

thm_2Ealignment_2Ealigned__numeric
(TMPmkjUdXZkxkR6sFBuTDnP2s73cCmUjMtN)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define c_2Ebool_2E21 to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a}))\ (\lambda V1P \in 2.V1P))\ (\lambda V2P \in 2.V2P))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1))$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (7)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (8)$$

Let $c_2Efcp_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Efcp_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (9)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Let $ty_2Efcp_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Efcp_2Efinite_image A0) \quad (11)$$

Definition 9 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$.

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V2t)))$

Definition 13 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then $(the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a))))$

Definition 15 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 16 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C A_27a P)))$

Definition 17 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 (A_27a^{ty_2Enum_2Enum})))$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efcp_2Ecart A0 A1) \quad (12)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efcp_2Edest_cart A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image A_27b)})^{(ty_2Efcp_2Ecart A_27a A_27b)}) \quad (13)$$

Definition 18 We define $c_2Efc_2Efc_index$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in (ty_2Efc_2Ecart\ A.27a)$

Definition 19 We define c_2Ebool_2ECOND to be $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.(ap$

Definition 20 We define $c_2Earithmetic_2EMIN$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 21 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in$

Definition 22 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 23 We define c_2Efc_2EFCP to be $\lambda A.27a : \iota.\lambda A.27b : \iota.(\lambda V0g \in (A.27a^{ty_2Enum_2Enum}).(ap$

Definition 24 We define $c_2Ewords_2Eword_slice$ to be $\lambda A.27a : \iota.\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum$

Definition 25 We define $c_2Ealignment_2Ealign$ to be $\lambda A.27a : \iota.\lambda V0p \in ty_2Enum_2Enum.\lambda V1w \in (ty_2Enum_2Enum$

Definition 26 We define $c_2Ealignment_2Ealigned$ to be $\lambda A.27a : \iota.\lambda V0p \in ty_2Enum_2Enum.\lambda V1w \in (ty_2Enum_2Enum$

Definition 27 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 28 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Definition 29 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Enum_2ESUC\ (ap$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (15)$$

Let $c_2Enumeral_2Eonecount : \iota$ be given. Assume the following.

$$c_2Enumeral_2Eonecount \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Enumeral_2Eexactlog : \iota$ be given. Assume the following.

$$c_2Enumeral_2Eexactlog \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}) \quad (17)$$

Definition 30 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2E_21\ 2)$

Definition 31 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_3E$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 32 We define $c_2Earithmetic_2EDIV2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_3E$

Let $c_2Enumeral_2Eexp_help : \iota$ be given. Assume the following.

$$c_2Enumeral_2Eexp_help \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (19)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (20)$$

Definition 33 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in (A_27b^{A_27a}). (\lambda V1x \in A_27b. \dots))$

Definition 34 We define $c_2Enumeral_2EiDUB$ to be $\lambda V0x \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2EODD) V0x)$

Definition 35 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (21)$$

Definition 36 We define $c_2Enumeral_2Einternal_mult$ to be $c_2Earithmetic_2E_2A$.

Definition 37 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2. \lambda V1n \in ty_2Enum_2Enum. (ap (ap (ap c_2Ebool_2ELET) V0b) V1n)$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum} \quad (22)$$

Definition 38 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2EfcP_2Ecart_2A_27a). (ap (ap c_2Esum_num_2ESUM) V0w)$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ewords_2Edimword A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (23)$$

Definition 39 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. \dots$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (24)$$

Definition 40 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. \dots$

Definition 41 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum. \lambda V1l \in ty_2Enum_2Enum. \dots$

Definition 42 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap c_2Ebit_2ESBIT) V0b$

Definition 43 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota. \lambda V0n \in ty_2Enum_2Enum. (ap c_2EfcP_2EFC) V0n$

Definition 44 We define $c_2Ewords_2Eword_2comp$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2EfcP_2Ecart_2A_27a). \dots$

Definition 45 We define $c_Ewords_Eword_add$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_2EfcP_2Ecart\ 2\ A_27a). \lambda V1$

Definition 46 We define $c_Ewords_Eword_sub$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_2EfcP_2Ecart\ 2\ A_27a). \lambda V1$

Definition 47 We define $c_Ewords_Eword_mul$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_2EfcP_2Ecart\ 2\ A_27a). \lambda V1$

