

thm_2Ealignment_2Ealigned__add__sub__prod
(TMKcdEzWzBLqmor-
JFK4gFheDaezh1PDs6zm)

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Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efc_2Efinite_image\ A0) \quad (1)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (2)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (3)$$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \quad (4)$$

Let $c_2Efc_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efc_2Edimindex\ A_27a \in (ty_2Eenum_2Eenum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (5)$$

Definition 2 We define c_2Ebool_2EET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a}))\ (\lambda V1Q \in 2.V1Q))\ (\lambda V2R \in 2.V2R))\ P)\ Q)$

Definition 4 We define c_2Ebool_2E2F to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 6 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E_7E))$

Definition 7 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in 2)))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num)$

Definition 9 We define $c_Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\lambda x.x \in A \wedge P x)$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_Emin_2E_40 A_27a))))$

Definition 11 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_Ebool_2E_2F_5C A_27a) P))$

Definition 13 We define $c_Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_Emin_2E_40 A_27a) (ty_2Enum_2Enum))$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efcp_2Ecart A0 A1) \quad (9)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efcp_2Edest_cart A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image A_27b)})^{(ty_2Efcp_2Ecart A_27a A_27b)}) \quad (10)$$

Definition 14 We define $c_Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart A_27a A_27b)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (11)$$

Definition 15 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 16 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (12)$$

Definition 17 We define $c_Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2E_2B))$

Definition 18 We define $c_Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (13)$$

Definition 19 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(c_Ebool_2ECOND A_27a t1 t2))))$

Definition 20 We define c_Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap c_Ebool_2ECOND)))$

Let $c_Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (14)$$

Definition 21 We define c_Ewords_2Ew2n to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Enum_2Enum^{(ty_2EfcP_2Ecart 2 A_27a)}).(ap (ap c_Esum_num_2ESUM))$

Let $c_Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Ewords_2Edimword A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)})^{ty_2Enum_2Enum} \quad (15)$$

Let $c_Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (16)$$

Definition 22 We define $c_Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2E_2D))$

Let $c_Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (17)$$

Definition 23 We define $c_Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2EDIV))$

Let $c_Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (18)$$

Definition 24 We define $c_Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2EMOD))$

Definition 25 We define c_Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V2n \in ty_2Enum_2Enum.(ap (ap (ap c_Ebit_2ESBIT)))$

Definition 26 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 27 We define c_2EfcP_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 28 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2EfcP_2EFCP$

Definition 29 We define $c_2Ewords_2Eword_2comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).$

Definition 30 We define $c_2Ewords_2Eword_2add$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V$

Definition 31 We define $c_2Ewords_2Eword_2sub$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V$

Definition 32 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 33 We define $c_2Earithmetic_2E_3C_2D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2$

Definition 34 We define $c_2Ewords_2Eword_2isl$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (19)$$

Definition 35 We define $c_2Ewords_2Eword_2mul$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V$

Definition 36 We define $c_2Earithmetic_2EMIN$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2E$

Definition 37 We define $c_2Ewords_2Eword_2slice$ to be $\lambda A_27a : \iota.\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2$

Definition 38 We define $c_2Ealignment_2Ealign$ to be $\lambda A_27a : \iota.\lambda V0p \in ty_2Enum_2Enum.\lambda V1w \in (ty_2$

Definition 39 We define $c_2Ealignment_2Ealigned$ to be $\lambda A_27a : \iota.\lambda V0p \in ty_2Enum_2Enum.\lambda V1w \in (ty_2$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0p \in ty_2Enum_2Enum.(\\ & \quad \forall V1a \in (ty_2EfcP_2Ecart\ 2\ A_27a).(\forall V2b \in (ty_2EfcP_2Ecart \\ & \quad 2\ A_27a).(p\ (ap\ (ap\ (c_2Ealignment_2Ealigned\ A_27a)\ V0p)\ V2b)) \Rightarrow \\ & \quad (((p\ (ap\ (ap\ (c_2Ealignment_2Ealigned\ A_27a)\ V0p)\ (ap\ (ap\ (c_2Ewords_2Eword_2add \\ & \quad A_27a)\ V1a)\ V2b))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ealignment_2Ealigned\ A_27a) \\ & \quad V0p)\ V1a))) \wedge ((p\ (ap\ (ap\ (c_2Ealignment_2Ealigned\ A_27a)\ V0p)\ (\\ & \quad ap\ (ap\ (c_2Ewords_2Eword_2sub\ A_27a)\ V1a)\ V2b))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ealignment_2Ealigned \\ & \quad A_27a)\ V0p)\ V1a)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0p \in ty_2Enum_2Enum.(\\ & \quad \forall V1w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(p\ (ap\ (ap\ (c_2Ealignment_2Ealigned \\ & \quad A_27a)\ V0p)\ (ap\ (ap\ (c_2Ewords_2Eword_2mul\ A_27a)\ (ap\ (ap\ (c_2Ewords_2Eword_2isl \\ & \quad A_27a)\ (ap\ (c_2Ewords_2En2w\ A_27a)\ (ap\ c_2Earithmetic_2ENUMERAL \\ & \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))\ V0p))\ V1w)))) \end{aligned} \quad (21)$$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (24)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (32)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{37}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{38}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{39}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0p \in ty_2Enum_2Enum. (\\
& \forall V1w \in (ty_2EfcP_2Ecart 2 A_27a). (\forall V2x \in (ty_2EfcP_2Ecart \\
& 2 A_27a). (((p (ap (ap (c_2Ealignment_2Ealigned A_27a) V0p) (ap \\
& (ap (c_2Ewords_2Eword_add A_27a) V1w) (ap (ap (c_2Ewords_2Eword_mul \\
& A_27a) (ap (ap (c_2Ewords_2Eword_lsl A_27a) (ap (c_2Ewords_2En2w \\
& A_27a) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO)))) V0p)) V2x))) \Leftrightarrow (p (ap (ap (c_2Ealignment_2Ealigned \\
& A_27a) V0p) V1w))) \wedge ((p (ap (ap (c_2Ealignment_2Ealigned A_27a) \\
& V0p) (ap (ap (c_2Ewords_2Eword_sub A_27a) V1w) (ap (ap (c_2Ewords_2Eword_mul \\
& A_27a) (ap (ap (c_2Ewords_2Eword_lsl A_27a) (ap (c_2Ewords_2En2w \\
& A_27a) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO)))) V0p)) V2x))) \Leftrightarrow (p (ap (ap (c_2Ealignment_2Ealigned \\
& A_27a) V0p) V1w))))))
\end{aligned}$$