2E}words_{.2E}En2w \\
& A_{.27a}) c_{.2E}num_{.2E}0)) \wedge (((ap (ap (c_{.2E}words_{.2E}word_{.mul} A_{.27a}) \\
& (ap (c_{.2E}words_{.2E}En2w A_{.27a}) (ap c_{.2E}arithm_{.2E}NUMERAL (ap \\
& c_{.2E}arithm_{.2E}BIT1 c_{.2E}arithm_{.2E}ZERO)))) V0v) = V0v) \wedge \\
& (((ap (ap (c_{.2E}words_{.2E}word_{.mul} A_{.27a}) V0v) (ap (c_{.2E}words_{.2E}En2w \\
& A_{.27a}) (ap c_{.2E}arithm_{.2E}NUMERAL (ap c_{.2E}arithm_{.2E}BIT1 \\
& c_{.2E}arithm_{.2E}ZERO)))) = V0v) \wedge (((ap (ap (c_{.2E}words_{.2E}word_{.mul} \\
& A_{.27a}) (ap (ap (c_{.2E}words_{.2E}word_{.add} A_{.27a}) V0v) (ap (c_{.2E}words_{.2E}En2w \\
& A_{.27a}) (ap c_{.2E}arithm_{.2E}NUMERAL (ap c_{.2E}arithm_{.2E}BIT1 \\
& c_{.2E}arithm_{.2E}ZERO)))) = V1w) = (ap (ap (c_{.2E}words_{.2E}word_{.add} \\
& A_{.27a}) (ap (ap (c_{.2E}words_{.2E}word_{.mul} A_{.27a}) V0v) V1w)) \wedge \\
& ((ap (ap (c_{.2E}words_{.2E}word_{.mul} A_{.27a}) V0v) (ap (ap (c_{.2E}words_{.2E}word_{.add} \\
& A_{.27a}) V1w) (ap (c_{.2E}words_{.2E}En2w A_{.27a}) (ap c_{.2E}arithm_{.2E}NUMERAL \\
& (ap c_{.2E}arithm_{.2E}BIT1 c_{.2E}arithm_{.2E}ZERO)))) = (ap (\\
& ap (c_{.2E}words_{.2E}word_{.add} A_{.27a}) V0v) (ap (ap (c_{.2E}words_{.2E}word_{.mul} \\
& A_{.27a}) V0v) V1w))))))) \\
\end{aligned} \tag{89}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0w \in (ty_{.2E}fc_{.2E}cart \\
& 2 A_{.27a}).((ap (c_{.2E}words_{.2E}word_{.2comp} A_{.27a}) V0w) = (ap (ap (\\
& c_{.2E}words_{.2E}word_{.mul} A_{.27a}) (ap (c_{.2E}words_{.2E}word_{.2comp} A_{.27a}) \\
& (ap (c_{.2E}words_{.2E}En2w A_{.27a}) (ap c_{.2E}arithm_{.2E}NUMERAL (ap \\
& c_{.2E}arithm_{.2E}BIT1 c_{.2E}arithm_{.2E}ZERO)))) V0w)))
\end{aligned} \tag{90}$$

Theorem 1

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0p \in ty_{.2E}enum_{.2E}enum.(\\
& \forall V1a \in (ty_{.2E}fc_{.2E}cart 2 A_{.27a}).(\forall V2b \in (ty_{.2E}fc_{.2E}cart \\
& 2 A_{.27a}).((p (ap (ap (c_{.2E}alignment_{.2E}aligned A_{.27a}) V0p) V2b)) \Rightarrow \\
& (((p (ap (ap (c_{.2E}alignment_{.2E}aligned A_{.27a}) V0p) (ap (ap (c_{.2E}words_{.2E}word_{.add} \\
& A_{.27a}) V1a) V2b))) \Leftrightarrow (p (ap (ap (c_{.2E}alignment_{.2E}aligned A_{.27a}) \\
& V0p) V1a))) \wedge ((p (ap (ap (c_{.2E}alignment_{.2E}aligned A_{.27a}) V0p) (\\
& ap (ap (c_{.2E}words_{.2E}word_{.sub} A_{.27a}) V1a) V2b))) \Leftrightarrow (p (ap (ap (c_{.2E}alignment_{.2E}aligned \\
& A_{.27a}) V0p) V1a)))))))
\end{aligned}$$