Assume the following.

$$((ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ c_2Enum_2E0)) \quad (26)$$

Assume the following.

$$((ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) \quad (27)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V0m)))) \quad (28)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n)\ V2p) = (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))\ V2p)))))) \quad (29)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Earithmetic_2E_2A\ c_2Enum_2E0)\ V0m) = c_2Enum_2E0) \wedge (((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ c_2Enum_2E0) = c_2Enum_2E0) \wedge (((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))\ V0m) = V0m) \wedge (((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))) = V0m) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ c_2Enum_2ESUC\ V0m))\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n))\ V1n)) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ (ap\ c_2Enum_2ESUC\ V1n)) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n)))))))))) \quad (30)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))\ V2p) = (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V2p))\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V1n)\ V2p)))))) \quad (31)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V2p) \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) = (ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap (ap c_2Earithmetic_2E_2A V2p) V0m)) (ap (ap c_2Earithmetic_2E_2A \\
& \quad \quad V2p) V1n))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
& \quad V0n) = (ap (ap c_2Earithmetic_2E_2B V0n) V0n)))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
& \quad V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& \quad V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& \quad \quad V1n)) V0m))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\
& \quad c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad \quad c_2Earithmetic_2EZERO))) V0n)))
\end{aligned} \tag{36}$$

Assume the following.

$$True \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& \quad V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))
\end{aligned} \tag{38}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\
& \quad A_27a. (p V0t)) \Leftrightarrow (p V0t)))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{42}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{43}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\
& A_27a.(((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2ET) \ V0t1) \\
& V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2EF \\
& V0t1) \ V1t2) = V1t2))))))
\end{aligned} \tag{46}$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Leftrightarrow ((p \ V0t) \Leftrightarrow False))) \tag{47}$$

Assume the following.

$$\begin{aligned}
& (((ap \ c_2Enum_2ESUC \ c_2Earithmetic_2EZERO) = (ap \ c_2Earithmetic_2EBIT1 \\
& \ c_2Earithmetic_2EZERO)) \wedge ((\forall V0n \in ty_2Enum_2Enum.((ap \\
& \ c_2Enum_2ESUC \ (ap \ c_2Earithmetic_2EBIT1 \ V0n)) = (ap \ c_2Earithmetic_2EBIT2 \\
& \ V0n))) \wedge (\forall V1n \in ty_2Enum_2Enum.((ap \ c_2Enum_2ESUC \ (ap \ c_2Earithmetic_2EBIT2 \\
& \ V1n)) = (ap \ c_2Earithmetic_2EBIT1 \ (ap \ c_2Enum_2ESUC \ V1n))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))))))))))))))))))))))))))
\end{aligned}$$

(50)

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((c_2Earithmic_2EZERO = (ap c_2Earithmic_2EBIT1 V0n)) \Leftrightarrow False) \wedge \\
& (((ap c_2Earithmic_2EBIT1 V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((c_2Earithmic_2EZERO = (ap c_2Earithmic_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap c_2Earithmic_2EBIT2 V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap c_2Earithmic_2EBIT1 V0n) = (ap c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap c_2Earithmic_2EBIT2 V0n) = (ap c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap c_2Earithmic_2EBIT1 V0n) = (ap c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap c_2Earithmic_2EBIT2 V0n) = (ap c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))))) \\
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmic_2E_3C_3D c_2Earithmic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D \\
& (ap c_2Earithmic_2EBIT2 V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))))))))) \\
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (((ap c_2Eprim_rec_2EPRE c_2Earithmic_2EZERO) = c_2Earithmic_2EZERO) \wedge \\
& (((ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)) = \\
& c_2Earithmic_2EZERO) \wedge ((\forall V0n \in ty_2Enum_2Enum. ((ap \\
& c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 (ap c_2Earithmic_2EBIT1 \\
& V0n))) = (ap c_2Earithmic_2EBIT2 (ap c_2Eprim_rec_2EPRE (ap \\
& c_2Earithmic_2EBIT1 V0n)))))) \wedge ((\forall V1n \in ty_2Enum_2Enum. \\
& ((ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 (ap c_2Earithmic_2EBIT2 \\
& V1n))) = (ap c_2Earithmic_2EBIT2 (ap c_2Earithmic_2EBIT1 \\
& V1n)))) \wedge ((\forall V2n \in ty_2Enum_2Enum. ((ap c_2Eprim_rec_2EPRE \\
& (ap c_2Earithmic_2EBIT2 V2n)) = (ap c_2Earithmic_2EBIT1 V2n))))))))) \\
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT1 \\
& \quad V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiDUB\ V0n))) \wedge \\
& \quad (((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT2\ V0n)) = (ap \\
& \quad \quad c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT1\ V0n))) \wedge ((ap \\
& \quad \quad c_2Enumeral_2EiDUB\ c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO)))) \\
& \hspace{15em} (54)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p\ (ap\ c_2Earithmetic_2EVEN\ c_2Earithmetic_2EZERO)) \wedge \\
& \quad ((p\ (ap\ c_2Earithmetic_2EVEN\ (ap\ c_2Earithmetic_2EBIT2\ V0n))) \wedge \\
& \quad ((\neg(p\ (ap\ c_2Earithmetic_2EVEN\ (ap\ c_2Earithmetic_2EBIT1\ V0n)))) \wedge \\
& \quad \quad ((\neg(p\ (ap\ c_2Earithmetic_2EODD\ c_2Earithmetic_2EZERO))) \wedge ((\\
& \quad \quad \neg(p\ (ap\ c_2Earithmetic_2EODD\ (ap\ c_2Earithmetic_2EBIT2\ V0n)))) \wedge \\
& \quad \quad (p\ (ap\ c_2Earithmetic_2EODD\ (ap\ c_2Earithmetic_2EBIT1\ V0n)))))))))) \\
& \hspace{15em} (55)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0acc \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& \quad (((ap\ (ap\ c_2Enumeral_2Etex_help\ c_2Earithmetic_2EZERO\ V0acc) = \\
& \quad (ap\ c_2Earithmetic_2EBIT2\ V0acc)) \wedge (((ap\ (ap\ c_2Enumeral_2Etex_help \\
& \quad (ap\ c_2Earithmetic_2EBIT1\ V1n))\ V0acc) = (ap\ (ap\ c_2Enumeral_2Etex_help \\
& \quad \quad (ap\ c_2Eprim_rec_2EPRE\ (ap\ c_2Earithmetic_2EBIT1\ V1n)))\ (ap \\
& \quad \quad c_2Earithmetic_2EBIT1\ V0acc))) \wedge ((ap\ (ap\ c_2Enumeral_2Etex_help \\
& \quad (ap\ c_2Earithmetic_2EBIT2\ V1n))\ V0acc) = (ap\ (ap\ c_2Enumeral_2Etex_help \\
& \quad \quad (ap\ c_2Earithmetic_2EBIT1\ V1n))\ (ap\ c_2Earithmetic_2EBIT1\ V0acc)))))) \\
& \hspace{15em} (56)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Enumeral_2Eonecount \\
& \quad c_2Earithmetic_2EZERO\ V0x) = V0x)) \wedge ((\forall V1n \in ty_2Enum_2Enum. \\
& \quad (\forall V2x \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Enumeral_2Eonecount \\
& \quad (ap\ c_2Earithmetic_2EBIT1\ V1n))\ V2x) = (ap\ (ap\ c_2Enumeral_2Eonecount \\
& \quad \quad V1n)\ (ap\ c_2Enum_2ESUC\ V2x)))))) \wedge ((\forall V3n \in ty_2Enum_2Enum. \\
& \quad (\forall V4x \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Enumeral_2Eonecount \\
& \quad (ap\ c_2Earithmetic_2EBIT2\ V3n))\ V4x) = c_2Earithmetic_2EZERO)))))) \\
& \hspace{15em} (57)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (((ap\ c_2Enumeral_2Exactlog\ c_2Earithmic_2EZERO) = c_2Earithmic_2EZERO) \wedge \\
& \quad ((\forall V0n \in ty_2Enum_2Enum. ((ap\ c_2Enumeral_2Exactlog\ (\\
& \quad \quad ap\ c_2Earithmic_2EBIT1\ V0n)) = c_2Earithmic_2EZERO)) \wedge (\forall V1n \in \\
& \quad \quad ty_2Enum_2Enum. ((ap\ c_2Enumeral_2Exactlog\ (ap\ c_2Earithmic_2EBIT2 \\
& \quad \quad \quad V1n)) = (ap\ (ap\ (c_2Ebool_2ELET\ ty_2Enum_2Enum\ ty_2Enum_2Enum) \\
& \quad \quad \quad (\lambda V2x \in ty_2Enum_2Enum. (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ ty_2Enum_2Enum) \\
& \quad \quad \quad \quad (ap\ (ap\ (c_2Emin_2E_3D\ ty_2Enum_2Enum)\ V2x)\ c_2Earithmic_2EZERO)) \\
& \quad \quad \quad \quad \quad c_2Earithmic_2EZERO)\ (ap\ c_2Earithmic_2EBIT1\ V2x))))\ (ap \\
& \quad \quad \quad \quad \quad \quad (ap\ c_2Enumeral_2Eonecount\ V1n)\ c_2Earithmic_2EZERO))))))
\end{aligned} \tag{58}$$

Assume the following.

$$(\forall V0x \in ty_2Enum_2Enum. ((ap\ c_2Earithmic_2EDIV2\ (ap\ c_2Earithmic_2EBIT1\ V0x)) = V0x)) \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1x \in ty_2Enum_2Enum. (\\
& \forall V2y \in ty_2Enum_2Enum. (((ap (ap c_2Earithmic_2E_2A c_2Earithmic_2EZERO) \\
& V0n) = c_2Earithmic_2EZERO) \wedge (((ap (ap c_2Earithmic_2E_2A \\
& V0n) c_2Earithmic_2EZERO) = c_2Earithmic_2EZERO) \wedge ((ap \\
& (ap c_2Earithmic_2E_2A (ap c_2Earithmic_2EBIT1 V1x)) (ap \\
& c_2Earithmic_2EBIT1 V2y)) = (ap (ap c_2Enumeral_2Einternal_mult \\
& (ap c_2Earithmic_2EBIT1 V1x)) (ap c_2Earithmic_2EBIT1 V2y)))) \wedge \\
& (((ap (ap c_2Earithmic_2E_2A (ap c_2Earithmic_2EBIT1 V1x)) \\
& (ap c_2Earithmic_2EBIT2 V2y)) = (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum \\
& ty_2Enum_2Enum) (\lambda V3n \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2ECOND \\
& ty_2Enum_2Enum) (ap c_2Earithmic_2EODD V3n)) (ap (ap c_2Enumeral_2Eexp_help \\
& (ap c_2Earithmic_2EDIV2 V3n)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 \\
& V1x)))) (ap (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmic_2EBIT1 \\
& V1x)) (ap c_2Earithmic_2EBIT2 V2y)))))) (ap c_2Enumeral_2Eexactlog \\
& (ap c_2Earithmic_2EBIT2 V2y)))) \wedge (((ap (ap c_2Earithmic_2E_2A \\
& (ap c_2Earithmic_2EBIT2 V1x)) (ap c_2Earithmic_2EBIT1 V2y)) = \\
& (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum ty_2Enum_2Enum) (\lambda V4m \in \\
& ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
& (ap c_2Earithmic_2EODD V4m)) (ap (ap c_2Enumeral_2Eexp_help \\
& (ap c_2Earithmic_2EDIV2 V4m)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 \\
& V2y)))) (ap (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmic_2EBIT2 \\
& V1x)) (ap c_2Earithmic_2EBIT1 V2y)))))) (ap c_2Enumeral_2Eexactlog \\
& (ap c_2Earithmic_2EBIT2 V1x)))) \wedge ((ap (ap c_2Earithmic_2E_2A \\
& (ap c_2Earithmic_2EBIT2 V1x)) (ap c_2Earithmic_2EBIT2 V2y)) = \\
& (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum ty_2Enum_2Enum) (\lambda V5m \in \\
& ty_2Enum_2Enum. (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum ty_2Enum_2Enum) \\
& (\lambda V6n \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
& (ap c_2Earithmic_2EODD V5m)) (ap (ap c_2Enumeral_2Eexp_help \\
& (ap c_2Earithmic_2EDIV2 V5m)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT2 \\
& V2y)))) (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap c_2Earithmic_2EODD \\
& V6n)) (ap (ap c_2Enumeral_2Eexp_help (ap c_2Earithmic_2EDIV2 \\
& V6n)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT2 V1x)))) \\
& (ap (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmic_2EBIT2 \\
& V1x)) (ap c_2Earithmic_2EBIT2 V2y)))))) (ap c_2Enumeral_2Eexactlog \\
& (ap c_2Earithmic_2EBIT2 V2y)))) (ap c_2Enumeral_2Eexactlog \\
& (ap c_2Earithmic_2EBIT2 V1x))))))))) (60)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap (ap c_2Enumeral_2Einternal_mult c_2Earithmetic_2EZERO) \\
V0n) = c_2Earithmetic_2EZERO) \wedge (((ap (ap c_2Enumeral_2Einternal_mult \\
& V0n) c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO) \wedge (((ap \\
& (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmetic_2EBIT1 \\
V0n) V1m) = (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B \\
& (ap c_2Enumeral_2EiDUB (ap (ap c_2Enumeral_2Einternal_mult \\
& V0n) V1m))) V1m))) \wedge ((ap (ap c_2Enumeral_2Einternal_mult (ap \\
& c_2Earithmetic_2EBIT2 V0n) V1m) = (ap c_2Enumeral_2EiDUB (ap \\
c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Enumeral_2Einternal_mult \\
& V0n) V1m)) V1m)))))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0m \in ty_2Enum_2Enum. (\\
& \forall V1n \in ty_2Enum_2Enum. ((ap (ap (c_2Ewords_2Eword_add \\
A_27a) (ap (c_2Ewords_2En2w A_27a) V0m)) (ap (c_2Ewords_2En2w \\
& A_27a) V1n)) = (ap (c_2Ewords_2En2w A_27a) (ap (ap c_2Earithmetic_2E_2B \\
& V0m) V1n))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0m \in ty_2Enum_2Enum. (\\
& \forall V1n \in ty_2Enum_2Enum. ((ap (ap (c_2Ewords_2Eword_mul \\
A_27a) (ap (c_2Ewords_2En2w A_27a) V0m)) (ap (c_2Ewords_2En2w \\
& A_27a) V1n)) = (ap (c_2Ewords_2En2w A_27a) (ap (ap c_2Earithmetic_2E_2A \\
& V0m) V1n))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0v \in (ty_2EfcP_2Ecart \\
& 2 A_27a). (\forall V1w \in (ty_2EfcP_2Ecart 2 A_27a). (\forall V2x \in \\
& (ty_2EfcP_2Ecart 2 A_27a). ((ap (ap (c_2Ewords_2Eword_add A_27a) \\
V0v) (ap (ap (c_2Ewords_2Eword_add A_27a) V1w) V2x)) = (ap (ap (\\
& c_2Ewords_2Eword_add A_27a) (ap (ap (c_2Ewords_2Eword_add \\
& A_27a) V0v) V1w)) V2x))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0v \in (ty_2EfcP_2Ecart \\
& 2 A_27a). (\forall V1w \in (ty_2EfcP_2Ecart 2 A_27a). ((ap (ap (c_2Ewords_2Eword_add \\
& A_27a) V0v) V1w) = (ap (ap (c_2Ewords_2Eword_add A_27a) V1w) V0v))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0v \in (ty_2EfcP_2Ecart \\
& \quad 2\ A_27a).(\forall V1w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(\forall V2x \in \\
& \quad (ty_2EfcP_2Ecart\ 2\ A_27a).((ap\ (ap\ (c_2Ewords_2Eword_mul\ A_27a) \\
& \quad V0v)\ (ap\ (ap\ (c_2Ewords_2Eword_add\ A_27a)\ V1w)\ V2x)) = (ap\ (ap\ (\\
& \quad c_2Ewords_2Eword_add\ A_27a)\ (ap\ (ap\ (c_2Ewords_2Eword_mul \\
& \quad A_27a)\ V0v)\ V1w))\ (ap\ (ap\ (c_2Ewords_2Eword_mul\ A_27a)\ V0v)\ V2x))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0v \in (ty_2EfcP_2Ecart \\
& \quad 2\ A_27a).(\forall V1w \in (ty_2EfcP_2Ecart\ 2\ A_27a).((ap\ (c_2Ewords_2Eword_2comp \\
& \quad A_27a)\ (ap\ (ap\ (c_2Ewords_2Eword_mul\ A_27a)\ V0v)\ V1w)) = (ap\ (ap \\
& \quad (c_2Ewords_2Eword_mul\ A_27a)\ (ap\ (c_2Ewords_2Eword_2comp \\
& \quad A_27a)\ V0v))\ V1w))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2EfcP_2Ecart \\
& \quad 2\ A_27a).((ap\ (c_2Ewords_2Eword_2comp\ A_27a)\ V0w) = (ap\ (ap\ (\\
& \quad c_2Ewords_2Eword_mul\ A_27a)\ (ap\ (c_2Ewords_2Eword_2comp\ A_27a) \\
& \quad (ap\ (c_2Ewords_2En2w\ A_27a)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap \\
& \quad c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))\ V0w)))
\end{aligned} \tag{68}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27a0.nonempty\ A.27a0 \Rightarrow \\
& \forall A.27a1.nonempty\ A.27a1 \Rightarrow \forall A.27a10.nonempty\ A.27a10 \Rightarrow \\
& \forall A.27a11.nonempty\ A.27a11 \Rightarrow \forall A.27a12.nonempty\ A.27a12 \Rightarrow \\
& \forall A.27a13.nonempty\ A.27a13 \Rightarrow \forall A.27a14.nonempty\ A.27a14 \Rightarrow \\
& \forall A.27a15.nonempty\ A.27a15 \Rightarrow \forall A.27a16.nonempty\ A.27a16 \Rightarrow \\
& \forall A.27a17.nonempty\ A.27a17 \Rightarrow \forall A.27a18.nonempty\ A.27a18 \Rightarrow \\
& \forall A.27a19.nonempty\ A.27a19 \Rightarrow \forall A.27a2.nonempty\ A.27a2 \Rightarrow \\
& \forall A.27a20.nonempty\ A.27a20 \Rightarrow \forall A.27a21.nonempty\ A.27a21 \Rightarrow \\
& \forall A.27a22.nonempty\ A.27a22 \Rightarrow \forall A.27a23.nonempty\ A.27a23 \Rightarrow \\
& \forall A.27a24.nonempty\ A.27a24 \Rightarrow \forall A.27a25.nonempty\ A.27a25 \Rightarrow \\
& \forall A.27a26.nonempty\ A.27a26 \Rightarrow \forall A.27a27.nonempty\ A.27a27 \Rightarrow \\
& \forall A.27a3.nonempty\ A.27a3 \Rightarrow \forall A.27a4.nonempty\ A.27a4 \Rightarrow \\
& \forall A.27a5.nonempty\ A.27a5 \Rightarrow \forall A.27a6.nonempty\ A.27a6 \Rightarrow \\
& \forall A.27a7.nonempty\ A.27a7 \Rightarrow \forall A.27a8.nonempty\ A.27a8 \Rightarrow \\
& \forall A.27a9.nonempty\ A.27a9 \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \\
& \forall A.27c.nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow \forall A.27e. \\
& nonempty\ A.27e \Rightarrow \forall A.27f.nonempty\ A.27f \Rightarrow \forall A.27g.nonempty \\
& A.27g \Rightarrow \forall A.27h.nonempty\ A.27h \Rightarrow \forall A.27i.nonempty\ A.27i \Rightarrow \\
& \forall A.27j.nonempty\ A.27j \Rightarrow \forall A.27k.nonempty\ A.27k \Rightarrow \forall A.27l. \\
& nonempty\ A.27l \Rightarrow \forall A.27m.nonempty\ A.27m \Rightarrow \forall A.27n.nonempty \\
& A.27n \Rightarrow \forall A.27o.nonempty\ A.27o \Rightarrow \forall A.27p.nonempty\ A.27p \Rightarrow \\
& \forall A.27q.nonempty\ A.27q \Rightarrow \forall A.27r.nonempty\ A.27r \Rightarrow \forall A.27s. \\
& nonempty\ A.27s \Rightarrow \forall A.27t.nonempty\ A.27t \Rightarrow \forall A.27u.nonempty \\
& A.27u \Rightarrow \forall A.27v.nonempty\ A.27v \Rightarrow \forall A.27w.nonempty\ A.27w \Rightarrow \\
& \forall A.27x.nonempty\ A.27x \Rightarrow \forall A.27y.nonempty\ A.27y \Rightarrow \forall A.27z. \\
& nonempty\ A.27z \Rightarrow ((\forall V0x \in ty.2Enum.2Enum.(p\ (ap\ (ap\ (c.2Ealignment.2Ealigned \\
& A.27a)\ (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1 \\
& (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO))))))\ (ap\ (c.2Ewords.2En2w \\
& A.27a)\ (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT2 \\
& (ap\ c.2Earithmetic.2EBIT1\ (ap\ c.2Earithmetic.2EBIT1\ V0x))))))\ \wedge \\
& ((\forall V1x \in ty.2Enum.2Enum.(p\ (ap\ (ap\ (c.2Ealignment.2Ealigned \\
& A.27b)\ (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT2 \\
& c.2Earithmetic.2EZERO))))\ (ap\ (c.2Ewords.2En2w\ A.27b)\ (ap\ c.2Earithmetic.2ENUMERAL \\
& (ap\ c.2Earithmetic.2EBIT2\ (ap\ c.2Earithmetic.2EBIT1\ V1x))))))\ \wedge \\
& ((\forall V2x \in ty.2Enum.2Enum.(p\ (ap\ (ap\ (c.2Ealignment.2Ealigned \\
& A.27c)\ (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1 \\
& c.2Earithmetic.2EZERO))))\ (ap\ (c.2Ewords.2En2w\ A.27c)\ (ap\ c.2Earithmetic.2ENUMERAL \\
& (ap\ c.2Earithmetic.2EBIT2\ V2x))))))\ \wedge ((\forall V3x \in ty.2Enum.2Enum. \\
& (p\ (ap\ (ap\ (c.2Ealignment.2Ealigned\ A.27d)\ (ap\ c.2Earithmetic.2ENUMERAL \\
& (ap\ c.2Earithmetic.2EBIT1\ (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO)))))) \\
& (ap\ (c.2Ewords.2Eword.2comp\ A.27d)\ (ap\ (c.2Ewords.2En2w\ A.27d) \\
& (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT2\ (ap\ c.2Earithmetic.2EBIT1 \\
& (ap\ c.2Earithmetic.2EBIT1\ V3x))))))\ \wedge ((\forall V4x \in ty.2Enum.2Enum. \\
& (p\ (ap\ (ap\ (c.2Ealignment.2Ealigned\ A.27e)\ (ap\ c.2Earithmetic.2ENUMERAL \\
& (ap\ c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO))))\ (ap\ (c.2Ewords.2Eword.2comp \\
& A.27e)\ (ap\ (c.2Ewords.2En2w\ A.27e)\ (ap\ c.2Earithmetic.2ENUMERAL \\
& (ap\ c.2Earithmetic.2EBIT2\ (ap\ c.2Earithmetic.2EBIT1\ V4x))))))\ \wedge \\
& ((\forall V5x \in ty.2Enum.2Enum.(p\ (ap\ (ap\ (c.2Ealignment.2Ealigned \\
& A.27f)\ (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1 \\
& c.2Earithmetic.2EZERO))))\ (ap\ (c.2Ewords.2Eword.2comp\ A.27f) \\
& (ap\ (c.2Ewords.2En2w\ A.27f)\ (ap\ c.2Earithmetic.2ENUMERAL\ (ap \\
& c.2Earithmetic.2EBIT2\ V5x))))))\ \wedge ((\forall V6x \in A.27g.(\forall V7y \in \\
& (ty.2EfcP.2Ecart\ 2\ A.27h).(\forall V8f \in (ty.2Enum.2Enum)^{A.27g}). \\
& ((p\ (ap\ (ap\ (c.2Ealignment.2Ealigned\ A.27h)\ (ap\ c.2Earithmetic.2ENUMERAL \\
& (ap\ c.2Earithmetic.2EBIT1\ (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO)))))) \\
& (ap\ (ap\ (c.2Ewords.2Eword.2add\ A.27h)\ V7y)\ (ap\ (c.2Ewords.2En2w \\
& A.27h)\ (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1 \\
& (ap\ c.2Earithmetic.2EBIT1\ (ap\ c.2Earithmetic.2EBIT1\ (ap\ V8f\ V6x))))))\ \Leftrightarrow \\
& (p\ (ap\ (ap\ (c.2Ealignment.2Ealigned\ A.27h)\ (ap\ c.2Earithmetic.2ENUMERAL \\
& (ap\ c.2Earithmetic.2EBIT1\ (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO))))))
\end{aligned}$$